THE GEOMETRIC SCALAR GRAVITY [MAIN STEPS] (1)

1905 - 1912

NORDSTRÖM - EINSTEIN - ABRAHAM...

$\phi_N(\vec{x}) \longrightarrow \phi(t, \vec{x})$ (1) {FROM NEWTONIAN POTENTIAL IN 3-D TO SCALAR FIELD IN 4-D

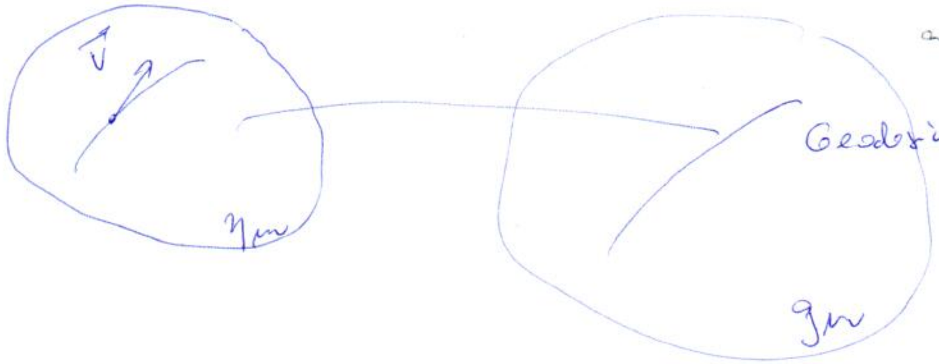
$\nabla^2 \phi \sim \rho \longleftrightarrow \Box \phi \sim T$

(2) (NEWTON'S EQ.) (NORDSTRÖN-EINSTEIN DYNAMICS)

$\phi \longrightarrow g_{\mu\nu} = \phi\, \eta_{\mu\nu}$

(3) [THE RESTRICTED CONFORMAL METRIC : Einstein/Nordström (d'Allembert)

(4) [THE GEOMETRIC EQUIVALENCE : elimination of forces by a metric change]

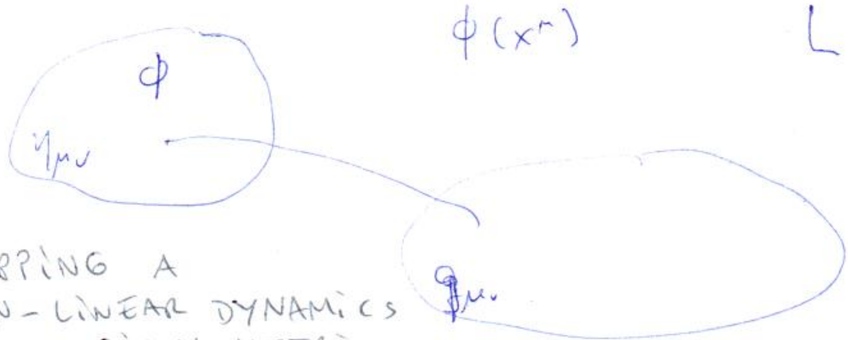


η_{00}

$g_{00} \longrightarrow 1 + f(\phi_N)$ (5) The Einstein analysis of a static low velocity body requires a unique function $g_{00}[\Phi_N]$.

$g_{\mu\nu}(x^\alpha)$

$\phi(x^\mu)$ $\qquad L = V(\phi)\, \partial_\mu \phi\, \partial^\mu \phi$



$\Box \phi = 0$ (9)

[DYNAMICAL BRIDGE]

(6) [MAPPING A NON-LINEAR DYNAMICS IN A GIVEN METRIC INTO AN EQUIVALENT ONE IN ANOTHER METRIC]

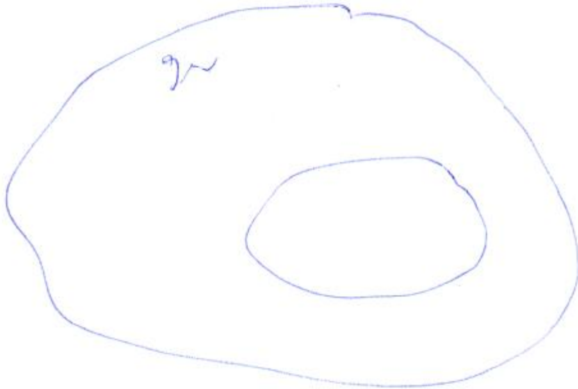
$\Box_m \phi + \frac{1}{2} \frac{V'}{V} \partial_\mu \phi\, \partial^\mu \phi = 0$

$$q^{\mu\nu} = \alpha(\phi)\,\eta^{\mu\nu} + \beta(\phi)\,\frac{\partial^\mu\phi\,\partial^\nu\phi}{(\partial_\lambda\phi)^2}$$

