(Multi-)Galileon Dualities

Based on work in collaboration with: James Scargill and Pedro Ferreira
arXiv: 1311.7009, 1410.7774, 1503.02700 + work in progress

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SW9, Cargese, 27/04/15

Galileon Dualities

Galileons:

$$\mathcal{L}_n \quad = \quad \pi \pi^{[a_1}_{a_1} \cdots \pi^{a_n]}_{a_n} \equiv \pi \mathcal{L}^{(n)}_{\rm TD},$$

$$\pi^{[a_1}_{a_1} \cdots \pi^{a_n]}_{a_n} \equiv \frac{1}{n!} \epsilon_{b_1 \cdots b_n} \epsilon^{a_1 \cdots a_n} \pi^{b_1}_{a_1} \cdots \pi^{b_n}_{a_n} .$$

There exists an invertible, non-linear, non-local field re-definition $\pi \to \sigma$, that leaves the physics described invariant.

Nicolis, Rattazzi, Trincherini '09, de Rham, Fasiello, Tolley '13

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There exists an invertible, non-linear, non-local field re-definition $\pi \to \sigma$, that leaves the physics described invariant.

$$\sigma = -\pi + \sum_{n=2}^{\infty} \frac{1}{2(n-1)!} \sum_{i=0}^{n-2} (-1)^i \binom{n-2}{i} \tilde{D}_{(i)} \left(\pi^\mu \pi_\mu \mathcal{L}^{\mathrm{TD}}_{(n-2-i)}(\pi)\right),$$

where $\tilde{D}_{(n)}(X) = \partial_{\nu_1\cdots\nu_n}\left(\pi^{\nu_1}\cdots\pi^{\nu_n}X\right)$ for some Lorentz scalar X.

$$\begin{aligned}\sigma = \quad & - \quad \pi + \frac{1}{2}\pi_a\pi^a - \frac{1}{2}\pi^a\pi^b\pi_{ab} + \frac{1}{2}\pi^a\pi^b{\pi_a}^c\pi_{bc} + \frac{1}{6}\pi^a\pi^b\pi^c\pi_{abc} \\ & - \quad \frac{1}{2}\pi^a\pi^b{\pi_a}^c{\pi_b}^d\pi_{cd} - \frac{1}{2}\pi^a\pi^b\pi^c{\pi_a}^d\pi_{bcd} - \frac{1}{24}\pi^a\pi^b\pi^c\pi^d\pi_{abcd} + \mathcal{O}(\pi^6).\end{aligned}$$

JN, Scargill '15

Why care?

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Strong/Weak Coupling Duality

(or, in general, a mapping between different strongly coupled theories)

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Duality mapping:

$$\int d^D x \sum_{n=1}^{D} c_{n+1} \pi \mathcal{L}^n_{\rm TD}[\pi] \longrightarrow \int d^D x \sum_{n=1}^{D} d_{n+1} \sigma \mathcal{L}^n_{\rm TD}[\sigma],$$

$$d_2 = c_2, \quad d_3 = 2c_2 - c_3, \quad d_4 = \frac{3}{2} c_2 - \frac{3}{2} c_3 + c_4, \quad d_5 = \frac{1}{5} \left(2c_2 - 3c_3 + 4c_4 - 5c_5\right) .$$

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de Rham, Fasiello, Tolley '13

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Free field Dual:

$$\mathcal{L}_2[\pi] = -\frac{1}{2}\pi_\mu\pi^\mu \to -\frac{1}{2}\sigma_\mu\sigma^\mu - \frac{1}{6}\mathcal{L}_3[\sigma] - \frac{1}{8}\mathcal{L}_4[\sigma] - \frac{1}{30}\mathcal{L}_5[\sigma]$$

Strong/Weak Coupling Duality

(or, in general, a mapping between different strongly coupled theories)

Why care?

$$\int d^D x \sum_n c_{(n)} \pi_{(1)} U_{(n)}[\Pi_{(2)}] \xrightarrow{\mathcal{D}_{\pi(1)}} -\int \mathrm{d}^D x \sum_n c_{(n)} \det(1+\Sigma_{(1)})(\sigma_{(1)} + \frac{1}{2}\sigma^{\gamma}_{(1)}\sigma^{(1)}_{\gamma})$$
$$\times \quad U_{(n)}\left[\left[(1+\Sigma_{(1)})^{-1}\right]^{\alpha}_{\mu} \partial_\alpha \left(\left[(1+\Sigma_{(1)})^{-1}\right]^{\beta}_{\nu} \pi^{(2)}_{\beta}\right)\right]$$

Healthy higher-derivative *eoms*

(for multi-field cases)

JN, Scargill '15, JN in progress

Why care?

