Spectral distortions in the cosmic microwave background polarization

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Cargèse. Hot Topics in Modern Cosmology
28.04.2015

Worked on:

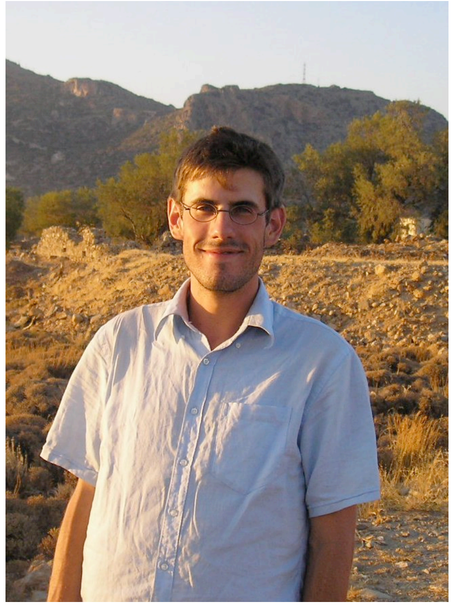
- Inflation: multifield effects, string-inspired models, primordial non-Gaussianities (review to appear for French Academy of Science)
- Modified Gravity: Massive Gravity on de-Sitter, Vainshtein mechanism
- New subject: Spectral distortions

Outline

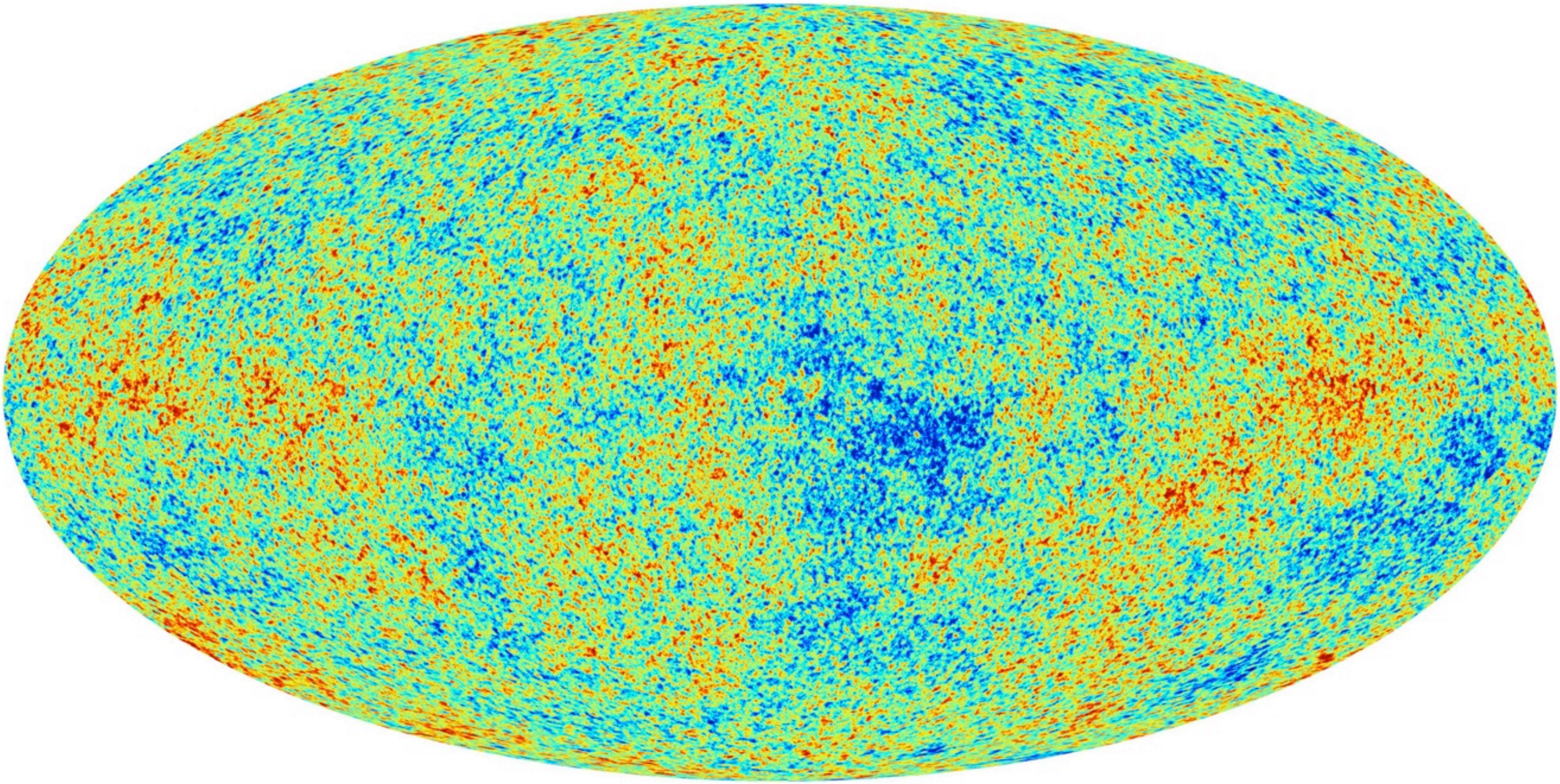
1. Spectral distortions

2. Our work

JCAP 03 (2014) 033
SRP, C. Fidler (Portsmouth), C. Pitrou (IAP), G. Pettinari (Sussex)



Cosmic Microwave Background temperature fluctuations



Planck all sky map

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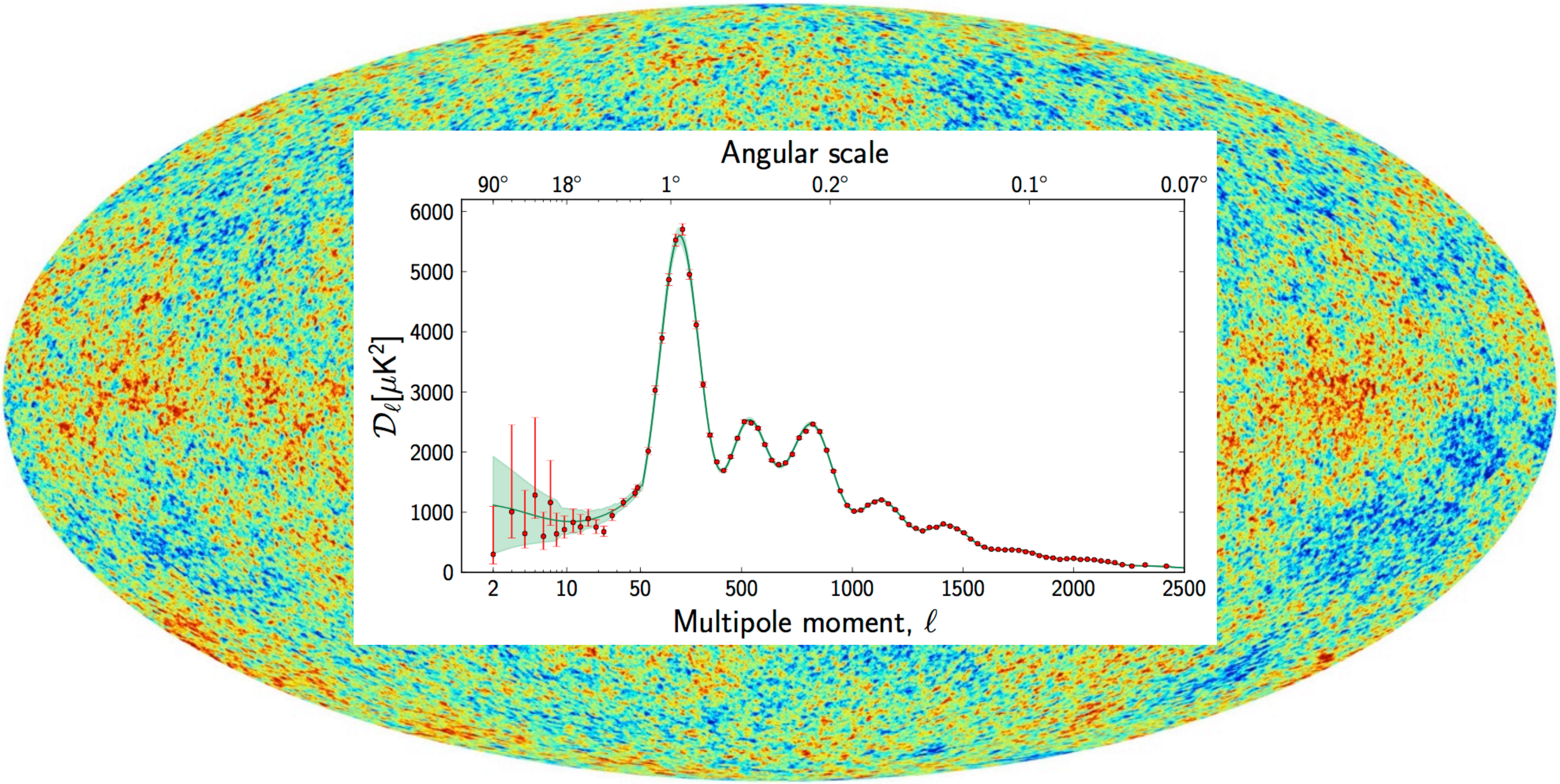

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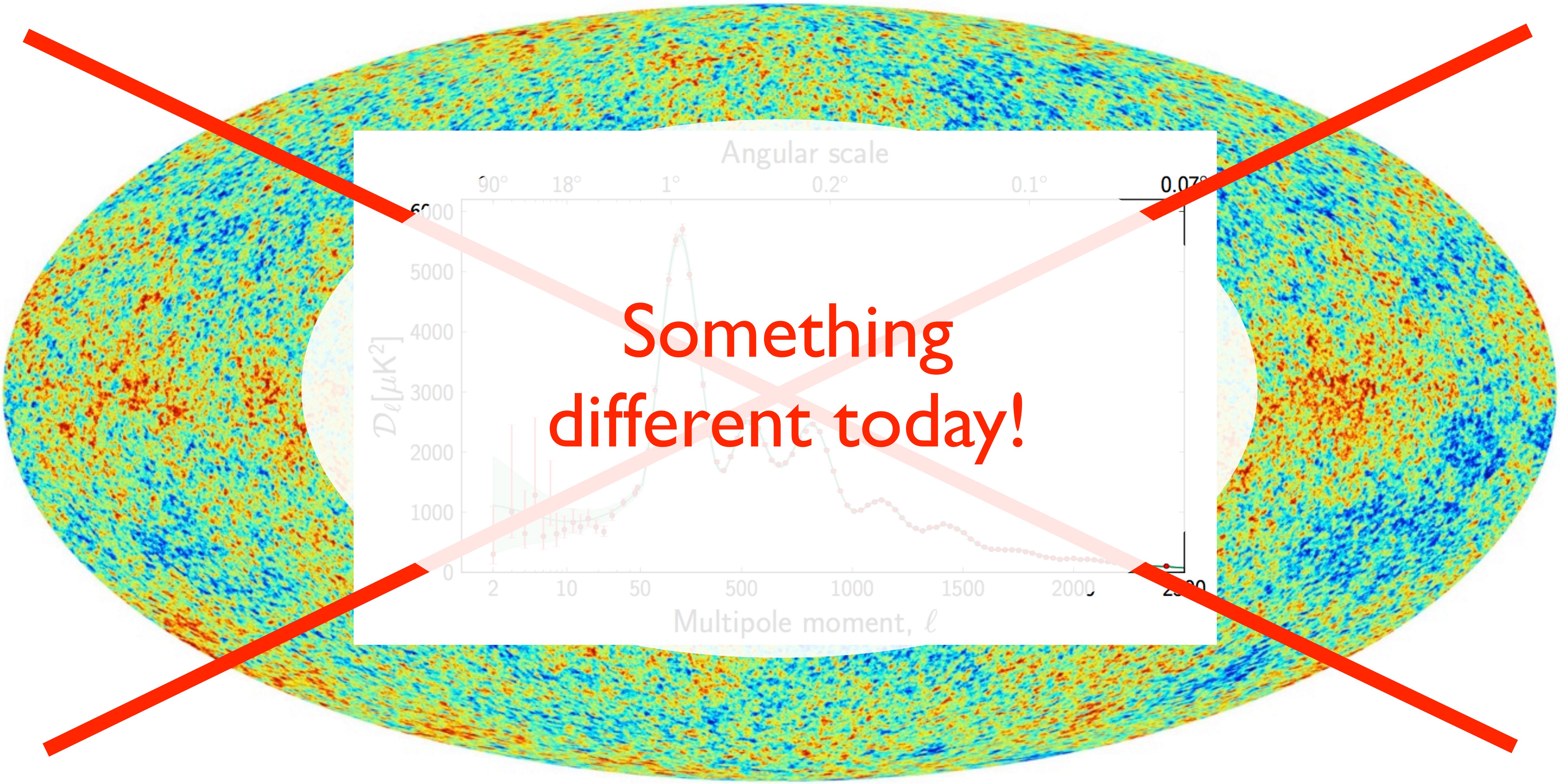


Planck all sky map

Energy dependence

- Previous picture assumed: $$I_{BB}(E,\hat{n}) = \frac{2}{e^{\frac{E}{T(\hat{n})}} - 1}$$

- Blackbody (BB) distribution of the CMB intensity with direction-dependent temperature.

- **But**: no full thermodynamic equilibrium throughout the universe history

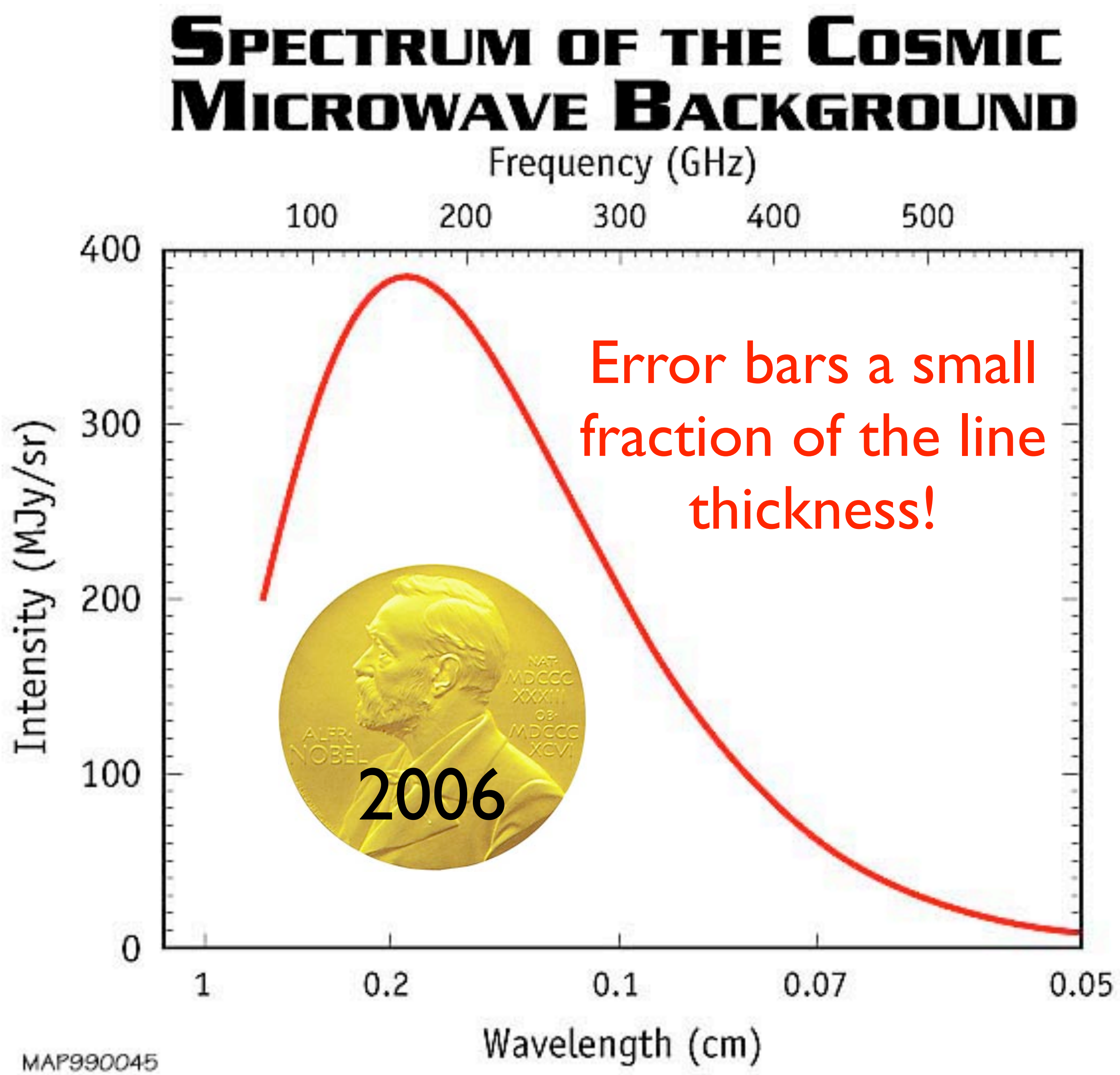
→

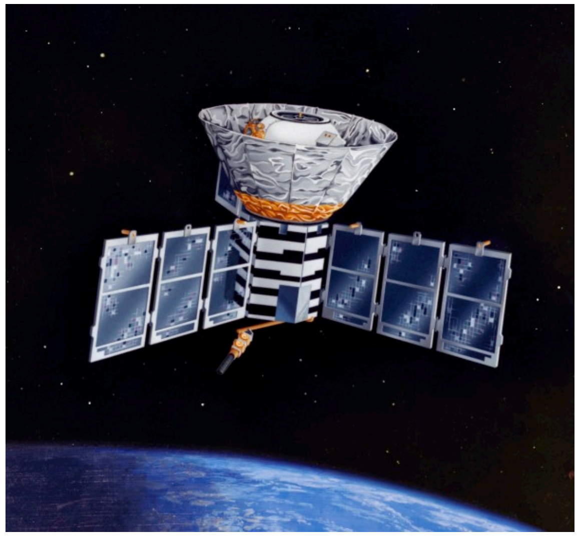
- The energy dependence **is** more complicated

- The temperature is not enough to characterize the CMB signal. Its spectral dependence contains another independent piece of information.