(7) [THE GENERAL FORMULA FOR THE EQUIVALENT METRIC]

$$V = \frac{\alpha+\beta}{\alpha^3}$$

(8) [UNIQUE RESTRICTION AMONG α, β and V]



[Solar System, Cosmology] [NEWTONIAN ATTRACTIONS]

(9) [DESCRIBED BY A UNIQUE SCALAR FIELD ϕ]

GR	GSG
Test particle follow geodesics $\longleftrightarrow$	
photon null geodesics ($g_{\mu\nu}$)	photon follow null geodesic ($q_{\mu\nu}$)
	$T^{\mu\nu}\ \partial_\lambda\phi\,\partial\phi$
Dynamics	
$G_{\mu\nu} = 0$	$\Box\phi = 0$ (q)

(10) [GR AND GSG ARE BASED ON A UNIQUE SET OF FUNDAMENTAL HYPOTHESIS [except for the DYNAMICS, they are the same]]

$$\frac{1}{\sqrt{-q}}\,\partial_\mu\left(\sqrt{-q}\,q^{\mu\nu}\,\partial_\nu\phi\right) = 0$$

$$ds^2 = A(r)\,dt^2 - B(r)\,dr^2 - r^2 d\Omega^2$$

$$= \left(1 - \frac{r_H}{r}\right) dt^2 - \frac{1}{1 - \frac{r_H}{r}} dr^2 - r^2 d\Omega^2$$

(14) THE SOLUTION OF GSG DYNAMICS FOR STATIC SPHERICALLY SYMMETRIC CONFIGURATION

(12) DYNAMICS OF FREE GRAVITATIONAL FIELD Φ

$$\Box_q \phi = 0$$

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(13) NON-NEWTONIAN ATTRACTIONS; GRAVITATIONAL WAVES ROTATING STARS, .. RECQUIRES TWO SCALAR FIELDS

TWO SCALAR FIELDS

$$(\phi, \xi)$$

COSMOLOGY $\qquad \phi(t)$

$$ds^2 = q_{\mu\nu}\,dx^\mu dx^\nu$$

$$= \left[\frac{1}{\alpha}\eta_{\mu\nu} - \frac{\beta}{\alpha(\alpha+\beta)} \frac{\partial_\mu\phi\,\partial_\nu\phi}{(\partial_\lambda\phi)^2}\right] dx^\mu dx^\nu$$

$$= dt^2 - A^2(t)\, d\vec{x}\cdot d\vec{x}$$

$$\begin{cases} \alpha = e^{-2\phi} \\ \beta = \frac{1}{4}(\alpha - 1)(\alpha - 9) \end{cases}$$

(14) FROM THE MOTION OF EARTH AND THE NEWTONIAN LIMIT FOLLOW SUCH (non-unique) CHOICE

$f(r)\, dt\, d\varphi$

(15) [ROTATING CONFIGURATIONS]

$(\varphi + \xi)$

(16) THE SECOND FIELD ξ ENTERS ONLY IN THE GRADIENT PART OF THE METRIC AS THE SUM

$\alpha(\phi) \qquad \beta(\phi)$

(17) TWO SCALAR FIELDS GRAVITY: THE ξ FIELD IS A "MOVING MATTER" EFFECT

$$\phi = 0 \longrightarrow \begin{vmatrix} \alpha = 1 \\ \beta = 0 \end{vmatrix}$$

$$g^{\mu\nu} = \alpha\, \eta^{\mu\nu} + \beta\, \frac{\partial^\mu(\phi+\xi)\, \partial^\nu(\phi+\xi)}{[\partial_\lambda(\phi+\xi)]^2}$$

$$\downarrow$$

$$g^{\mu\nu} = \eta^{\mu\nu}$$

(18) NOTE: $\alpha = \alpha(\phi)$, $\beta = \beta(\phi)$

(19) THE FORM OF THE GRAVITATIONAL METRIC FOR THE GENERAL CASE OF TWO FIELDS

$$g^{\mu\nu} = \alpha\, \eta^{\mu\nu} + \beta\, \frac{\partial^\mu(\phi+\xi)\,(\partial^\nu(\phi+\xi))}{[\partial_\lambda(\phi+\xi)]^2}$$

$$\begin{cases} \underset{M}{\Box}\phi + \frac{1}{2}\frac{V'}{V}(\partial_\lambda\phi)^2 = 0 \\ \underset{M}{\Box}\xi = 0 \end{cases}$$

