New Fate of the FRW Spacetime in a 5D Brane World

Dark energy from Brane-world gravity [without effective cosmological constant]

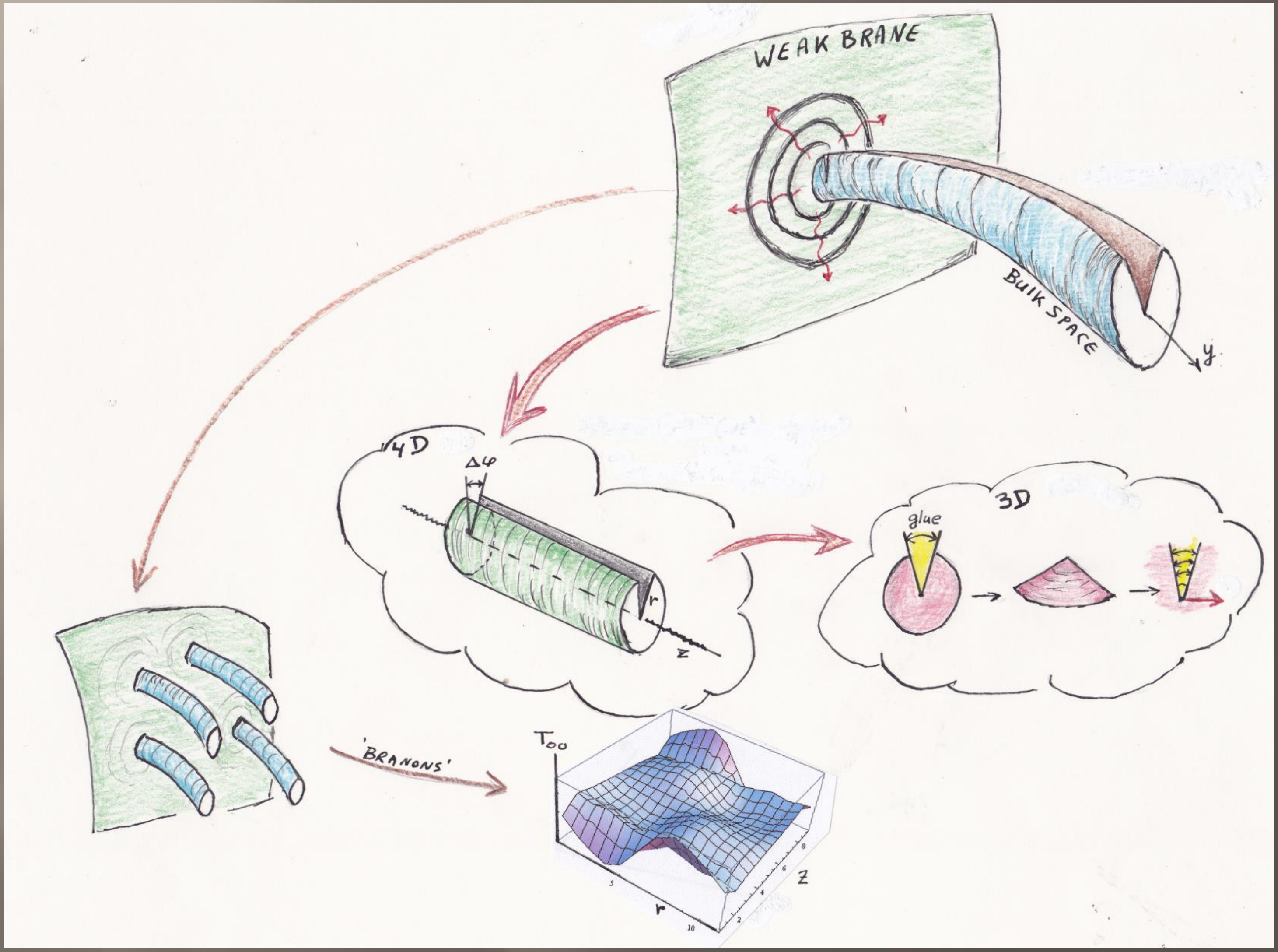
Reinoud Jan Slagter **Asfyon** The Netherlands

Subm: **Fo of Ph**
Coop: S.Pan
Jadapur univ

Hot topics in Modern Cosmology Spontaneous Workshop IX
2015 Cargèse

Artist impression

[after my talk, you will understand (I hope)]



Summary of talk

- Consider **warped 5D FRW model**:

empty bulk + effective $\Lambda = 0$ + U(1) scalar gauge field in the brane

- **"Branons"** can be formed: they **interact** with SM matterfields
- The **late-time behavior** could be significant deviate from the standard evolution of the FRW.
- Description is covariant, so no ad-hoc modifications in Friedmann eq.
- The effect is triggered by the **time-dependent warp factor** $W(t,r,y) = W_1.W_2$

$$W_1(t,r) = \frac{\pm 1}{\sqrt{\tau r}}\sqrt{(c_1 e^{\sqrt{2\tau}t} + c_2 e^{-\sqrt{2\tau}t})(c_3 e^{\sqrt{2\tau}r} + c_4 e^{-\sqrt{2\tau}r})} \qquad W_2(y) \sim e^{\sqrt{-\Lambda_5}(y-y_0)}$$

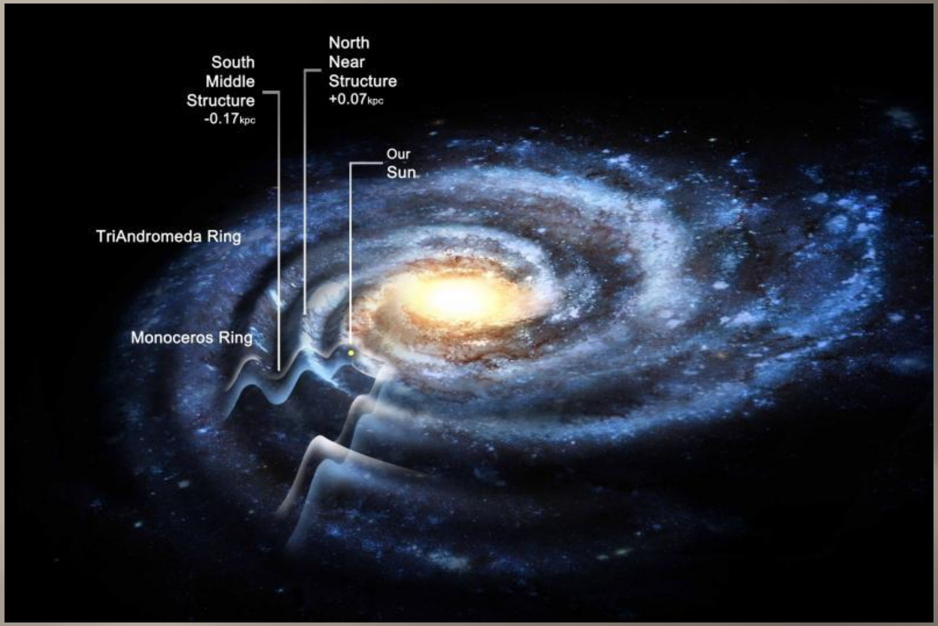
- **In 4D FRW:** large disturbances are rapidly damped as the expansion proceed

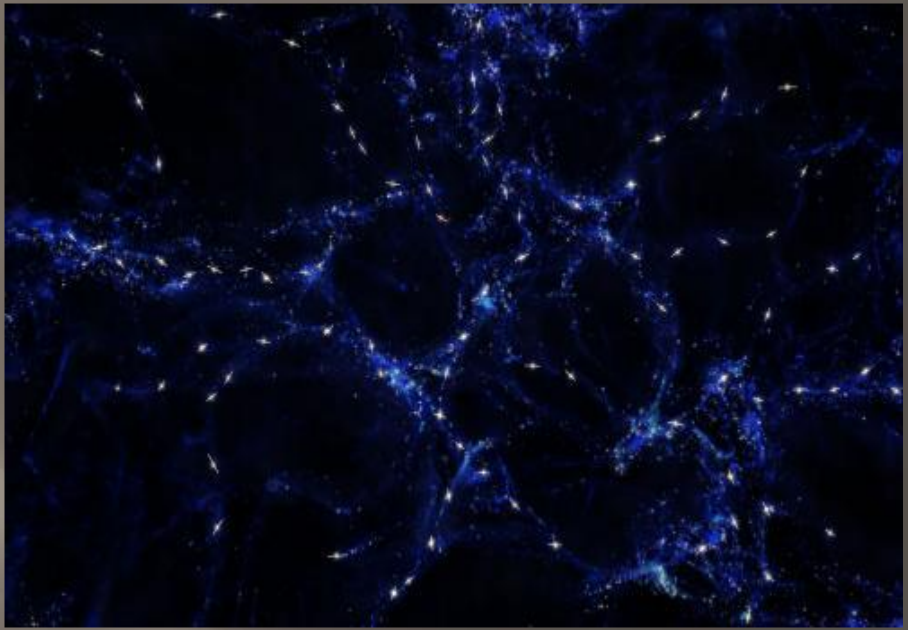
In warped 5D FRW: **they survive**

- Because **gravity** can propagate in the bulk, the cosmic string can build up a huge angle deficit (or mass per unit length) by the warp factor
- **Disturbances** in the spatial components of the stress-energy tensor cause cylindrical symmetric waves, amplified due to the presence of the bulk space and warp factor
- No conflict with observ.: effect 4D ST is non-conical. In 4D counterpart models, i.e., Einstein-Rosen type ST's, C-energy related to angle deficit asymptotically

$$ds^2 = \cdots (1 - 4\pi G C_\infty)^2 r^2 d\varphi^2$$ [**unwanted**]

- This long range effect could also explain the recently found **spooky alignment** of **quasars** in vast structures in the cosmic web
- Explanation of the **wave-like ripples** in galaxies?





"Traveling" waves in the galaxy disk

"Spooky" alignment quasars

Explainable?

Present State of our Universe

- The expansion of our universe is **accelerating.**
- LCDM in agreement with **Planck 2015** results [.... **A&A** , 2015]
- One needs **dark energy** with an effectively negative pressure, $p < -\frac{1}{3}\rho$

  LCDM: w = -1 **[Planck 2015: w > −1 ?]**
- We should live now in the cosmological constant dominated era (and approx.)

$$\Omega_\Lambda = 0.73 \qquad \Omega_M = \Omega_{DM}\,(= 0.23) + \Omega_B\,(= 0.046)$$

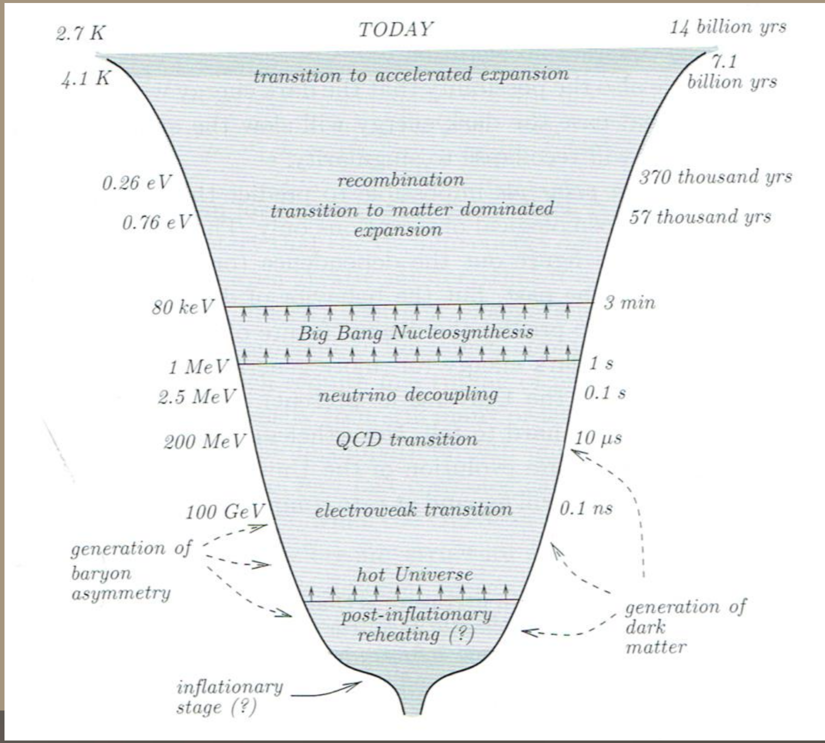
- The main parameters: **size: 10^{28} cm**

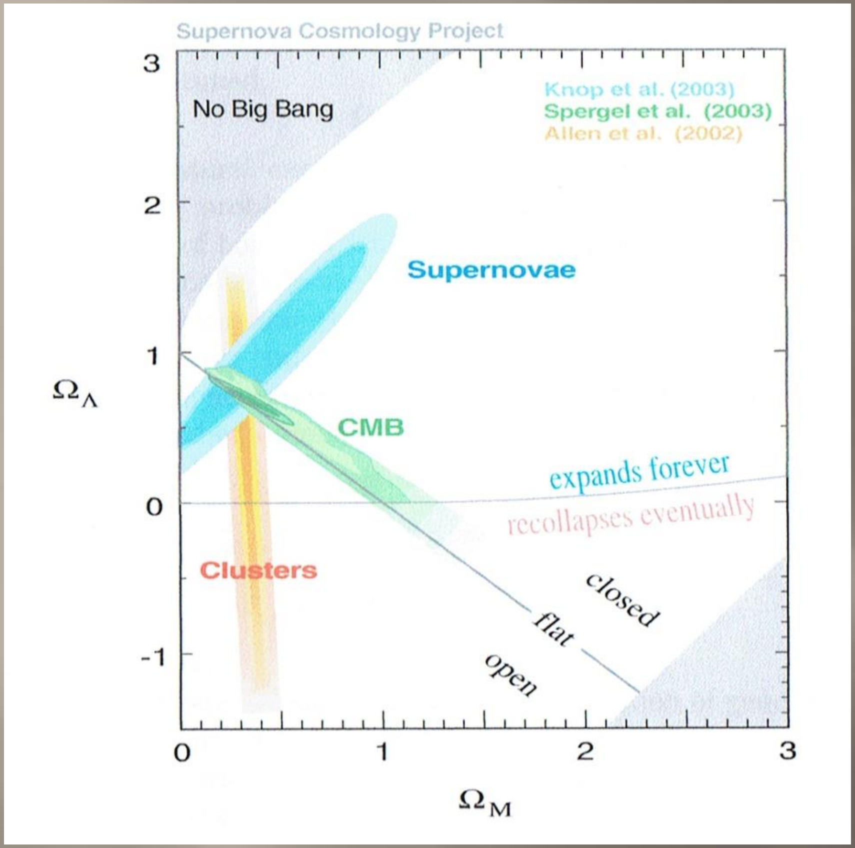
$H_0 = 72 \frac{km}{sMpc}$ **age: 13-14 Gyr**

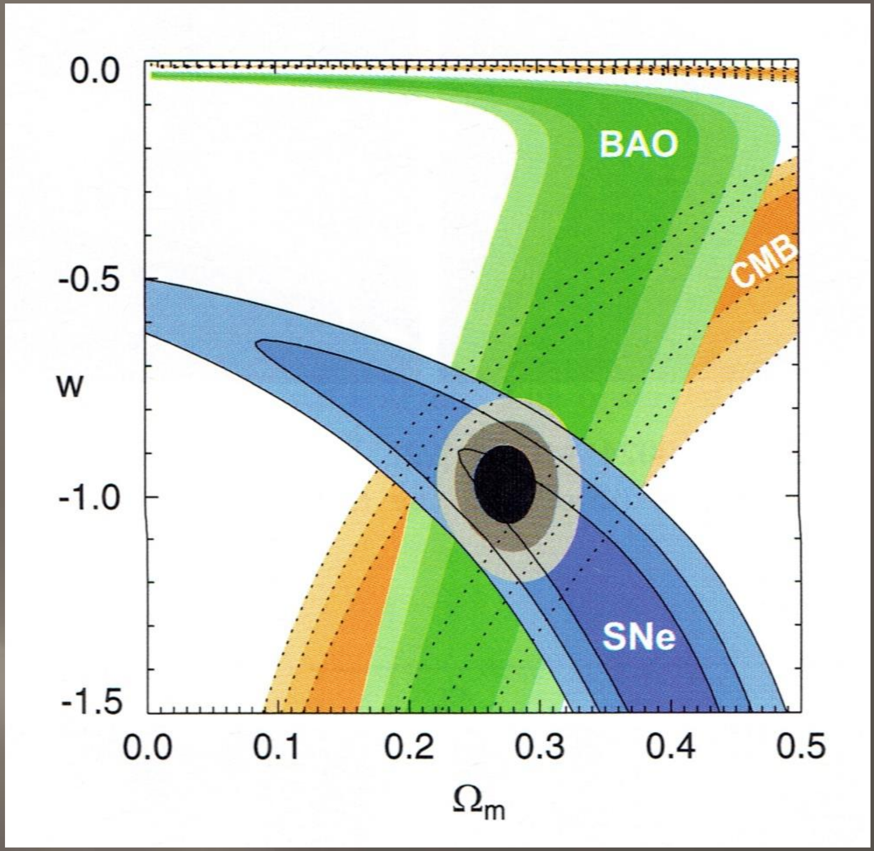
- The planck mass in natural units($8\pi G=1$)

$M_{pl} = 2,4\ 10^{18} GeV$

- The curvature fraction: $\Omega_K < 0.07$
- The CMB radiation with **T=2.726 K**
- The ratio of baryons to photons: $\frac{\eta_b}{\eta_\gamma} = 10^{-10}$
- Primordial helium abundance **0.2465**
- Deuterium abundance **2.53**
- Angular anisotropy: $\frac{\delta T}{T} = 10^{-4}$
- Dipole: $\delta T_{dip} = 3.346\ mK$







Observational constraints in $(\Omega_\Lambda - \Omega_M) - plane$

Observational constraints in the (w-Ω_M) $- \, plane$ for the dark energy EoS

The 4D Dynamics of Cosmological Expansion

- We have the FLRW metric

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right)\right]$$

- Friedmann eq: $\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} + \frac{1}{3}\Lambda$

- Conservation eq: $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$
- EoS: $P(\rho) = w.\rho$

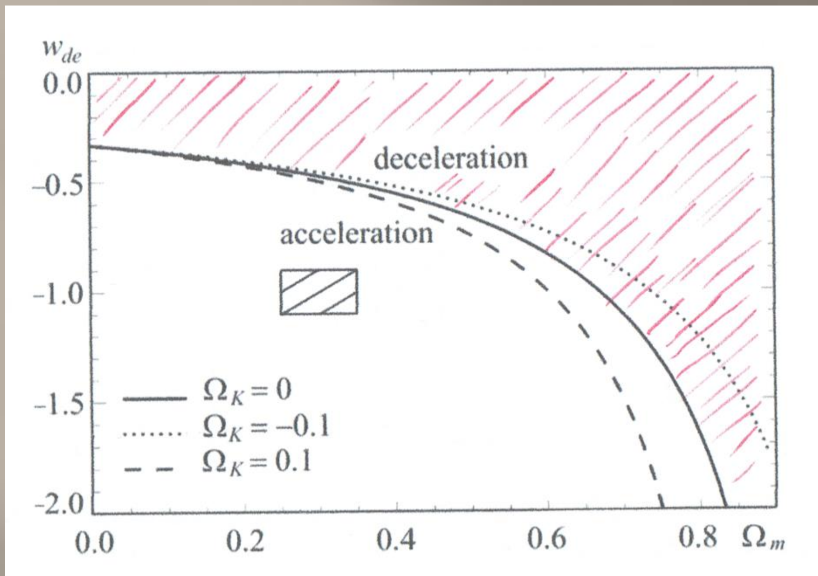
We can write $H^2 = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda + \rho_r + \rho_{curv})$ [crit dens: $\rho_c(t_0) = \frac{3H_0^2}{8\pi G}$]

