The Cosmological Constant Problem (and its sequester)

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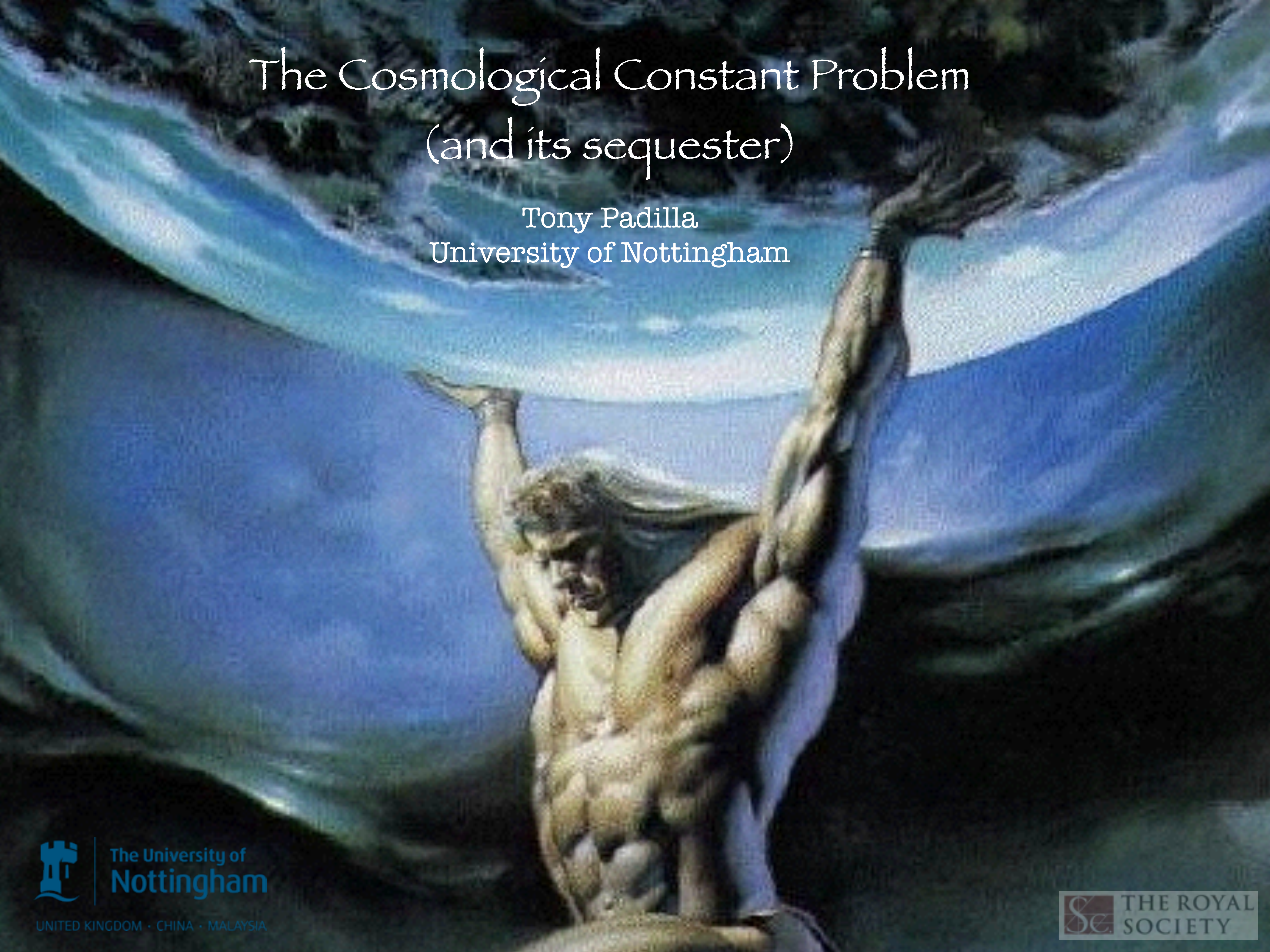

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THE ROYAL SOCIETY

with Nemanja Kaloper

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see also
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The Cosmological Constant Problem