Current spectral distortions constraints

COBE/FIRAS (**F**ar **I**nfra**R**ed **A**bsolute **S**pectrophotometer)





Compton y-distortion:

$$|y| \leq 1.5 \times 10^{-5}$$

Chemical potential mu-distortion:

$$|\mu| \leq 9 \times 10^{-5}$$

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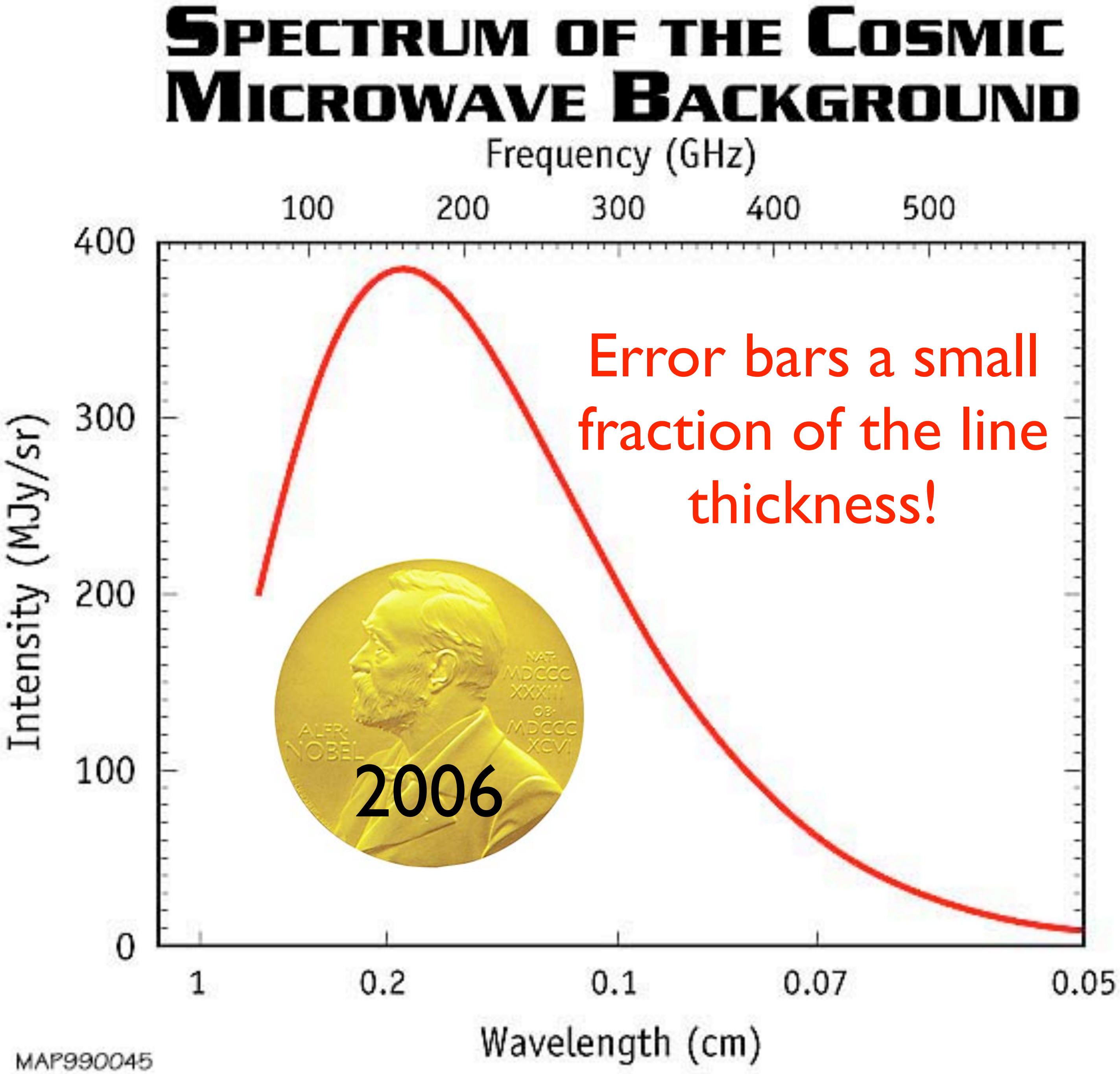

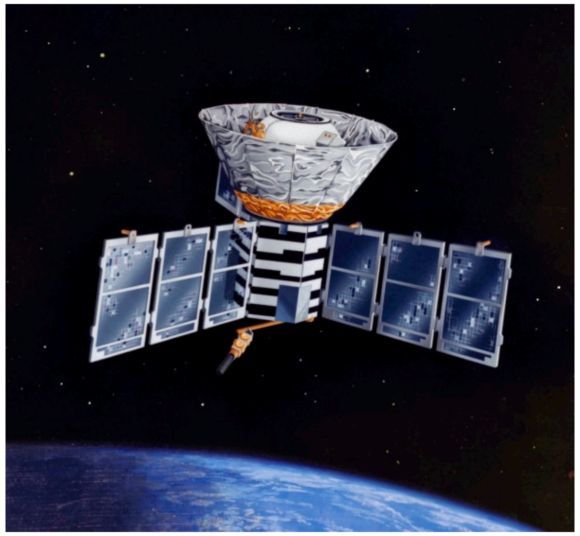

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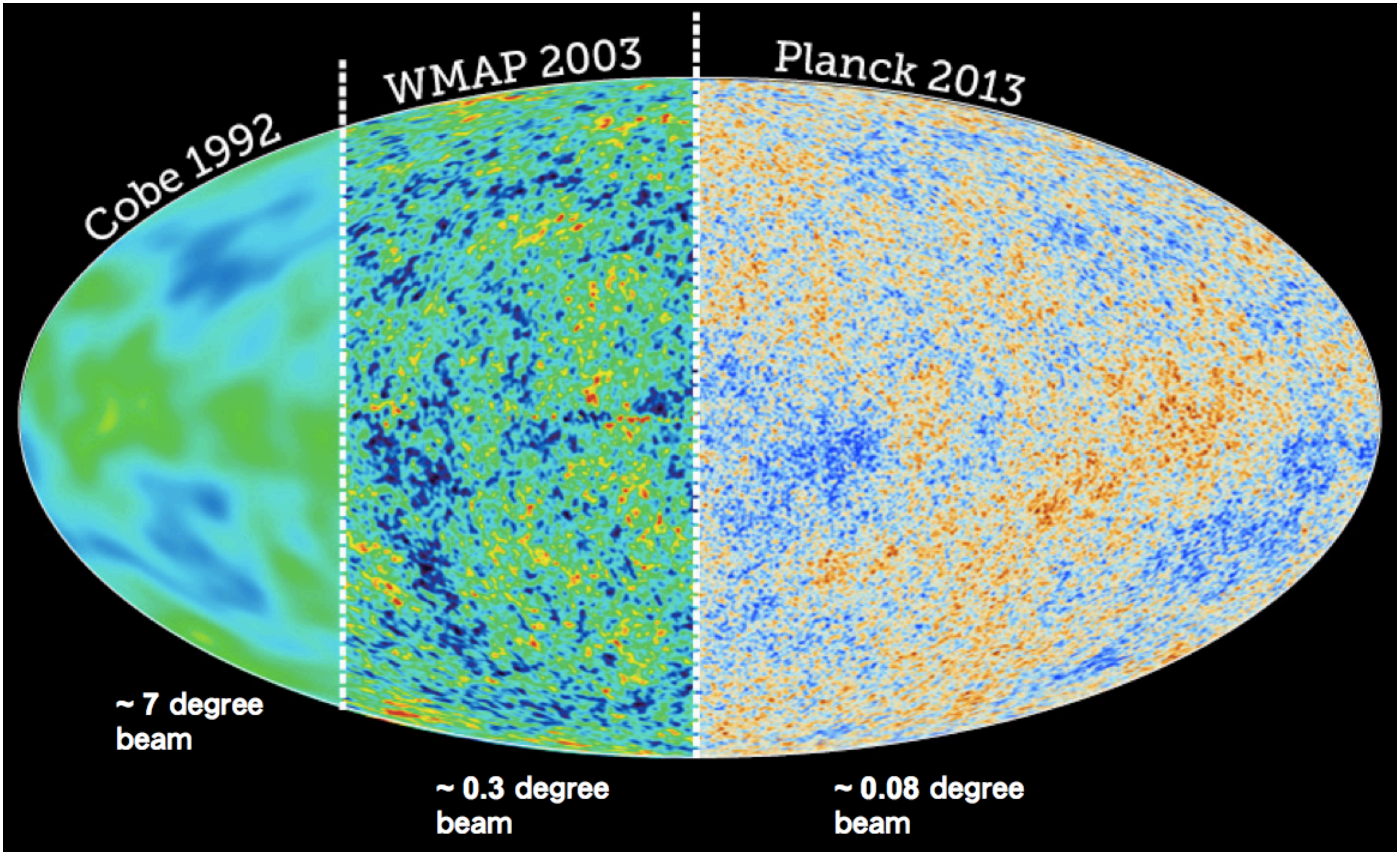
Only very small distortions of the CMB spectrum are allowed

Dramatic improvement in angular resolution and sensitivity in the past decades



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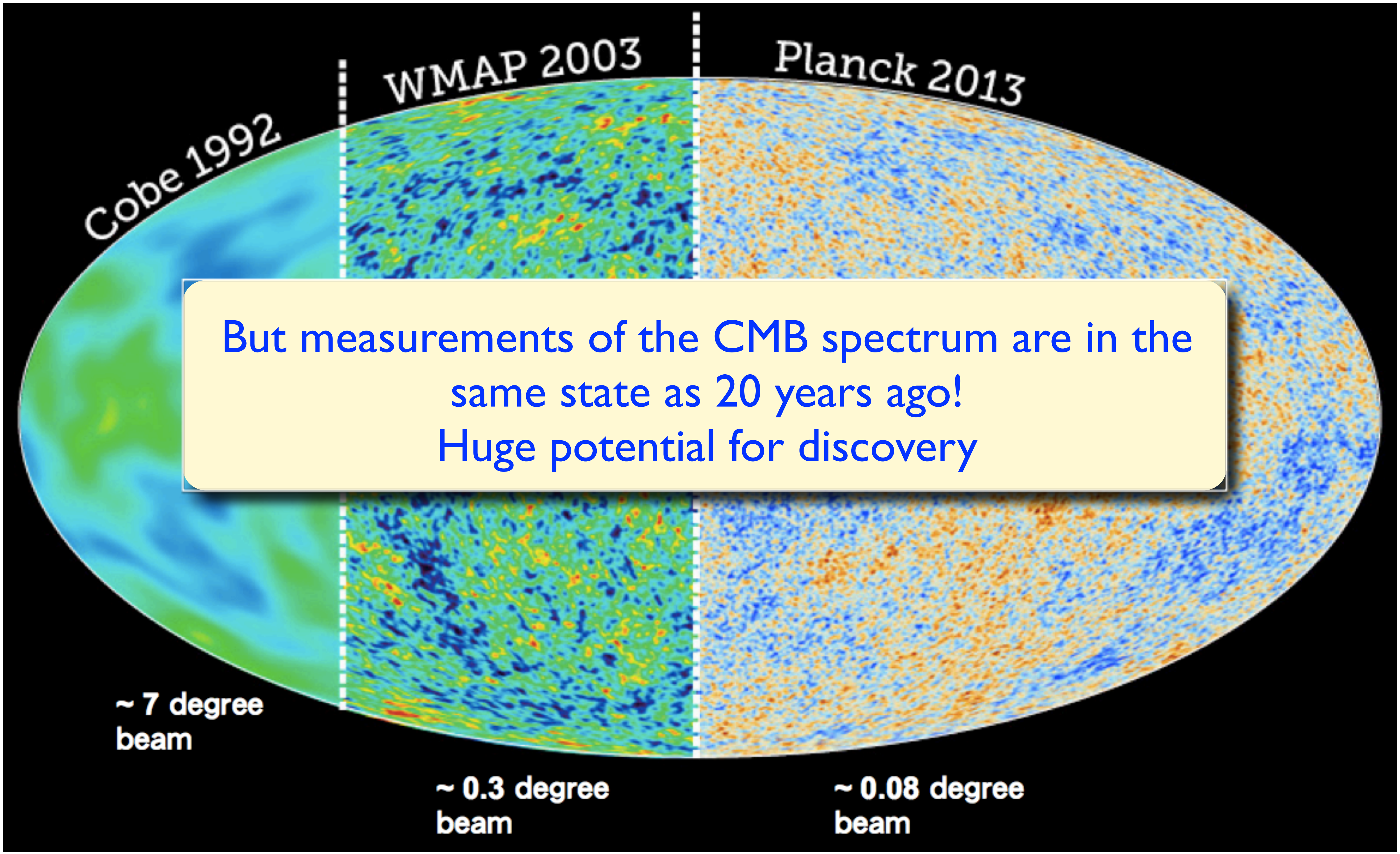


But measurements of the CMB spectrum are in the same state as 20 years ago!
Huge potential for discovery

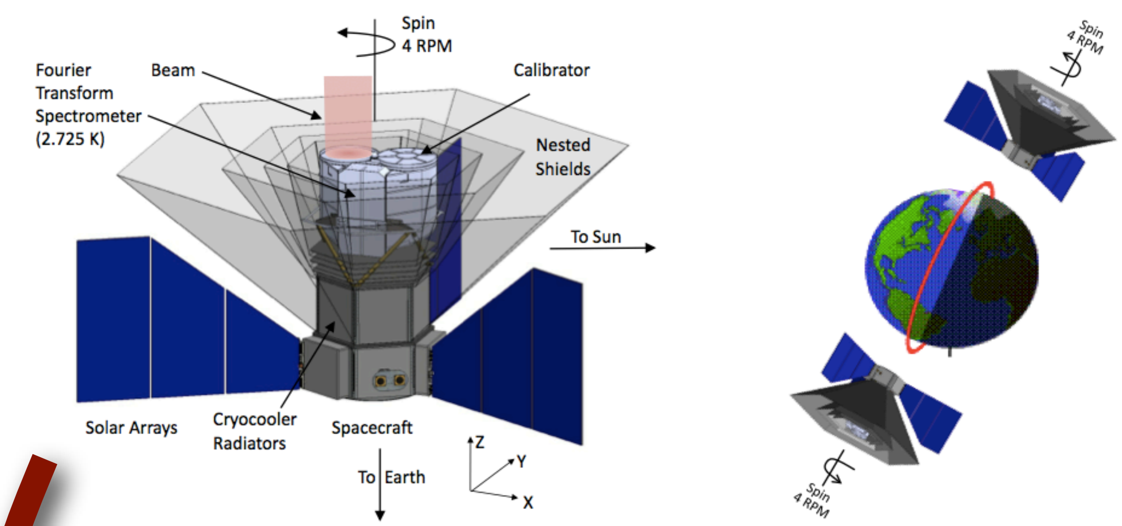
Future expected constraints

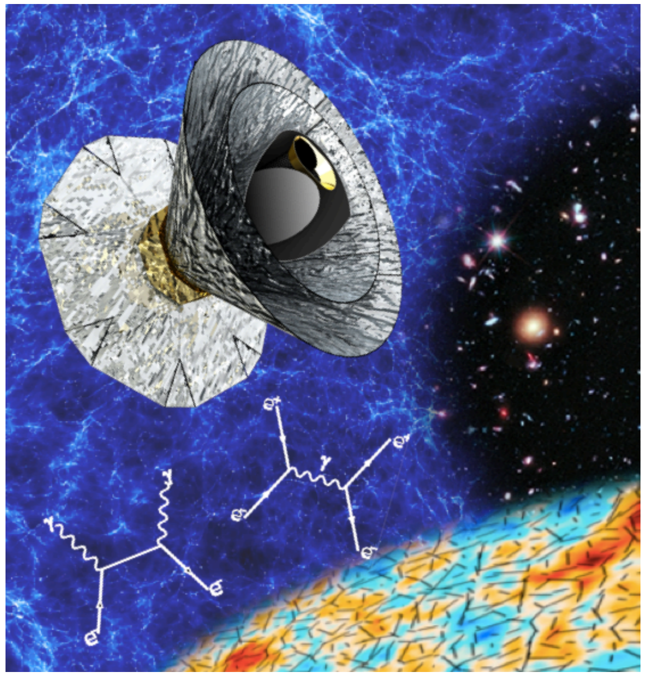
PIXIE

arXiv:1105.2044

COrE/PRISM?

