Gravity Modification

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References

work in collaboration with D. Comelli, M. Crisostomi and F. Nesti

Comelli-Crisostomi-Nesti-LP PRD86 101502 (2012)
Comelli-Crisostomi-Nesti-LP JHEP 1203 (2012) 067
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Comelli-Nesti-LP JCAP 05 036 (2014)

Modified Gravity ?

A single free parameter: $G \sim 1/M_{\rm pl}^2$

- Weak Equivalence principle (10^{-13})
- Post Newtonian solar system tests (weak field) ($10^{-3} - 10^{-5}$)
- Indirect GWs emission test: binary pulsar (10^{-3})
- GR is an effective field theory:

  quantum corrections small and under control when
  $E << \Lambda_{\text{cut-off}} \sim \left(10^{-33}\,\text{cm}\right)^{-1} \sim M_{pl} \sim 10^{19}$ GeV

Then why ?

- Theoretical motivation: is GR an isolated theory ?
  In gauge theories we can give mass to gauge bosons ($W^{\pm}$, Z) effectively controlling the interaction range:

  $$\frac{1}{r} \qquad \text{vs} \qquad \frac{e^{-m\,r}}{r}$$

  Is GR gauge theory alike and a massive gravity phase exists ?
- Cosmological motivation:
  The universe is accelerating
  GR requires a tiny cosmological constant $\Lambda \sim \left(10^{-3}\,eV\right)^4$
  CC is equivalent to a fluid with $p = -\rho$, $w = -1$
  Quantum Field Prediction is way out observed value
  $\Lambda_{\text{QFT}} \geq 10^{60}\,\Lambda_{\text{obs}}$
  Perhaps $w \neq 1$ and a Dark Energy model is needed

Modify what and at what scale ?

$$\int d^4x\sqrt{g}\left[M_{pl}^2 R(g) + \mathcal{L}_{matter}(g,\phi)\right]$$

Where ?

Dark energy scale $H_0^{-1} \approx 4.2$ Gpc or $H_0 \approx 10^{-33}$ eV

Modification in the infrared: large distance and low energy

What ?

- Modify the way matter couple to gravity is modified

  tough ... equivalence principle is well established
- New "gravitational" fields that couple with $g_{\mu\nu}$ are introduced Scalars, vectors, tensors ...
- $g_{\mu\nu}$ is still the only "gravitational" field but $R(g)$ is modified
- Add non derivative terms for $g_{\mu\nu}$

Warm up: Massive Electrodynamics

- Massless case $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu$

  one gauge invariance $A_\mu \to A_\mu + \partial_\mu f$

  $\partial^\nu F_{\mu\nu} = -J_\mu$

  #DOF: $2 = 4 - 2 \times 1$ the two elicity states of the photon

- Massive case $L_m = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + A_\mu J^\mu$

  $\partial^\nu F_{\mu\nu} + m^2 A_\mu = J_\mu \Rightarrow \partial^\mu A_\mu = 0$

  #DOF: $3 = 4 - 1$ massive spin 1 particle $\to$ three spin states

  no gauge invariance, broken by the mass term

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Stuckelberg trick, gauge inv. as redundant description

Does really a massive photon break gauge invariance ?

add a scalar ϕ by the following field redefinition $A_\mu = \widetilde{A_\mu} + \partial_\mu \phi$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + j^\mu A_\mu = -\frac{1}{4}\widetilde{F_{\mu\nu}}\widetilde{F^{\mu\nu}} - \frac{m^2}{2}\left(\widetilde{A_\mu} + \partial_\mu \phi\right)^2 + J^\mu \widetilde{A_\mu}$$

Massive ED plus ϕ is gauge invariant again

$\widetilde{A_\mu} \to \widetilde{A_\mu} + \partial_\mu f$ and $\phi \to \phi - f$ #DoF counting: $4 + 1 - 2 \times 1 = 3$!