General Covariance & Equivalence Principle ⇒ Vacuum Energy Gravitates

$$-V_{vac}\int\sqrt{-g}d^4x \implies T_{\mu\nu} = -V_{vac}g_{\mu\nu}$$

The Cosmological Constant Problem

General Covariance & Equivalence Principle ⇒ Vacuum Energy Gravitates

$$-V_{vac}\int\sqrt{-g}d^4x \implies T_{\mu\nu} = -V_{vac}g_{\mu\nu}$$

…add a bare cosmological constant…

$$-(V_{vac}+\Lambda_{bare})\int\sqrt{-g}d^4x \implies T_{\mu\nu} = -\Lambda_{tot}g_{\mu\nu}$$

where $\Lambda_{tot} = V_{vac} + \Lambda_{bare} \lesssim (meV)^4$

Estimating the vacuum energy

$$V_{vac} \supset \quad \sum_m \int d^3k \frac{1}{2} \hbar \sqrt{k^2 + m^2}$$

$$\sim c_\nu m_\nu^4 + c_e m_e^4 + c_\mu m_\mu^4 + \ldots + M_{\text{cut-off}}^4$$

+ vacuum loops involving virtual gravitons

+ counterterms

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=

+ …..

+ counterterms

Quantum Gravity cut-off — $(10^{18}\ GeV)^4$ — fine tuning to 120 decimal places

SUSY cut-off — $(TeV)^4$ — fine tuning to 60 decimal places

EW phase transition — $(200\ GeV)^4$ — fine tuning to 56 decimal places

QCD phase transition — $(0.3\ GeV)^4$

muon — $(100 MeV)^4$ — fine tuning to 44 decimal places

electron — $(MeV)^4$ — fine tuning to 36 decimal places

$(meV)^4$ — observed value

Why is the cosmological constant so small?????

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Within EFT, renormalised couplings of relevant operators CANNOT be predicted, must be measured!

Big, small, who cares? Just measure them.

But NATURALNESS requires measured couplings to be RADIATIVELY STABLE

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But NATURALNESS requires measured couplings to be RADIATIVELY STABLE

naturalness ensures that low energy EFTs agree on low energy couplings.

More on NATURALNESS

electron mass should be larger than its EM energy of electron

but if EM energy ~ α/r, suggests electron is larger than nucleus!

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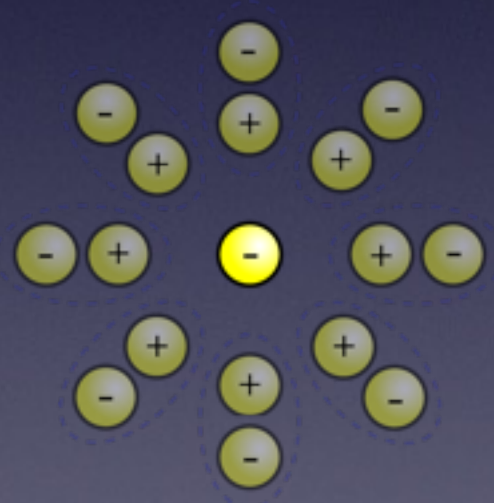

positrons help preserve naturalness

EM energy ~ $\alpha m \log(mr)$

The cosmological constant is radiatively UNstable (and badly so)

At one loop, $V_{vac} = V_{vac}^{tree} + V_{vac}^{1loop} \sim M_{UV}^4 \gtrsim (TeV)^4$

tune Λ_{bare} to great precision

At two loops, $V_{vac} = V_{vac}^{tree} + V_{vac}^{1loop} + V_{vac}^{2loop}, \qquad V_{vac}^{2loop} \sim V_{vac}^{1loop}$

REtune Λ_{bare} to same precision

At three loops, ...

How can we make the cosmological constant radiatively stable?

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Look to gravity:
perhaps the radiative corrections are there, but they simply don't gravitate.


NO GO AREA!