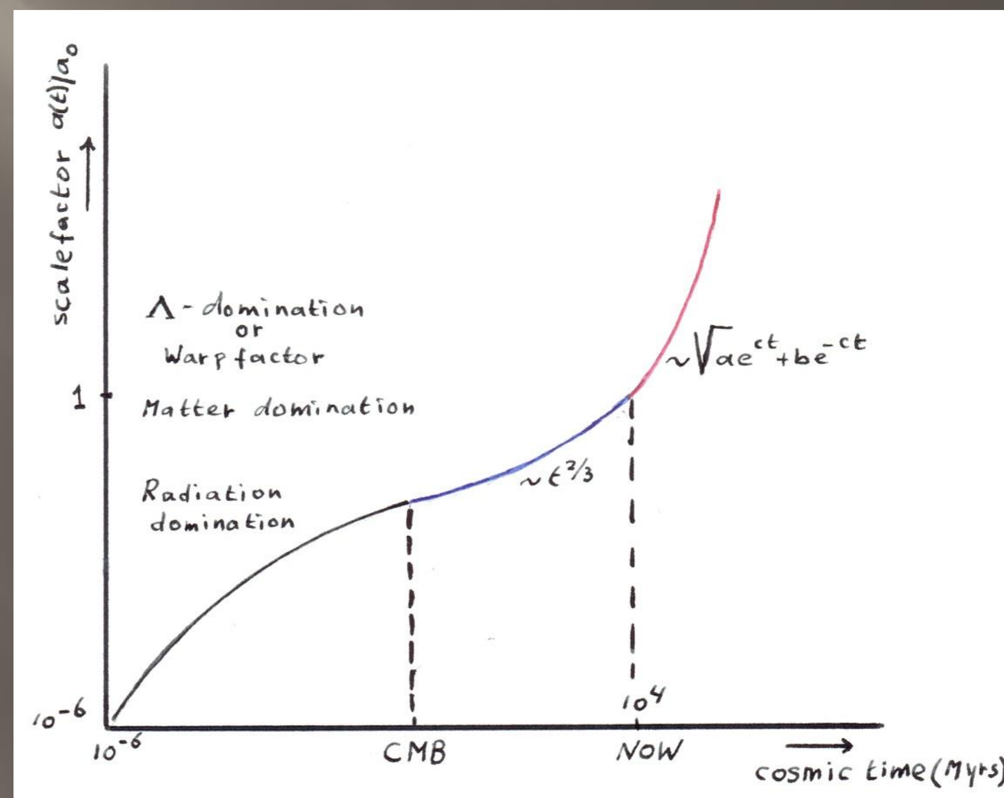
Or, [dimensionless Ω=$\frac{\rho(t)}{\rho_c(t)}$] $H^2 = H_0\left(\Omega_r\frac{a_0^4}{a^4} + \Omega_m\frac{a_0^3}{a^3} + \Omega_K\frac{a_0^2}{a^2} + \Omega_{de}\left(\frac{a_0}{a}\right)^{3(1-w_{de})}\right)$

Deceleration parameter: $q = \frac{H_0^2}{H^2}\left(\Omega_r\frac{a_0^4}{a^4} + \frac{1}{2}\Omega_m\frac{a_0^3}{a^3} + \frac{1}{2}(1+3w_{de})\Omega_{de}\left(\frac{a_0}{a}\right)^{3(1-w_{de})}\right)$

At current epoch t=t_0 : $\Omega_r + \Omega_m + \Omega_K + \Omega_{de} = 1$

The 4D Dynamics of Cosmological Expansion





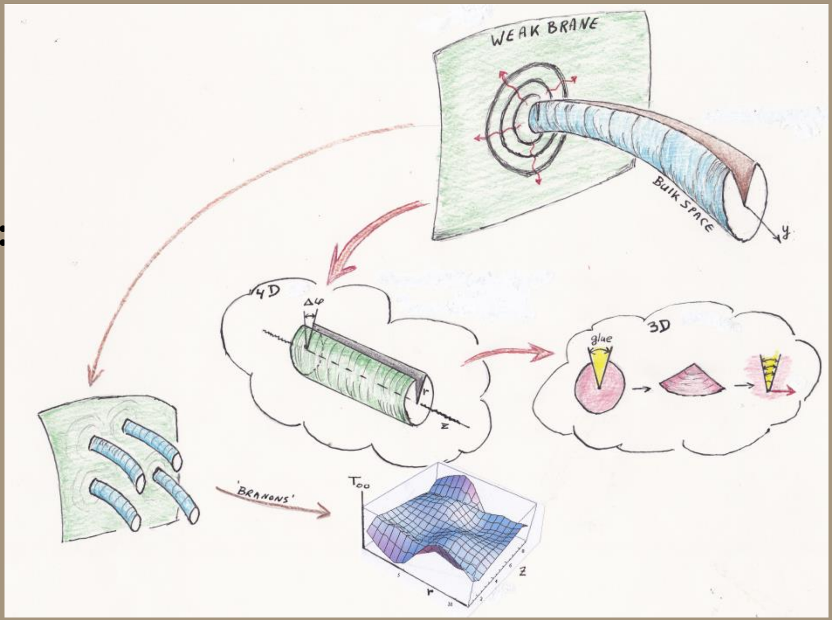
Why Cosmological Cosmic Strings?

- We should like to impose bounds on the amount of **cylindrical** GW from CS in expanding universe
- will they be **detectable**?
- **However:** C-energy ~ $\frac{r_{cs}}{R_H}$ **extreme small** [Gregory, 1989]
- **Expected disturbances fade away during expansion [“]**

NOW: Disturbances in the spatial components of the stress-energy tensor cause cylindrical symmetric waves, amplified due to the presence of the **bulk space** with **warp factor**

- The effective 4D spacetime of the CS in agreement with GUT;
- **The U(1) scalar gauge field has lived up to his reputation! : triggered inflation,**
  GL theory, Nielsen-Olesen vortex
- **The most simple brane world model:**
  Dvali-Gabadadse-Porrati vacuum FRW model:

However: ** no seeds for disturbances
**** bulk is 5D infinite Minkowski(no zero mode of graviton**
**** cross-over scale needed**
**** no “room” for effects like**
large scale alignment quasars
traveling waves in galaxies
**** “Ghost”-problem [Gorbunov,2005]**



Why Warped 5D Space times?

•The explanation of acceleration phenomenon with a cosmological constant is rather **problematic:**

► **Coincidence-problem:** $\Omega_\Lambda \sim \Omega_M$

► **Finetuning-problem:** $\rho_{\Lambda,obs} \sim 10^{-57} GeV^4 \quad \rho_{\Lambda,theor} \sim 1\, TeV^4$

► **Ad hoc modifications:** of the **Friedmann** equation risky, specially when considering density perturbations: do it **covariantly**

► **Disturbances don't survive in 4D models** : at least some of them are needed for the observed large-scale structures; in **warped 5D model**: they do survive

So modify GR : D-branes. 1. Dvali-Gabadadze- Porrati (DGP)
2. Randall-Sundrum (RS)

By-product: hierarchy problem solved [for example in RS-1 model]

Modify Matter or Modify Gravity?

►Many phenomenological models for dark energy. For example:

Quintessence: Scalar field + potential $w_{de} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$

► Find the V(ϕ) for which we have acceleration: **slow roll**

Phantom, …..

So: $$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + T_{\mu\nu}^{dark})$$

Alternatives:

I.►"Infrared"- modification of GR: $G_{\mu\nu} + G_{\mu\nu}^{dark} = 8\pi G T_{\mu\nu}$ **[DGP[covariantly]**

in stead of ad hoc: $H^2 = \frac{8\pi G}{3} f(\rho)$

gravity "leaks" of the brane in the bulk at large scales

at small scales gravity bound to brane

but: cross-over scale needed for transition from 4-5D behavior

II. ►Modify GR at high energies [RS}: curved bulk

Low energy: zero mode of graviton dominate on brane : GR recovered

High energy: massive modes of graviton dominate; gravity on the brane behaves increasingly in a 5D way

"Ultraviolet" modifications: $H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{2\pi G L^2}{3}\rho\right) + \frac{\Lambda}{3}$

At low energy: we recover Friedmann

At high energy: gravity "leaks" off the brane; different types of expansion possible; modification of inflation?

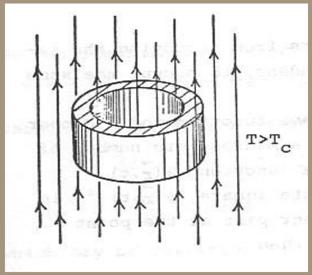
Basics of Cosmic Strings

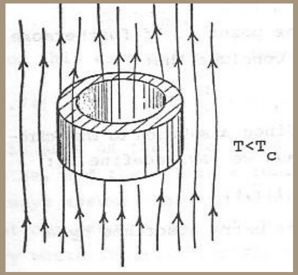
► **Superconductivity type II:**

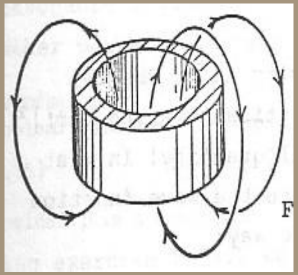
There is the well-known Meissner effect : the magnetic field is trapped at $T<T_c$ Pairs of electrons in supercond. state: **Cooper-pairs** are formed, whose properties are described by an **order** parameter:

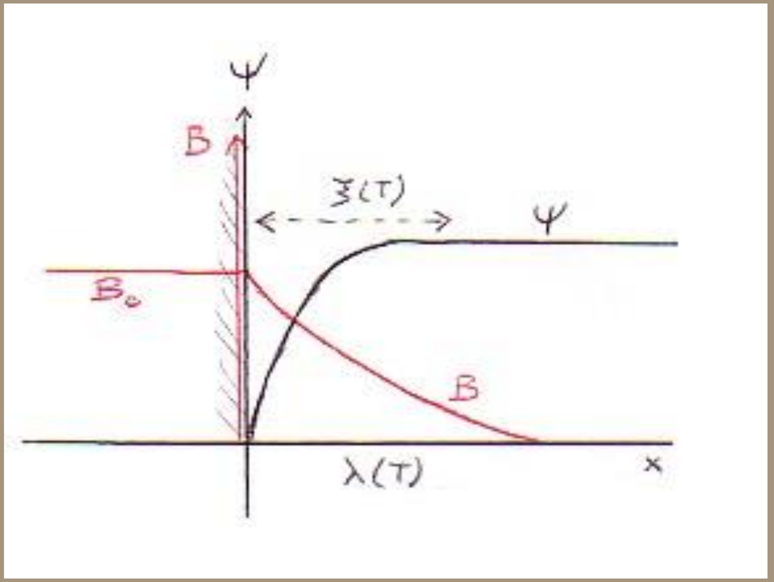
$$\Psi(r,t)=\rho(r,t).e^{i\varphi(r,t)}$$

The magnetic flux is quantized: $\Phi=\int A.dr=2\pi n/q$









1. **φ makes a jump of $2\pi n$!**
2. **Solution of Ginzberg-Landau eq.**
3. **Ψ min coupled to gauge field A**
4. **Exterior deriv. replaced by gauge- cov. $d-i\, q/\hbar\, A$**
5. **Two characterisic lengths: coherence length + penetration length**

Followup: GL equations

► The GL equations: $D^2\,\Psi=\Psi(\alpha+\beta\Psi^2)$

► Potential: $U(\Psi)=-\gamma+\frac{1}{2}\alpha\Psi^2+\frac{1}{4}\Psi^4$

► Vacuum $\Psi=0$ no Cooperpairs

► Degenerate vacuum: superconducting state of $\Psi=\Phi(r).e^{in\varphi}$

► **GL equations:** $r\left(\frac{A'}{r}\right)'=-\frac{1}{\lambda}\Phi^2(1-A)$ $\quad (r\Phi')'=-\alpha(\Phi^2-1)\Phi+n^2\frac{(1-A)^2}{r^2}\Phi^2$

► In the superconducting vacuum,B has to vanish and is expelled from any region where the order parameter is in the SC vacuum (Meissner effect)

► **Ansatz:** A=nh/q A(r).dφ

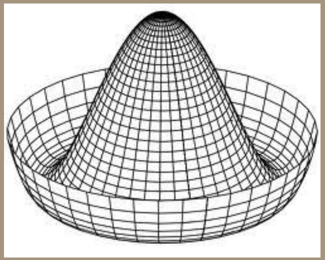
►So either **B** vanishes or **Ψ**. In de normal state **Ψ,** the magnetic field can propagate freely.

Nielsen-Olesen vortex

►The first example of string-like solution in classical field theory: NO-string:
complex scalar Higgs field minimal coupled to Maxwell field.

►Mexican-hat potential

$$U = \lambda/4(\Phi^2 - \eta^2)^2 \qquad [\eta \ \text{VEV}]$$



► The gauge covariant field eq:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F^{\mu\sigma}) = \frac{ie}{2}(\Phi\overline{D^\sigma\Phi} - \bar{\Phi}D^\sigma\Phi)$$

$$\frac{1}{\sqrt{-g}}D_\upsilon(\sqrt{-g}D^\upsilon\Phi) = \lambda(\Phi^2 - \eta^2)\Phi$$

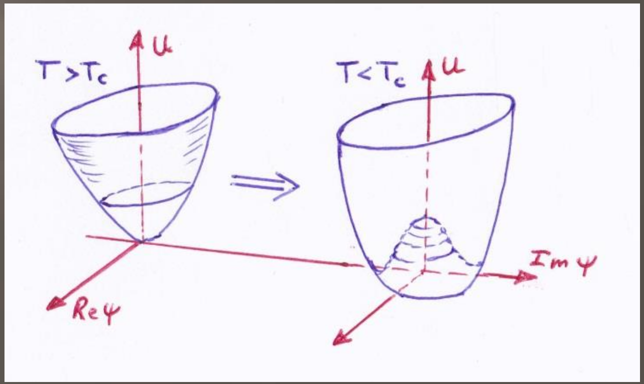
►without electric field, we obtain for A and Φ exactly the GL-super-conductivity model! [with $e= q/h$, $\lambda= \beta$ $\mu^2 =-\alpha$]

Spontaneous symmetry breaking (SSB)

- Suppose U temperature dependent:

model is invariant under U(1) of global phase transitions $\Phi = e^{i\alpha}\,\Phi$

- minima lie on a circle $|\Phi| = \eta$ so the true vacuum is $<0|\Phi|0> = \eta\, e^{i\theta}$
- A phase transformation changes $\theta \rightarrow \theta + \alpha$ so $|0>$ is not invariant $\rightarrow$ SSB.
- The unbroken $<0|\Phi|0> = 0$ is unstable



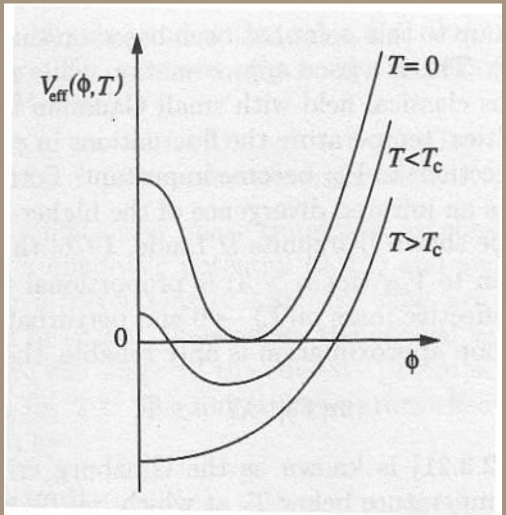
Cosmological 1-st and 2-nd order Phase Transitions

► In reality, Φ is a quantum field, so $U(\Phi)$ must be modified due to radiative corrections. For the Goldstone model, the second order phase transition is described by the high-temperature effective potential

$$U_{eff}(\Phi,T) = m(T)^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4, m^2 = \frac{\lambda}{12}(T^2 - 6\eta^2)$$

In Hot Big Bang model the universe starts at very high temperature.When universe cools down below T_c, Φ develops an expectation value:

$|\Phi| = (T_c^2 - T^2)^{1/2}$



► **The phase φ takes again different values at different regions of space.**

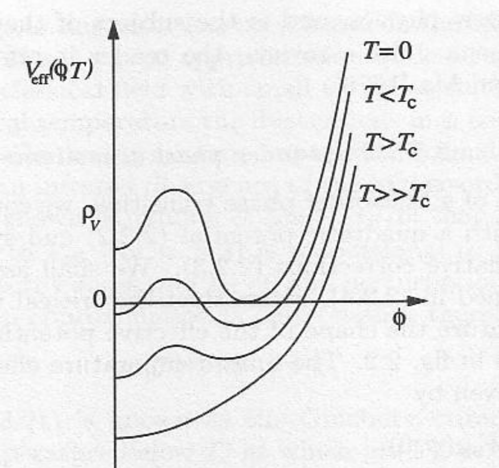
Consider now the first order effective potential

$$U_{eff} = m(T)^2|\Phi|^2 + \frac{3e^2}{16\pi^2}|\Phi|^4 ln(\frac{|\Phi|^2}{\sigma^2}),$$

$m^2 = \mu_0^2 + \frac{1}{4}e^2T^2$

difference: symmetric phase below T_c remains meta-stable if $\mu_0^2 < 0$

application: Inflation



Topological defect: Cosmic String

►**Consider:** vacuum manifold M

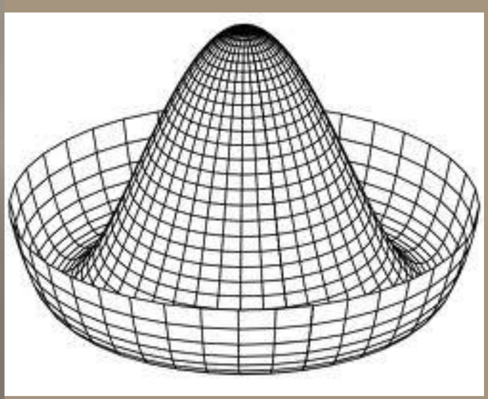
►**Exp value** **$<0|\Phi|0>=\eta\, e^{in\theta}$** winding number n

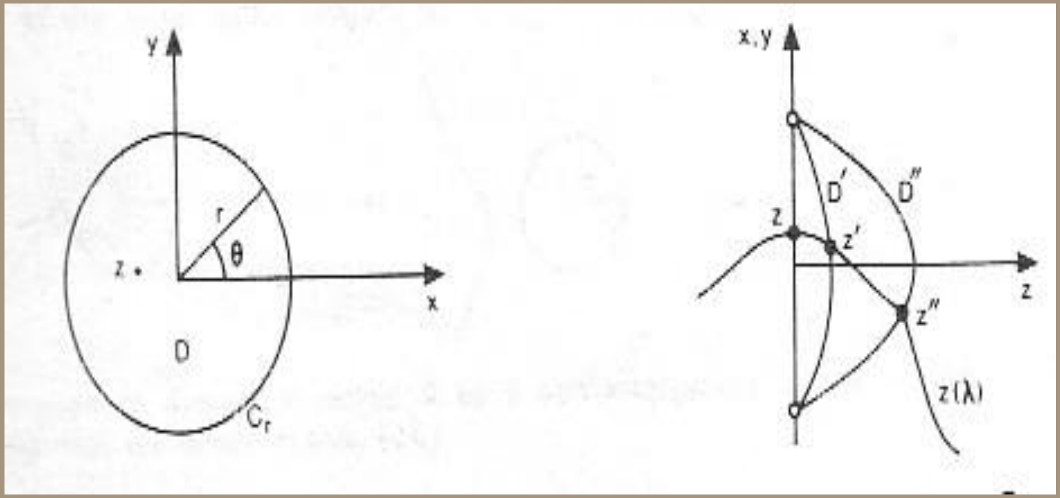
►**Φ** winds once around the vacuum manifold M as **x** winds one time around the circle in position space.

When Φ is continuous and one demands single-valuedness of Φ:

►**<u>then</u>** Φ cannot remain in M everywhere on the disk **D** bounded by **C_r** . If we shrink the circle to **r → 0**, we obtain the contradiction with **single- valuedness.**

So there must be at least one point **z** on the disk D where $\Phi_z = 0$. So Φ rises to the top of the potential.