arXiv:1310.1554



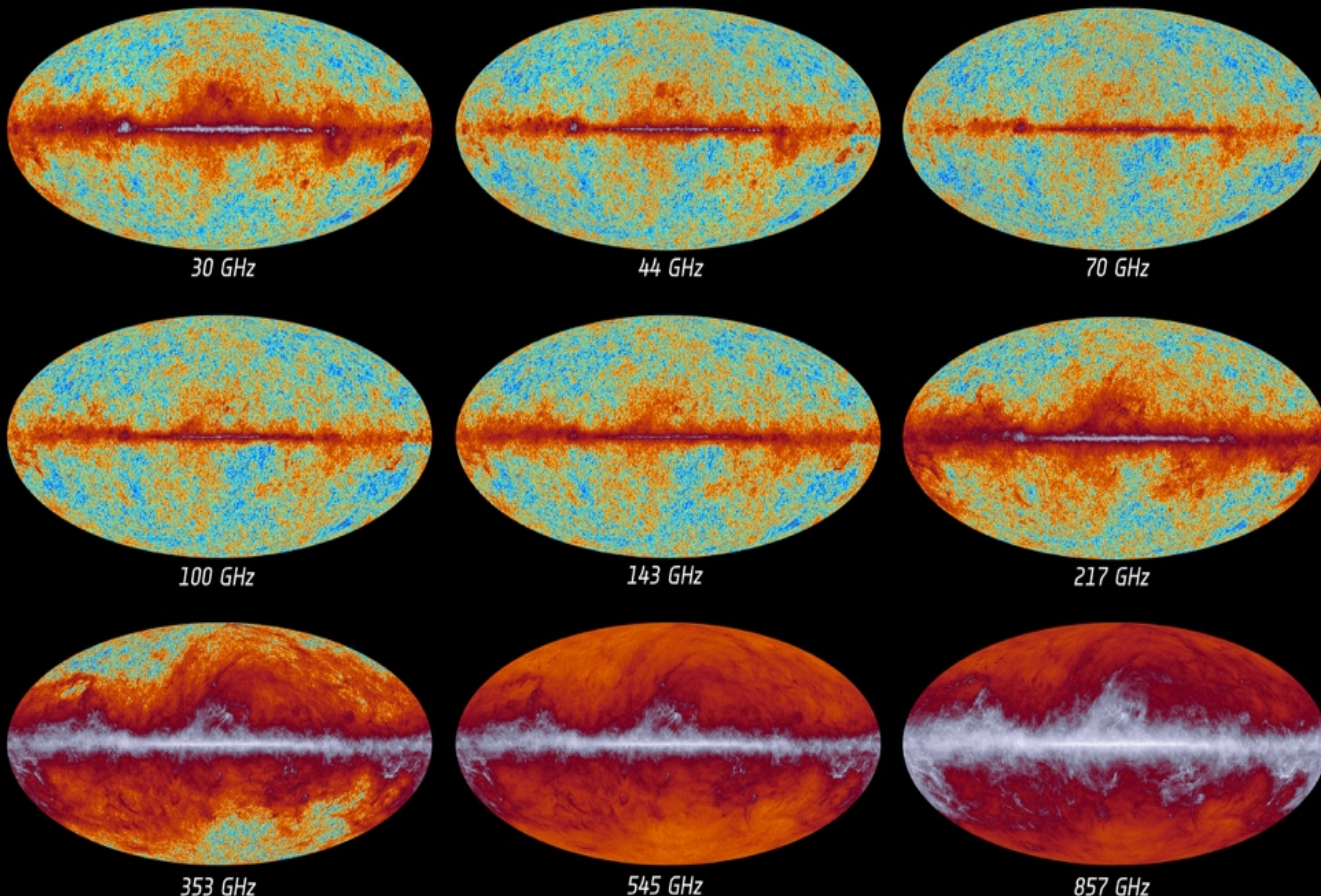


- 400 spectral channels in the frequency range 30 GHz - 6 THz (9 for Planck)
- About 1000 times more sensitive than COBE/FIRAS
- Improved limits on mu and y by 3 orders of magnitude!

planck

The sky as seen by Planck

esa



Physical mechanisms that lead to spectral distortions

- Energy injection in the primordial plasma at $z <$ few x 10^6

- Heating by decaying or annihilating relic particles

- Dissipation of primordial acoustic waves (window into small scale power spectrum)

- Cosmological recombination

Les Houches lecture notes, Chluba 13

- SZ effect from galaxy clusters, effects of reionization ...

Lots of effects within the reach of future experiments

The field of CMB spectral distortions is observationally and theoretically very promising.

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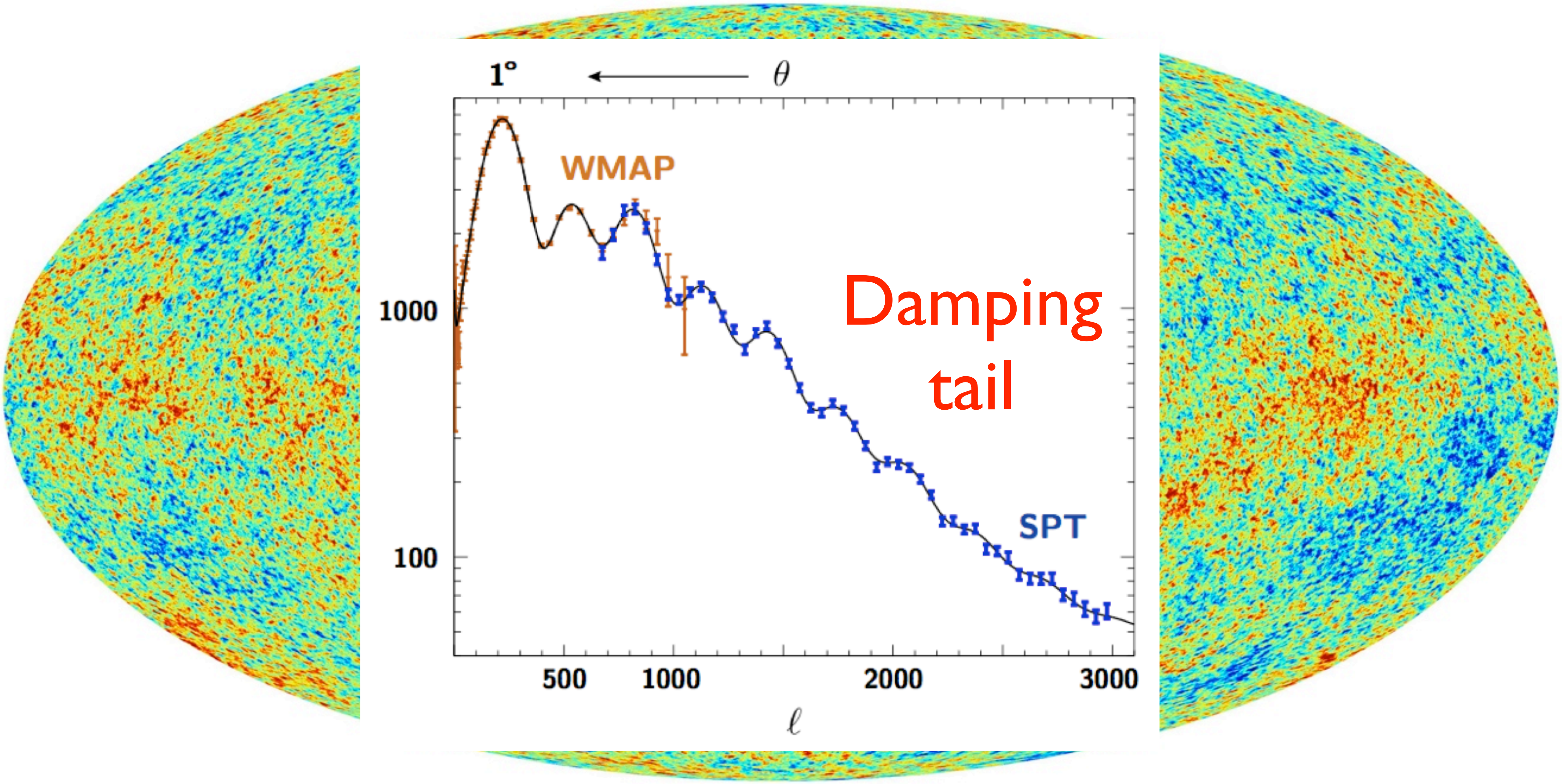
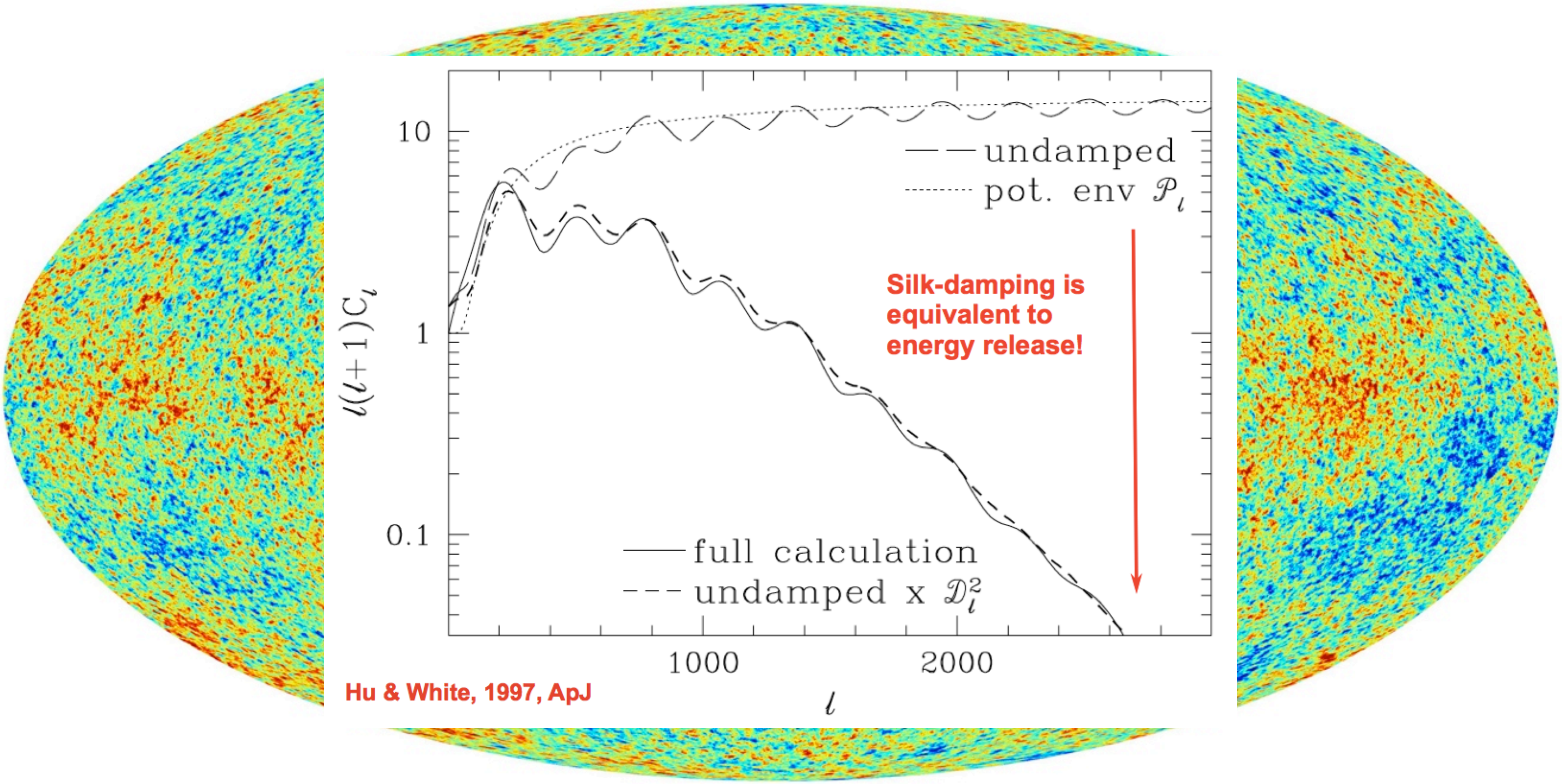
Dissipation of small-scale acoustic modes



Dissipation of small-scale acoustic modes



Dissipation of small-scale acoustic modes

Dependencies on:

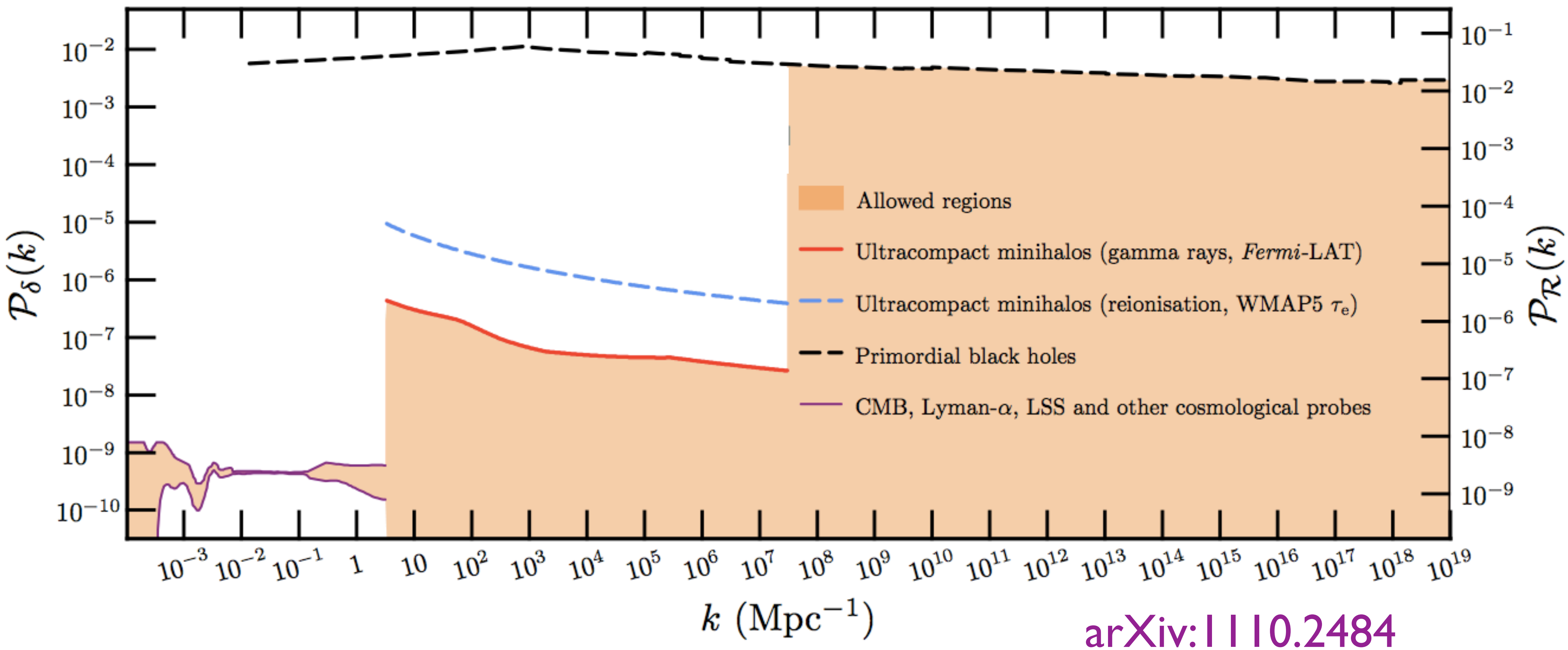
- Amplitude of the power spectrum
- Shape of the power spectrum
- Primordial non-Gaussianities in the squeezed limit

Pajer and Zaldarriaga 2012, Gang and Komatsu 2012

- Type of the perturbations (adiabatic vs isocurvature)

Dent et al 2012, Chluba and Grin 2012

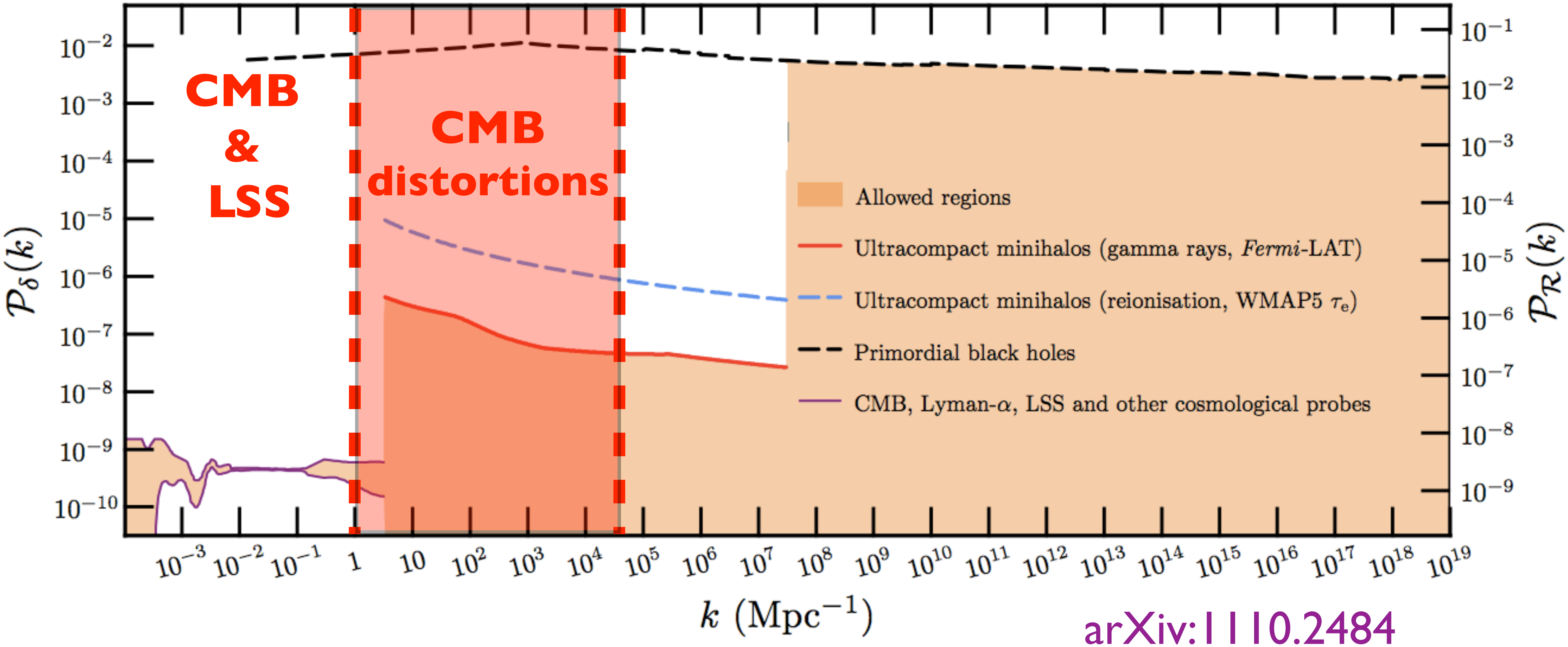
Power spectrum constraints



arXiv:1110.2484

- Amplitude of the power spectrum rather uncertain at k > 3 Mpc^{-1}
- Improving limits at smaller scales would constrain inflationary models

Power spectrum constraints



arXiv:1110.2484

- Amplitude of the power spectrum rather uncertain at k > 3 Mpc^{-1}
- Improving limits at smaller scales would constrain inflationary models
- CMB spectral distortions allows us to probe a total of 17 e-folds of inflation

Our work

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• The field of CMB spectral distortions is still in its infancy

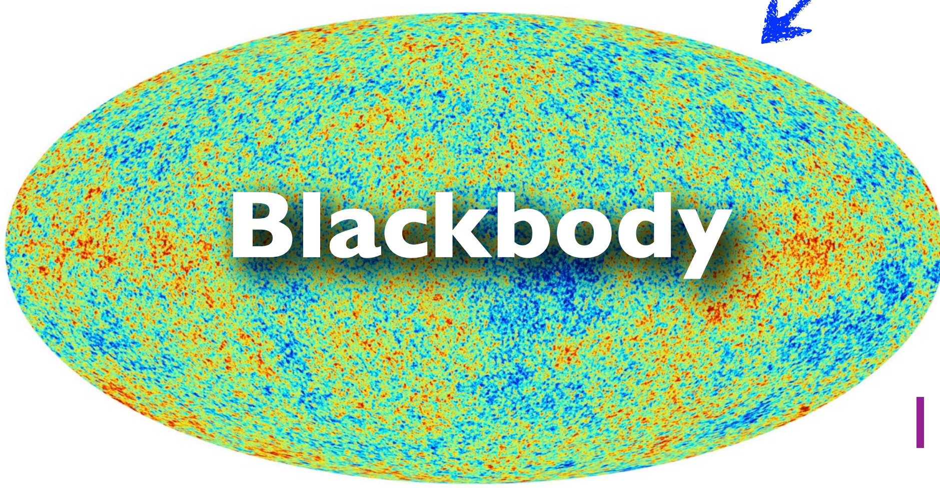
• Most work to date concentrate on the CMB intensity, and its monopole

• But future experiments will characterize the spectrum of the CMB anisotropies, both in intensity and polarization.