The Sequester

[redacted]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \Lambda - \mathcal{L}(g^{\mu\nu}, \Psi) \right]$$

[redacted]

Introduce global dynamical variables Λ

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[redacted]

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λ sets the hierarchy between matter scales and M_{pl}

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

[redacted]

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Equations of motion

$$\Lambda \text{ equation} \quad : \quad \frac{\sigma'}{\lambda^4\mu^4} = \int d^4x\sqrt{g}$$

$$\lambda \text{ equation} \quad : \quad 4\Lambda\frac{\sigma'}{\lambda^4\mu^4} = \int d^4x\sqrt{g}\,\lambda^4\,\tilde{T}^{\mu}{}_{\mu}$$

$$g_{\mu\nu} \text{ equation} \quad : \quad M_{pl}^2 G^{\mu}_{\nu} = -\Lambda\delta^{\mu}_{\nu} + \lambda^4\tilde{T}^{\mu}_{\nu}$$

$$\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta\tilde{g}^{\mu\nu}}\int d^4x\sqrt{-\tilde{g}}\mathcal{L}(\tilde{g}^{\mu\nu},\Psi)$$

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Equations of motion

Λ equation :

λ equation :

$$\Lambda = \frac{1}{4}\langle T^{\alpha}{}_{\alpha}\rangle, \qquad \langle Q\rangle = \frac{\int d^4x Q\sqrt{g}}{\int d^4x\sqrt{g}}$$

$g_{\mu\nu}$ equation : $M_{pl}^2 G^{\mu}_{\nu} = -\Lambda\delta^{\mu}_{\nu} + T^{\mu}_{\nu}$

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Equations of motion

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$$M_{pl}^4 G^\mu_\nu = -\frac{1}{4}\langle \tau^\alpha_\alpha \rangle \delta^\mu_\nu + \tau^\mu_\nu$$

Vacuum energy drops out at each and every loop order

No hidden equations — this is everything!

Residual cosmological constant $\Lambda_{eff} = \frac{1}{4}\langle \tau^\alpha_\alpha \rangle$

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Residual cosmological constant $\Lambda_{eff} = \frac{1}{4}\langle \tau^\alpha_\alpha \rangle$

Λ_{eff} has nothing to do with vacuum energy

It is radiatively stable

OK to fix it empirically

How big is Λ_{eff}?

For standard matter, space-time integrals dominated by time when universe is largest

$$\int d^4x\sqrt{-g} \sim \frac{1}{H_{age}^4}$$

where lifetime $t_{age} \sim \frac{1}{H_{age}} \gtrsim 13.7$ Gyrs

$$\langle \tau_\alpha^\alpha \rangle \sim \rho_{age} \sim \text{energy density at largest size} < \rho_c$$

$\Rightarrow \Lambda_{eff}$ *is not dark energy … too small!*

Observational consequences?

Universe has finite spacetime volume

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$$\frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 \sqrt{g}$$

space-time volume must be finite or else $\lambda \to 0$

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

if $\lambda \to 0$ particle masses go to zero

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Ends in a crunch

w=-1 is transient

Ω_k>0

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$\Omega_k > 0$

circles in the sky?
possible correlation
between 1+w and Ω_k

COLLAPSE TRIGGER

DARK ENERGY

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Linear potential $V = m^3\varphi$

form protected by shift symmetry, size of m^3 *technically natural*

If $\varphi_{in} > M_{pl}$*, then when scalar dominates, does so in SLOW ROLL until collapse time*

$$t_{collapse} \sim \sqrt{\frac{M_{pl}}{m^3}}$$

Radiatively stable choice of collapse time?

Radiatively stable choice of φ_{in}?

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Yes, thanks to m^3

Radiatively stable choice of φ_{in}?

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Yes, thanks to m^3

Radiatively stable choice of φ_{in}?

Yes, thanks to shift symmetry

But its not even a “choice”…. <R>=0 picks out precisely those solutions with $\varphi_{in} > M_{pl}$!!!!!!!

WHY NOW?

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Because the radiatively stable parameter
$m^3 \sim M_{pl} H_0^2$

Prediction:

WHY NOW?

Because the end is nigh!!!

Why is it nigh?