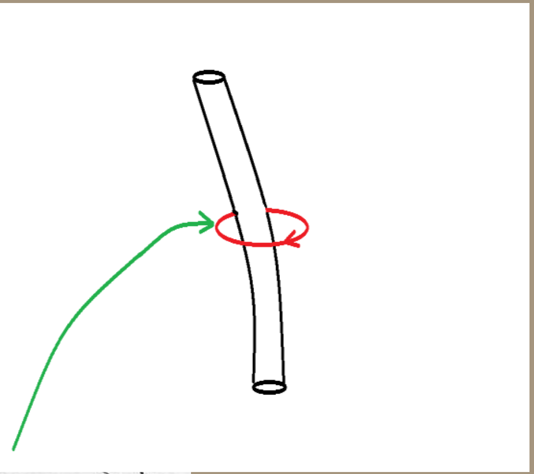
The jump in θ is again related to the magn flux in the string $\Phi = n\frac{2\pi}{e}$

Trapped energy of false vacuum

► Stoke's : within the circle $n \neq 0$ due to the flux

► Trapped energy of the false vacuum

► One " Higgs-pencil" cannot follow the symmetry in the plane: if it lays down, symmetry will be broken. At this point there is a lot of potential energy stored in the scalar field configuration



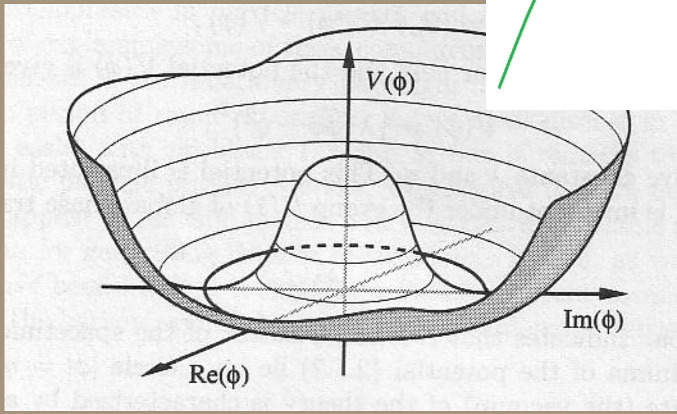
►For large r we have

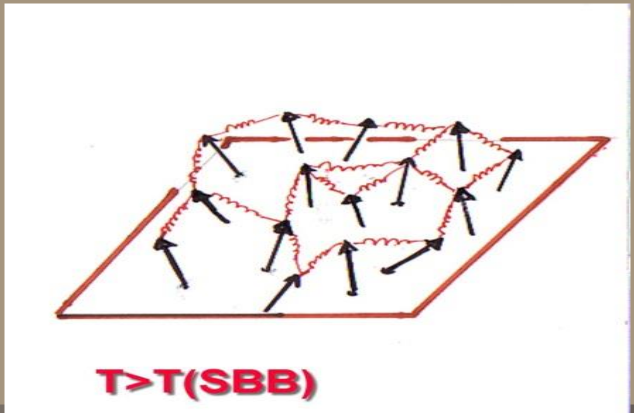
$\Phi = \eta e^{in\varphi}$ $A_\mu = 1/ie\, \partial_\mu \Phi$

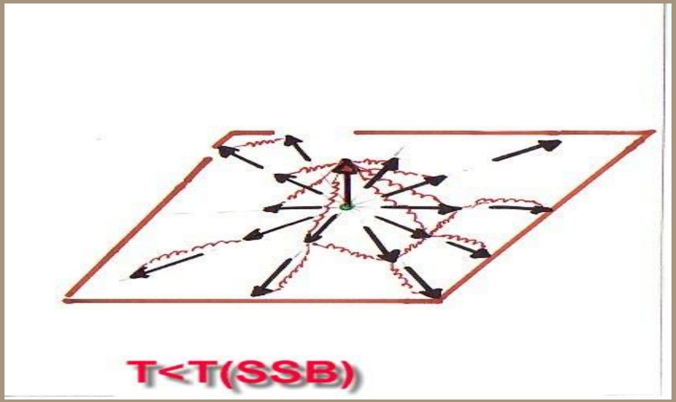
and we obtain a quantized magnetic flux: $\Phi = \int A dr = 2\pi n/e$

see:

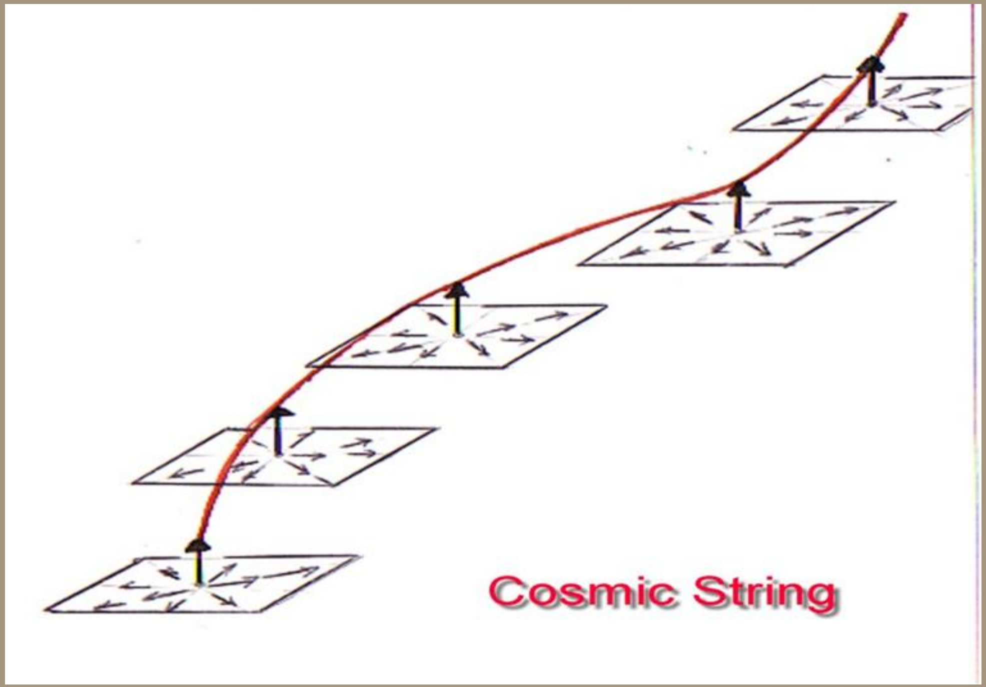
type II superconductivity!!







The cosmic string has formed.



Collection of point where the field is the the false vacuum

General Relativistic Cosmic Strings

- Strings have stress energy, so they couple to gravity.

- Solve **simultaneously** the coupled Einstein-Scalar-Gauge field equations.
- Space time with 3 Killing

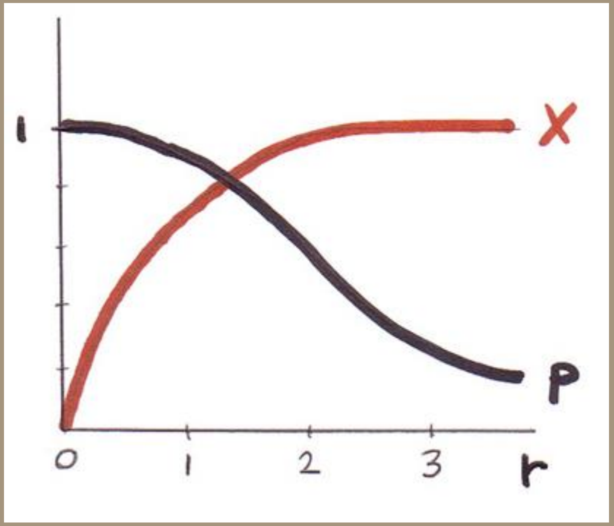
$$\boxed{ds^2 = -e^A dt^2 + e^B dz^2 + dr^2 + e^C d\varphi^2}$$

- To restore boost symmetry: A=B
- The system becomes

$$G_{\mu\nu} = \kappa^2{}_4 T_{\mu\nu} \qquad D_\mu D^\mu \Phi - 2\frac{\partial U}{\partial \Phi^*} = 0 \qquad \nabla^\mu F_{\mu\nu} - \frac{1}{2} ie[\Phi(D_\nu\Phi)^* - \Phi^*(D_\nu\Phi)] = 0$$

- Now take $\theta=\varphi$ and $A_\nu = \frac{(P-1)}{e}\nabla_\nu\varphi$ and $\Phi = X e^{i\varphi}$
- On **Minkowski** with $X \to X/\eta$ and $r \to r/\eta\sqrt{\beta}$

numerical solution:



Where did we see this picture before?

The general solution

► On curved space time we have the equations:

$$\partial_{rr}K = \frac{1}{2}\kappa_4{}^2\eta^2[-\frac{3}{4}K(X^2-1)^2 - 2e^{2A}\frac{X^2P^2}{K} + \frac{e^{2A}}{\alpha K}(\partial_r P)^2]$$

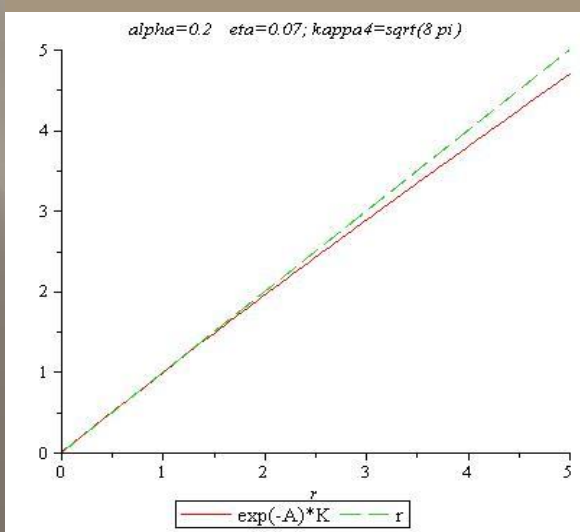
$$\partial_{rr}A = -\frac{\partial_r A\partial_r K}{K} + \kappa_4{}^2\eta^2[-\frac{1}{4}(X^2-1)^2 + \frac{e^{2A}}{\alpha K^2}(\partial_r P)^2]$$

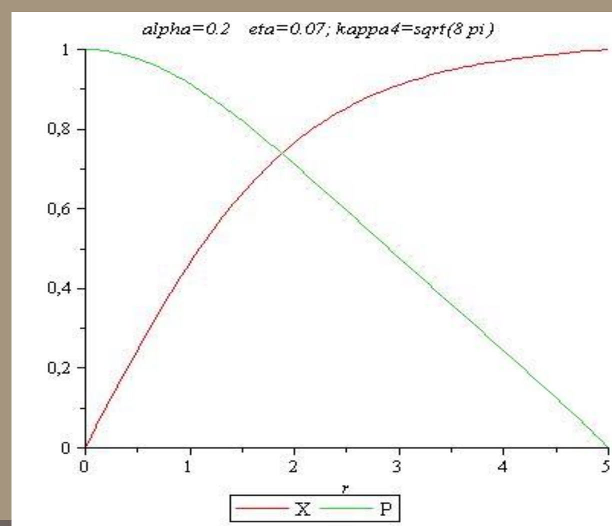
$$\partial_{rr}X = -\frac{\partial_r X\partial_r K}{K} + \frac{1}{2}X(X^2-1)^2 + \frac{e^{2A}}{K^2}XP^2$$

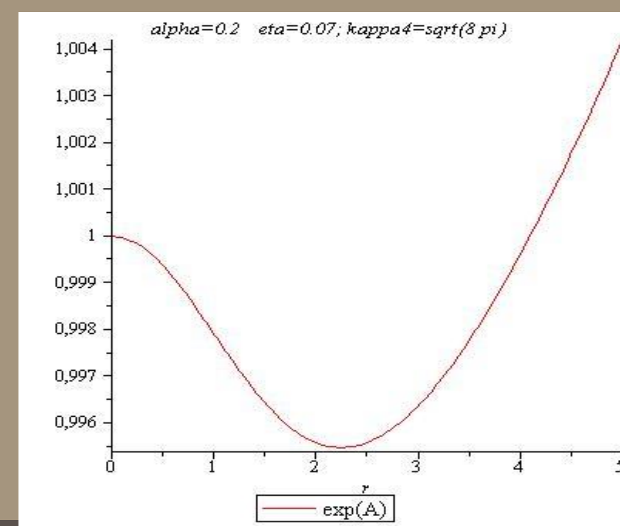
$$\partial_{rr}P = -2\partial_r P\partial_r A + \frac{\partial_r P\partial_r K}{K} + \alpha X^2 P$$

Only numerical solutioins:

►Note: X and P as GL solution!! Look at $g_{\varphi\varphi}$ component: **angle deficit**







Surprise: Conical spacetime: Angle deficit

► Following **Garfinkle (1987)** one defines

$$\Theta_1 \equiv K\partial_r A \qquad \Theta_2 \equiv \partial_r K$$

Then the eq. can be written as

$$\partial_r \Theta_1 = \kappa_4{}^2 \mathrm{K}(\rho_\varphi + \rho_r) \qquad \partial_r \Theta_2 = \kappa_4{}^2 \mathrm{K}(\rho_\varphi + 3\rho_r - 2\sigma)$$

With $T_{\mu\nu} = \sigma k_t k_t + \rho_z k_z k_z + \rho_\varphi k_\varphi k_\varphi + \rho_r k_r k_r$ [k orthonormal vectors]

($\sigma > |\rho_r|$)

► We have conservation of **energy-stress**:

$$k_\mu \nabla_\nu T^{\mu\nu} = \partial_r(K\rho_r) - \rho_\varphi \partial_r K + (\sigma + \rho_\varphi) K \partial_r A = 0$$

► Then one can write

$$\boldsymbol{\partial_r \left[\Theta_1\left(\Theta_2 - \tfrac{3}{4}\Theta_1\right)\right] = \partial_r[\kappa_4{}^2 \mathrm{K}^2 \rho_r]}$$

► Let us assume: $\int_0^\infty K^2\, \sigma$ conv $\qquad \lim_{r\to\infty} K^2 \sigma = 0$

Conical spacetime+Angle deficit

- Then we obtain

$$\boxed{k_1\left(k_2-\frac{3}{4}k_1\right)=0}$$

With k_1 and k_2 the asymptotic values of Θ_1 and Θ_2

- So we have two solutions: $k_1 = 0$ or $k_2 = \frac{3}{4} k_1$ [Kasner metric]
- For $k_1 = 0$, the asymptotic metric then becomes:

$$\boxed{ds^2=-e^{a_0}(dt^2-dz^2)+dr^2+e^{-2a_0}(k_2r+a_2)^2\,d\varphi^2}$$

- This metric can be brought to **Minkowski** by the change of variables

$$ds^2=-(dt^2-dz^2)+dr^2+d\varphi'^2$$

- However:

$$\mathbf{0\le\varphi'\le 2\pi e^{-a_0}k_2<2\pi}$$

Conical spacetime+Angle deficit

►So we have an angle deficit

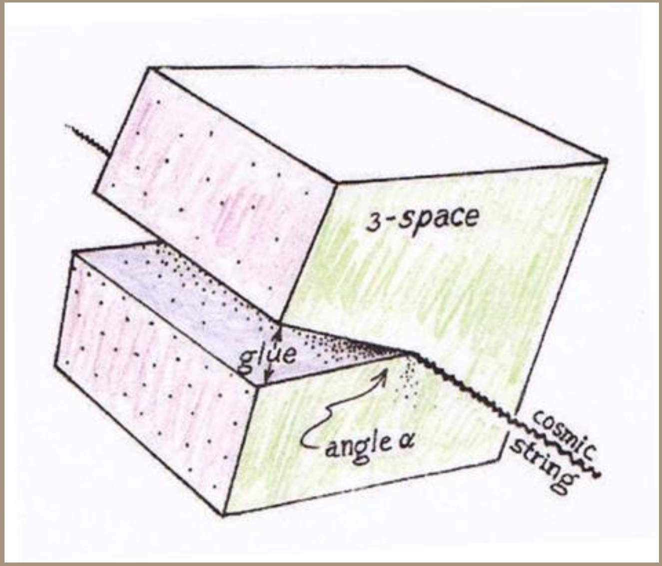
$\Delta\theta = 2\pi(1 - e^{-a_0}k_2)$ [k_2 determined by η, m_A / m_Φ

► On proves:

$\Delta\theta = \kappa_4{}^2\,\mu + \frac{\pi}{2}\int_0^\infty e^{-A}K\,(\frac{dA}{dr})^2\,\mathrm{dr}$

With $\mu \sim \eta^2$ the linear energy density

$$\mu = 2\pi\int_0^\infty e^{-A}K\sigma dr$$



►The angle deficit will increase with the energy scale of symmetry breaking. Further, for GUT scale, $\eta \sim 10^{16}$ GeV, so the mass per unit length is $G\mu \sim 10^{-6}$
Numerical analysis of super massive cosmic
strings, shows that the solution becomes singular at finite distance of the string or the angle deficit becomes greater than 2π [angle surplus]

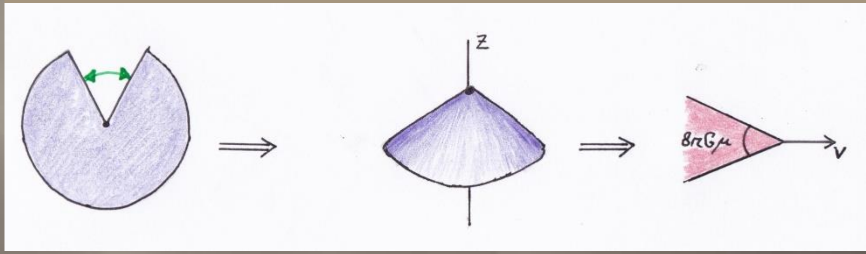
Time-machines?

- Some physicists believe in timemachines around CS:

gott ctc 2.wmv

't Hooft [1990-1994]: NO

<u>However:</u> In 2+1 dimensions: "cosmons "
example of self-gravitating particles
quantizable? ['t Hooft 1990]



NOTE:
(interesting)

Gravitational waves and non-abelian CS: [**slagter**, Class.Quantum Grav,2002]
Strings on Einstein-Rosen background [**slagter**, Gen. Rel. Grav. 1991]

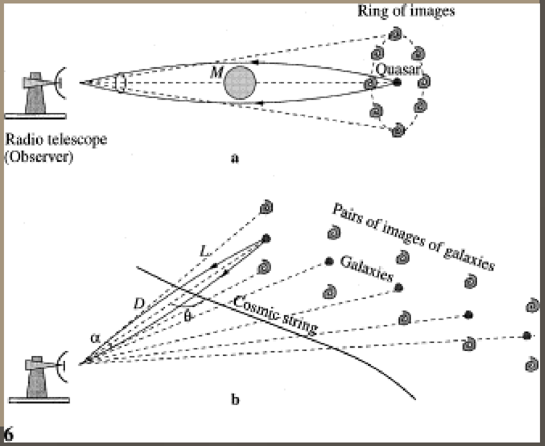
Problems for Cosmic Strings from Observations

- ► density perturbations : $\frac{\delta\rho}{\rho} \sim G\mu = \eta^2 / M_p^{\ 2} \sim 10^{-6}$ for GUT scale

- ► in first instance correct with observations

- ► Now: inconsistencies with new CBM power spectrum COBE, WMAP
- ► They cannot provide a satisfactory explanation for the magnitude of the initial density perturbations
- ► How to handle **super-massive CS** with $G\mu >>1$ [phase transition at energy much larger than GUT

- ► where is the axially symmetric gravitational lensing-effect?



- ► Cosmological CS: late-time conical residu [unwanted] [Gregory, 1989]

exit CS?

Rescue of CS

reborn CS → Go to warped 5D RS model

*** **in the brane**: unobservable angle deficit

*** **asymptotically**: no conical space time [**Slagter**, 2012;IJMPD]

*** **So no conflict with:**

1. CMB-spectrum
2. Absence of axially symm. double images

→ **CS can be produced in superstring theory**

→ **Super massive CS with Gμ >> 1 will be warped down to GUT scale(10^{-7})**

►► **Mass:** $\mu = 2\pi F(y) \int_0^\infty e^{-A} K\sigma dr$ **with F the WARPFACTOR**

SO: building up a huge mass in the bulk : KK-modes on brane

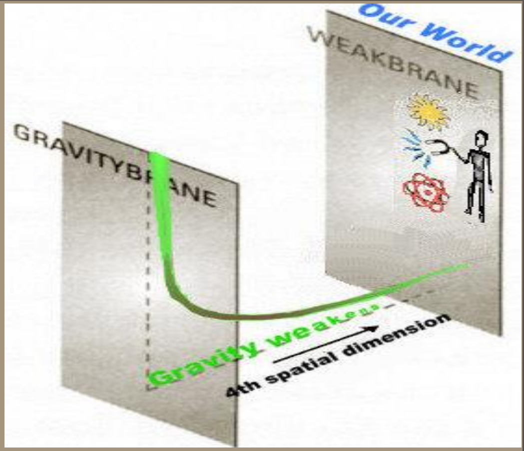
►► **Test of RS type models against observational constraint possible !**

Cern: KK-particles detectable?

Brane world models of Randall-Sundrum

► Large extra dimension [no curled-up tiny-dim.]