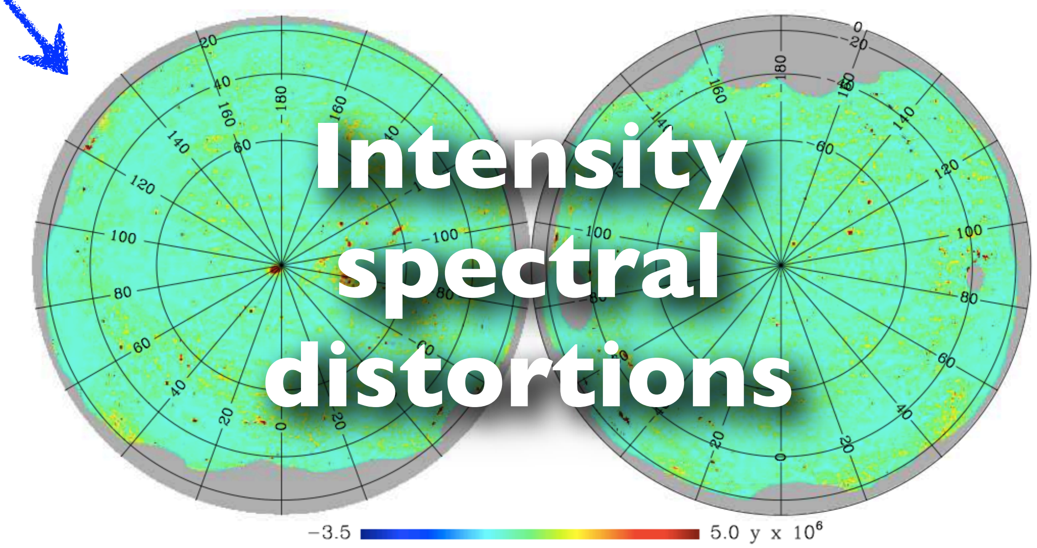
• In 1312.4448, we computed the unavoidable spectral distortions of the CMB polarization induced by non-linear effects in the Compton interactions between CMB photons and the flow of intergalactic electrons (non-linear kinetic Sunyaev Zel'dovich, kSZ^2)

CMB spectral distortions

$$I\left(E,\hat{n}\right) = I_{\text{Planck}}\left(E;T(\hat{n})\right) + y(\hat{n}) \times \left(\begin{matrix}\text{Other}\\ \text{spectral dependence}\end{matrix}\right)$$



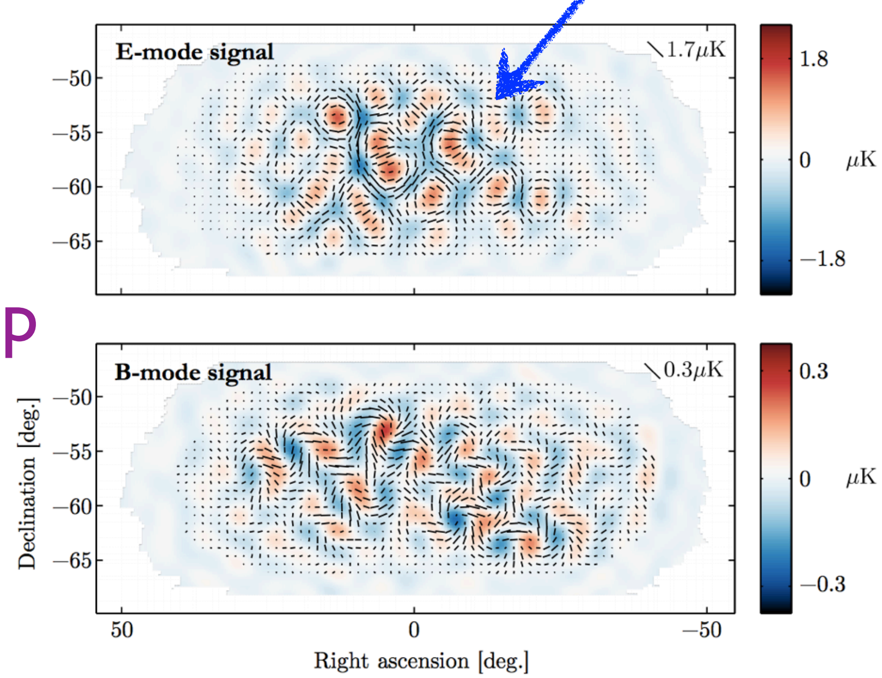
Planck
1303.5062



Planck 1303.5081

$$P_{\mu\nu}\left(E,\hat{n}\right) = \text{Standard polarization} + \textbf{?}$$

Bicep



Polarization spectral distortions

Statistical description of polarized radiation

- Boltzmann equation better formulated in a tetrad basis $e_{(a)}{}^{\mu} \; (a = 0, 1, 2, 3)$

- Photon momentum projected onto the set of tetrads $p^{\mu} = p^{(a)} e_{(a)}{}^{\mu}$

$$p^{(0)} = E \,, \qquad p^{(i)} = E\, n^{(i)} \quad (i = 1, 2, 3)$$

Physical energy

photon direction

- Hermitian tensor-valued distribution function $f_{\mu\nu}(\eta, \mathbf{x}, p^{(i)})$

$$\epsilon^{\mu} \, \epsilon^{*\nu} \, f_{\mu\nu}(\eta, \mathbf{x}, p^{(i)})$$

number density in phase space of photons at $(\eta, \mathbf{x}, p^{(i)})$ with polarization state vector ϵ^{μ}

Distribution function

- Direction 4-vector of photons $n^{\mu} \equiv n^{(i)} e_{(i)}{}^{\mu}$
- Projection operator, or screen projector $S_{\mu\nu} \equiv g_{\mu\nu} + e^{(0)}{}_{\mu} e^{(0)}{}_{\nu} - n_{\mu} n_{\nu}$
- Decomposition of the distribution function

$$f_{\mu\nu} \equiv \frac{1}{2}\left(I\, S_{\mu\nu} \;+\; P_{\mu\nu} + i\epsilon_{\rho\mu\nu\sigma} e_{(0)}{}^{\rho} n^{\sigma} V\right)$$

trace part

symmetric traceless part

antisymmetric part

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trace part

symmetric traceless part

antisymmetric part

intensity

linear polarization

circular polarization

Intensity y-type distortions

$$I\Big(E,\hat{n}\Big) = I_{\rm BB}\left(\frac{E}{T(\hat{n})}\right) + y(\hat{n})\,\mathcal{D}_E^2\,I_{\rm BB}\left(\frac{E}{T(\hat{n})}\right)$$

Direction dependent blackbody

y-Compton parameter

y-type distortion

$$\mathcal{D}_E^2 \equiv E^{-3}\frac{\partial}{\partial \ln E}\left(E^3\frac{\partial}{\partial \ln E}\right) = \frac{\partial^2}{\partial \ln E^2} + 3\,\frac{\partial}{\partial \ln E}$$

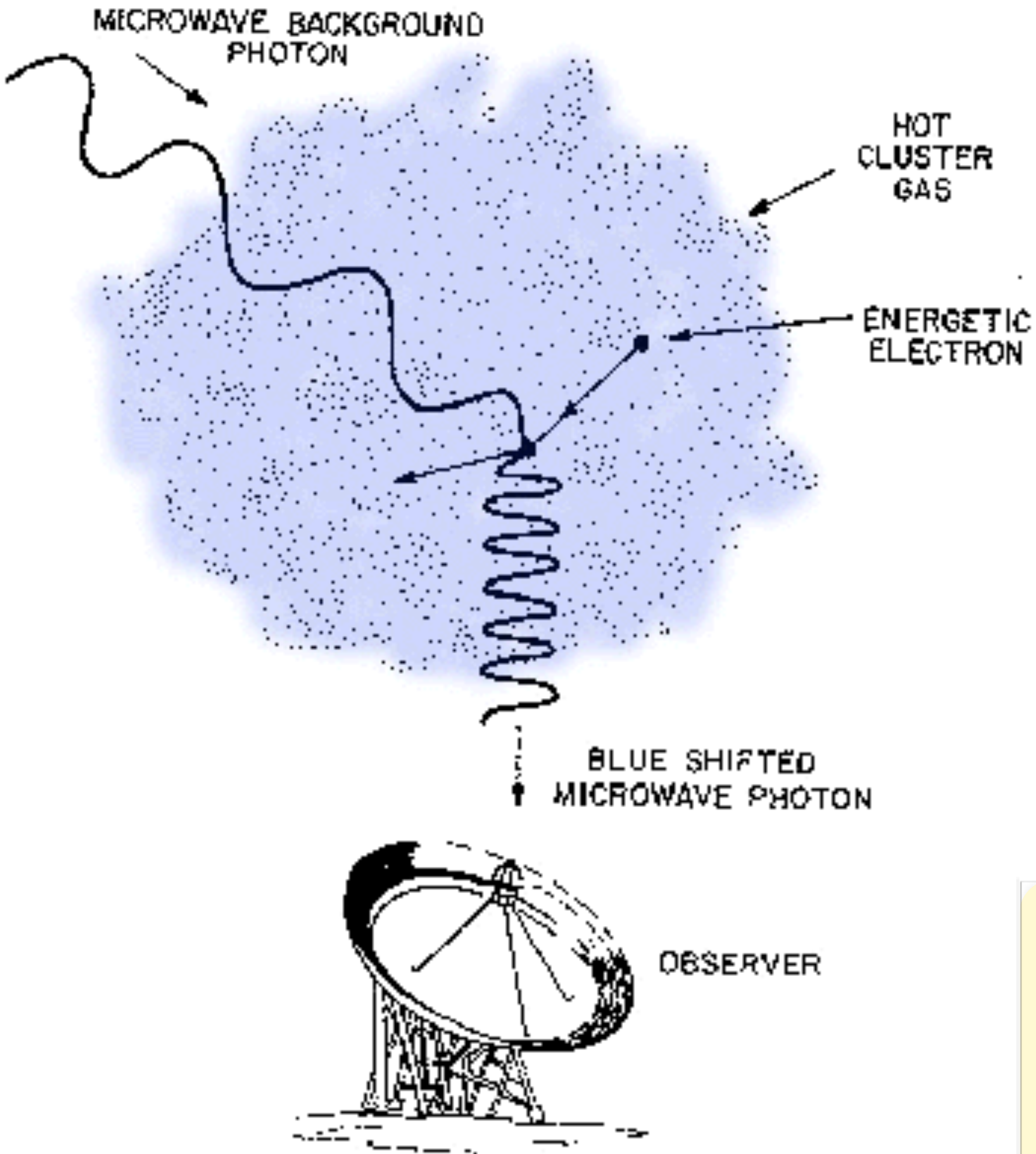
Number density of photons:

$$n \propto \int I\,E^3\,\mathrm{d}\ln E$$

T: temperature of a blackbody that would have the same number density

see Pitrou, Stebbins, 1402.0968

Sunyaev-Zel'dovich effect

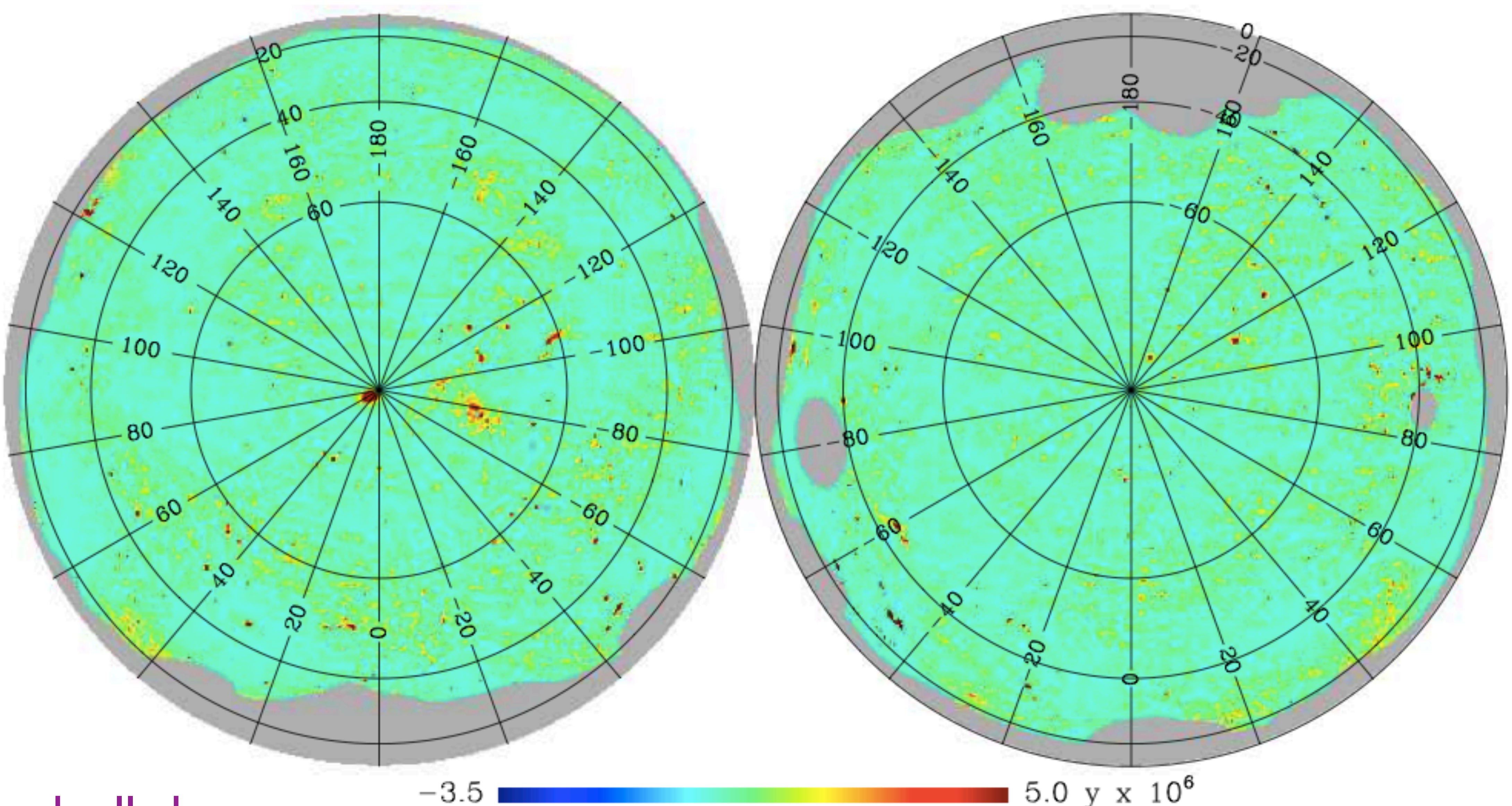


Compton interactions between CMB photons and electrons in clusters of galaxies.