Because the radiatively stable parameter
$m^3 \sim M_{pl} H_0^2$

Prediction: $1+w \sim \Omega_\kappa^2$

Stuff I don't have time to talk about?

Symmetries
Phase transitions
Inflation
Particle Physics Phenomenology
Corrections to Planck mass

and plenty more........

Summary

Found a way to cancel SM vacuum energy at any order in loops

Locally theory looks just like GR with a small Λ

Small Λ is radiatively stable & determined by a global boundary condition

Universe is doomed


The End
(of the talk)

"Fine tuning" and renormalization

$$V_{vac}^{\phi,\text{1-loop}} = -\frac{m^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \text{finite} + \ln\left(\frac{M_{UV}^2}{m^2}\right)\right]$$

Divergent vacuum energy

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Divergent vacuum energy

$$\Lambda_{bare} = \frac{m^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \ln\left(\frac{M_{UV}^2}{\mathcal{M}^2}\right)\right]$$

Bare counterterm

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Bare counterterm

Renormalized vacuum energy:

$$\Lambda_{ren} = V_{vac}^{\phi,\text{1-loop}} + \Lambda_{bare} = \frac{m^4}{(8\pi)^2}\left[\ln\left(\frac{m^2}{\mathcal{M}^2}\right) - \text{finite}\right]$$

Depends on arbitrary subtraction scale so cannot be predicted...must be measured!

Symmetries?

Approximate scaling

$$\delta_\epsilon \lambda = \epsilon\lambda,\ \delta_\epsilon \Lambda = 4\epsilon\Lambda,\ \delta_\epsilon \left(\eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{pl}}\right) = -2\epsilon\eta_{\mu\nu} - \epsilon\frac{h_{\mu\nu}}{M_{pl}}$$

$$\delta_\epsilon S = \mathcal{O}\left(\frac{1}{M_{pl}}\right)$$

Approximate shift

$$\delta_\alpha \Lambda = \alpha m^4 \lambda^4,\ \delta_\alpha \mathcal{L}_m = \alpha m^4$$

$$\delta_\alpha S = \alpha \frac{m^4}{\mu^4}\sigma'$$

$$\frac{\Lambda_{eff}}{M_{pl}^2} \sim \frac{1}{4}\frac{\langle\tau\rangle}{M_{pl}^2}$$

→ 0 as $M_{pl} \to \infty$

→ 0 as $\mu \to \infty$ (for fixed volume)

conformal limit, vanishing λ

Phase transitions?

$$\tau^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle \tau^{\alpha}{}_{\alpha}\rangle = \begin{cases} -\langle V_{before} - V\rangle \delta^{\mu}{}_{\nu} & t < t_* \,, \\ -\langle V_{after} - V\rangle \delta^{\mu}{}_{\nu} & t > t_* \,. \end{cases}$$

$$\langle V_{after} - V\rangle = -\Delta V \frac{\int_{t_{bang}}^{t_*} dt a^3}{\int_{t_{bang}}^{t_{crunch}} dt a^3} \sim \mathcal{O}(1)\rho_{age}\frac{\Delta V}{M_{Pl}^2 H_*^2}\left(\frac{H_{age}}{H_*}\right)^{\frac{1-w}{1+w}} \ll \rho_{age} < \rho_{now}$$

negligible effect today

$\langle V_{before} - V\rangle \sim \mathcal{O}(1)\Delta V$ *short burst of inflation in build up to a transition*

Inflation?

In slow roll approximation,

$$3M_{Pl}^2 H^2 = \frac{1}{2}m^2\varphi^2 + \Lambda_{\rm eff}\,, \qquad \Lambda_{\rm eff} = -\frac{m^2}{2}\frac{\int dt a^3\varphi^2}{\int dt a^3}\,,$$

$$\int dt a^3\varphi^2 \sim \int_0^{t_{end}} dt a^3\varphi^2 100 M_{Pl}^2 \int_0^{t_{end}} dt a^3 \sim 100 M_{Pl}^2 e^{3N}/H^4$$