(20) DYNAMICS OF ϕ AND ξ. THEY DO NOT INTERACT.
IN THE CASE OF $\phi = 0$ THE ξ-FIELD DOES NOT GENERATE GRAVITATIONAL FIELD (There is no negative mass)

$$\int L(\phi) + L_M$$

(21) INCLUDING MATTER THROUGH MINIMAL COUPLING. THE FIELDS (ϕ, ξ) INTERACTS WITH MATTER <u>ONLY</u> THROUGH THE METRIC $g^{\mu\nu}$

$$\delta \int \sqrt{-g}\, L_M = \int \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$$

$$= \int \sqrt{-g}\, T_{\mu\nu}\, \delta\left[\alpha\, \eta^{\mu\nu} + \beta\, \frac{\partial^\mu\phi\,\partial^\nu\phi}{(\partial_\lambda\phi)^2}\right]$$

(22) SOLVING THE PROBLEMS OF NORDSTRÖM-EINSTEIN ILL EQUATIONS OF MOTION

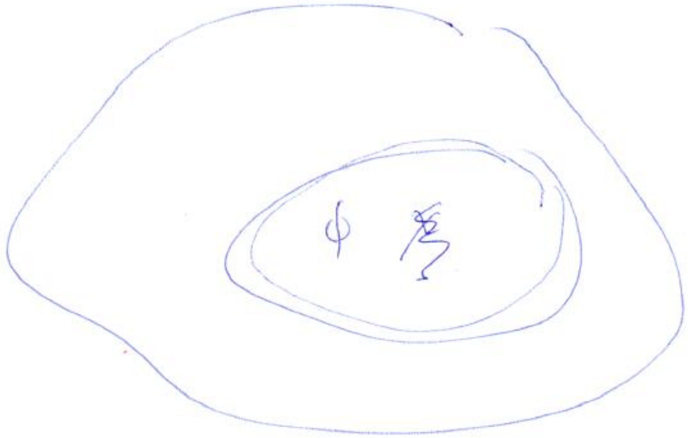
$$\boxed{\underset{(g)}{\Box}\phi = \kappa\,\chi}$$

T

ϕ

$T^{\mu\nu}\phi_{,\nu}$

$T^{\mu\nu}\phi_{,\mu}\phi_{\nu}$



$$\Box \phi = \kappa \chi$$

<u>GR</u>

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$ <u>EXACT!</u>

$$g_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu} + h_\mu{}^\lambda h_{\lambda\nu} + \dots$$

Feynman, Gupta, Deser, ...

$$G^{\mu\nu} = -\kappa T^{\mu\nu}$$

$$\overset{(L)}{G}{}^{\mu\nu} + \tau^{\mu\nu} = -\kappa T^{\mu\nu}$$

$$\overset{(L)}{G}_{\mu\nu} = -\kappa \left(T_{\mu\nu} + \tau_{\mu\nu} \right)$$

(23) [THE ENERGY OF GRAVITATIONAL FIELD. NO PROBLEM WITH ~~THE~~ EQUIVALENCE PRINCIPLE [once the metric does not have a proper dynamics, but hamitates the dynamics of the ϕ field]]

$$\Box_M \phi = \kappa(\chi + \Psi)$$

$$\Box_g \phi = 0$$

$$ds^2 = \alpha_0 \, dt^2 - \frac{1}{\beta_0} dr^2 - r^2 \, d\Omega^2$$

A PARTICULAR SOLUTION FOR CONSTANT ϕ FIELD.
NOTE $R \sim \frac{1}{r^2}$

$$R \neq 0$$

BINARY PULSAR

[internal stars, gravitational waves, gravitational radiation $\Rightarrow$ next seminar]

$$\delta\phi$$

Cargese 29/April/2015
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