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$



► At low energy: gravity localized at the brane: GR recovered. Modification to the weak field eq. Negative bulk Λ prevents gravity to leak into extra dimensions (squeezes gravity closer to the weak brane)

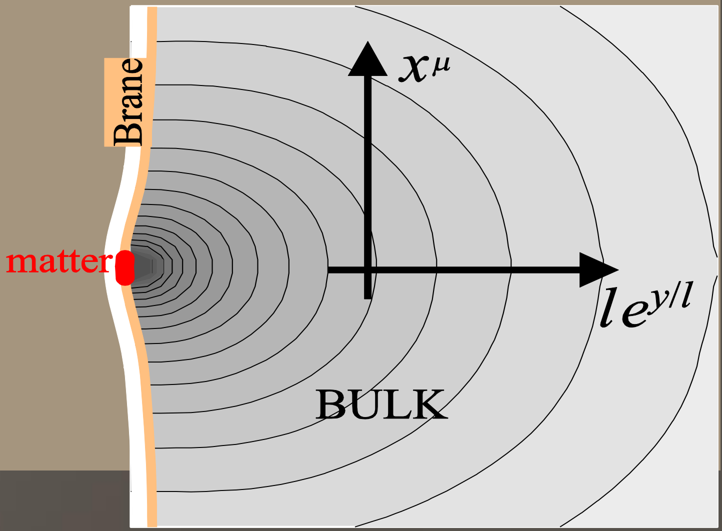
► At high energy: gravity "leaks" into the bulk

► Solves hierarchy problem

► The 5D graviton effects (KK modes) **detectable?**

► Because of the exponential warping is the effective scale on visible brane at y=L:

$$M_p^2 = M_5^3\,(1-e^{-2kL})/k$$



Randall-Sundrum I model

- RS-I model: $L\rightarrow\infty$ and $M_5^3 = M_p^2/l$
- The contribution to the 5D volume is however finite because of the warpfactor
- Higher-dim graviton has massive 4D modes **felt on the brane** **[KK modes]**
- Fine-tuning : $\lambda_4 = \frac{3M_p}{4\pi l^2}$
- RS-action: $$S=\int dyd^4x\sqrt{-g}\left(\frac{1}{2}M_5^3R-\Lambda\right)+\sum_1^2 d^4x\sqrt{-h^i}\left(\Lambda_i - L^i_{mat}\right)$$
- Contribution of the massive KK-modes sums up to a correction of the 4D potential

$$V(r) \approx \frac{4\pi GM}{r}\left(1+\frac{2l^2}{3r^2}\right)$$

The warped U(1) gauge CS in 5D

- The action of the model under consideration is

$$S = \int d^5x\sqrt{-{}^5g}\big[\, \frac{1}{2\kappa^2{}_5}({}^5R - \Lambda_5) + S_{bulk}\big] + \int d^4x\sqrt{-{}^4g}\big[\frac{1}{\kappa^2{}_5}\lambda_4 + S_{brane}\big]$$

- Λ_5 the cosmological constant in the bulk
- λ_4 the brane tension
- S_{brane} the effective 4D Lagrangian, which is given by a generic functional of the brane metric and matter fields on the brane and will also contain the extrinsic curvature corrections due to the projection of the 5D curvature.
- If there is a bulk scalar field (no coupling to a 5D gauge field A as in the 4D case) then we have for the 5D equations

$${}^5G_{\mu\nu} = -\Lambda_5\, {}^5g_{\mu\nu} + \kappa_5^2\, {}^5T_{\mu\nu} \qquad {}^5\nabla_\mu {}^5\nabla^\mu\, {}^5\Phi = 0$$

- We consider first the time-independent warpfactor F(y)

The **5D metric of the cosmic string** **[**Slagter,IJMPD,2012**]**

$$ds^2 = F(y)\left[e^{A(r,t)}\left(-dt^2 + dz^2\right) + dr^2 + K(r,t)^2 e^{-2A(r,t)} d\varphi^2\right] + dy^2$$

- Constraint equation: $(\partial_r A)^2 = \frac{4}{3}\frac{\partial_r K \partial_r A}{K} - \frac{2}{3}\frac{\partial_{rr} K}{K} - c_1$

The Brane Induced Field Equations

$$ {}^4G_{\mu\nu} = -\Lambda_{eff}\, {}^4g_{\mu\nu} + \kappa_4^2\, {}^4T_{\mu\nu} + \kappa_5\, {}^4S_{\mu\nu} - E_{\mu\nu} + 2/3\kappa_5\, {}^4F_{\mu\nu} $$

► $\Lambda_{eff} = ½(\Lambda_5 + \kappa_4^2 \lambda_4)$ [=0 for RS finetuning]

► $S_{\mu\nu}$ is the quadratic term in the energy-momentum tensor arising from the extrinsic curvature terms in the projected Einstein tensor.

► $E_{\mu\nu}$ represents the 5D graviton effects, i.e., Kaluza-Klein modes in the linearized theory and is a part of the 5D Weyl tensor. Carries information of the gravitational field outside de **brane** and is constrained by the motion of the matter on the brane, i.e., the **Codazzi** equation.

► We take F=0 (no bulk matter)

► We again try to find solutions for: metric comp. A, K

scalar Higgs and gauge field X and P

5D-corrected equations

- $$\partial_r \Phi_2 = \frac{6}{11} K[\kappa_4{}^2 (3\rho_r - 2\sigma + \rho_\varphi) + \kappa_5{}^4 (3\xi_r - 2\xi_t + \xi_\varphi) + \frac{3c_1}{4} - 6\Lambda_{eff}]$$
- $$4\partial_r \Phi_1 + \partial_r \Phi_2 = 6K[\kappa_4{}^2 (\rho_r + \rho_\varphi) + \kappa_5{}^4 (\xi_r + \xi_\varphi) + \frac{c_1}{4} - 2\Lambda_{eff}]$$
- Equations for X and P are the same as in 4D
- For $\mathbf{c_1 = 8\Lambda_{eff}}$ we see: on right hand side same combination of the $T_{\mu\nu}$ and $S_{\mu\nu}$ as is the 4D case !!
- Again we evaluate : $\frac{d}{dr}(\kappa_4{}^2 K^2 \rho_r)$ using conservation of T and S:

$$\boxed{\frac{d}{dr}[\kappa_4{}^2 K^2 \rho_r] = \frac{d}{dr}[\frac{2}{3}\Theta_1\Theta_2 + \frac{1}{12}\Theta_2{}^2 - \frac{1}{2}\Theta_1{}^2]}$$

- Using the same boudary conditions as in 4D, then

$$\frac{2}{3}\Theta_1\Theta_2 + \frac{1}{12}\Theta_2{}^2 - \frac{1}{2}\Theta_1{}^2 = 0$$

Note: for warped FRW we need $\mathbf{\Lambda_{eff} = 0}$

Asymptotic behaviour and angle deficit ?

- NOW asymptotical: $\Theta_1 = 0$ no option! $\longrightarrow$ NO conical space time
- Solution

$$\bar{\Theta}_1 = \frac{1}{-4 \pm \sqrt{22}} \bar{\Theta}_2$$

So we obtain for the asymptotic metric:

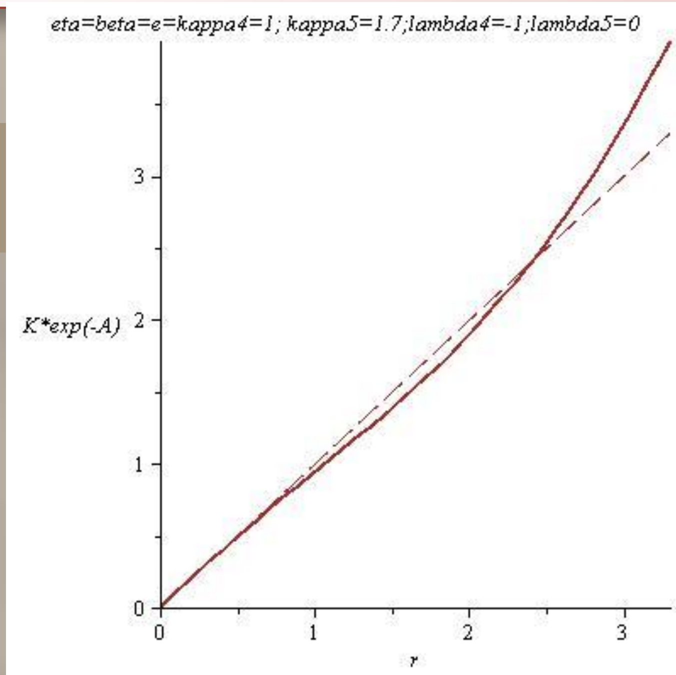
$$\bar{K} = k_2 r + a_2 \qquad \bar{A} = \frac{\ln(k_2 r + a_2)}{-4 \pm \sqrt{22}} + a_0$$

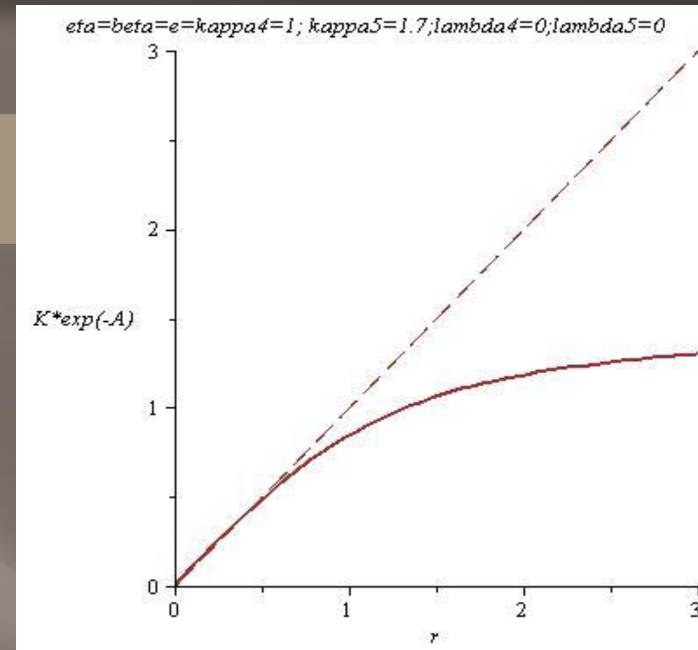
- Our metric becomes:

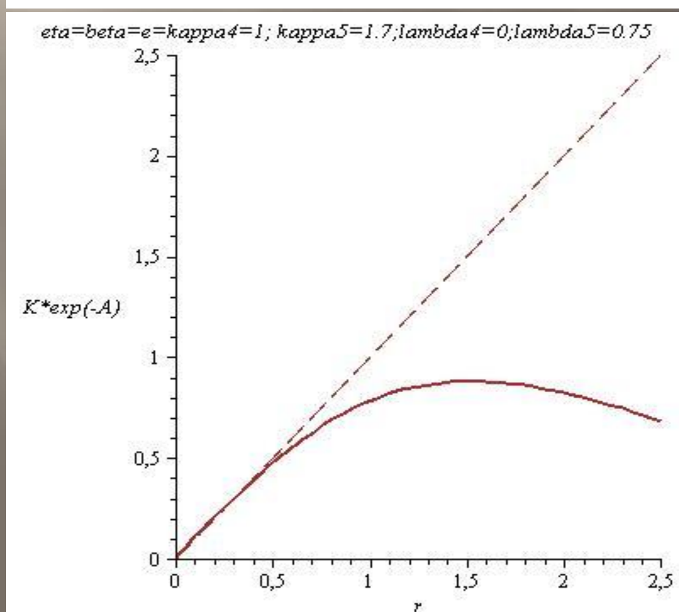
$$ds^2 = F(y)\left[e^{a_0}(k_2 r + a_2)^{\frac{1}{-4\pm\sqrt{22}}}\left[-dt^2 + dz^2\right] + dr^2 + e^{-2a_0}(k_2 r + a_2)^{2+\frac{1}{-4\mp\sqrt{22}}} d\varphi^2\right]$$

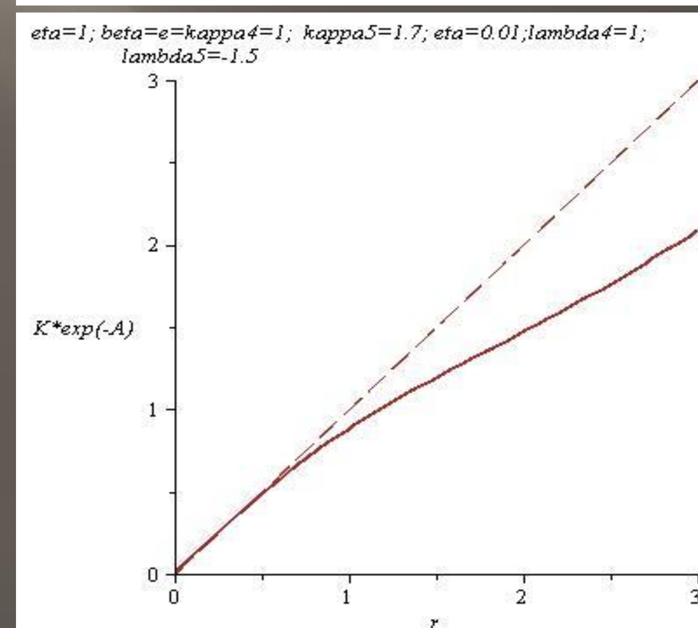
- Kasner-like solution! Behaviour dependent of $\frac{k_2}{a_2}$
- angle deficit: $\Delta\theta = \kappa_4{}^2 \mu - 2\pi \int_0^\infty \left[\frac{e^{-A}}{K}\left(\frac{2}{3}\Theta_1\Theta_2 - \frac{1}{6}\Theta_1{}^2 + \frac{1}{12}\Theta_2{}^2\right) + \frac{1}{6}\frac{d(e^{-A}\Theta_2)}{dr} - \frac{2}{3}\frac{d(e^{-A}\Theta_1)}{dr}\right] dr$
- correction terms now not of order η^4 as in the 4D case!

Some numerical sol of $g_{\varphi\varphi}$







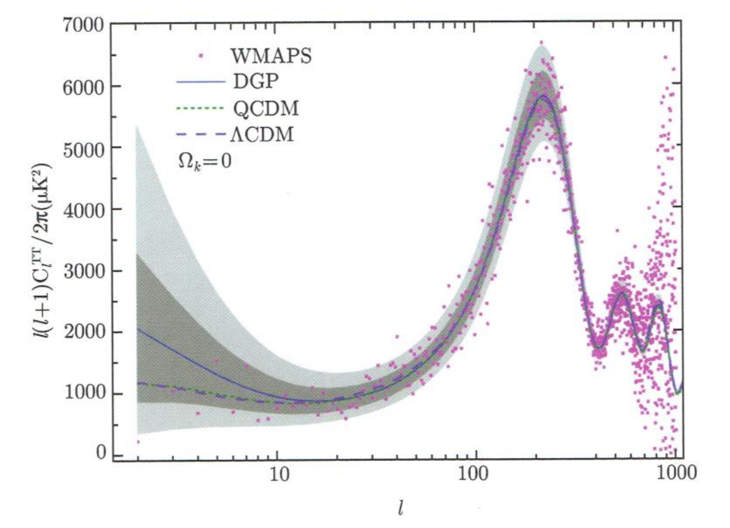


Large-Scale modification of GR on FRW model

- Self-acceleration at late time possible without Λ?

$\longrightarrow$ DGP-5D model: Attempt to explain the late-time acceleration as **self-acceleration without an inflaton field.** [Dvali et al, 2000]

However: ** DGP has 5D Minkowski bulk, so infinite volume
** There is across-over scale needed; Gost?
** Disfavored by observations?



Slow leakage of gravity off our 3D universe.

$$H^2 - \frac{H^\alpha}{r_{co}^{2-\alpha}} = \frac{8\pi G}{3}\rho_m; \quad r_{co} = \frac{1}{H_0(1-\Omega_{m0})}$$

In general:
Gravity leakage at late-times initiates acceleration, due to weakening of gravity on the brane – not due to any negative pressure field.
4D gravity is recovered at high energy via the lightest KK modes of the graviton

Here: **We will use the generalized time-dependent FRW model from the cyl sym ST in the RS-model:** [Slagter, Pan, 2015]

$$ds^2 = W(t,r,y)^2\left[e^{2\gamma(t,r)-2\psi(t,r)}(-dt^2+dr^2) + e^{2\psi(t,r)}dz^2 + r^2 e^{-2\psi(t,r)}d\varphi^2\right] + dy^2$$

Warpfactor W and y the bulk space

The Warped 5D FRW model: Relation with 4D results

- It is problematic to embed U(1) CS in FRW model due to **boost** invariance.
- However, for **radiating cyl symm** spacetimes one can analyze the gravitating CS [for spherical spacetimes: Birkoff's theorem: no dynamical degree of freedom]
- Importance of **cyl symm grav waves** was already noticed by Einstein-Rosen[1936]
- U(1) CS can be embedded into a flat 4D FRW along the polar axis [Gregory, 1989]
- The approx spacetime becomes (conical):

$$ds^2 = a(t)^2[-dt^2 + dr^2 + K(r)^2dz^2 + (1 - 4\pi G\mu)^2S(r)^2d\varphi^2]$$

and can be matched on the well known FRW spacetime by suitable transformation

$$ds^2 = a(t)^2\left[-dt^2 + \frac{dR^2}{1 - kR^2} + R^2d\theta^2 + (1 - 4\pi G\mu)^2R^2sin^2\theta d\varphi^2\right]$$

- Result: No contribution from the gravitation waves from the CS because

$$\frac{r_{CS}}{R_H} \sim \frac{\dot{a}}{a} \sim \mathbf{10^{-20}}$$

- Disturbances are damped rapidly by $(\frac{r_{CS}}{R_H})^2$; They look like scaled version of CS
- Asymptotic conical ST (angle deficit) is **problematic**. Also found in radiative cyl. Einstein-Rosen ST: C-energy related to angle deficit [just as mass is related to angle deficit for CS].

So: Surviving disturbances must be very small (otherwise conflict with observ)

The Warped 5D FRW model

On 5D warped spacetime: **we found that the eff. 4D spacetime is non-conical [Slagter,2013]**

we WILL find: surviving disturbances + modified expansion

In fact two factors:

1. **Source from the bulk Weyl tensor: effective stress-tensor of some radiation "fluid"(dark-radiation)**
2. **Disturbances from the U(1) scalar gauge field by the warped amplification.**

The Warped 5D FRW model: The bulk equations

The 5D equations become: [Shiromizu et al , 2000]

$$ {}^{5}G_{\mu\nu} = -\Lambda_5 \, {}^{5}g_{\mu\nu} + \kappa_5^2 \delta(y)(-\Lambda_4 \, {}^{4}g_{\mu\nu} + {}^{4}T_{\mu\nu}) $$

We obtain: $W(t,r,y) = W_1(t,r) W_2(y)$

The equations for W_1 and W_2 become:

$$ \partial_{yy} W_2 = -\frac{(\partial_y W_2)^2}{W_2} - \frac{1}{3}\Lambda_5 - \frac{c_1}{W_2} \; ; \; (\partial_y W_2)^2 = -\frac{1}{6}\Lambda_5 W_2^2 + c_2 $$

with **simplified** solution $\mathbf{W_2 = e^{\sqrt{-\Lambda_5}(y-y_0)}}$ **[as in RS-model]**

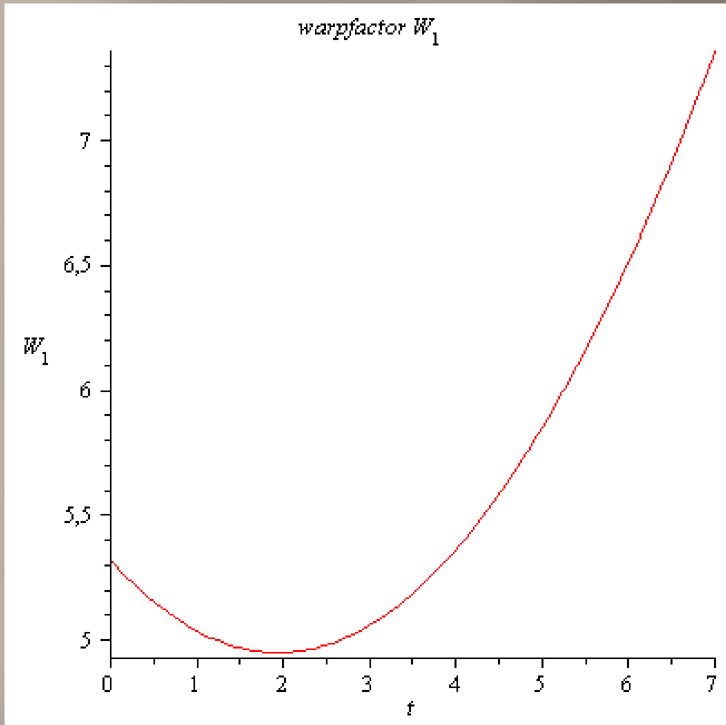
(there are some other possibilities)

The equation for W_1 becomes

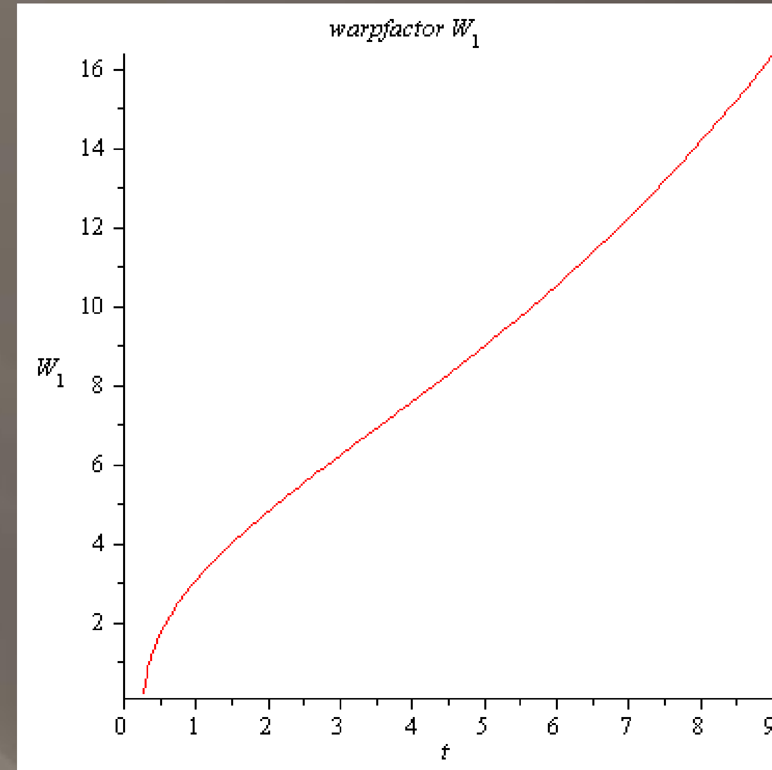
$$ \ddot{W}_1 = W_1'' + \frac{1}{W_1}\left(W_1'^2 - \dot{W}_1^{\,2}\right) + \frac{2}{r} W_1' $$

with solution

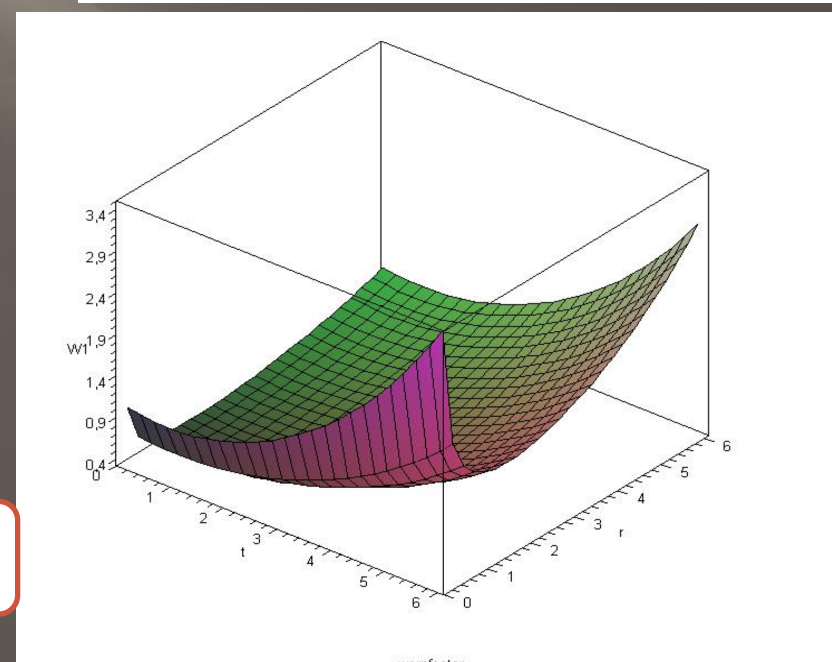
$$ \mathbf{W_1(t,r) = \frac{1}{\sqrt{\tau r}} \sqrt{\left(d_1 e^{\sqrt{2\tau} t} - d_2 e^{-\sqrt{2\tau} t}\right)\left(d_3 e^{\sqrt{2\tau} r} - d_4 e^{-\sqrt{2\tau} r}\right)}} $$



Plot of the warpfactor $W_1(t)$



Plot of the warpfactor W(t,r)



Problem: find relation between the integration constants τ and d_i and the physical parameters of the model!!