Special choice of gauge (unitary): $f = -\phi \Rightarrow$ back to massive ED

The limit $m \to 0$ is smooth, the extra degree of freedom decouples

Bottom line:
adding (Stuckelberg) fields any theory can be made gauge invariant

Gauge invariant is a very useful redundant description of physics

Massive gravity, linearized level

a mass term is the simplest infrared modification

- GR in the weak field limit

$$M_{pl}^2 E_{\mu\nu}^{(1)} = T_{\mu\nu}^{(1)} \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

#DOF: $10 - 2 \times 4 = 2$ 4 gauge modes $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

- Linear mGR theory Pauli and Fierz '39

$$L = M_{pl}^2 L_{\text{spin } 2} + M_{pl}^2 m^2 \left(a\, h_{\mu\nu} h^{\mu\nu} + b\, h^2\right) \qquad h = h_{\mu\nu}\eta^{\mu\nu}$$

$$E_{\mu\nu}^{(1)} - \frac{m^2}{4}\left(a\, h_{\mu\nu} + b\, h\, \eta_{\mu\nu}\right) = M_{pl}^{-2} T_{\mu\nu}^{(1)} \qquad \partial^\nu E_{\mu\nu}^{(1)} = 0$$

4 constraints #DOF: $10 - 4 = 6 = 5 + 1$

- The sixth mode is a ghost (Boulware-Deser) when $a + b \neq 0$.

$$h_{00} = \psi\,, \qquad h_{0i} = \partial_i v\,, \qquad h_{ij} = \partial_i \partial_j \sigma + \delta_{ij}\tau$$

$$L = M_{\rm pl}^2 k^2 \begin{pmatrix} \dot\sigma \\ \dot\tau \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & \frac{4}{a} \end{pmatrix} \begin{pmatrix} \dot\sigma \\ \dot\tau \end{pmatrix} + \ldots \qquad \partial_i \partial_i \to -k^2$$

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Discontinuity vs Ghost: Vainshtein Mechanism

tune $a + b = 0$ Pauli-Fierz (PF) theory

no ghost, however the $m \to 0$ limit is not smooth (vDVZ disc.)

an extra scalar mode still couples to matter when $m \to 0$

$\Rightarrow$ light bending is 25% off from the experimental value

linear FP theory fails to pass solar system tests

very clean using Stuckelberg formulation Arkani-Hamed-Georgi-Schwartz 03

Saving PF: linearized approximation must fail in the solar system

valid only $r > r_V = \left(M_\odot\, m^{-2}\, M_{pl}^{-2}\right)^{1/3} \sim 10^{16}$ Km

Vainshtein '72

Babichev-Deffayet-Ziour '09

Kaloper-Padilla-Tanahashi '11

the discontinuity is removed in non perturbative way: Vainshtein Mechanism

interesting but problematic for massive gravity

Sbisa,Niz,Koyama,Tasinato '12

Koyama, Niz,Tasinato '13

for a review Babichev-Deffayet '13

Avoiding the vDVZ discontinuity

Rubakov 04, Dubovsky 04

- There is a way to avoid the vDVZ discontinuity: Berezhiani-Comelli-Nesti-LP 07

  change the structure of mass terms

$$L_{\text{spin 2}} + \frac{1}{4}\left(m_0^2\, h_{00}^2 + 2m_1^2\, h_{0i}h_{0i} - m_2^2\, h_{ij}h_{ij} + m_3^2\, h_{ii}^2 - 2m_4^2\, h_{00}h_{ii}\right)$$

  different masses for each rotational invariant combination
  No Lorentz symmetry for mass terms

  No ghost when $m_0 = 0$

  $SO(3)$ invariant mass terms give a smooth $m \to 0$ limit

  $m_i \to 0 \quad m_i^2/m_j^2$ fixed, no discontinuity

- The matter coupling is standard, weak equivalence principle OK
- Weak field expansion works fine in the solar system Comelli-Nesti-LP 13

  Post Newtonian correction can be computed in controlled way

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Linear Theory: Summary

- PF Lorentz invariant massive GR:

  either ruled out or non-perturbative in the solar system

- Lorentz breaking massive GR:

  consistent and in agreement with basic weak-field tests

What happens beyond the linear level ?