Photon number is conserved, but energy is redistributed: spectral distortions.

$$y(\hat{n}) = \int n_e \frac{k_{\mathrm{B}} T_e}{m_e c^2} \sigma_{\mathrm{T}} \, \mathrm{d}s$$

Planck y-map



Planck all sky map
1303.5081

Detection of galaxy clusters through the thermal Sunyaev Zel'dovich effect

Polarization y-type distortions

$$P_{\mu\nu}\left(E,\hat{n}\right) = -\mathcal{P}_{\mu\nu}(\hat{n})\,\frac{\partial}{\partial \ln E} I_{\rm BB}\left(\frac{E}{T(\hat{n})}\right) + y_{\mu\nu}(\hat{n})\,\mathcal{D}_E^2\, I_{\rm BB}\left(\frac{E}{T(\hat{n})}\right)$$

Polarization tensor

'Standard polarization'

Polarization distortion

E and B modes

E^y and B^y modes

- Similarly to y, Compton scattering generates a non-zero polarization distortion only beyond first-order perturbation theory

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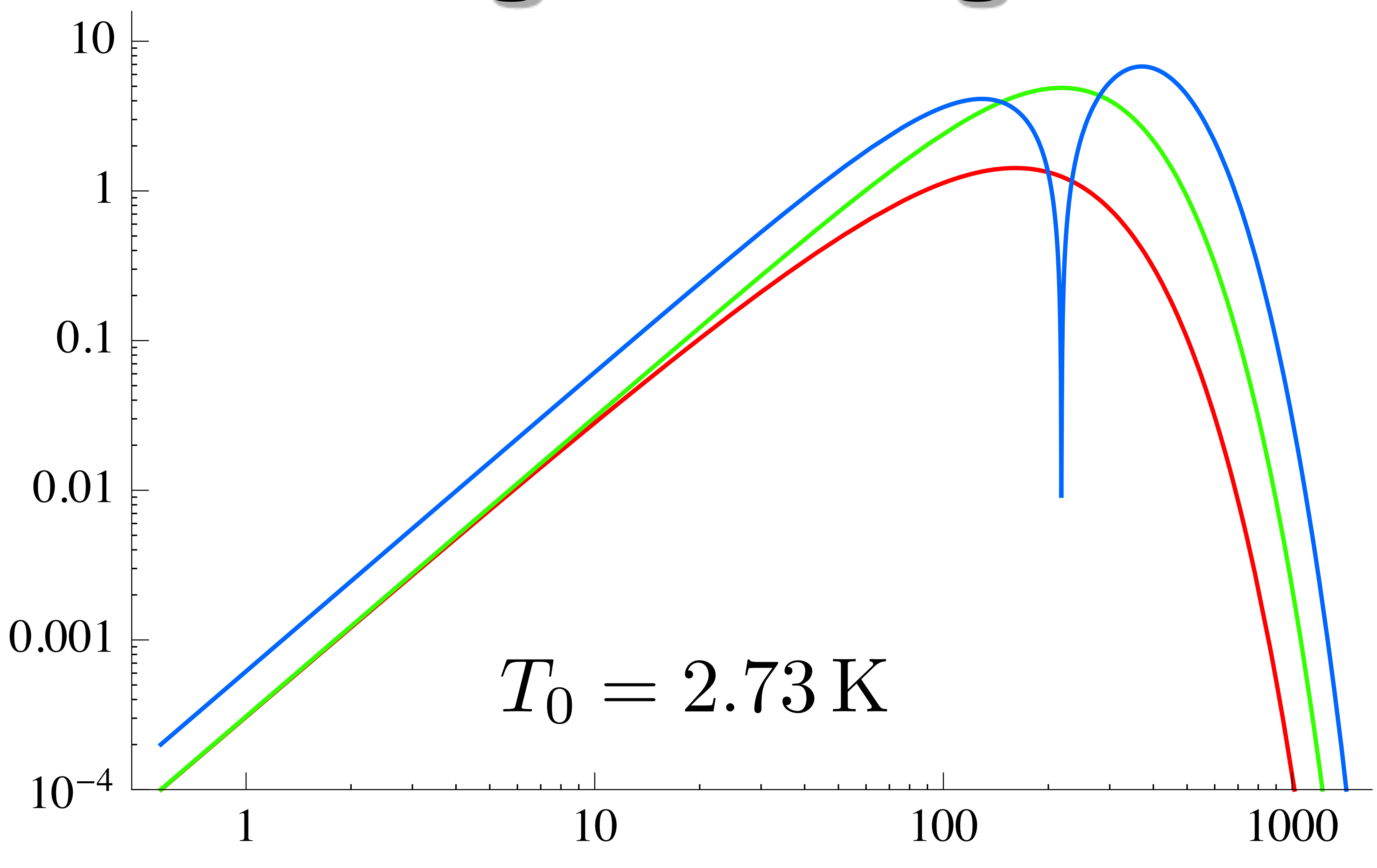
- Need for polarized Boltzmann equation at second order, with proper spectral dependence decomposition

Naruko, Pitrou, Koyama, Sasaki 1304.6929

Brightness signals



Blackbody spectrum $\left(\frac{E}{T_0}\right)^3 I_{\mathrm{BB}}(E/T_0)$

Standard polarization $\left(\frac{E}{T_0}\right)^3 \frac{\partial I_{\mathrm{BB}}(E/T_0)}{\partial \ln E}$

Polarization distortion $\left(\frac{E}{T_0}\right)^3 \mathcal{D}_E^2 I_{\mathrm{BB}}(E/T_0)$

Boltzmann equation for polarization distortion

Boltzmann equation:

$$y'_{(i)(j)} + n^{(l)}\partial_l y_{(i)(j)} = \tau'\left(-y_{(i)(j)} + C^y_{(i)(j)}\right)$$

Thomson interaction rate $\tau' \equiv a\,\overline{n}_e\,\sigma_{\rm T}$

Line of sight formal solution

$$r(\eta) \equiv \eta_0 - \eta$$

$$y_{ij}(\eta_0, k_i, n^i) = \int_{\eta_{\rm re}}^{\eta_0} {\rm d}\eta\,\tau'\,e^{-\tau}\,e^{-{\rm i}\,k_i\,n^i(\eta_0-\eta)}\,C^y_{ij}(\eta, k_i, n^i)$$

$$\frac{{\rm d}\tau(\eta)}{{\rm d}\eta} \equiv -\tau' \qquad \tau(\eta_0) = 0$$

Optical depth

Boltzmann equation for polarization distortion

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$$g(\eta) = \tau' e^{-\tau}$$

Visibility function

Boltzmann equation for polarization distortion

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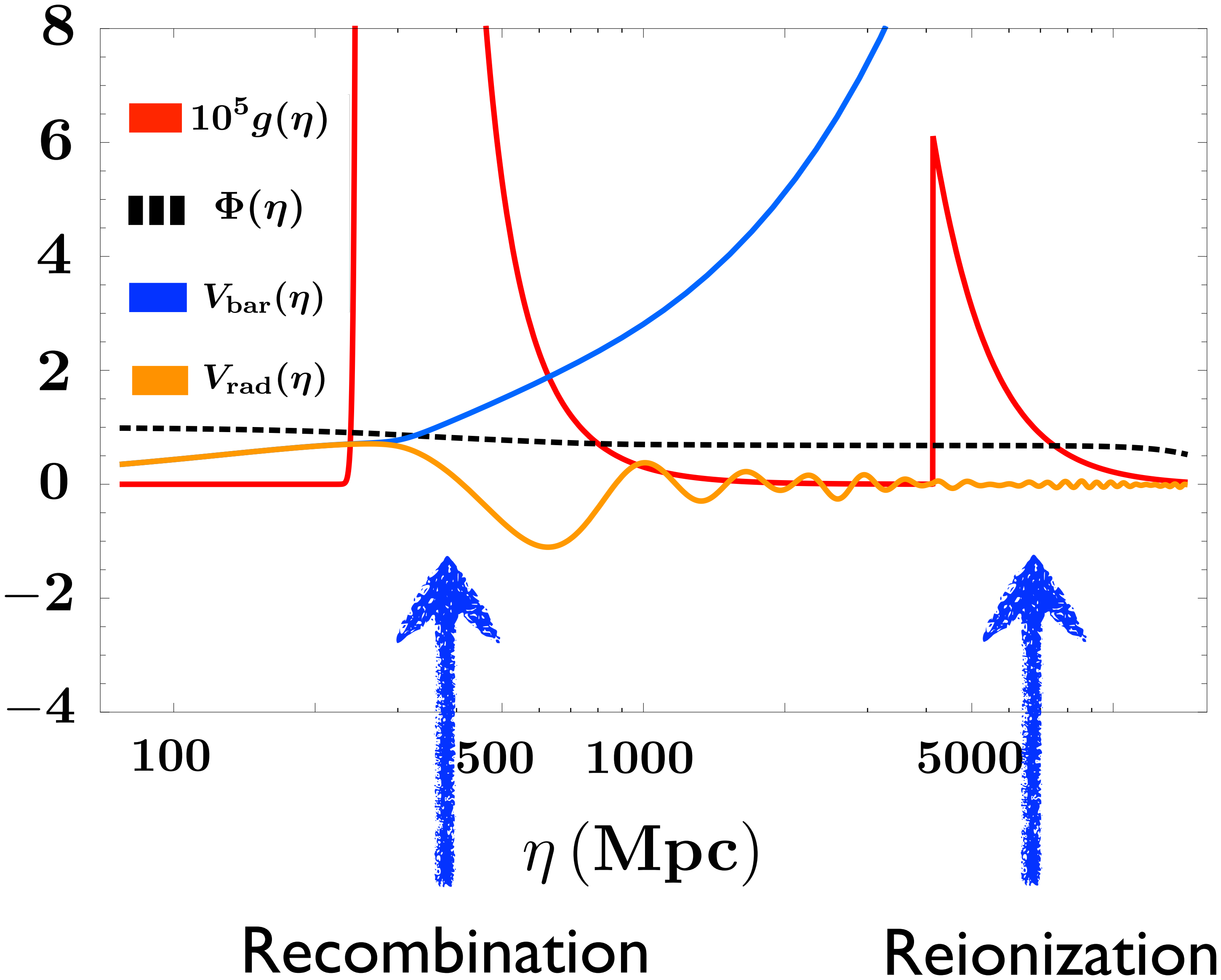
$$r(\eta) = \eta_0 - \eta$$ comoving distance from us

Non-linear kSZ effect (kSZ²)



Leading-order collision term:

$$C_{ij}^{y\,(\mathrm{L.O.})} = -\frac{1}{10}\,\left[v_i\,v_j\right]^{\mathrm{TT}}$$

$$\left[v_i\,v_j\right]^{\mathrm{TT}} \equiv \left[S_i{}^k S_j{}^l - \frac{1}{2} S^{kl} S_{ij}\right] v_k\,v_l$$

Difference between the first-order electron and photon velocities.

Grows after recombination.

Main signal originates from reionization (z < 15)

Multipolar expansion

the aim:

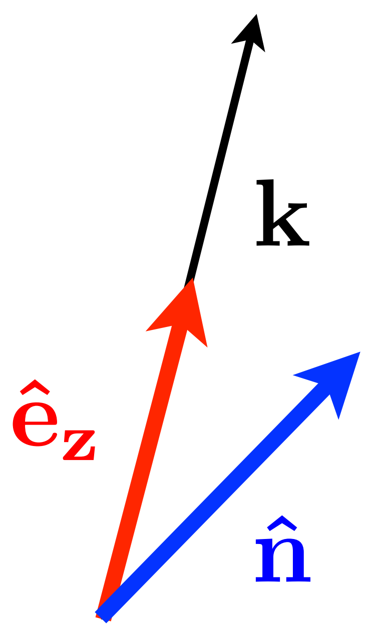
$$y_{ij}(\mathbf{k},\hat{\mathbf{n}}) = \sum_{\pm} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} \left[\, E^{y}_{\ell m}(\mathbf{k}) \pm i B^{y}_{\ell m}(\mathbf{k}) \,\right] \frac{Y^{\pm 2}_{\ell m}(\hat{\mathbf{n}})}{N_\ell}\, m_i^{\pm}\, m_j^{\pm}$$

$$N_\ell \equiv \mathrm{i}^\ell \sqrt{(2\ell+1)/(4\pi)}$$

Spin-2 spherical harmonics

Natural polarization basis

$$m_i^{\pm} \equiv (\hat{e}_i^{\theta} \mp i \hat{e}_i^{\phi})/\sqrt{2}$$



Multipolar expansion of the collision term

Leading-order collision term quadratic in:

$$v_i(\eta, \mathbf{k}) \;=\; -i\,\hat{k}_i\,F(k,\eta)\,\Phi(\mathbf{k})$$

transfer function of the baryon velocity

primordial potential

Convolution operator

$$\mathcal{K}\{\dots\} \;\equiv\; \int \frac{\mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2}{(2\pi)^3}\,\delta^3_{\mathrm{D}}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\,\dots$$

$$E[C^y]_{\ell m}(\mathbf{k}) = \delta^2_\ell\,\mathcal{K}\left\{ S_m(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2)\,F(k_1,\eta)\,F(k_2,\eta)\,\Phi(k_1)\Phi(k_2) \right\}$$

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$$(\ell=1)\otimes(\ell=1)$$

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$$(\ell=1)\otimes(\ell=1)$$

geometrical factor:

$$S_m(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2) = -\frac{\pi}{15}\sqrt{\frac{2}{3}}\sum_{n=-1}^{1}\alpha_{n,m}\left(Y_1^{m-n}(\hat{\mathbf{k}}_1)Y_1^n(\hat{\mathbf{k}}_2)\right)^*$$

$$\alpha_{0,m}\equiv\sqrt{(4-m^2)},\qquad \alpha_{\pm1,m}\equiv\sqrt{(2\pm m)(2\pm m-1)/2}$$

Analytic solution

Collision term → Polarization distortion

free-streaming

E-modes only → **E- and B-modes**

$$y_{ij}(\eta_0, k_i, n^i) \;=\; \int_{\eta_{\rm re}}^{\eta_0} \mathrm{d}\eta\, \tau'\, e^{-\tau}\, e^{-\mathrm{i}\, k_i\, n^i(\eta_0-\eta)}\, C^y_{ij}(\eta, k_i, n^i)$$

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Rayleigh formula to expand the exponential into spherical harmonics + Addition of spherical harmonics

Analytic solution

Collision term → Polarization distortion

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Rayleigh formula to expand the exponential into spherical harmonics + Addition of spherical harmonics

$$\frac{E^y_{\ell m}(\mathbf{k})}{2\ell+1} = \mathcal{K}\left\{\int_{\eta_{\rm re}}^{\eta_0} {\rm d}\eta\, g(\eta)\, \epsilon^{(m)}_\ell[kr(\eta)]\, S_m(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2)\, F(k_1,\eta)F(k_2,\eta)\, \Phi(k_1)\Phi(k_2)\right\}$$

$$\frac{B^y_{\ell m}(\mathbf{k})}{2\ell+1} = \mathcal{K}\left\{\int_{\eta_{\rm re}}^{\eta_0} {\rm d}\eta\, g(\eta)\, \beta^{(m)}_\ell[kr(\eta)]\, S_m(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2)\, F(k_1,\eta)F(k_2,\eta)\, \Phi(k_1)\Phi(k_2)\right\}$$

Analytic solution

Collision term → Polarization distortion

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Built out of spherical Bessel functions

$$\frac{B^y_{\ell m}(\mathbf{k})}{2\ell+1} = \mathcal{K}\left\{\int_{\eta_{\rm re}}^{\eta_0} {\rm d}\eta\, g(\eta)\, \beta^{(m)}_\ell[kr(\eta)]\, S_m(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2)\, F(k_1,\eta) F(k_2,\eta)\, \Phi(k_1)\Phi(k_2)\right\}$$

Angular power spectra

$$y_{ij}(\mathbf{x}, \hat{\mathbf{n}}) = \sum_{\pm} (Q^y \pm i\, U^y)(\mathbf{x}, \hat{\mathbf{n}})\, m_i^{\pm}\, m_j^{\pm}$$

Distortion Stokes parameters

$$(Q^y \pm iU^y)(\mathbf{x}, \hat{\mathbf{n}}) = \sum_{\ell=2}^{\infty} \sum_{m=-l}^{l} (e^y_{\ell m}(\mathbf{x}) \pm i\, b^y_{\ell m}(\mathbf{x}))\, Y^{\pm 2}_{\ell m}(\hat{\mathbf{n}}; \hat{\mathbf{e}})$$

$$C_\ell^{E^y} \equiv \langle |e^y_{\ell m}(\mathbf{x})|^2 \rangle \quad \text{and} \quad C_\ell^{B^y} \equiv \langle |b^y_{\ell m}(\mathbf{x})|^2 \rangle$$

The result

$$(2\ell+1)^2\, C_\ell^{E^y} = \frac{2}{\pi}\sum_{m=-2}^{2}\int \mathrm{d}k\, k^2\, \mathcal{Q}_{\ell m}^{E^y}(k)$$

with

$$\langle E^y_{\ell m}(\mathbf{k}) E^{y*}_{\ell m'}(\mathbf{k}')\rangle = (2\pi)^3\, \delta^3_{\mathrm{D}}(\mathbf{k}-\mathbf{k}')\, \mathcal{Q}_{\ell m}^{E^y}(k)\, \delta_{mm'}$$

$$\boxed{\begin{aligned}\mathcal{Q}_{\ell m}^{E^y}(k) = \frac{2(2\ell+1)^2}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}_1 P(k_1)P(k_2)\left|S_m(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2)\right|^2 \\ \times\left|\int_{\eta_{\rm re}}^{\eta_0}\mathrm{d}\eta\, g(\eta)\, \epsilon_\ell^{(m)}[kr(\eta)]F(k_1,\eta)F(k_2,\eta)\right|^2\end{aligned}}$$

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statistical isotropy truly 2-dimensional integrals

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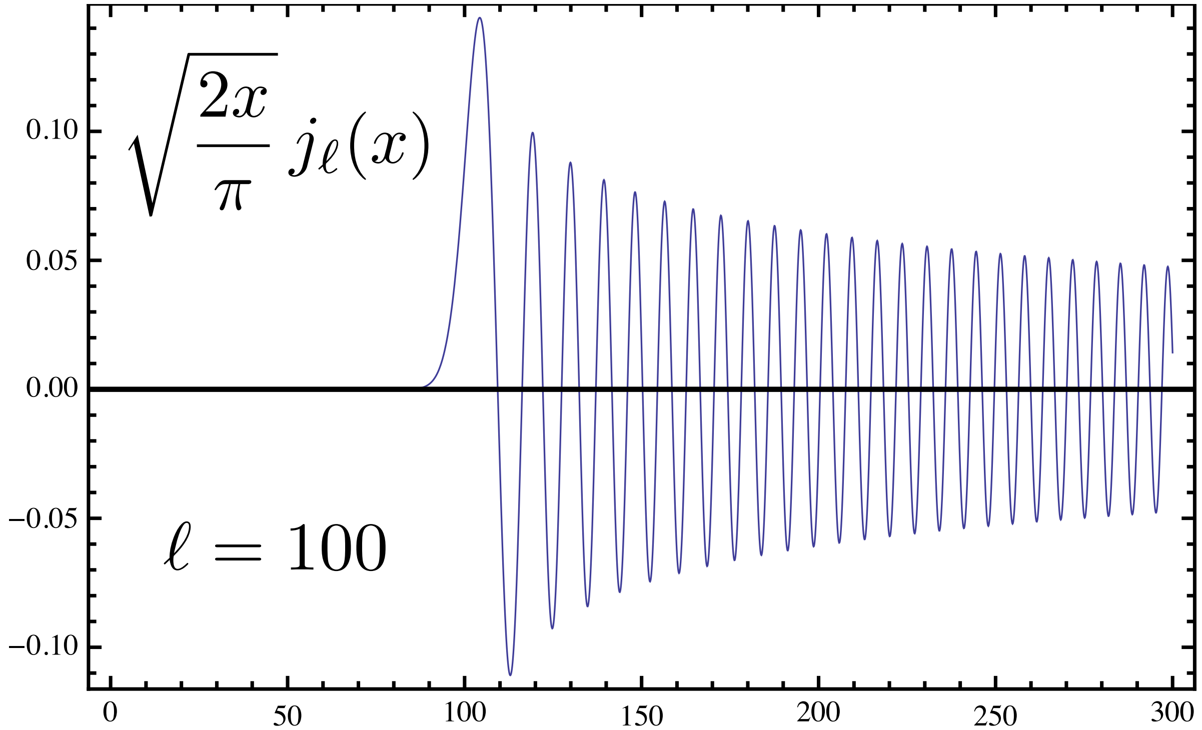
Limber approximation