W has extremum in t: $\boxed{-log(-\frac{d_1}{d_2})/\sqrt{\tau}}$

" r: $\boxed{\text{RootsOf[} d_3(X-1)e^{2X} + d_4(X+1) \text{]}/\sqrt{\tau}}$

The Warped 5D FRW model: The effective 4D eq

The field equations induced on the brane are:

$$^4G_{\mu\nu} = -\Lambda_{eff}\,{}^4g_{\mu\nu} + \kappa_4^2\,{}^4T_{\mu\nu} + \kappa_5\,{}^4S_{\mu\nu} - E_{\mu\nu}$$

[Λ_{eff} =½($\Lambda_5 + \kappa_4^2\,\Lambda_4$ =0 for RS fine-tuning]

$S_{\mu\nu}$ is the quadratic term in the energy momentum tensor arising from the extrinsic curv terms
$E_{\mu\nu}$ is a part of the Weyl tensor and carries information of the gravitational field outside the brane and is constrained by the Codazzi eq.

$$^4\nabla_\mu K^\mu_\nu - {}^4\nabla_\nu K = {}^5R_{\mu\rho}\,{}^4g^\mu_\nu n^\rho$$

Further:

$$E_{\mu\nu} = {}^5C_{\alpha\gamma\beta\delta} n^\gamma n^\delta\,{}^4g^\alpha_\mu\,{}^4g^\beta_\nu\,; \qquad K_{\mu\nu} = -\frac{1}{2}\kappa_5^2\left({}^4T_{\mu\nu} + \frac{1}{3}(\Lambda_4 - {}^4T)\,{}^4g_{\mu\nu}\right)$$

From the contracted 4D Bianchi equations and the 5D Einstein, we obtain the supplementtary equations

The Warped 5D FRW model: The effective 4D eq

$$\mathcal{L}_n K_{\mu\nu} = K_{\mu\alpha}K^{\alpha}_{\nu} - E_{\mu\nu} - \frac{1}{6}\Lambda_5\, {}^4\mathrm{g}_{\mu\nu}$$

$$\mathcal{L}_n E_{\mu\nu} = {}^4\nabla^{\alpha}\mathcal{B}_{\alpha(\mu\nu)} + \frac{1}{6}\Lambda_5(K_{\mu\nu} - {}^4\mathrm{g}_{\mu\nu}K) + K^{\alpha\beta}R_{\mu\alpha\nu\beta} + 3K^{\alpha}_{(\mu}E_{\nu)\alpha} - KE_{\mu\nu} + (K_{\mu\alpha}K_{\nu\beta} - K_{\alpha\beta}K_{\mu\nu})K^{\alpha\beta}$$

$$\mathcal{L}_n \mathcal{B}_{\mu\nu\alpha} = -2\,{}^4\nabla_{[\mu}E_{\nu]\alpha} + K^{\beta}_{\alpha}\mathcal{B}_{\mu\nu\beta} - 2\mathcal{B}_{\alpha\beta[\mu}K^{\beta}_{\nu]}$$

From these equations one obtains the **same solutions** for W_1 and W_2 !!
[**easily done in MAPLE**]
W can **not** be obtained from the 4D equations: so **a brane-bulk effect.**
The **4D scalar-gauge field** equations become

$$\ddot{P} - P'' = -\frac{P'}{r} + 2\left(P'\psi' - \dot{P}\dot{\psi}\right) - e^2 W_1^2 e^{2\gamma-2\psi} P X^2$$

$$\ddot{X} - X'' = \frac{X'}{r} + \frac{2}{W_1}(W_1'X' - \dot{W}_1\dot{X}) - \frac{e^{2\gamma}XP^2}{r^2} - \frac{1}{2}W_1^2 e^{2\gamma-2\psi}\beta X(X^2 - \eta^2)$$

We see the warpfactor W_1 entering the equations
We still have the **contracted Bianchi**:

$$ {}^4\nabla_{\nu}E^{\mu\nu} = \frac{6\kappa^2}{\lambda}\, {}^4\nabla_{\nu}S^{\mu\nu}$$

shows qualitatively how 1+3 spacetime variations in the matter-radiation on the brane can source KK modes. Very complicated to evaluate. TODO

The Warped 5D FRW model: The effective 4D eq

and the Einstein equations

$$\ddot{\psi} - \psi'' = \frac{\psi'}{r} - \frac{{W_1}'}{rW_1} + \frac{2}{W_1}\left(W_1'\psi' - \dot{W}_1\dot{\psi}\right) + \frac{3\kappa_4^2}{4r^2}\left(\frac{e^{2\psi}\left(\dot{P}^2 - {P'}^2\right)}{e^2 W_1^2} - X^2P^2e^{2\gamma}\right)$$
$$+ \frac{\kappa_5^4}{64r^2}\left(\beta(X^2-\eta^2)^2 - 4\frac{e^{2\psi}X^2P^2}{r^2W_1^2} - 8\frac{e^{2\psi-2}(\dot{X}^2 - {X'}^2)}{W_1^2}\right)\left(\frac{e^{2\psi}(\dot{P}^2 - {P'}^2)}{e^2W_1^2}\right.$$
$$\left. - X^2P^2e^{2\gamma}\right)$$

$$\dot{\gamma} = \frac{1}{\dot{W}_1}\left[\frac{{W_1}'}{r} - \gamma' W_1' - \frac{\gamma' W_1}{2r} - \frac{1}{2W_1}\left({W'}^2 + 3\dot{W}_1^2\right) + \frac{1}{2}W_1\left(\dot{\psi}^2 + {\psi'}^2\right) + W_1'' + W_1'\psi'\right.$$
$$+ \dot{W}_1\dot{\psi} + \frac{3\kappa_4^2}{8}\left(\frac{e^{2\psi}(\dot{P}^2 + {P'}^2)}{e^2r^2W_1} + W_1({X'}^2 + \dot{X}^2)\right)$$
$$+ \frac{\kappa_5^4}{128}\left(\beta(X^2-\eta^2)^2 + 8\frac{e^{2\psi}X^2P^2}{r^2W_1^2} + 4\frac{e^{2\psi-2\gamma}(\dot{X}^2 - {X'}^2)}{W_1^2}\right)\left(\frac{e^{2\psi}(\dot{P}^2 + {P'}^2)}{e^2r^2W_1}\right.$$
$$\left.\left. + W_1({X'}^2 + \dot{X}^2)\right)\right]$$

The Warped 5D FRW model:Einstein-Rosen waves

Compare with the Einstein-Rosen waves(vacuum):

$$\ddot{\psi} - \psi'' - \frac{\psi'}{r} = 0\,; \qquad \dot{\gamma} + \gamma' = 2r\psi'\dot{\psi} + r({\psi'}^2 + \dot{\psi}^2)$$

Investigations goes back a long time: Einstein-Rosen [1937]

A. Flat, **Milne**, and non-flat solutions (regular or conical) possible.
B. Non-vacuum dust collapse: emission of GW (self –similar), with two parameters [Nakao, 2009]

In 4D: **if magnetic part of Weyl** tensor is zero: only **static** solutions [Herrera, 2007]
In our model also : non-static solutions possible.

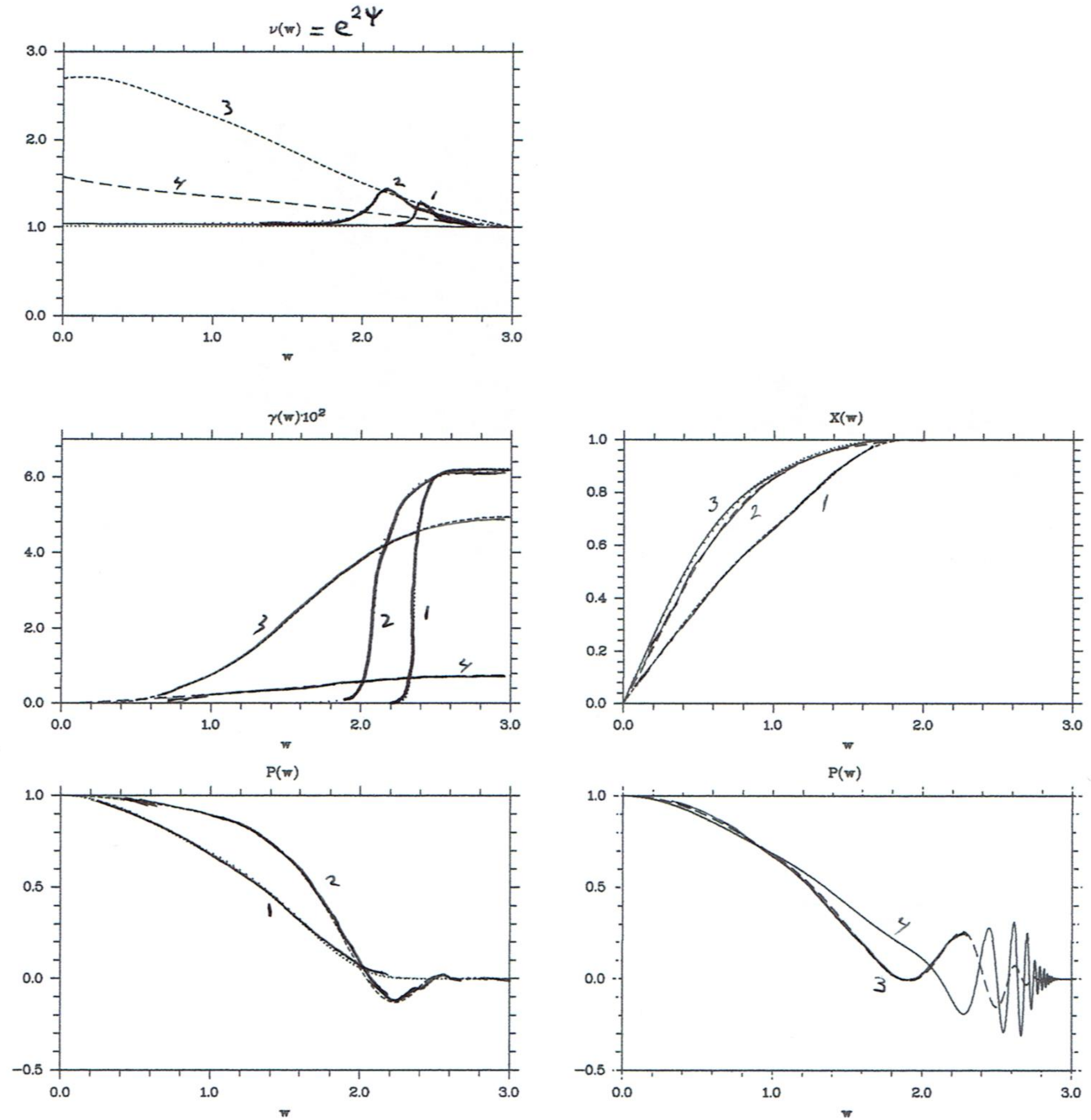
C. For **pulse-type** solution: inward radially directed flow of energy → **GW emission.**
Non-conicality constraint needed in expanding Kasner universe. [Gowdy, 2007]
D. Weber-Wheeler pulse solutions: interactions causes CS to oscillate with $f \sim \alpha$
correct asymptotic behavior.

Vickers,2001:
[4D]
Inwards moving pulse

Excites the CS

Reflected at the Origin

and travel outwards



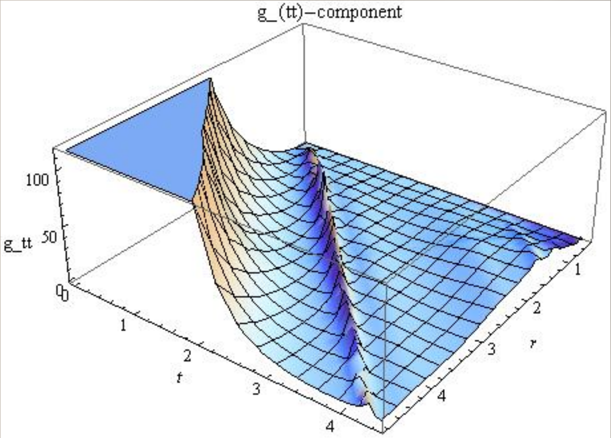
The Warped 5D FRW model: Numerical results

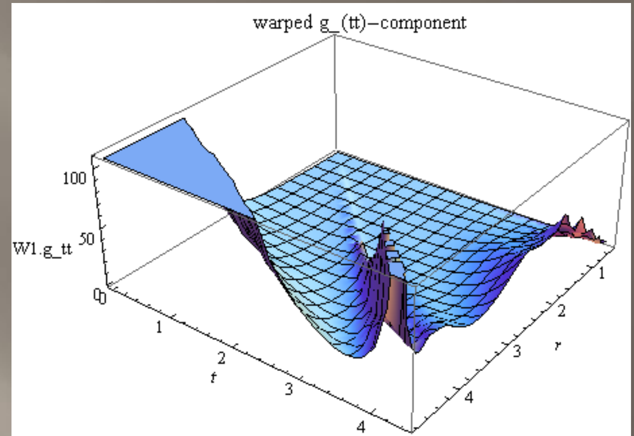
1. **Initial values: regular, no initial disturbances**

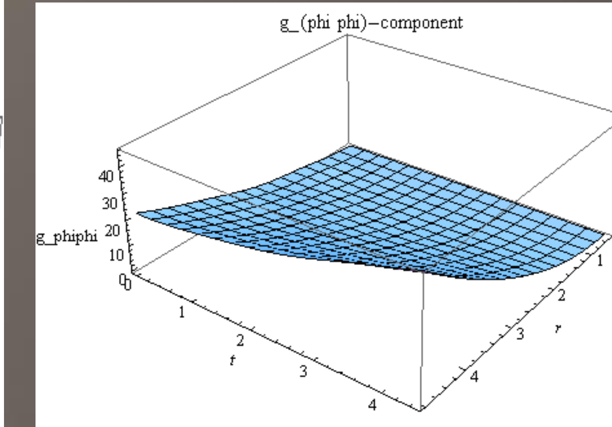
$$\gamma(r,t) = r \qquad \psi(r,t) = 0$$

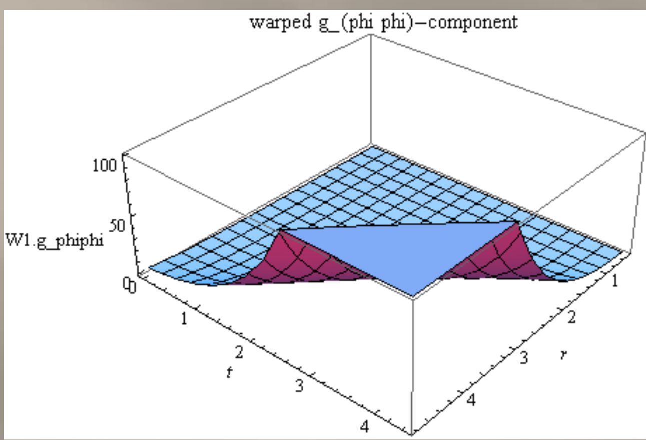
$$P(r,t)=e^{-ar} \qquad X(r,t)=1-e^{-br}$$