The duality transformation

$$\pi \to \sigma = -\pi + \sum_{n=2}^{\infty} \frac{1}{2(n-1)!} \sum_{i=0}^{n-2} (-1)^i \binom{n-2}{i} \tilde{D}_{(i)} \left(\pi^\mu \pi_\mu \mathcal{L}^{\rm TD}_{(n-2-i)}(\pi) \right)$$

is a symmetry (up to TD) of the following two-parameter set of (tadpole-free) Galileon theories $\int d^4x \sum_{n=1}^{4} c_{n+1} \pi \mathcal{L}^n_{\rm TD}[\pi]$

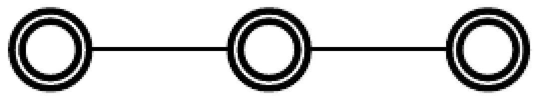
$$c_3 = c_2 \qquad\qquad c_5 = -\frac{1}{10} c_2 + \frac{2}{5} c_4 .$$

(Probably) no finite-order polynomial, non-linear symmetries exist.

Non-linear symmetries for Galileons

de Rham, Keltner, Tolley '14; Hinterbichler, Joyce '15, Deffayet, JN, Sivanesan, von Strauss in progress

Why care?



$$S \sim \int d^4x \sum_n \Big(\pi_{(1,2)} U_n(\Pi_{(1,2)}) + \sigma_{(2,1)} U_n(\Sigma_{(2,1)}) + \pi_{(2,3)} U_n(\Sigma_{(2,1)})$$

$$+ \sigma_{(2,1)} U_n(\Pi_{(2,3)}) + \pi_{(2,3)} U_n(\Pi_{(2,3)}) + \sigma_{(3,2)} U_n(\Sigma_{(3,2)}) \Big)$$

Decoupling limit of Bi- and Multi-Gravity

JN, Scargill '15, JN in progress

GR to Bigravity

①

$$\mathcal{S} = \int d^4x_{(1)}\sqrt{-g[x_{(1)}]}\,R[g(x_{(1)})]$$

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① ②

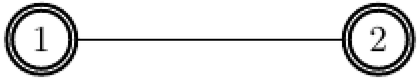
$$\mathcal{S} = \int d^4x_{(1)}\sqrt{-g[x_{(1)}]}\,R[g(x_{(1)})] + \int d^4x_{(2)}\sqrt{-f[x_{(2)}]}\,R[f[x_{(2)}]]$$

Two copies of general co-ordinate invariance GC_i.

$$g_{\mu\nu}(x_{(1)}) \xrightarrow{d_{(1)}} \partial_\mu d^\alpha_{(1)}\partial_\nu d^\beta_{(1)} g_{\alpha\beta}(d_{(1)}(x_{(1)})). \qquad f_{\mu\nu}(x_{(2)}) \xrightarrow{d_{(2)}} \partial_\mu d^\alpha_{(2)}\partial_\nu d^\beta_{(2)} f_{\alpha\beta}(d_{(2)}(x_{(2)})).$$

GR to Bigravity

$$\mathcal{S} = \int d^4x_{(1)} \sqrt{-g[x_{(1)}]}\, R[g(x_{(1)})]$$



$$\begin{aligned} \mathcal{S} &= \int d^4x_{(1)} \sqrt{-g[x_{(1)}]}\, R[g(x_{(1)})] + \int d^4x_{(2)} \sqrt{-f[x_{(2)}]}\, R[f[x_{(2)}]] \\ &+ m^2 \int d^4x \sqrt{-g[x_{(1)}]} \sum_{n=0}^{4} \beta_n\, e_n(\sqrt{g^{-1}[x_{(1)}] f[x_{(2)}]}) \end{aligned}$$

Two copies of general co-ordinate invariance GC_i, which get broken down to the diagonal subgroup by the interaction term.