For a slowly varying function with respect to the oscillations of the jl's

$$\sqrt{\frac{2x}{\pi}}\, j_\ell(x) \simeq \delta\left(x - \left(\nu = \ell + \frac{1}{2}\right)\right)$$

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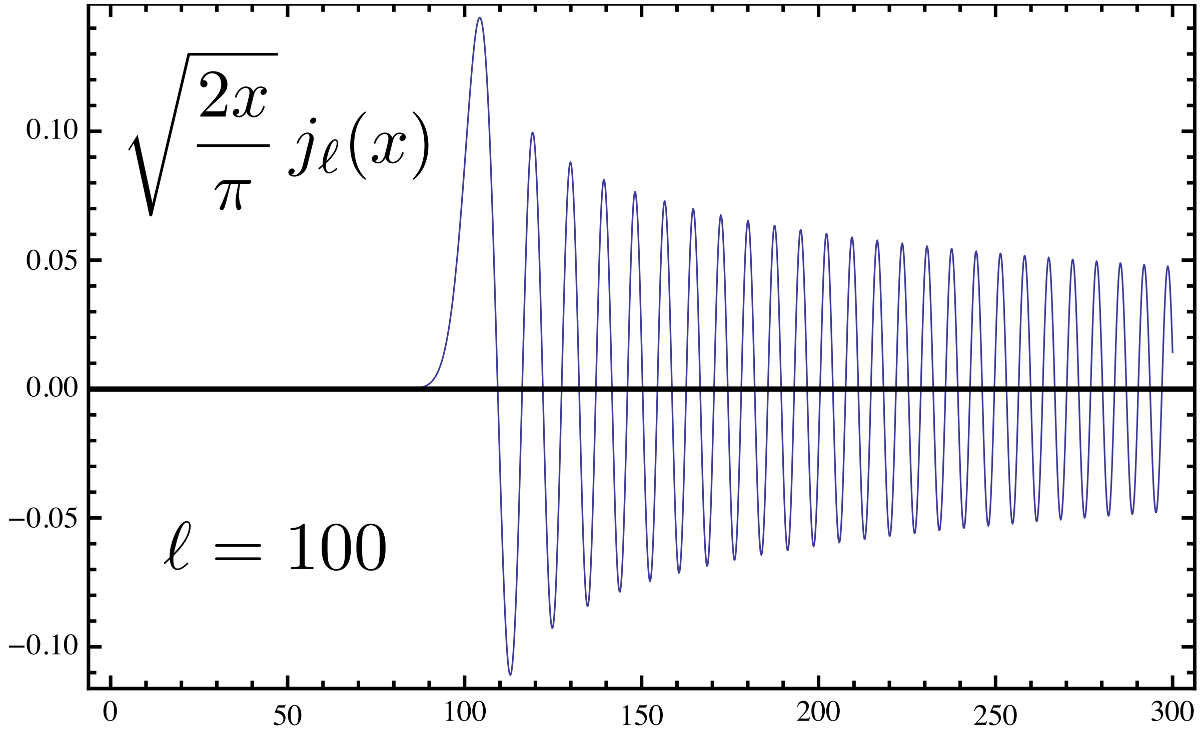

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3D integrals → 2D integrals

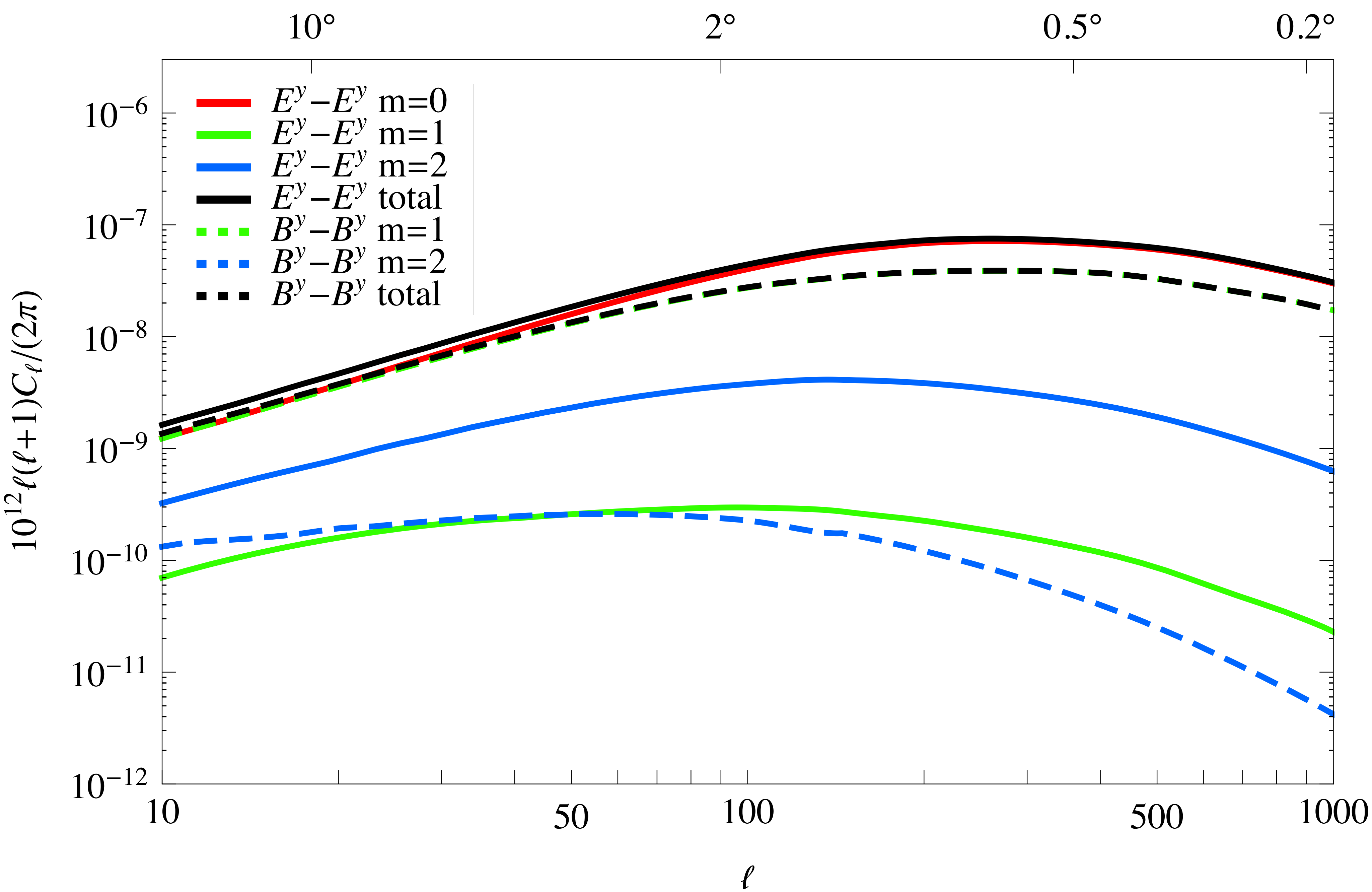
$$C^{E^y}_{\ell\,\mathrm{Limber}} = \frac{1}{4\,(2\pi)^2}\int_0^{r_{\mathrm{re}}} \frac{\mathrm{d}r}{r^2}\, k_1^2\,\mathrm{d}k_1 \sin\theta_{\mathbf{k}_1}\mathrm{d}\theta_{\mathbf{k}_1}\; P(k_1)P(k_2)\;\left[g(\eta)\,F(k_1,\eta)\,F(k_2,\eta)\right]^2 \times\left(3\,|S_0(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2)|^2 + |S_2(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2)|^2\right)$$

$$C^{B^y}_{\ell\,\mathrm{Limber}} = \frac{1}{(2\pi)^2}\int_0^{r_{\mathrm{re}}} \frac{\mathrm{d}r}{r^2}\, k_1^2\,\mathrm{d}k_1 \sin\theta_{\mathbf{k}_1}\mathrm{d}\theta_{\mathbf{k}_1}\; P(k_1)P(k_2)\;\left[g(\eta)\,F(k_1,\eta)\,F(k_2,\eta)\right]^2\;|S_1(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2)|^2$$

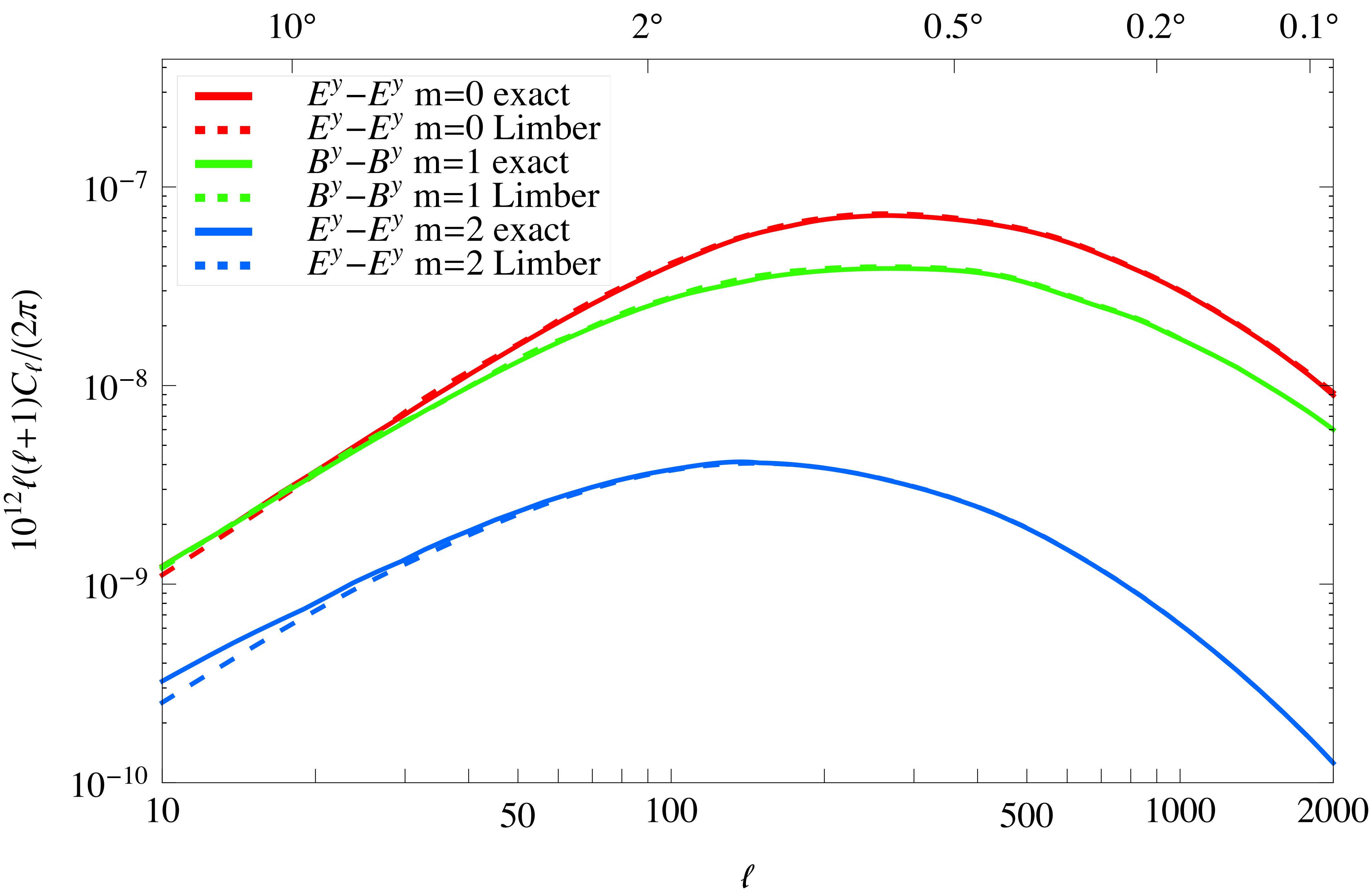
$$\mathbf{k_2} = \mathbf{k} - \mathbf{k_1}, \quad kr = \ell + \frac{1}{2}, \quad \eta = \eta_0 - r$$

Numerical results

Exact results with SONG, **Pettinari, Fidler** et al, 1302.0832

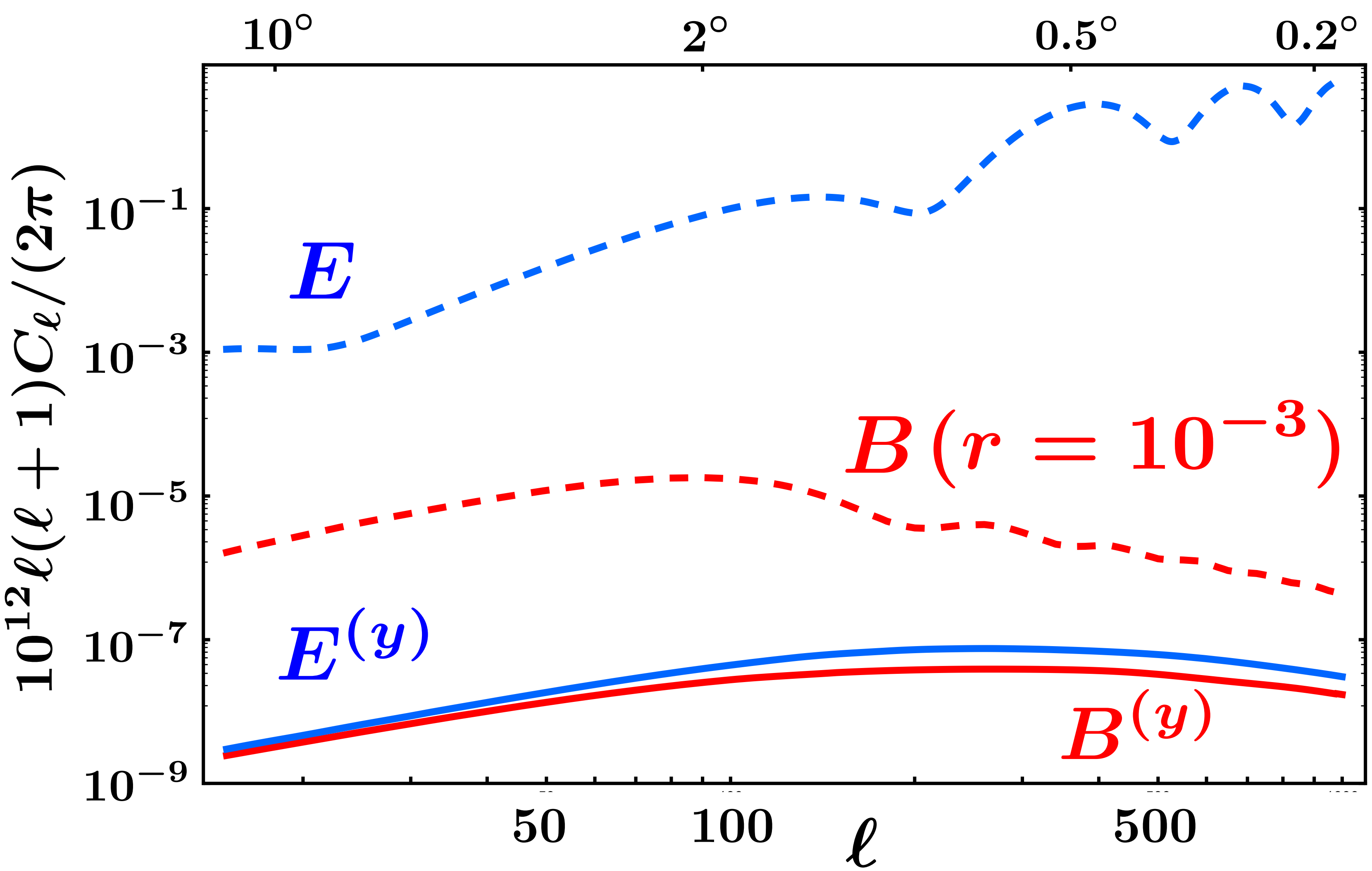


Exact vs Limber



Limber approximation is excellent

Numerical results



SONG, Pettinari, Fidler et al, 1302.0832

- E^y and B^y modes of similar magnitude (same sources)
- Smooth spectra (no acoustic oscillation structure)
- Naive suppression for a second-order effect mitigated by the growth of the electron velocity