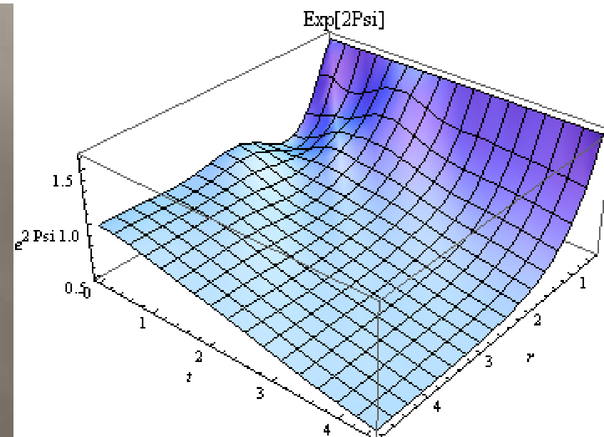
$$W=0.1\sqrt{\frac{e^{r+t}}{r}}$$

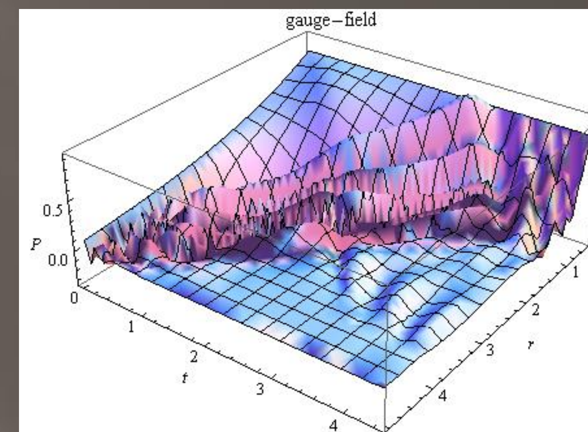

g_(tt)−component
g_tt
t
r

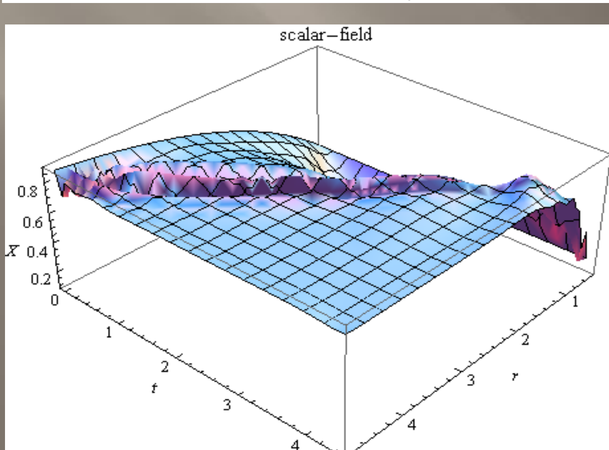

warped g_(tt)−component
W1.g_tt
t
r

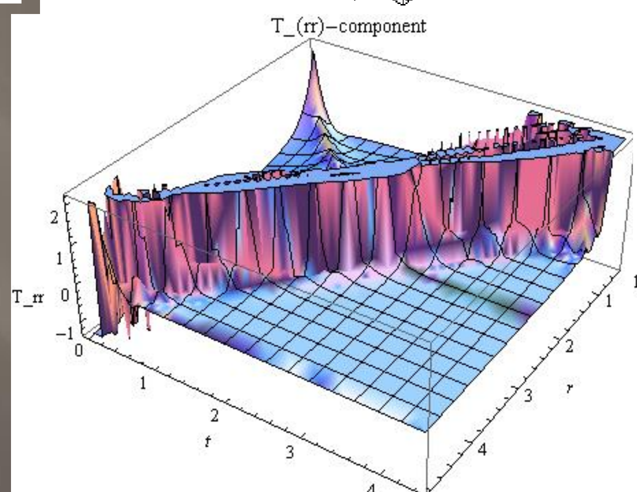

g_(phi phi)−component
g_phiphi
t
r

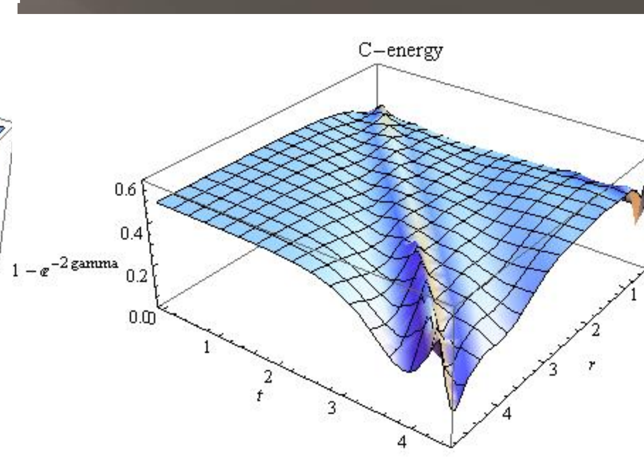

warped g_(phi phi)−component
W1.g_phiphi
t
r

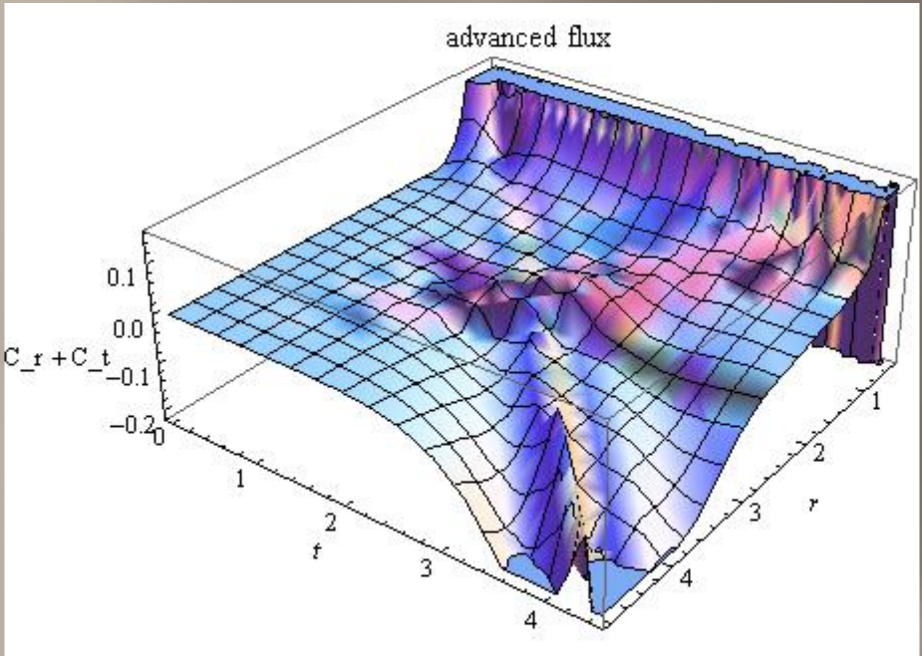

Exp[2Psi]
e^2 Psi
t
r

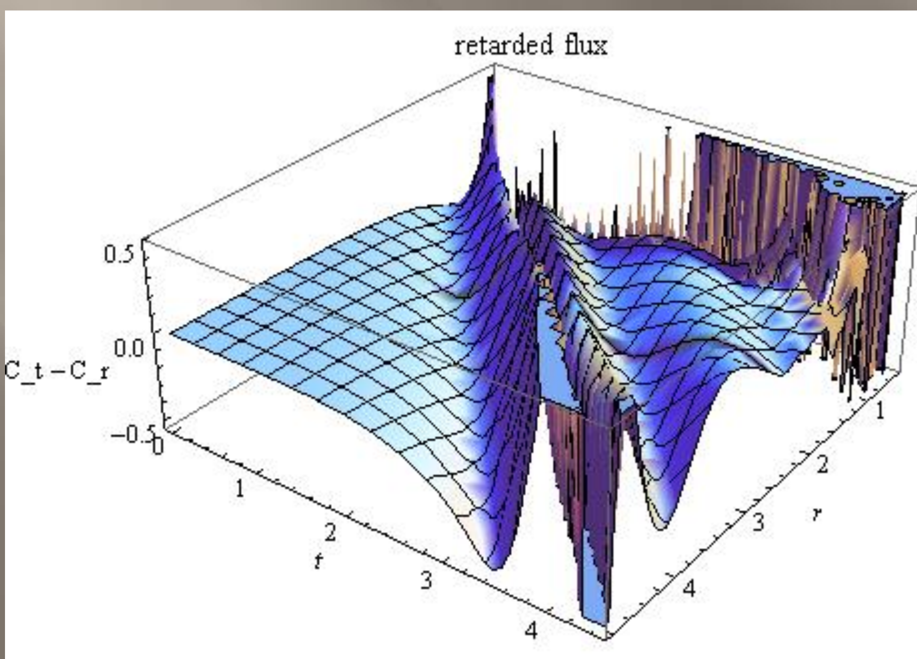

gauge−field
P
t
r


scalar−field
X
t
r


T_(rr)−component
T_rr
t
r


C−energy
1 − e^−2 gamma
t
r





General behavior:

► disturbances induced in P and X

► **collapse** of the scalar-gauge field

► emission of GW

► time dep warpfactor component triggers the acceleration

► **negative** advanced flux as it should be

The Warped 5D FRW model: **The $^4T_{\mu\nu}$ components**

We have:

$$^4T_{rr} = -\frac{1}{8}\beta W_1^2 e^{2\gamma-2\psi}(X^2-\eta^2)^2+\frac{1}{2}(X_t^2+X_r^2)+\frac{1}{2}\frac{e^{2\psi}}{e^2r^2W_1^2}(P_t^2+P_r^2)-\frac{1}{2}\frac{X^2P^2e^{2\gamma}}{r^2}$$

$$^4T_{zz} = -\frac{1}{8}\beta W_1^2 e^{2\psi}(X^2-\eta^2)^2-\frac{1}{2}e^{4\psi-2\gamma}(X_r^2-X_t^2)+\frac{1}{2}\frac{e^{6\psi-2\gamma}}{e^2r^2W_1^2}(P_t^2-P_r^2)-\frac{1}{2}\frac{X^2P^2e^{4\psi}}{r^2}$$

$$^4T_{\varphi\varphi} = -\frac{1}{8}\beta W_1^2 e^{-2\psi}(X^2-\eta^2)^2+\frac{1}{2}e^{-2\gamma}(X_t^2-X_r^2)-\frac{1}{2}\frac{e^{2\psi-2\gamma}}{e^2W_1^2}(P_t^2-P_r^2)-\frac{1}{2}X^2P^2$$

1. Two new terms: with W_1^2 and $\frac{1}{W_1^2}$: **will alternate with the behavior of w_1**
2. z-comp : **neglectable**

In static case: $(X_t^2+X_r^2)$ $(P_t^2+P_r^2)$ **: produces tention or pressure**

[in electrmagn: work is required to squeeze the lines of magn field towards axis]

3. Here they alternate

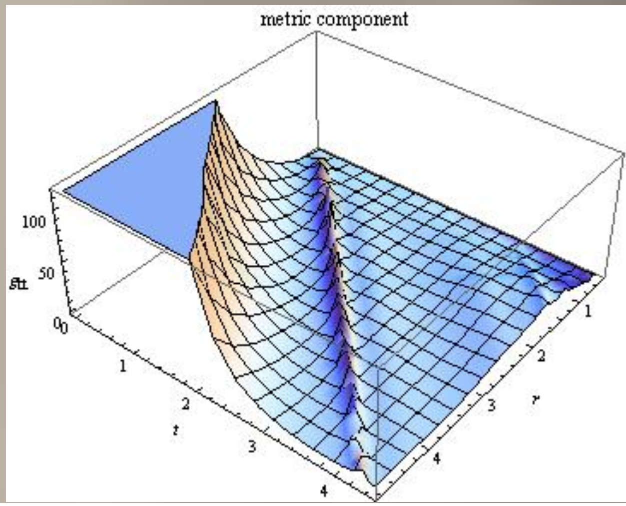
The Warped 5D FRW model: Numerical results

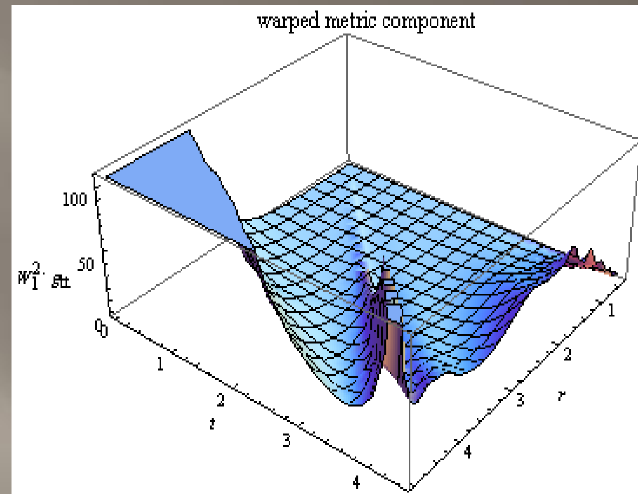
2. **Initial values: only fluctuating ψ in r**

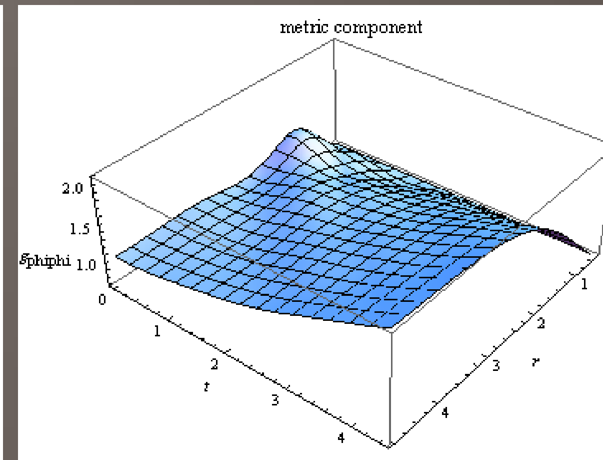
$$\gamma(r,t) = r \qquad \psi(r,t) = 0.4e^{-r}sin(3r)$$

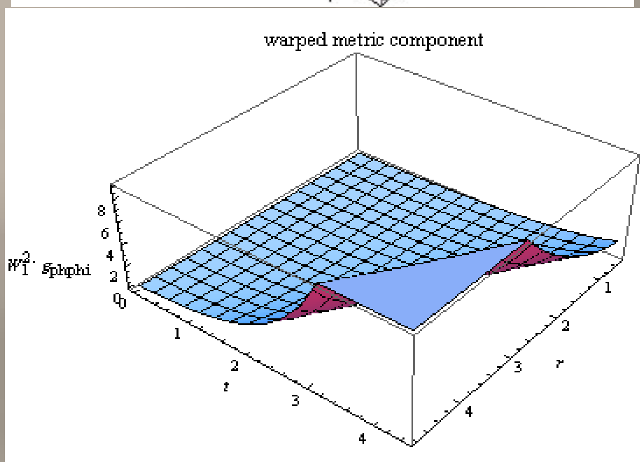
$$P(r,t)=e^{-ar} \qquad X(r,t)=1-e^{-br}$$

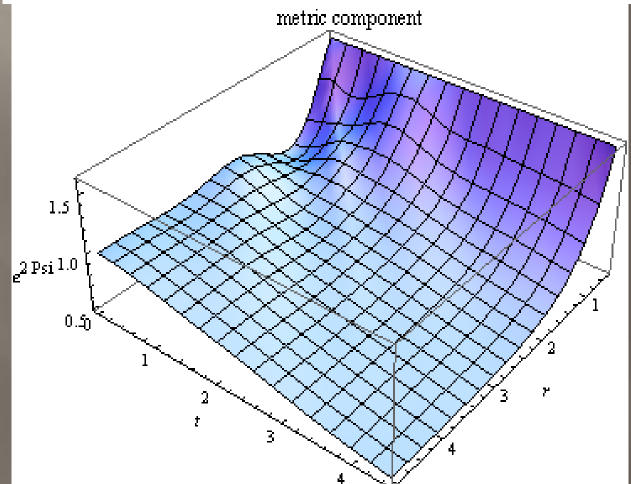
$$W=0.1\sqrt{\frac{e^{r+t}}{r}}$$

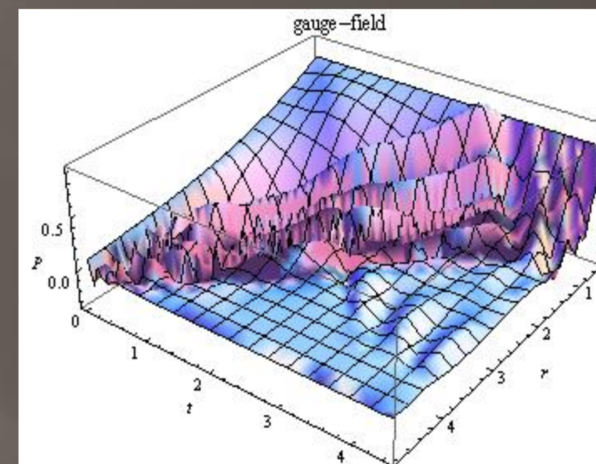

metric component
g_{tt}
t
r

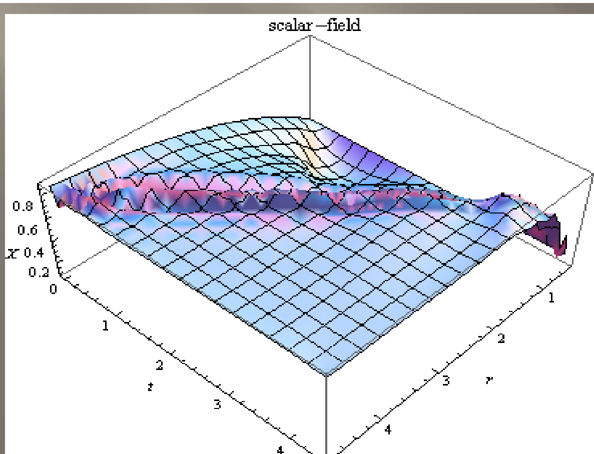

warped metric component
$W_1^2 \cdot g_{tt}$
t
r

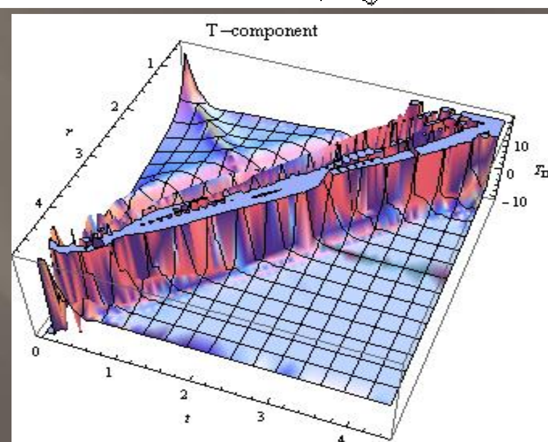

metric component
g_{phiphi}
t
r

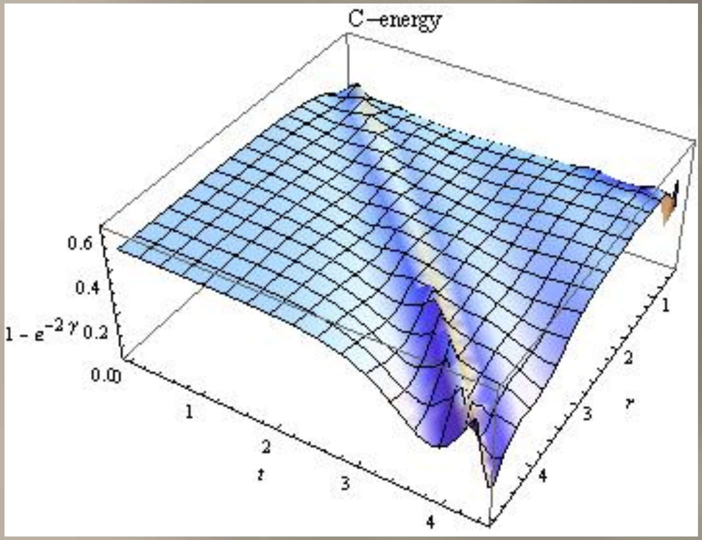

warped metric component
$W_1^2 \cdot g_{\text{phiphi}}$
t
r

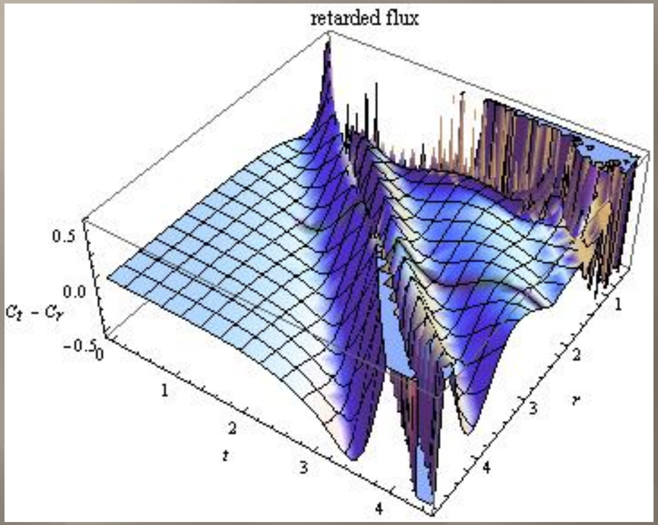

metric component
$e^{2\,\text{Psi}}$
t
r


gauge−field
P
t
r


scalar−field
X
t
r


T−component
T_{rr}
t
r





►**Note:** **significant change in behavior of $g_{\varphi\varphi}$ due to warpfactor**

►**Investigate the dependence of expansion on the parameters of the model**

Again: imploding pulse.