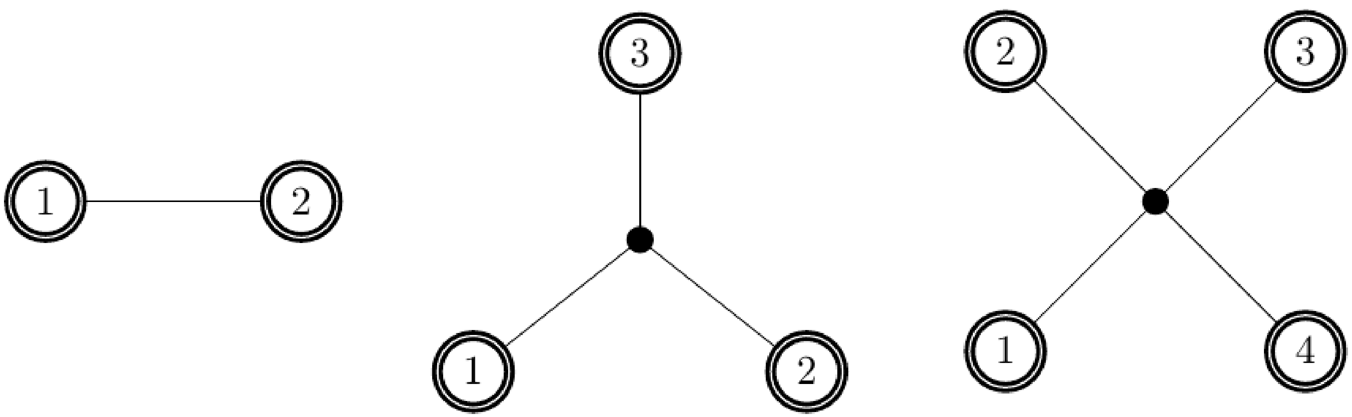
$$g_{\mu\nu}(x_{(1)}) \xrightarrow{d_{(1)}} \partial_\mu d^\alpha_{(1)} \partial_\nu d^\beta_{(1)} g_{\alpha\beta}(d_{(1)}(x_{(1)})). \qquad f_{\mu\nu}(x_{(2)}) \xrightarrow{d_{(2)}} \partial_\mu d^\alpha_{(2)} \partial_\nu d^\beta_{(2)} f_{\alpha\beta}(d_{(2)}(x_{(2)})).$$

The vielbein picture

$$\mathcal{S}_{\rm MG} = \sum_{(i)}^{N} \frac{M_{\rm Pl}^2}{4} \int \epsilon_{ABCD} \mathbf{E}^A_{(i)} \wedge \mathbf{E}^B_{(i)} \wedge \mathbf{R}^{CD}\left[E_{(i)}\right]$$

$$+ \sum_{(i,j,k,l)}^{N} \frac{m^2_{(i,j,k,l)}}{2} \beta_{(i,j,k,l)} \int \epsilon_{ABCD}\, \mathbf{E}^A_{(i)} \wedge \mathbf{E}^B_{(j)} \wedge \mathbf{E}^C_{(k)} \wedge \mathbf{E}^D_{(l)},$$

$$\mathbf{E}^A_{(i)} \equiv E^A_{\mu\ (i)} dx^\mu_{(i)}, \qquad g^{(i)}_{\mu\nu} = E^A_{\mu\ (i)} E^B_{\nu\ (i)} \eta_{AB}$$



Hinterbichler, Rosen '12, Deffayet, Mourad, Zahariade '12, Hassan, Schmidt-May, von Strauss '12

Goldstone bosons

$$\mathcal{S}_{\mathrm{MG}} = \sum_{(i)}^{N} \frac{M_{\mathrm{Pl}}^2}{4} \int \epsilon_{ABCD} \mathbf{E}^A_{(i)} \wedge \mathbf{E}^B_{(i)} \wedge \mathbf{R}^{CD}\left[E_{(i)}\right]$$

$$+ \sum_{(i,j,k,l)}^{N} \frac{m^2_{(i,j,k,l)}}{2} \beta_{(i,j,k,l)} \int \epsilon_{ABCD}\, \mathbf{E}^A_{(i)} \wedge \mathbf{E}^B_{(j)} \wedge \mathbf{E}^C_{(k)} \wedge \mathbf{E}^D_{(l)},$$

Stückelberg replacement:

$$E^A_{\mu\,(i)}[x] \to \tilde{E}^A_{\mu\,(j)(i)}[Y(x)] = \Lambda^A_{B\,(j,i)}[Y(x)] E^B_{\nu\,(i)}[Y(x)] \partial_\mu Y^\nu_{(j,i)}[x]$$

'Link fields' Y:

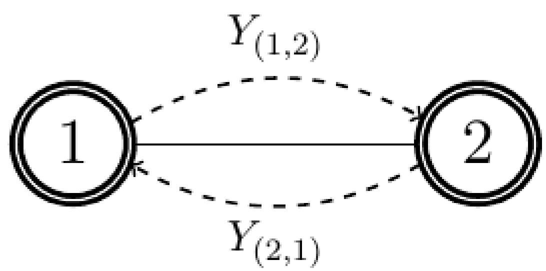
$$Y^\nu_{(i,j)} = x^\nu_{(i)} + \hat{B}^\nu_{(i,j)} + \partial^\nu \hat{\pi}_{(i,j)}.$$

Λ_3 Decoupling Limit:

$$M_{\mathrm{Pl}} \to \infty, \qquad m_{(i,j,k,l)} \to 0, \qquad \Lambda_{3\,(i,j,k,l)} \text{ fixed}, \qquad \hat{\beta}_{(i,j,k,l)} \text{ fixed}.$$

Hinterbichler, Rosen '12; Ondo, Tolley '13; JN in progress

Link fields



Co-ordinate transformation:

$$x^{\mu}_{(2)} = Y^{\mu}_{(1,2)}[x_{(1)}] = x^{\mu}_{(1)} + \partial_{(1)}\pi[x_{(1)}]$$

Gauge invariance:

$$\mathcal{S}_I = \int d^D x \sqrt{g_{(1)}} f(\mathbf{g}_{(1)}, \mathbf{g}_{(2)}) \to \int d^D x \sqrt{g_{(1)} \circ Y_{(1,2)}} f(\mathbf{g}_{(1)} \circ Y_{(1,2)}, \mathbf{g}_{(2)},)$$

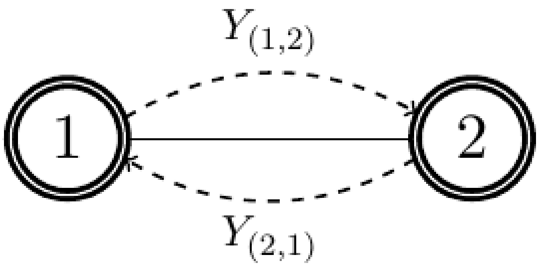
$$\mathcal{S}_{II} = \int d^D x \sqrt{g_{(1)}} f(\mathbf{g}_{(1)}, \mathbf{g}_{(2)}) \to \int d^D x \sqrt{g_{(1)}} f(\mathbf{g}_{(1)}, \mathbf{g}_{(2)} \circ \tilde{Y}_{(2,1)})$$

Field relations:

$$(x + \partial\pi)^{\mu} + \frac{\partial}{\partial(x + \partial\pi)_{\mu}} \sigma(x + \partial\pi) = x^{\mu}$$

Arkani-Hamed, Georgi, Schwartz '02; JN, Scargill, Ferreira'13; JN, Scargill '15

Galileon Dualities



Duality maps:

$$\mathcal{D}_\pi : \begin{cases} \pi(x) \longrightarrow \sigma(\tilde{x}) = -\pi(x) - \frac{1}{2}(\partial\pi(x))^2 , \\ \partial_\mu \pi(x) \longrightarrow \tilde{\partial}_\mu \sigma(\tilde{x}) = -\partial_\mu \pi(x) , \\ \Pi^\nu_\mu(x) \longrightarrow \Sigma^\nu_\mu(\tilde{x}) = -\left[1^\alpha_\nu + \Pi^\alpha_\nu(x)\right]^{-1} \Pi^\alpha_\mu(x) \end{cases}$$