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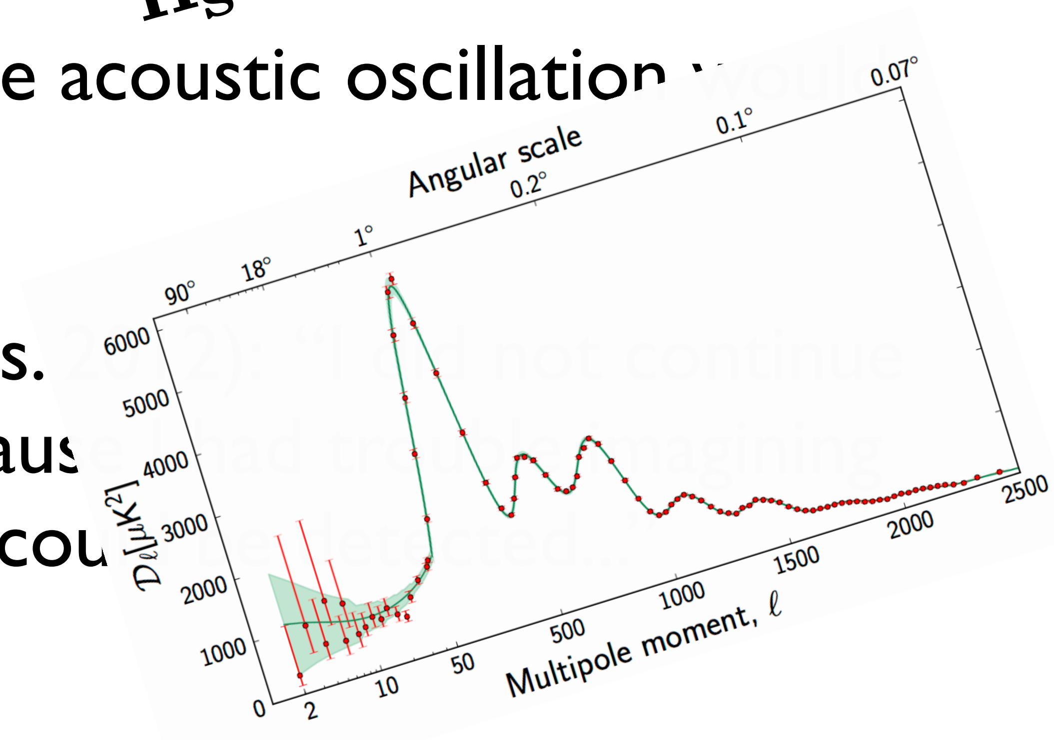

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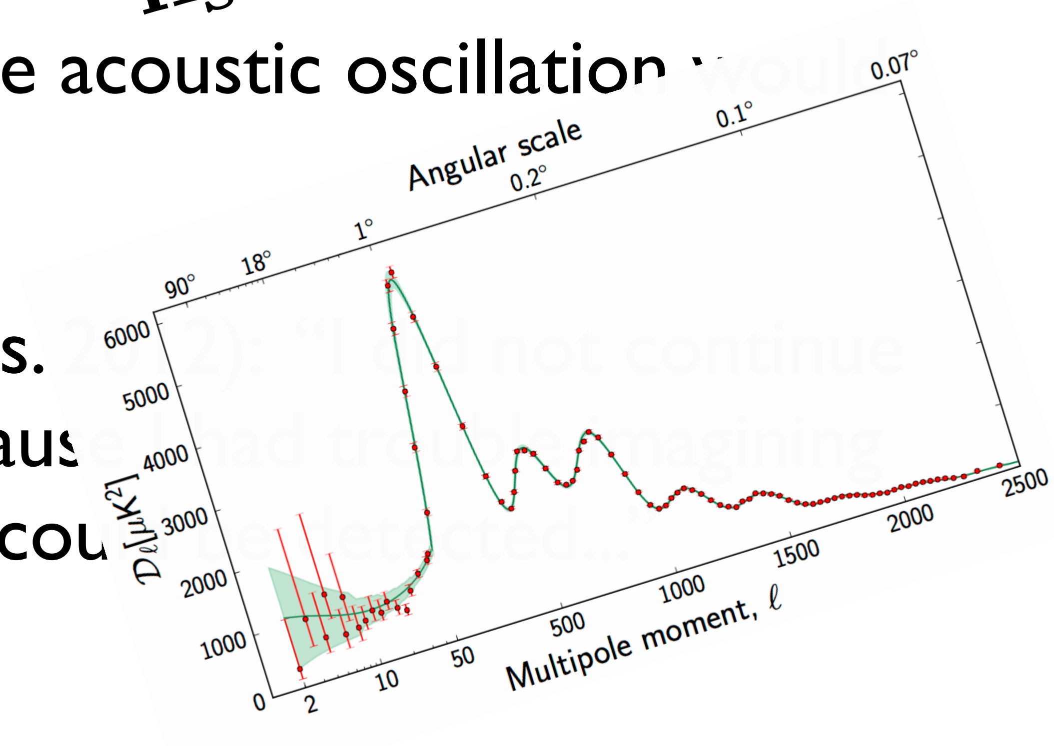

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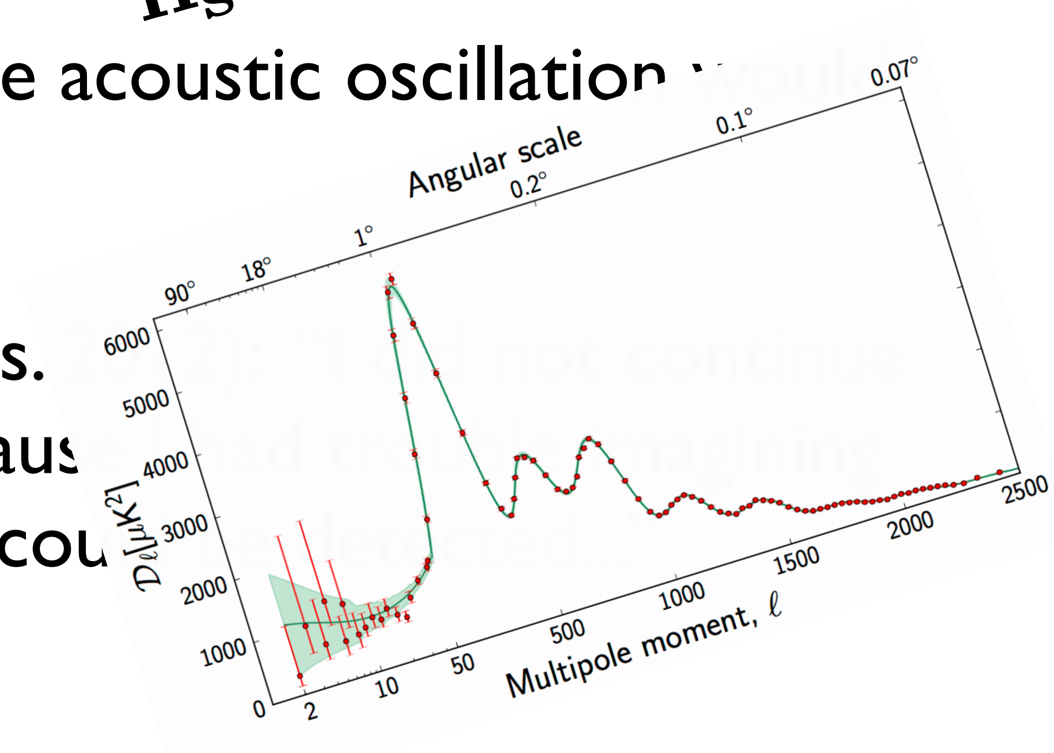

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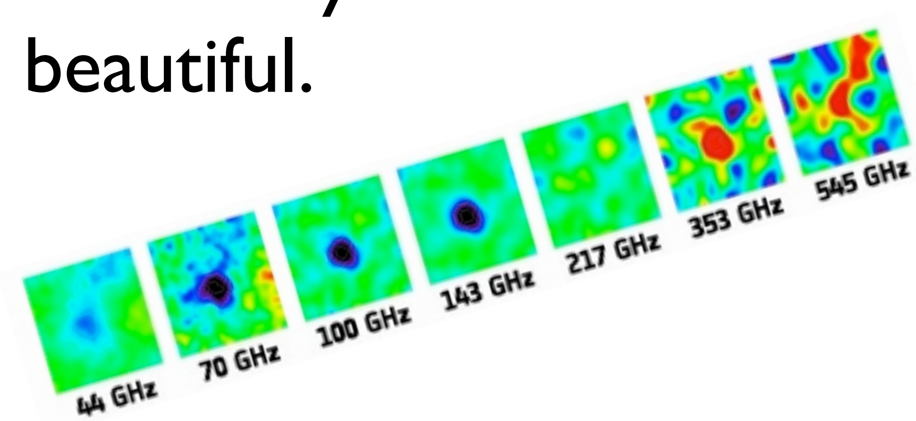
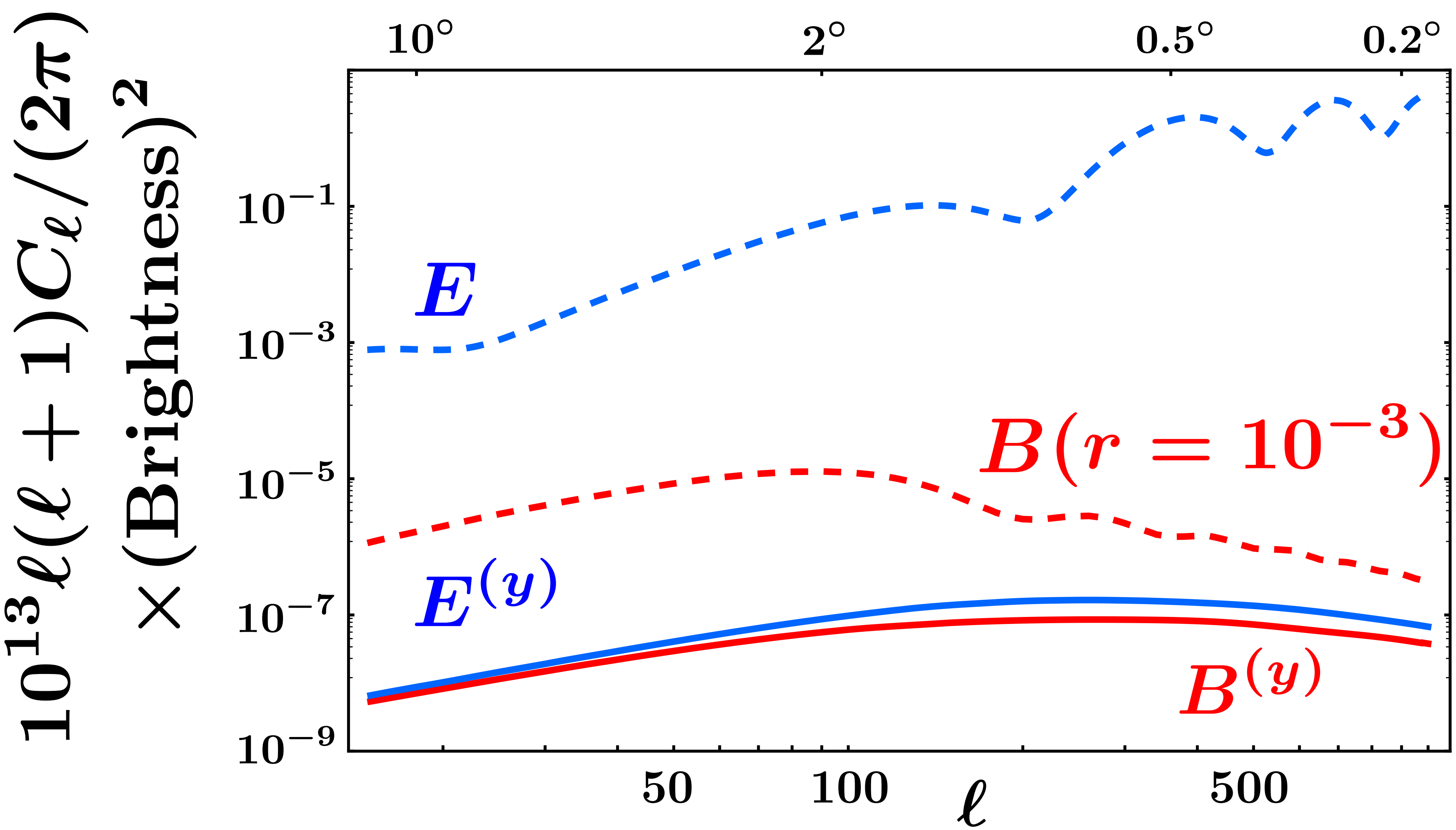


Total signal: angular times energy dependence



@ 70 GHz

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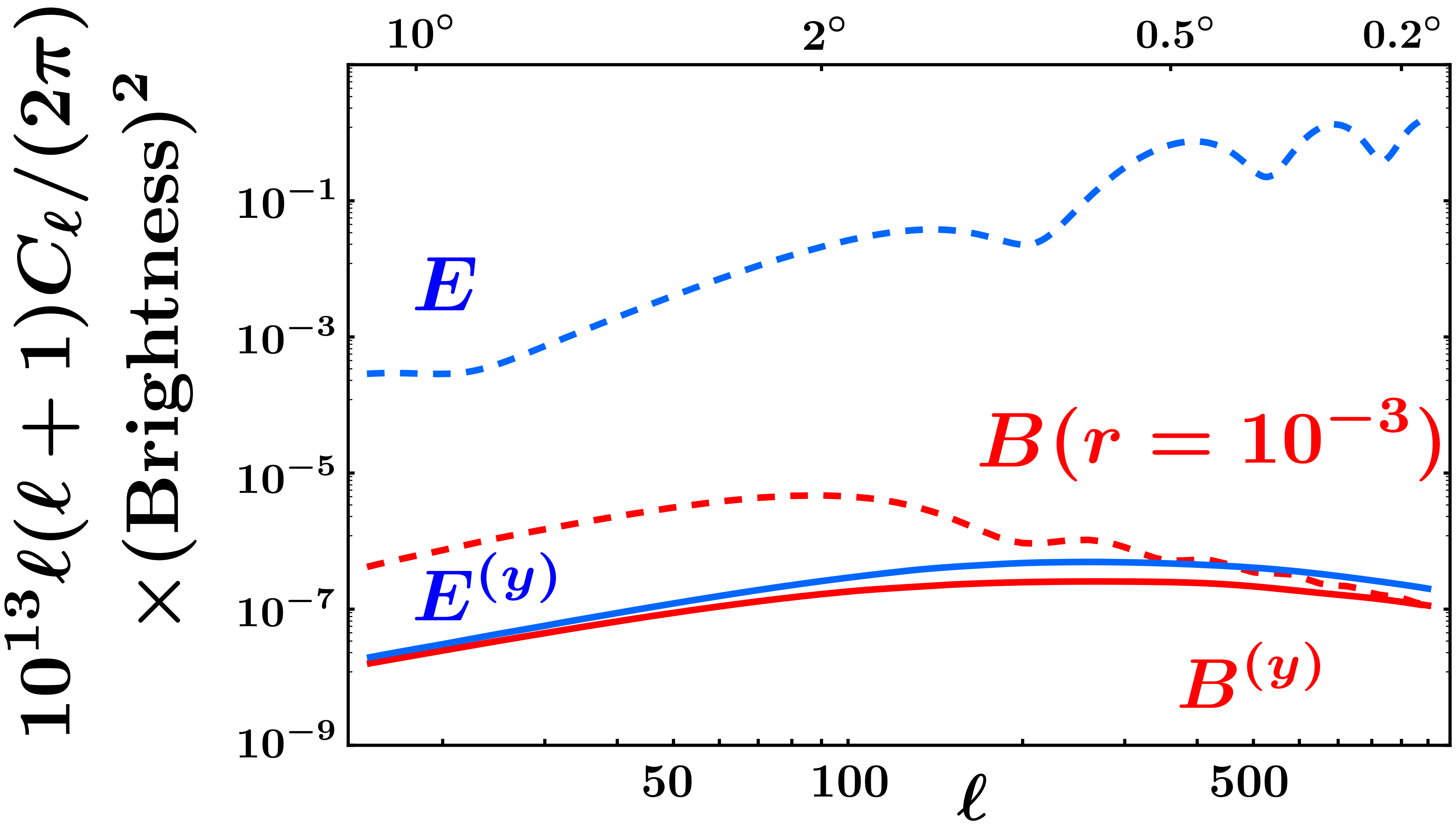

@ 512 GHz

Non-linear kSZ effect from clusters

- The same effect is discussed in the context of galaxy clusters

astro-ph/0307293, astro-ph/0208511 ...

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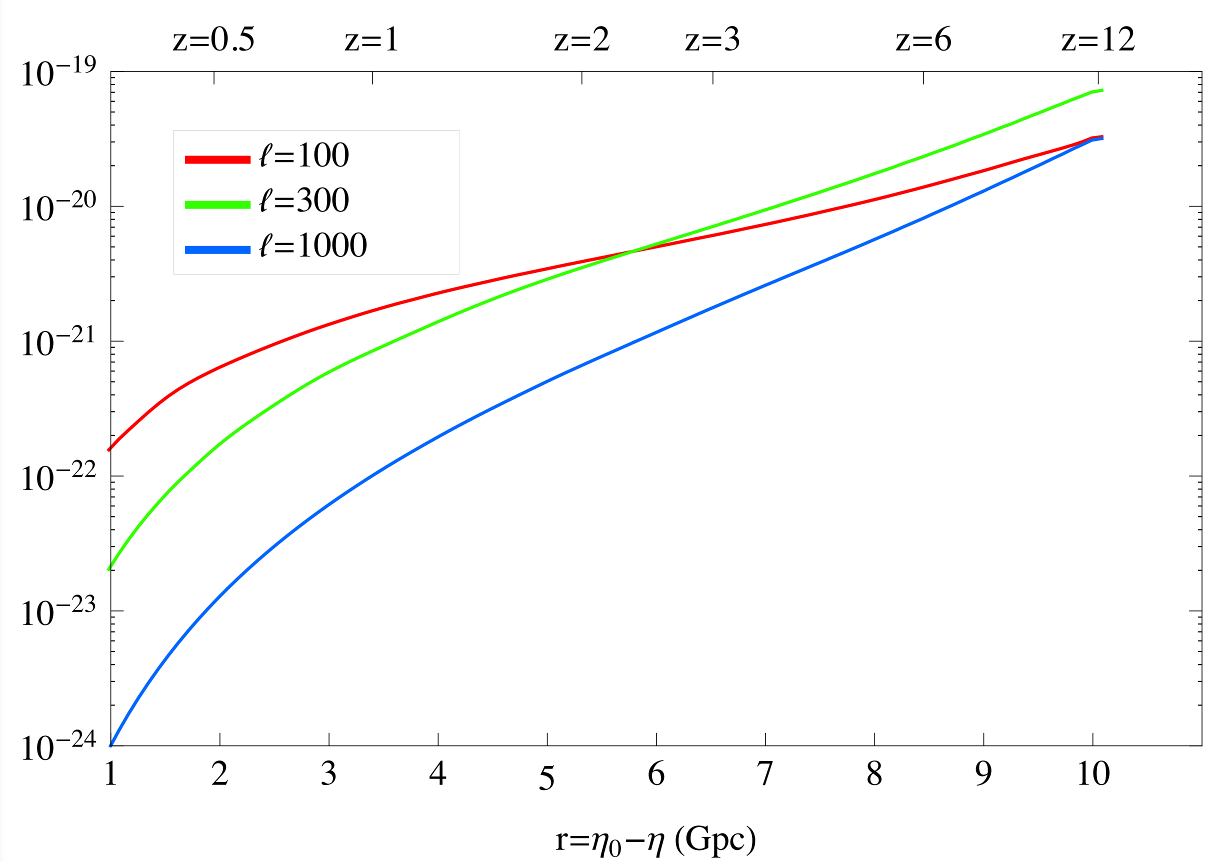
- on angular scales at which clusters are unresolved, $\ell \lesssim 500$, linear description is enough to model the electron number density

- ***additional contribution pre-formation of clusters***, for $2 \lesssim z \lesssim 12$, when the visibility function is the largest.