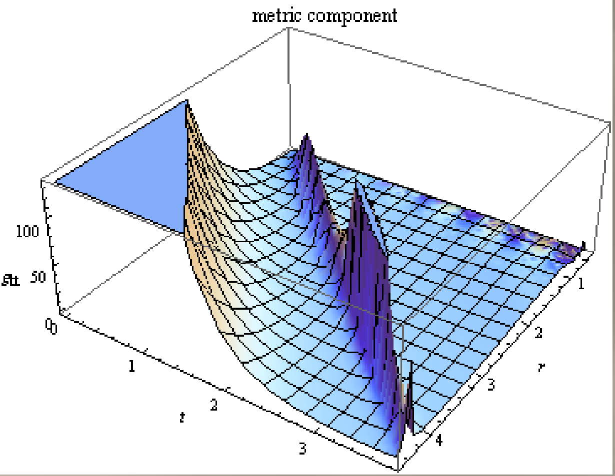
The Warped 5D FRW model: Numerical results

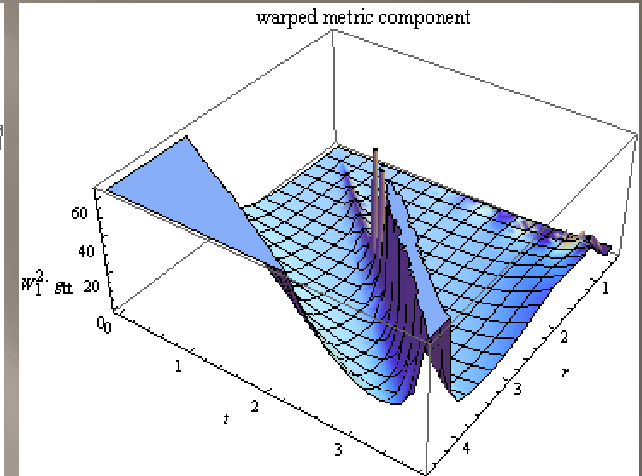
3. **Initial values: only simple pulse in ψ**

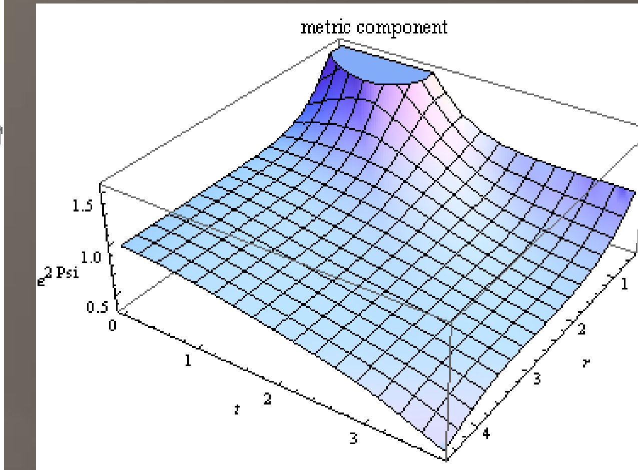
$$\gamma(r,t) = r \qquad \psi(r,t) = 0.4e^{-(r-t)^2}$$

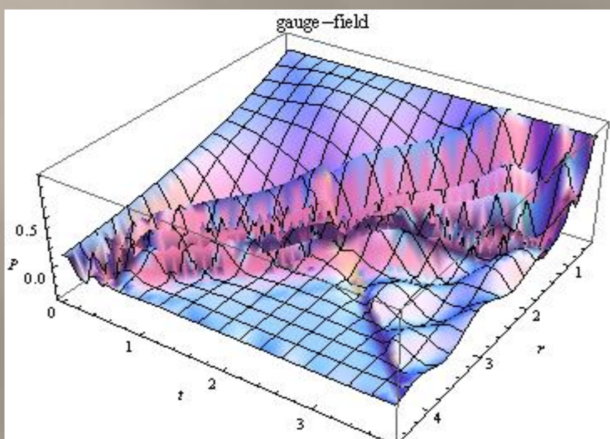
$$P(r,t)=e^{-ar} \qquad X(r,t)=1-e^{-br}$$

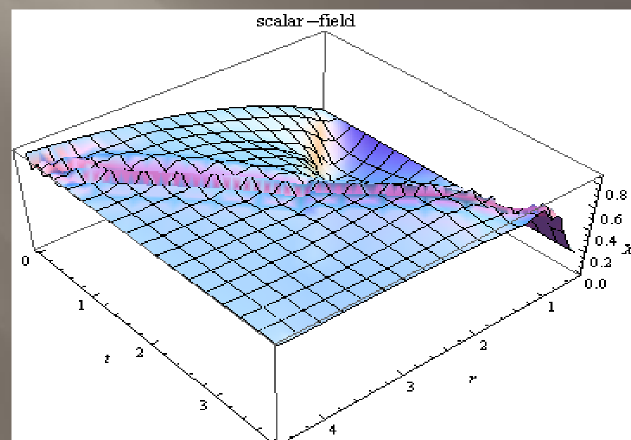
$$W=0.1\sqrt{\frac{e^{r+t}}{r}}$$

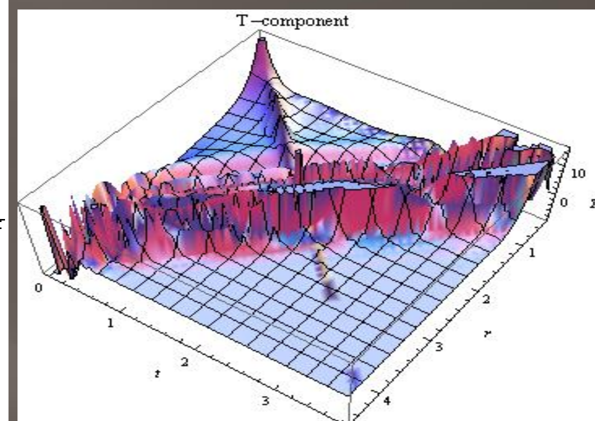

metric component
gtt
t
r

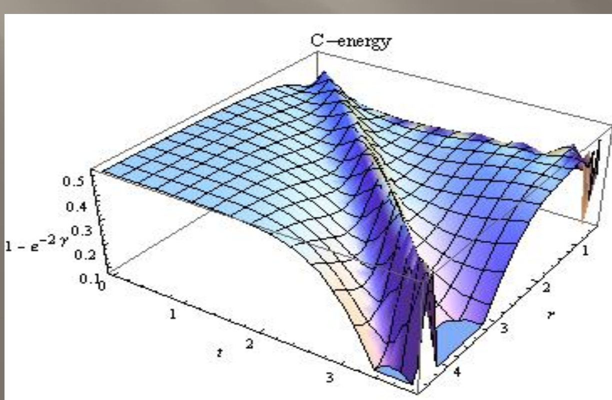

warped metric component
t
r

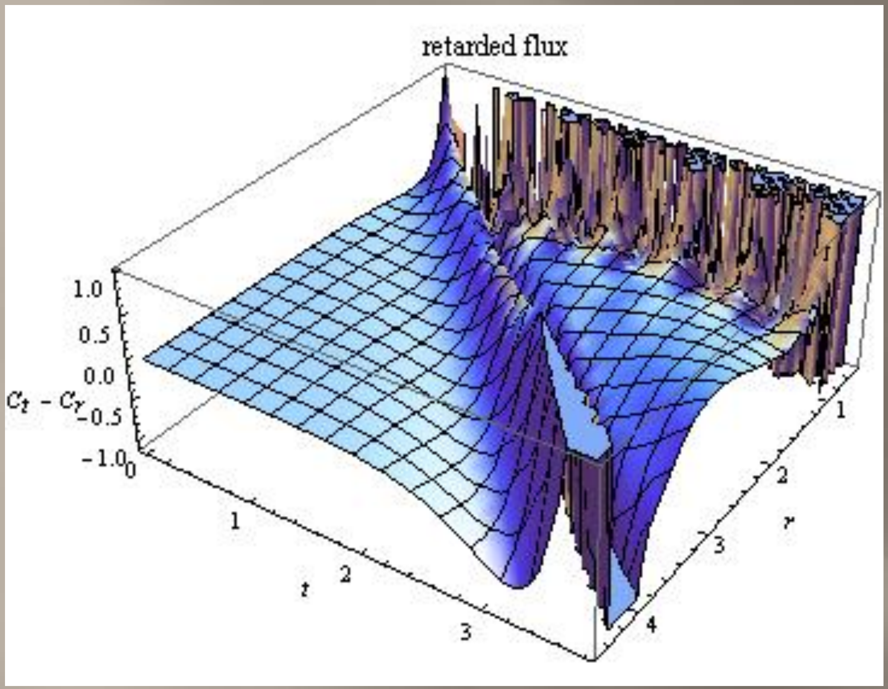

metric component
e2 Psi
t
r

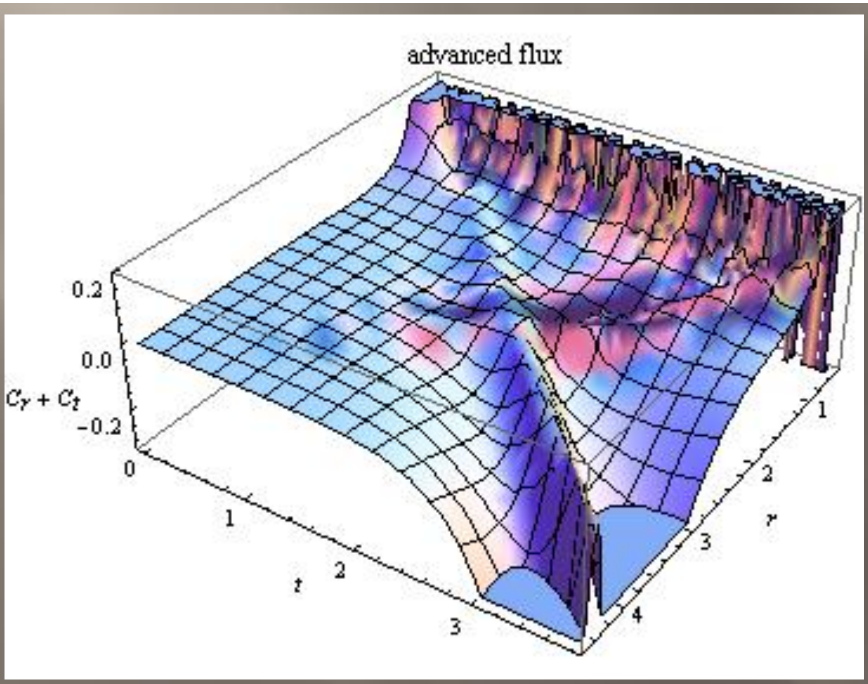

gauge −field
P
t
r


scalar −field
X
t
r


T −component
Trr
t
r


C −energy
1 - e−2 γ
t
r





- ►CS starts to **oscillate**;
- ►Frequency ~ $\frac{M_{gauge}}{M_{scalar}}$
- ►Returns to their **original state** closer to r=0 ?

  [found in 4D model:
- ►**Negative** advanced flux [**losing energy**]

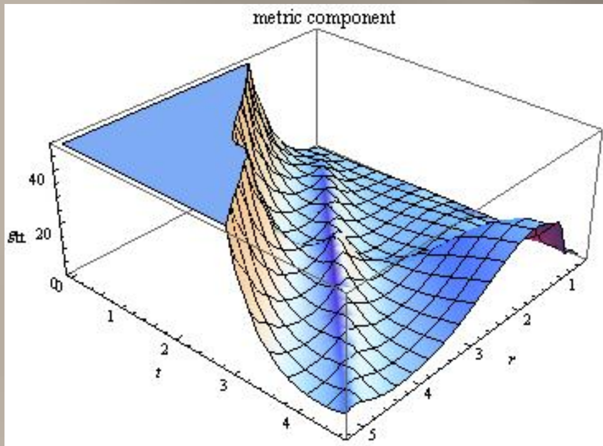
The Warped 5D FRW model: **Numerical results**

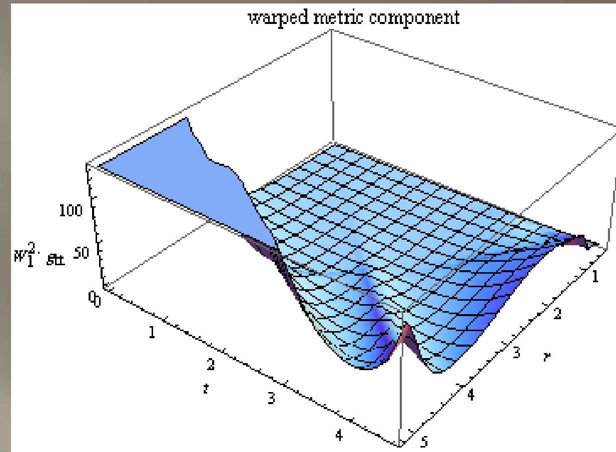
4. Initial values:

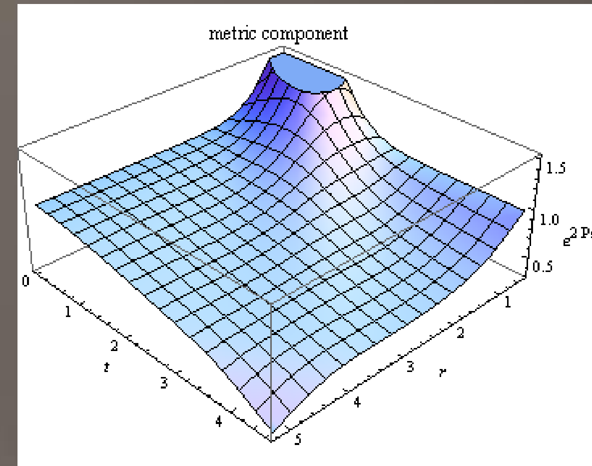
$$\gamma(r,t) = r \qquad \psi(r,t) = 0.4\, e^{-(r-t)^2}$$

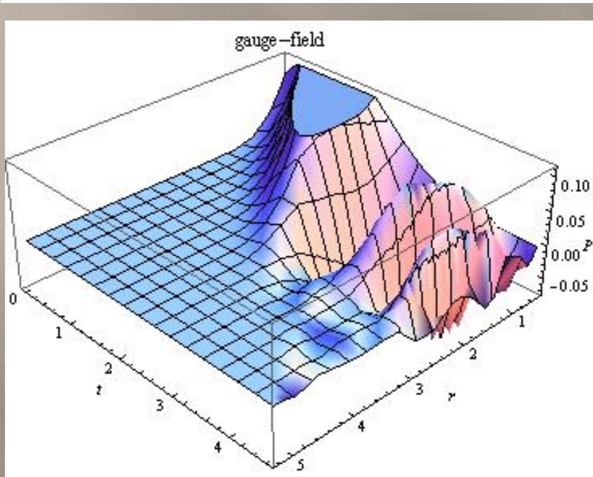
$$P(r,t) = 0.15\, e^{-2(r-0.5t)^2} \qquad X(r,t) = 1\text{-}e^{-br}$$

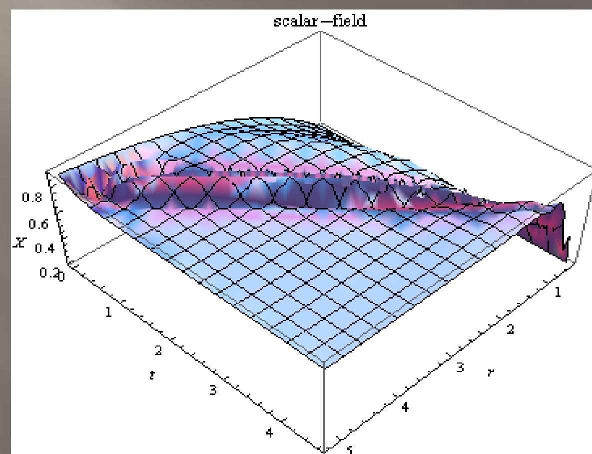
$$W = 0.1\sqrt{\frac{e^{r+t}}{r}}$$

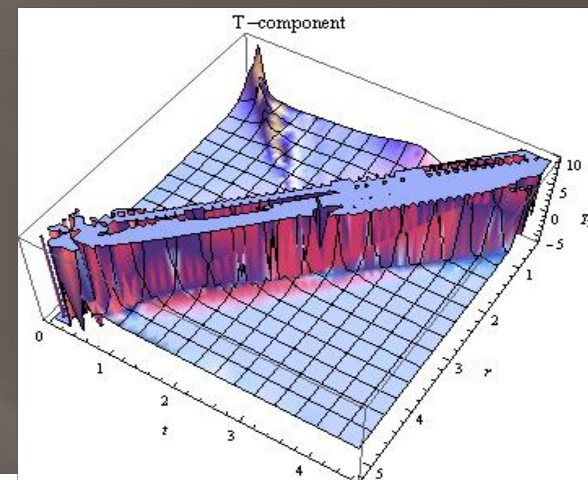

metric component
g_{tt}
t
r

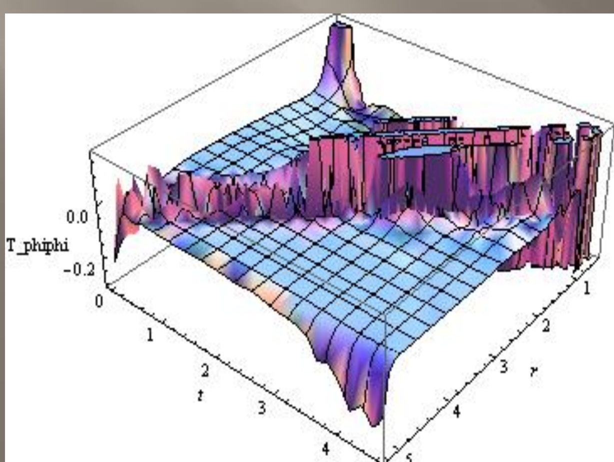

warped metric component
$W_1^2 \cdot g_{tt}$
t
r

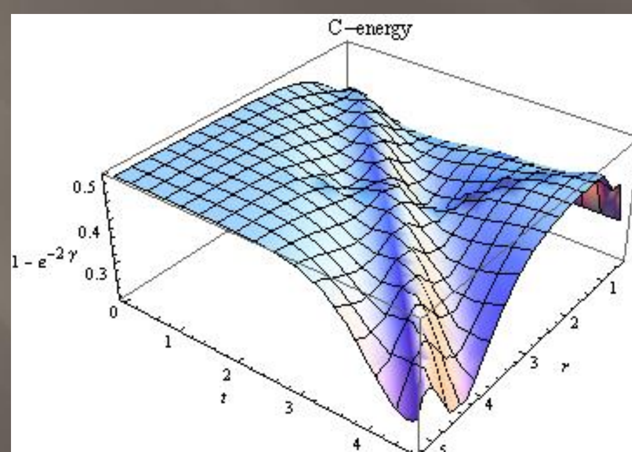

metric component
$e^{2\,\mathrm{Psi}}$
t
r

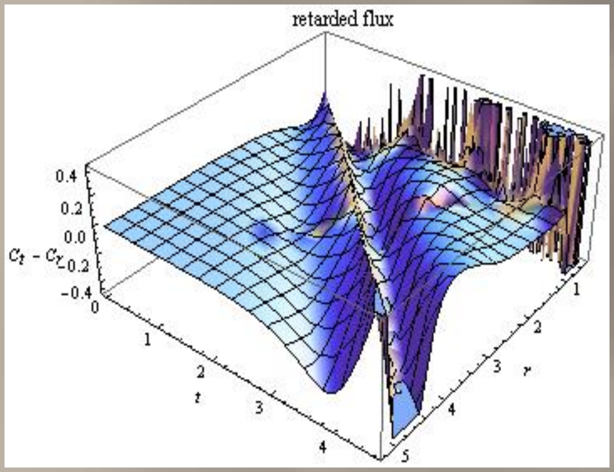

gauge−field
P
t
r

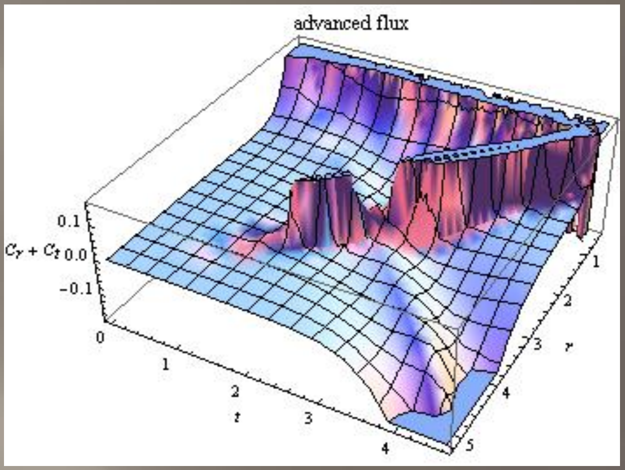

scalar−field
X
t
r

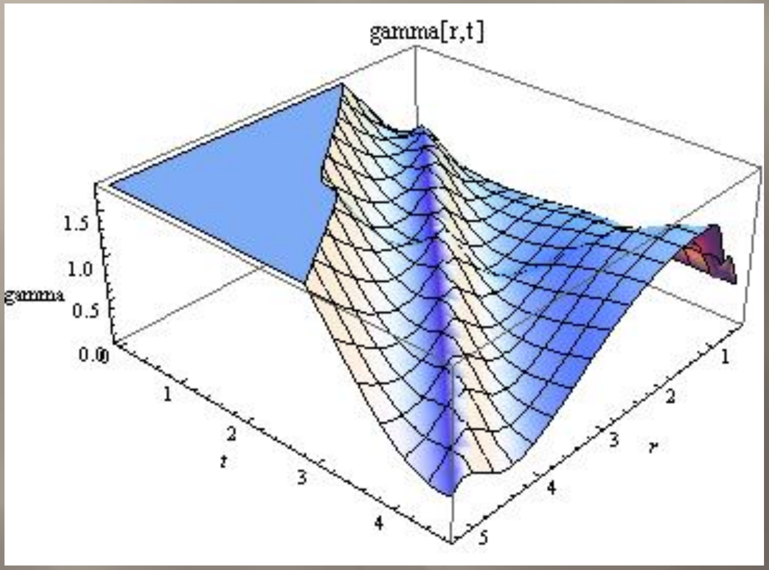

T−component
T_{rr}
t
r


T_phiphi
t
r


C−energy
$1 - e^{-2\gamma}$
t
r







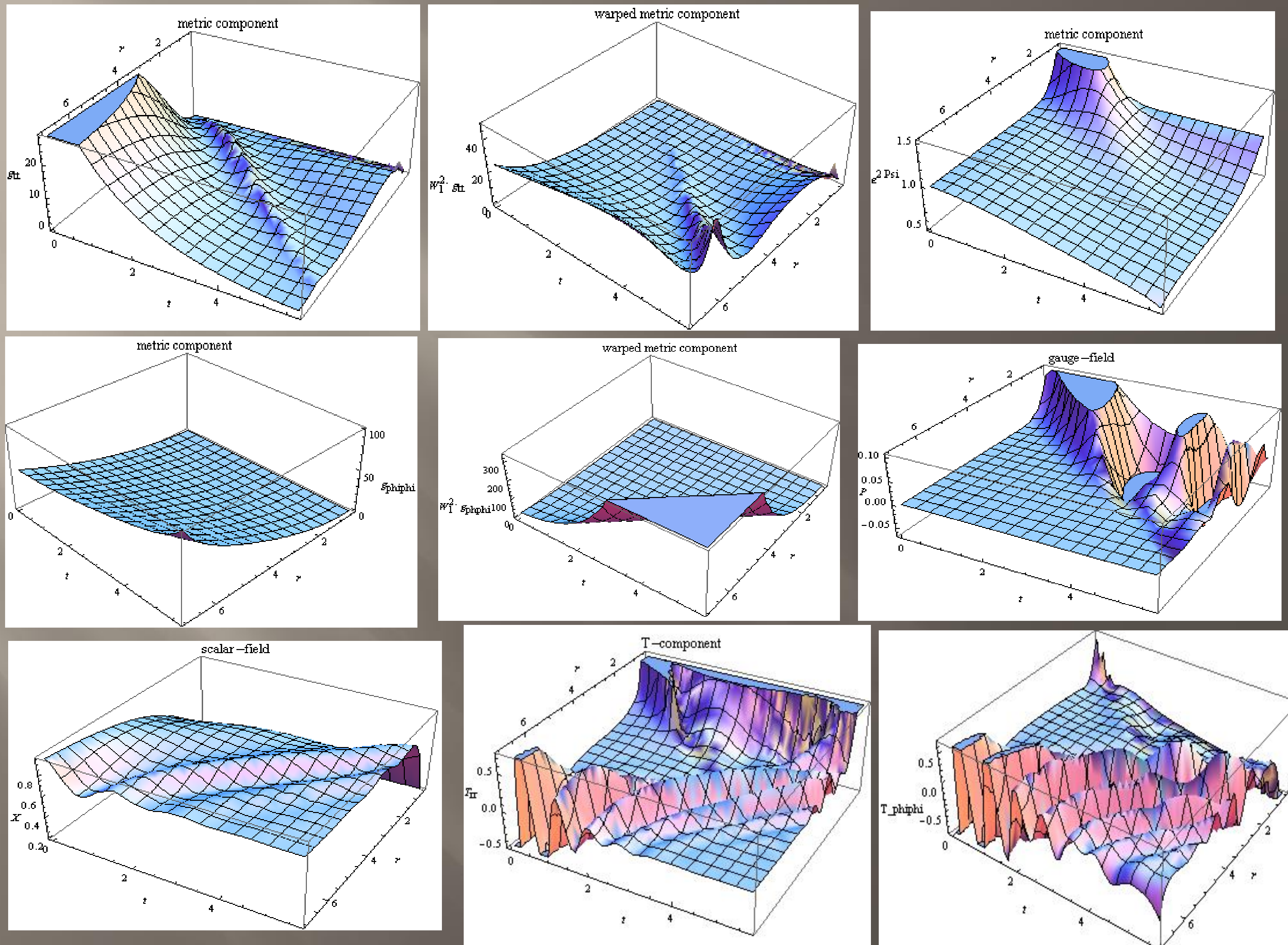
Note: **Induced retarded and advanced waves**