Nonlinear Massive Gravity I

- Add to the GR Lagrangian an extra piece V that depends on the metric field (no derivatives allowed)

$$\sqrt{g}\left(R - m^2 V\right)$$

such that when $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\sqrt{g}\left(R - m^2 V\right) = \mathcal{L}_{\text{spin 2}} + \frac{1}{4}\left(m_0^2\, h_{00}^2 + 2m_1^2\, h_{0i}h_{0i} - m_2^2\, h_{ij}h_{ij} + m_3^2\, h_{ii}^2 - 2m_4^2\, h_{00}h_{ii}\right) + \cdots$$

- Lorentz invariant mass term when V is such that:

$$m_0^2 = a + b\,, \quad m_1^2 = -b\,, \quad m_2^2 = -a\,, \quad m_4^2 = b\,, \quad m_3^2 = b$$

$$-m^2\, V = m^2\, \left(a\, h_{\mu\nu} h^{\mu\nu} + b\, h^2\right) + \ldots$$

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Nonlinear Massive Gravity II

- To build a nontrivial V we need extra stuff
  There is no "scalar" function of the metric itself
- option 1: Introduce an extra **non-dynamical metric** $\tilde{g}_{\mu\nu}$
- option 2: $\tilde{g}_{\mu\nu}$ is a **dynamical metric**

  we end up in a **bimetric theory**: bigravity   not in this talk
- A Lorentz invariant example: take $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$

  Scalars made out of $X^{\mu}_{\nu} = g^{\mu\alpha}\eta_{\alpha\nu}\,,\qquad \tau_n = \mathrm{Tr}(X^n)$

$$a\,(\tau_1 - 4)^2 + b\,(\tau_2 - 2\tau_1 + 4) = \left(a\,h_{\mu\nu}h^{\mu\nu} + b\,h^2\right) + O(h)^3$$

  Similiar in the $SO(3)$ invariant case

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Nonlinear Massive Gravity: Stuckelberg

- Diff restored by four Stuckelberg fields ϕ^A Arkani-Hamed-Georgi-Schwartz 03

$$\eta_{\mu\nu} \to \tilde{g}_{\mu\nu} = \partial_\mu \phi^A \partial_\nu \phi^B \eta_{AB}$$

  The Stuckelberg are “coordinates” of a fictitious flat space and $e^A = d\phi^A$ are the tetrads with $de^A = 0$

- Adapting the coordinates such that $\tilde{g}_{\mu\nu}$ is the Minkowski metric

  (unitary gauge) $\partial_\mu \phi^A = \delta^A_\mu$

- Unitary gauge coordinates represents a preferred frame to be specified

  For instance: the frame where the sun is at rest or where the CMB is almost isotropic

- Similar Stuckelberg construction in the Lorentz breaking case

  Dubovsky 04, Rubakov-Tnyakov 08, Comelli-Nesti-Pilo 13

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Nonlinear Massive Gravity III

- The presence of $\tilde{g}_{\mu\nu}$ breaks inevitably local Lorentz (grav. sector)

$$g_{ab} = \mathrm{diag}(-1, 1, 1, 1)\,, \qquad \tilde{g}_{ab} = \mathrm{diag}(-\alpha_0, \alpha_1, \alpha_2, \alpha_3)$$

  Accidental Lorentz symm. of V (unitary gauge) is different from local Lorentz symmetry of the equivalence principle

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Taming the zoo of Massive Gravity: non-linear analysis

How to choose V ? How many DoF propagate ?

Non-perturbative analysis needed

Canonical analysis is best suited $\begin{cases} \text{nonperturbative} \\ \text{background independent} \end{cases}$

ADM splitting of spacetime

(spatial) 3-metric γ_{ij} of $t = const$ hypersurface
normal $n^\mu = (N^{-1}, N^{-1}\, N^i)$ of the hypersurface
$N \to$ Lapse function, $N^i \to$ shift vector

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j \gamma_{ij} & \gamma_{ij} N^j \\ \gamma_{ij} N^j & \gamma_{ij} \end{pmatrix} \qquad N^A = (N, N^i)$$

ADM language $\qquad \mathcal{V} = m^2 \sqrt(g)\, V = m^2\, N\, \gamma^{1/2}\, V(N^A, \gamma_{ij})$