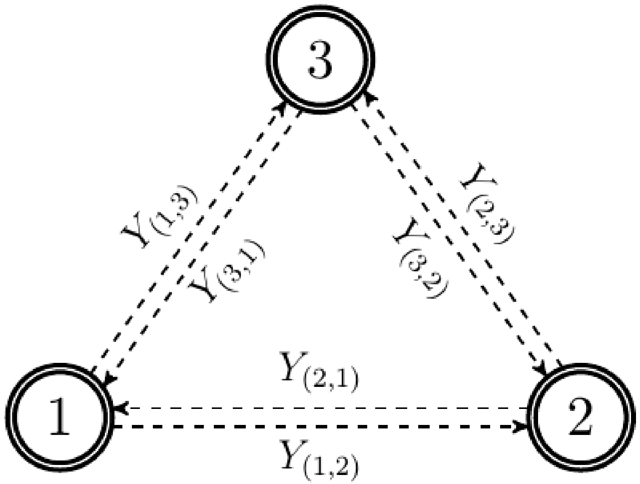
$$\mathcal{D}_\pi : \begin{cases} \chi(x) \to \tilde{\chi}(\tilde{x}) = \chi(x) , \\ \partial_\mu \chi(x) \to \tilde{\partial}_\mu \tilde{\chi}(\tilde{x}) = \frac{\delta x^\nu}{\delta \tilde{x}^\mu} \partial_\nu \chi(x) = \left[1 + \Sigma(\tilde{x})\right]^\nu{}_\mu \partial_\mu \chi(x) , \\ \partial_\mu \partial_\nu \chi(x) \to \tilde{\partial}_\mu \tilde{\partial}_\nu \tilde{\chi}(\tilde{x}) = \left[1 + \Sigma(\tilde{x})\right]^\alpha_\mu \partial_\alpha \left(\left[1 + \Sigma(\tilde{x})\right]^\beta_\nu \partial_\beta \chi(x) \right) \end{cases}$$

(Single) Galileon Duality:

$$\mathcal{S}^{\rm Gal}_{(n)}(\pi) \xrightarrow{\mathcal{D}_\pi} \int \mathrm{d}^D x \sum_{k=n}^{D} c_{(n)} (-1)^{n+1} \frac{(D-n)!}{(k-n)!(D-k)!} (\sigma + \frac{1}{2}\sigma^\mu \sigma_\mu) U_{(k)}[\Sigma(x)]$$

Fasiello, Tolley '13; de Rham, Keltner, Tolley '14, JN, Scargill '15

(Multi-)Galileon Dualities I



Constraints:

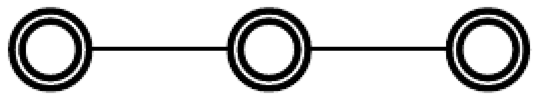
$$Y_{(1,3)}[x_{(1)}] \circ Y_{(3,1)} = x_{(1)}$$
$$Y_{(1,3)}[x_{(1)}] \circ Y_{(3,2)} \circ Y_{(2,1)} = x_{(1)}$$

Explicit field relations:

$$(x+\partial\pi)^\mu + \frac{\partial}{\partial(x+\partial\pi)_\mu}\sigma(x+\partial\pi) + \ldots = x^\mu \qquad x_{(3)} = x_{(1)} + \partial_{(1)}\pi[x_{(1)}] + \partial_{(2)}\phi[x_{(2)}] + \ldots$$

JN, Scargill '15, JN in progress

(Multi-)Galileon Dualities II



Decoupling Limit:

$$S \sim \int d^4x \sum_n \Big(\pi_{(1,2)} U_n(\Pi_{(1,2)}) + \sigma_{(2,1)} U_n(\Sigma_{(2,1)}) + \pi_{(2,3)} U_n(\Sigma_{(2,1)})$$

$$+ \sigma_{(2,1)} U_n(\Pi_{(2,3)}) + \pi_{(2,3)} U_n(\Pi_{(2,3)}) + \sigma_{(3,2)} U_n(\Sigma_{(3,2)}) \Big)$$

Dual higher-derivative theories:

$$\int d^Dx \sum_n c_{(n)} \pi_{(1)} U_{(n)}[\Pi_{(2)}] \xrightarrow{\mathcal{D}_{\pi(1)}} -\int \mathrm{d}^Dx \sum_n c_{(n)} \det(1+\Sigma_{(1)})(\sigma_{(1)} + \frac{1}{2}\sigma^{\gamma}_{(1)}\sigma^{(1)}_{\gamma})$$

$$\times \quad U_{(n)}\left[\left[(1+\Sigma_{(1)})^{-1}\right]^{\alpha}_{\mu}\partial_\alpha\left(\left[(1+\Sigma_{(1)})^{-1}\right]^{\beta}_{\nu}\pi^{(2)}_{\beta}\right)\right]$$

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Conclusions

- Duality exists in the form of an invertible, non-linear, non-local field redefinition
- May have interesting consequences for strong coupling scale, superluminalities, UV completion, ...
- Can be understood as a consequence of gauge invariance + be generalised to multi-field cases in this way
- Duality generically relates multi-galileons with healthy higher-derivative theories
- Important in order to understand low-energy limit of Multi-Gravity theories

Thank you!