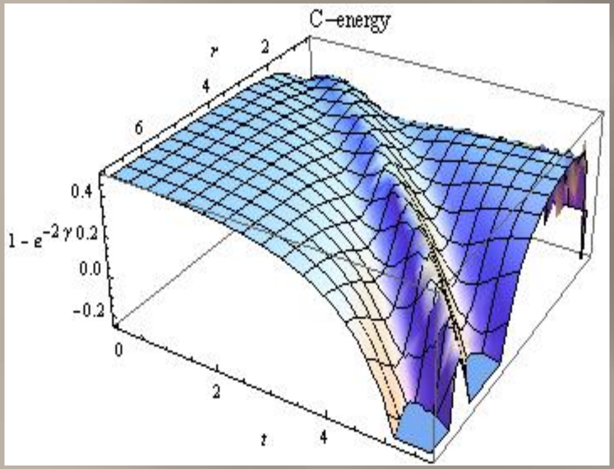
The Warped 5D FRW model: **Numerical results**

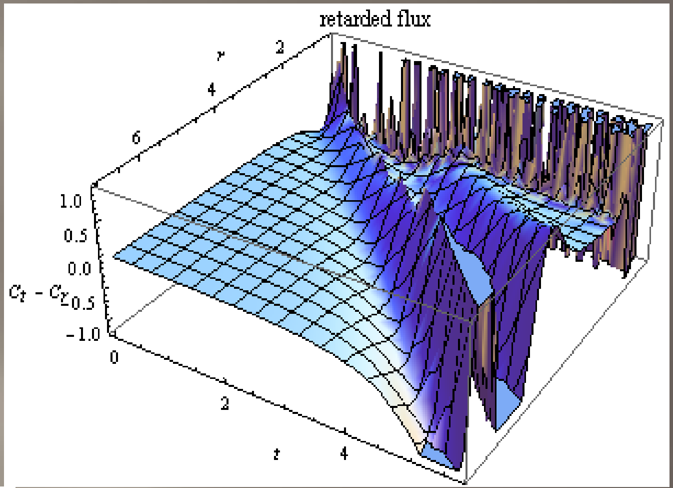
5. **Initial values:** **Weber-Wheeler pulse of $\gamma(r,t)$ and $\psi(r,t)$**

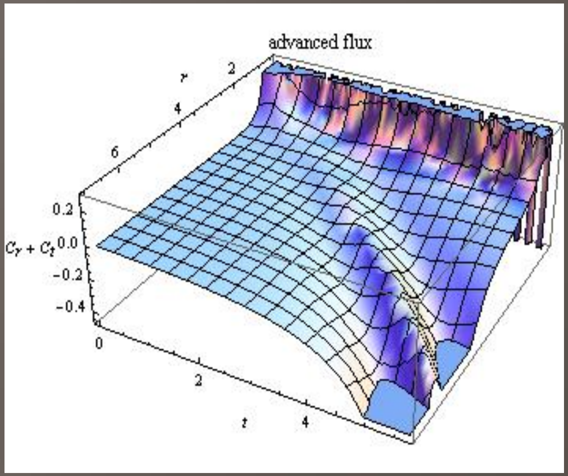
$$P(r,t) = 0.15\, e^{-2(r-0.5t)^2} \qquad X(r,t) = 1-e^{-br}$$

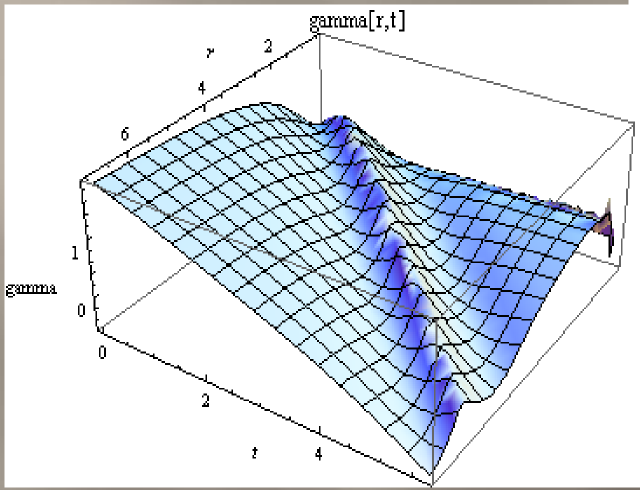
$$W = 0.1\sqrt{\frac{e^{r+t}}{r}}$$


metric component
g_tt
warped metric component
W_1^2 · g_tt
metric component
e^2 Psi
metric component
g_phiphi
warped metric component
W_1^2 · g_phiphi
gauge −field
P
scalar −field
X
T−component
T_rr
T_phiphi
t
r









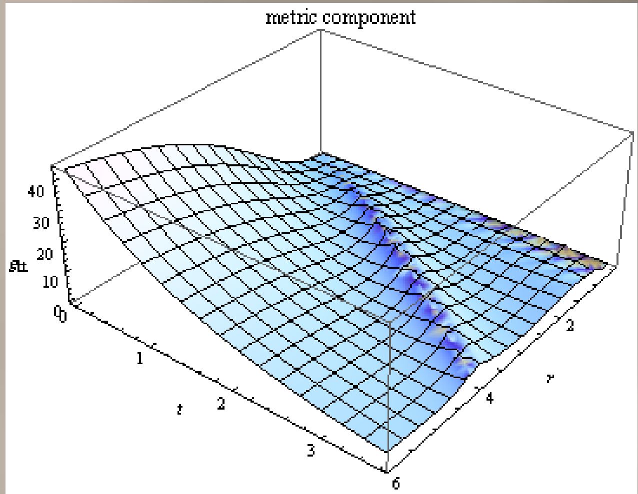
Note the more profound behavior of the T_{rr} *and* $T_{\varphi\varphi}$ components: as expected

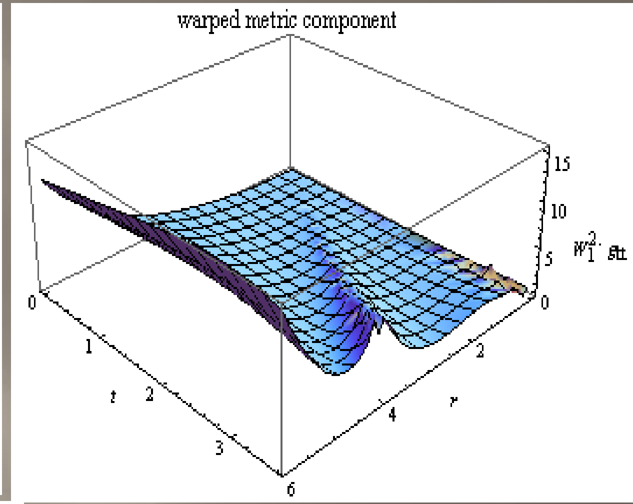
The Warped 5D FRW model: Numerical results

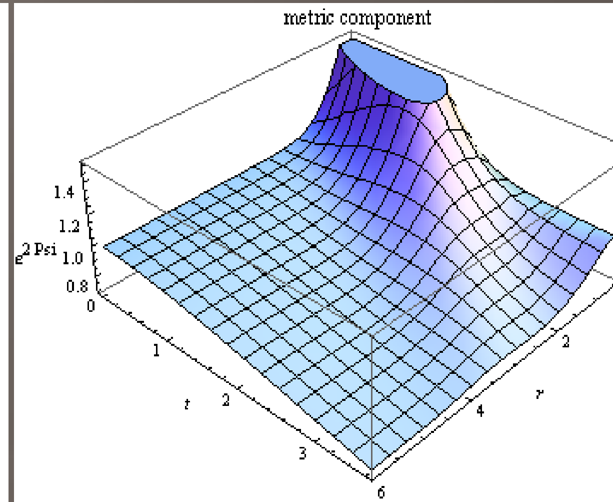
5. **Initial values:** Weber-Wheeler pulse of $\gamma(r,t)$ and $\psi(r,t)$

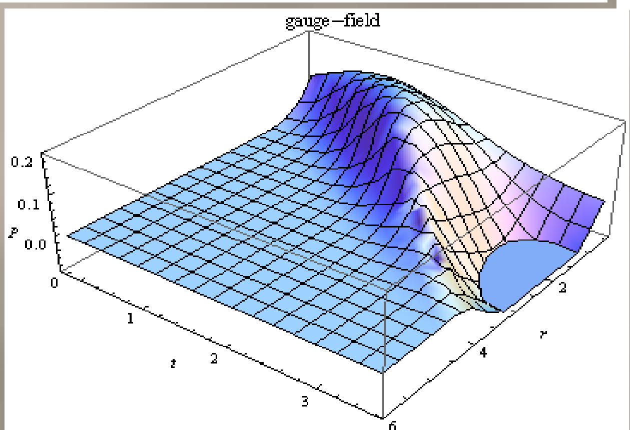
P(r,t) = 0.15 $e^{-2(r-0.5t)^2}$ X(r,t) = 1-$e^{-b(r-0.2t)}$

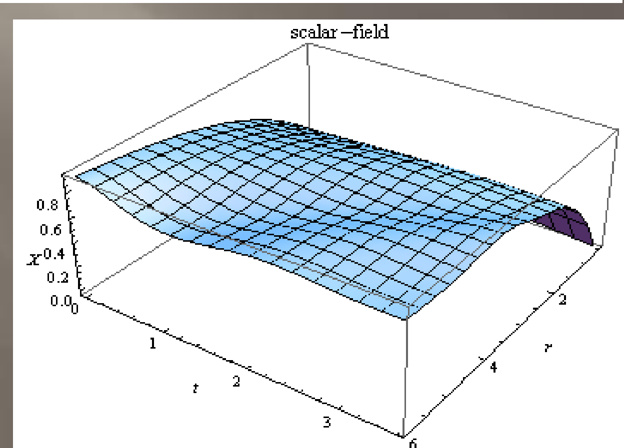
W= 0.1$\sqrt{\frac{e^{r+t}}{r}}$

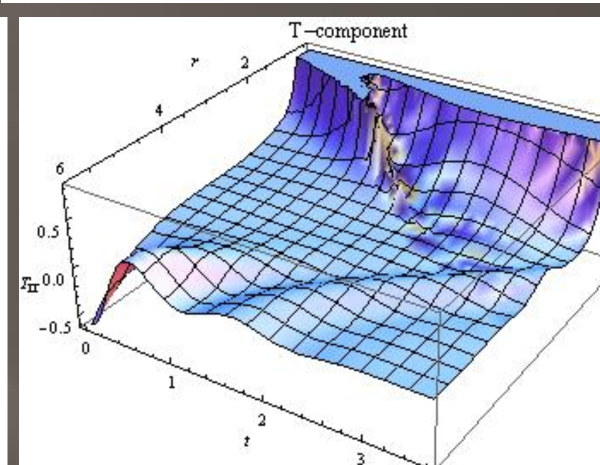

metric component
gtt
t
r

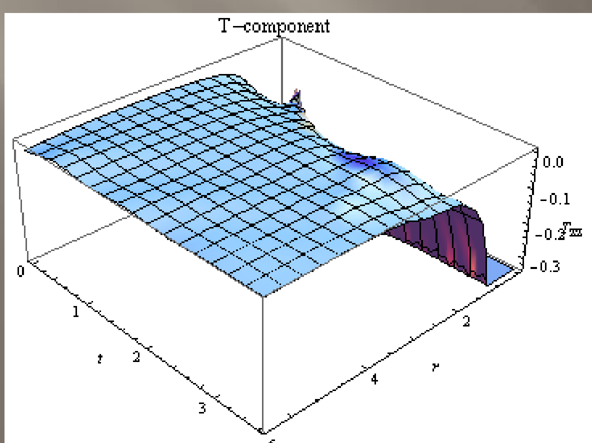

warped metric component
t
r

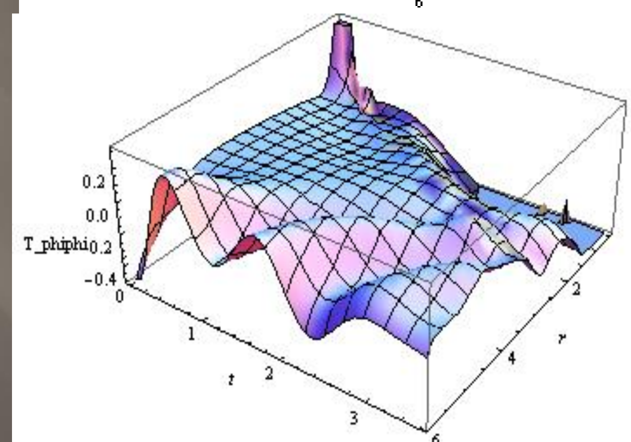

metric component
e^2 Psi
t
r

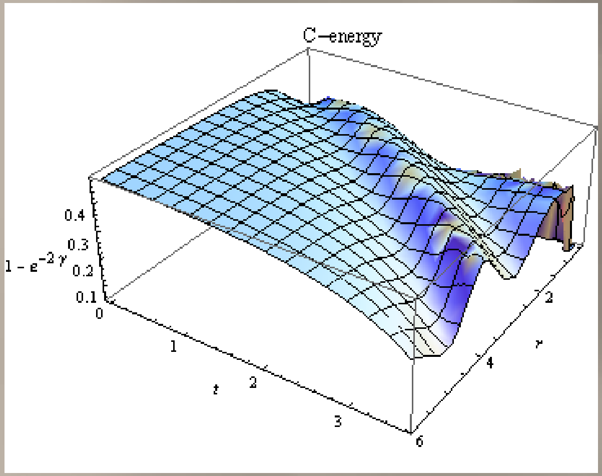

gauge−field
P
t
r

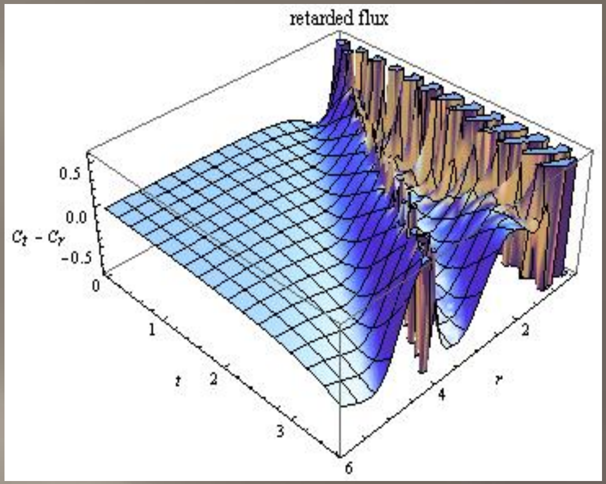

scalar −field
X
t
r

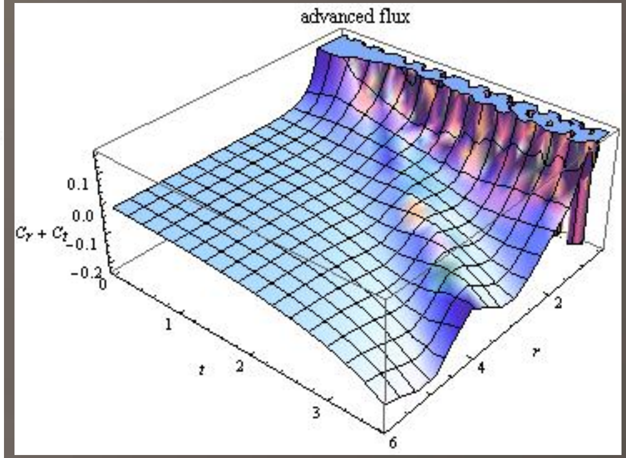

T−component
t
r

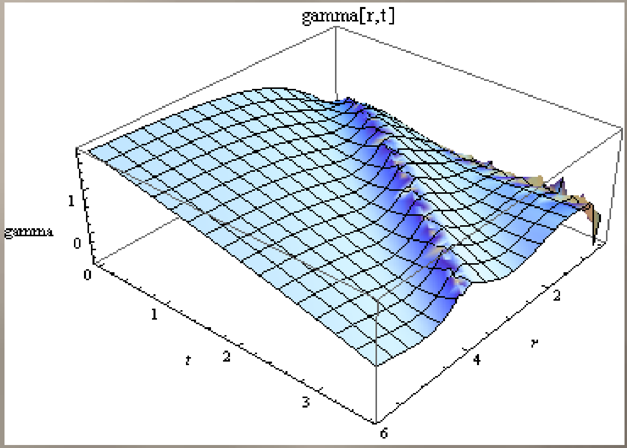

T−component
t
r


T_phiphi
t
r


C−energy
$1 - e^{-2\gamma}$
t
r


retarded flux
$C_t - C_r$
t
r


advanced flux
$C_r + C_t$
t
r


gamma[r,t]
gamma
t
r

Conclusions

- **I** **If we live in a RS brane world model: consider cosmological CS**

$\boldsymbol{\Lambda_{eff} = 0}$ ***Standard model fields confined to brane***: $\boldsymbol{U(1)}$ ***scalar − gauge field***

- **II** **SM fields interact via bulk Weyl tensor: brane-fluctuations("branons")**

  Covariant treatment (no ad-hoc mod of Friedmann]
- **III** **Induces pulse-like behavior in scalar-gauge fields**

  Survive expansion [in contrast with 4D results]
- **IV** **" self-acceleration" due to warpfactor-component**

  Solves fine-tuning and smallness of Λ (and hierarchy)

$$\boldsymbol{W_1(t,r) = \frac{1}{\sqrt{\tau r}}\sqrt{(c_1 e^{\sqrt{2\tau} t} + c_2 e^{-\sqrt{2\tau} t})(c_3 e^{\sqrt{2\tau} r} + c_4 e^{-\sqrt{2\tau} r})} \qquad W_2(y) \sim e^{\sqrt{-\Lambda_5}(y-y_0)}}$$

- ***V*** **Higher-dimensional graviton has massive KK-modes felt on the brane**

  Huge warp-factor amplifies the angle-deficit (or mass/unit length) of CCS
- ***VI*** *No conflict with observations: a. in 4D: conical residu at late-time in ST*

  in 5D warped: NO

  b. No axially lens-effect
- ***VI*** *Num solutions premature: what is the relation between* $\boldsymbol{c_i}$, $\boldsymbol{\tau}$ **and the phys param.**

ToDo: **1. scale-invariance of perturbations**

via high-freq 2-timing method [see R.Slagter , Astrphys. Journ., 1983,'84]

2. How enter the parameters of the model in W(t,r,y) Contracted Bianchi

3. explanation of traveling wave-ripples in galaxies?