using $\varphi \le$ few $\times\, M_{Pl}$ during the last 60 efolds. Thus $|\Lambda_{\rm eff}| \lesssim 50\frac{m^2 M_{Pl}^2}{H^4\int dt a^3}e^{3N}$. But the integral in denominator involves full cosmic history, and is $\simeq 1/H_{age}^4$. This means,

$$|\Lambda_{\rm eff}| \lesssim 50\frac{m^2 M_{Pl}^2 H_{age}^4 e^{3N}}{H^4}\,.$$

So relative to the scale of inflation, $\frac{|\Lambda_{\rm eff}|}{M_{Pl}^2 H^2} \lesssim 50\frac{m^2 H_{age}^4 e^{3N}}{H^6}$. For slow roll inflation $m < H$. To solve the horizon problem, $H_0 e^N/H \le 1$. Also $H_{age} \le H_0$. All of this shows that

$$\frac{|\Lambda_{\rm eff}|}{M_{Pl}^2 H^2} \ll 1\,.$$

Particle Physics Phenomenology

Do not solve hierarchy problem, but compatible with other solutions

To avoid hierarchy between physical masses and bare masses, desire $\lambda \sim \mathcal{O}(1)$. Then if we take $\mu = |V_{vac}|^{1/4} r^{-1}$ for $\sigma(z) \sim e^z$ we have

$$\lambda \sim e^{\mathcal{O}(r^4)} \frac{H_{age}}{|V_{vac}|^{1/4}} r$$

So generally, for large $V_{vac} \gtrsim$ TeV, we only require $r \sim$ few to get $\lambda \sim \mathcal{O}(1)$.

Corrections to Planck mass?

$$\mathrm{M}_{Pl}^2 \to M_{Pl}^2 + \mathcal{O}(1) \times \mathcal{M}_{phys}^2 + \mathcal{O}(1) \times m_{phys}^2 \ln\left(\frac{\mathcal{M}_{phys}}{m_{phys}}\right) + \mathcal{O}(1) \times m_{phys}^2 + \ldots$$

Physical subtraction scale: $\mathcal{M}_{phys} = \lambda\mathcal{M}$, giving corrected action

$$S = \int d^4x\sqrt{g}\left[\frac{M_{Pl}^2 + \lambda^2 M^2}{2} R - \Lambda - \lambda^4 \mathcal{L}(\lambda^{-2} g^{\mu\nu}, \Phi)\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right) .$$

λM is total finite renormalization of M_{pl} which remains after subtracting the infinities.

...the larger and older the universe, the smaller λ, so easy to ensure $\lambda M < M_{pl}$

Weinberg No Go

$$S[\pi, g_{\mu\nu}] = \int d^4x\sqrt{-g}R + \Delta L(\pi, g_{\mu\nu}, \text{derivatives})$$

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Assume translation invariant solution for ANY vacuum energy:

$$\text{On shell field eqns: } \left.\frac{\partial \Delta L}{\partial g_{\mu\nu}}\right|_{g,\pi=\text{const}} = 0, \ \left.\frac{\partial \Delta L}{\partial \pi}\right|_{g,\pi=\text{const}} = 0$$

Scalar eqn $\implies$ trace of gravity eqn

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If $g_{\mu\nu}$ and π are constant then ΔL is invariant under

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where we define $\phi = \int \frac{d\pi}{f(\pi)}$

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Then $\Delta L = \sqrt{-\hat{g}}\mathcal{L}(\hat{g}_{\mu\nu}, \text{derivatives})$ where $\hat{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$

$$\implies \left.\frac{\partial \Delta L}{\partial g_{\mu\nu}}\right|_{g,\pi=\text{const}} = \frac{1}{2} g^{\mu\nu} \Delta L\Big|_{g,\pi=\text{const}}$$

Recall $\Delta L_{g,\pi=\text{const}} = -V_0\sqrt{-\hat{g}}$

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Fine tuning

$$\Longrightarrow \left.\frac{\partial \Delta L}{\partial g_{\mu\nu}}\right|_{g,\pi=\mathrm{const}} = \frac{1}{2} g^{\mu\nu} \Delta L\Big|_{g,\pi=\mathrm{const}}$$

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Fine tuning

Scale invariance