Comelli-Crisostomi-Nesti-Pilo '12, Comelli-Nesti-Pilo '13 & '14

Results from Canonical Analysis

1. No condition on $V \Rightarrow$ 6 DoF propagate
   Around flat space : 2 tensors + 2 vectors + 1+ 1 scalars
   One of the scalars is a the Boulware-Deser ghost. No good
2. 5 DoFs propagate if and only if

$$\det\left(\frac{\partial^2 V}{\partial N^A \partial N^B}\right) \equiv \det(V_{AB}) = 0 \quad \text{and} \quad \mathrm{rank}(V_{AB}) = 3$$ Monge-Ampere eq.

   plus another condition on V involving γ_{ij} not shown

3. $\mathrm{rank}(V_{AB}) < 3 \Rightarrow$ #DoF$= 2, 3$ see Comelli-Nesti-Pilo 2014

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Taming the zoo of Massive Gravity: bottom line

Construction of **all** mGR with (5 DOF)

infinitely many in terms of two functions !!

$$V(N, N^i, \gamma_{ij}) = \mathcal{U} + N^{-1} \left(\mathcal{E} + \mathcal{Q}^i\, \mathcal{U}_{\xi^i}\right)$$

$$\mathcal{U}(\mathcal{K}^{ij})\,, \quad \mathcal{E}(\gamma_{ij},\ \xi^i) \qquad \mathcal{K}^{ij} = \gamma^{ij} - \xi^i\,\xi^j$$

ξ^i is defined by $N^i - N\,\xi^i = \left(\frac{\partial^2 \mathcal{U}}{\partial \xi^i \partial \xi^j}\right)^{-1} \frac{\partial \mathcal{E}}{\partial \xi^j} \equiv \mathcal{Q}^i(\gamma^{ij}, \xi^i)$

- Interesting example:
  $\mathcal{E} \equiv \mathcal{E}(\gamma_{ij}) \;\Rightarrow\; \xi^i = N^i/N \quad \Rightarrow \quad \mathcal{K}^{ij} = \gamma^{ij} - N^{-2}\,N^i\,N^j \equiv g^{ij}$

  $$V = \mathcal{U}(g^{ij}) + N^{-1}\,\mathcal{E}(\gamma_{ij})$$

- The nonlinear version the Pauli-Fierz mass term (Lorentz invariant) (de Rham-Gabadze-Tolley) is recovered

  $$V \sim \mathrm{Tr}(X^{1/2}), \quad X^{\mu}_{\nu} = g^{\mu\alpha}\eta_{\alpha\nu}$$

Taming the zoo of Massive Gravity: bottom line

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Pauli-Fierz Pauli Status

- The FP tuning can extended at the nonlinear level de Rham-Gabadaze-Tolley '11
- In the Solar system nonperturbative physics required (Vainshtein)

  post Newtonian corrections are very difficult to compute

  see however Avilez-Lopez-Padilla-Saffin-Skordis '15

  Vainshtein mechanism is problematic in massive gravity
- No spatially flat FRW cosmology
- As an effective field theory the cutoff is very low

  $$\Lambda_3 = \left(m^2 M_{pl}\right)^{1/3} \sim \left(10^3 \text{ Km}\right)^{-1} \sim 10^{-13} \text{ eV}$$

  Even the static gravitation potential of two masses one meter apart might get large quantum corrections

  see conflicting papers: Burrage-Kaloper-Padilla '14 and N. Brouzakis and N. Tetradis '14

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Massive Gravity Cosmology: general results I

Comelli-Nesti-LP 14

- The most general ansatz compatible with homogeneity CMB isotropy frame $\equiv$ massive GR preferred frame

$$ds^2 = -N^2 dt^2 + a(t)^2 \, \delta_{ij} \, dx^i dx^j$$

zero spatial curvature for simplicity

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$\mathcal{U}$ enters in the total Hamiltonian as $N\mathcal{U}$ likewise GR as required by time reparametrization $\Rightarrow \mathcal{U}$ part automatically conserved

Only the $\mathcal{E}$ part is constrained

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- Non-perturbative construction of all mGR theories with 5 DoF
- mGR theory favours Lorentz breaking theories in the grav. sector

  - 5 DOF and no vDVZ discontinuity

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General Picture and Summary