Contribution(z) to $\ell(\ell+1)C_{\ell\,\mathrm{Limber}}^{E^y}$



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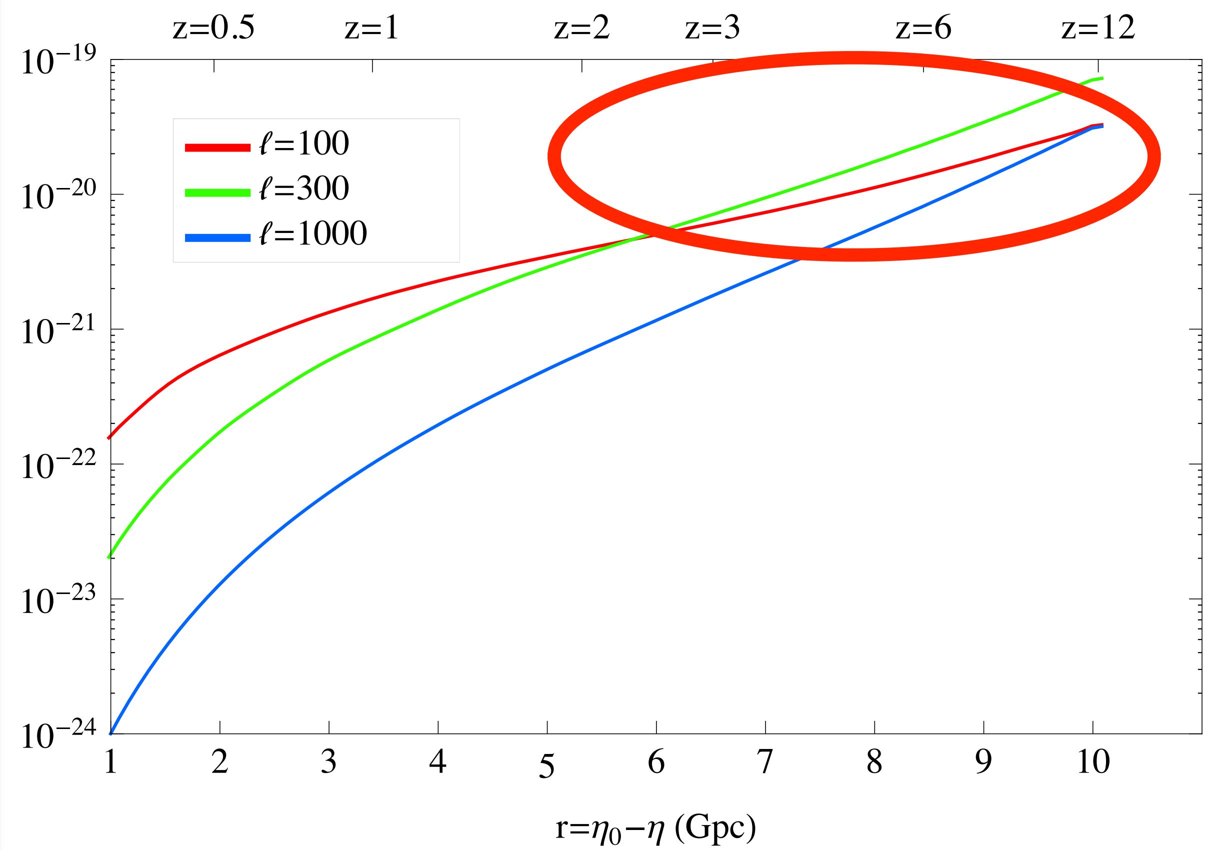
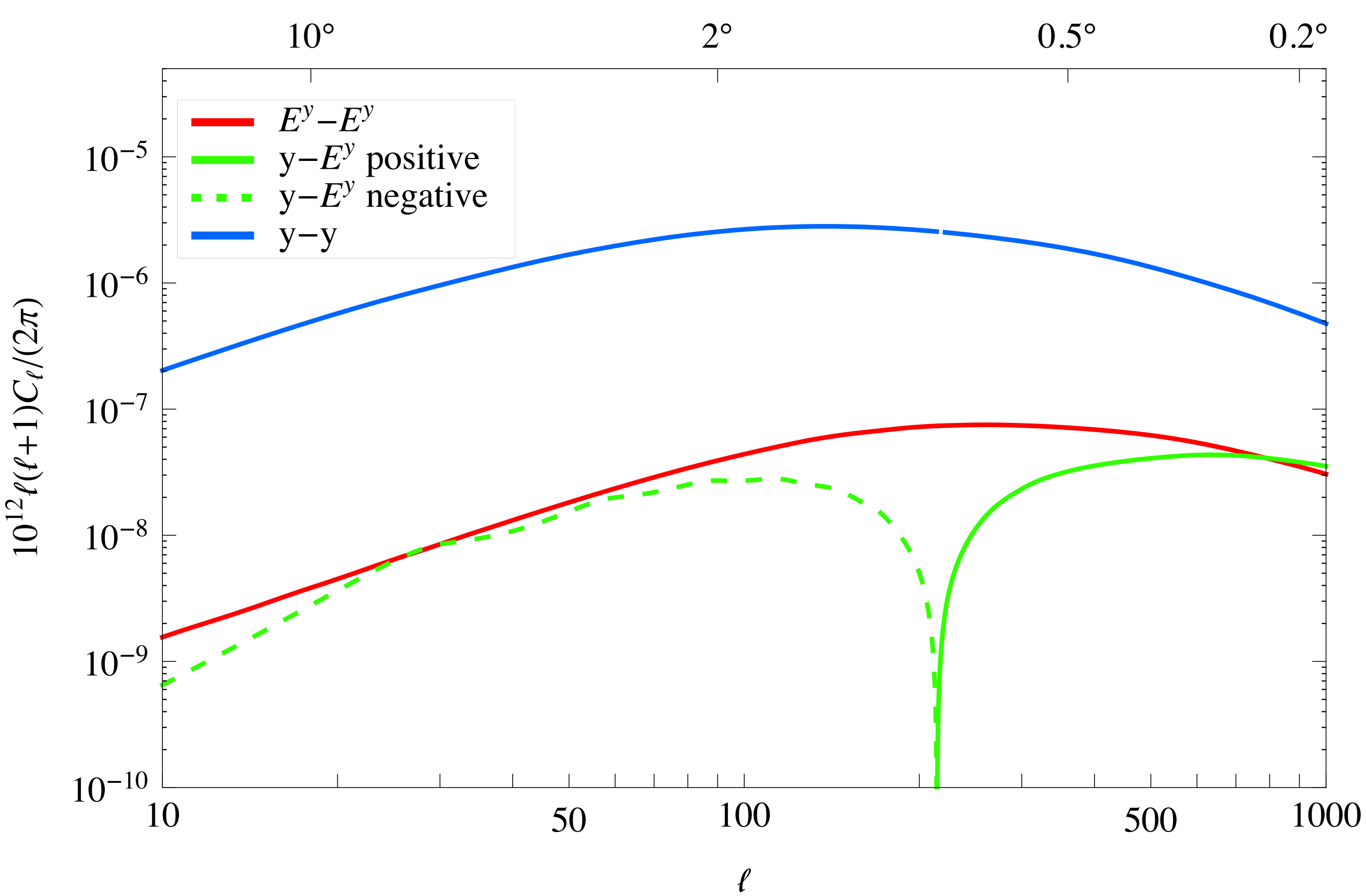


Improving the detectability with cross-correlations

- ***Standard polarization*** has a similar contribution

$$\mathcal{P}_{\mu\nu} = (\mathcal{P}_{\mu\nu})_{\rm linear} + 4\,(y_{\mu\nu})_{kSZ} \quad \Rightarrow \quad \begin{array}{l} \langle E^{\rm st} E^{y*}\rangle = 4\langle E^{y} E^{y*}\rangle \\ \langle B^{\rm st} B^{y*}\rangle = 4\langle B^{y} B^{y*}\rangle \end{array}$$

- Correlation with the ***y-type intensity distortion*** (sourced by tSZ effect + kSZ^2 effect)

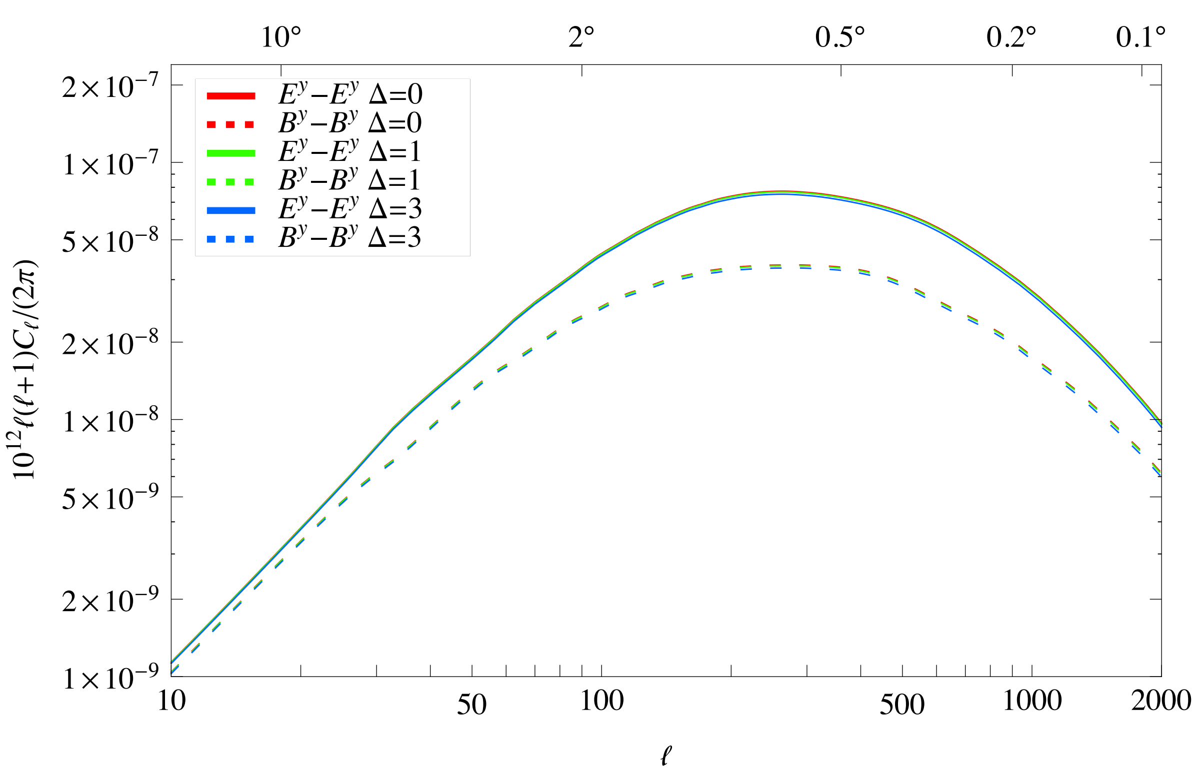


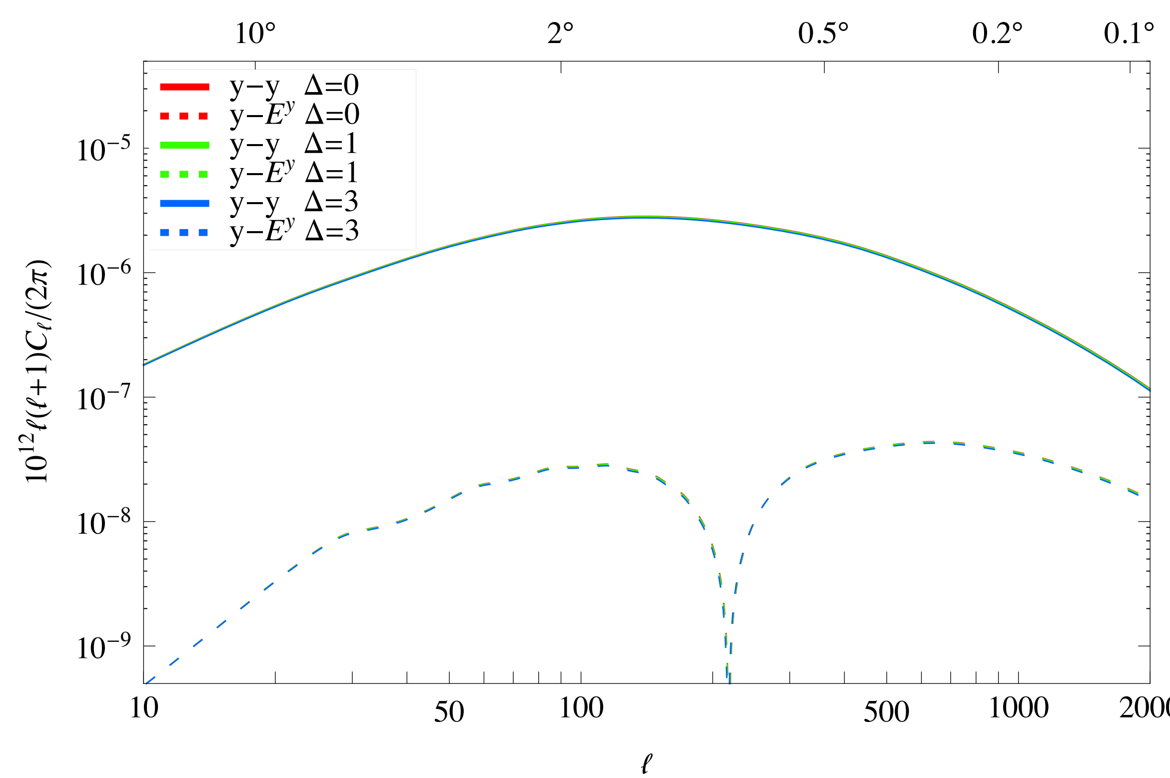
Effects of an extended period of reionization

- Reionization history is unknown but is necessarily more complicated than the simple scenario of instantaneous reionization (patchy etc).

$$x_e(z) \equiv \frac{n_e(z)}{n_H(z)} = \frac{1}{2}\left\{1 + \tanh\left[\frac{(1+z_r)^{3/2} - (1+z)^{3/2}}{\Delta}\right]\right\}$$

built such that total optical depth independent of Delta.





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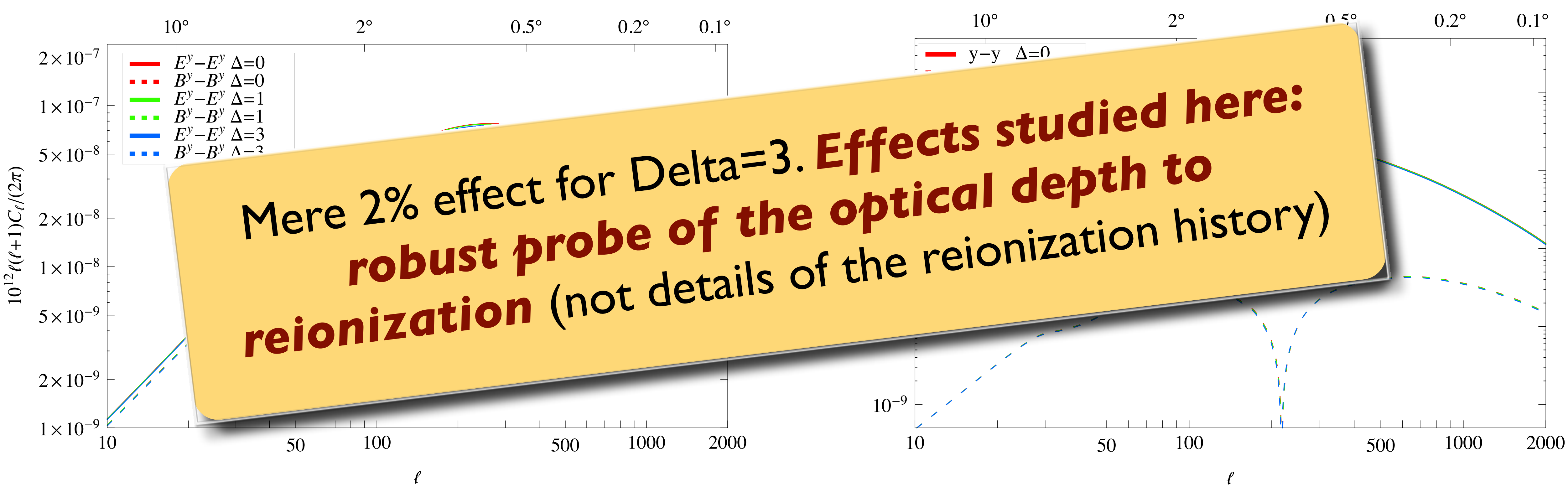

Mere 2% effect for Delta=3. **Effects studied here: *robust probe of the optical depth to reionization*** (not details of the reionization history)

Conclusions

• CMB spectral distortions: new promising observational window in cosmology

• Probe of the thermal history of the universe, inflation, dark matter, reionization...

• It should be studied at the level of the anisotropies of the intensity and polarization

• First step in this direction: unavoidable contribution to diffuse polarization distortion generated by non-linear kSZ effect from reionization. Larger than contribution from clusters.

• Guaranteed signal in the vanilla cosmological model.
Worth studying for extensions.