Details of inflation from recent CMB data

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Present status of the inflationary scenario

Physical scales related to inflation

Visualizing small differences in the duration of inflation

Model reconstruction from observational data

Gravitational waves from inflation and other effects

Conclusions

Main epochs of the Universe evolution

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

> 60 ~ 55 8 0.3

Main advantages of inflation

1. Aesthetic elegance
Inflation – hypothesis about an almost maximally symmetric (quasi-de Sitter) stage of the evolution of our Universe in the past, before the hot Big Bang. If so, preferred initial conditions for (quantum) inhomogeneities with sufficiently short wavelengths exist – the adiabatic in-vacuum ones. In addition, these initial conditions represent an attractor for a much larger compact open set of initial conditions having a non-zero measure in the space of all initial conditions.

2. Predictability, proof and/or falsification
Given equations, this gives a possibility to calculate all subsequent evolution of the Universe up to the present time and even further to the future. Thus, any concrete inflationary model can be proved or disproved by observational data.

3. Naturalness of the hypothesis

Remarkable qualitative similarity between primordial and present dark energy.

4. Relates quantum gravity and quantum cosmology to astronomical observations

Makes quantum gravity effects observable at the present time and at very large – cosmological – scales.

5. Produces (non-universal) arrow of time for our Universe

Origin – initial quasi-vacuum fluctuation with a fantastically large correlation radius.

Present status of inflation

Now we have numbers.
P. A. R. Ade et al., arXiv:1502.01589

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \zeta^2(\mathbf{r}) >= \int \frac{P_\zeta(k)}{k} dk, \quad P_\zeta(k) = \left(2.21^{+0.07}_{-0.08}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.005$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$.

From "proving" inflation to using it as a tool

Simple (one-parameter, in particular) models may be good in the first approximation (indeed so), but it is difficult to expect them to be absolutely exact, small corrections due to new physics should exist (indeed so).

Present status of inflation: transition from "proving" it in general and testing some of its simplest models to applying the inflationary paradigm to investigate particle physics at super-high energies and the actual history of the Universe in the remote past using real observational data on $n_s(k) - 1$ and $r(k)$.

The reconstruction approach – determining curvature and inflaton potential from observational data.

The most important quantities:
1) for classical gravity – $H, \dot{H}$
2) for super-high energy particle physics – m^2_{infl}.

Physical scales related to inflation

I. Curvature scale

$$H \sim \sqrt{P_\zeta} M_{Pl} \sim 10^{14}\mathrm{GeV}$$

II. Inflaton mass scale

$$|m_{infl}| \sim H\sqrt{|1-n_s|} \sim 10^{13}\mathrm{GeV}$$

New range of mass scales significantly less than the GUT scale.

Often another energy scale $E = (\hbar^3 c^3 V)^{1/4} \sim \sqrt{HM_{Pl}}$ is introduced which is indeed of the order of the GUT scale. But is this quantity physical?

Let us apply the same method to water and discover the characteristic energy scale of water:

$$E = (1 \frac{\mathrm{g}}{\mathrm{cm}^3} \times c^2)^{1/4} = 45\,\mathrm{keV}.$$

Completely misleading (but instructive) result showing that one has to be cautious applying such an estimate to "cold" physical systems.

Outcome of inflation

In the super-Hubble regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

Quantum-to-classical transition

In fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta,\ g$).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

Visualizing small differences in the number of e-folds

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different is different point of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\zeta(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982) in the case of one-field inflation.

CMB temperature anisotropy

$$T_\gamma = (2.72548 \pm 0.00057)\mathrm{K}$$

$$\Delta T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$< a_{\ell m} a_{\ell' m'} >= C_\ell \delta_{\ell \ell'} \delta_{m m'}$$

Theory: averaging over realizations.
Observations: averaging over the sky for a fixed ℓ.

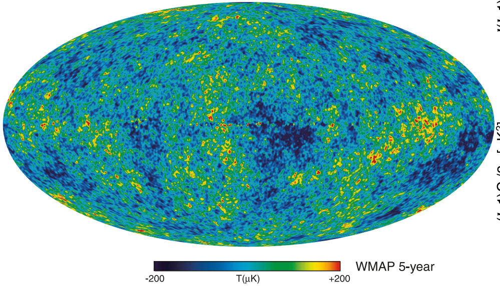
For scalar perturbations, generated mainly at the last scattering surface (the surface or recombination) at $z_{LSS} \approx 1090$ (the Sachs-Wolfe, Silk and Doppler effects), but also after it (the integrated Sachs-Wolfe effect).
For GW – only the ISW works.

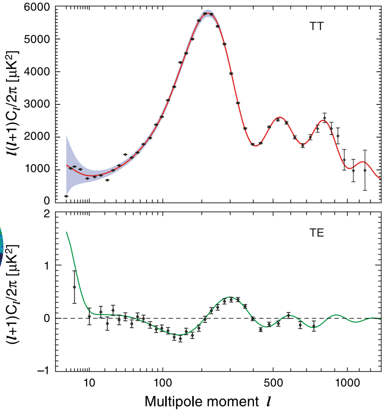
For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\zeta(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1$,

$$\ell(\ell+1)C_{\ell,s} = \frac{2\pi}{25}P_\zeta$$





Accuracy: with $\frac{\Delta T}{T} \sim 10^{-6}$, $\delta N \sim 5 \times 10^{-6}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, $\delta t \sim t_{Pl}$!

FLRW dynamics with a scalar field

In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|$, $\dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V$, $|V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \quad \dot{\phi} \approx -\frac{V'}{3H}, \quad N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln \frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.

The spectral slope

$$n_s(k) - 1 \equiv \frac{d \ln P_\zeta(k)}{d \ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d\ln P_g(k)}{d\ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50 - 60)$.

Inflation in $f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G} \int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu$$

Here $f''(R)$ is not identically zero. Usual matter described by the action S_m is minimally coupled to gravity.

Vacuum one-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Metric variation is assumed everywhere. Palatini variation leads to a different theory with a different number of degrees of freedom.

Field equations

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right) = -\left(T_{\mu\,(vis)}^\nu + T_{\mu\,(DM)}^\nu + T_{\mu\,(DE)}^\nu\right) \ ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T_{\mu\,(DE)}^\nu = F'(R)\,R_\mu^\nu - \frac{1}{2}\,F(R)\delta_\mu^\nu + \left(\nabla_\mu\nabla^\nu - \delta_\mu^\nu\nabla_\gamma\nabla^\gamma\right)F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R)\ .$$

The special role of $f(R) \propto R^2$ gravity: admits de Sitter solutions with any curvature.

Transformation to the Einstein frame and back

In the Einstein frame, free particles of usual matter do not follow geodesics and atomic clocks do not measure proper time.

From the Jordan (physical) frame to the Einstein one:

$$g^E_{\mu\nu} = f' g^J_{\mu\nu}, \quad \kappa\phi = \sqrt{\frac{3}{2}} \ln f', \quad V(\phi) = \frac{f'R - f}{2\kappa^2 f'^2}$$

where $\kappa^2 = 8\pi G$.

Inverse transformation:

$$R = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 4\kappa^2 V(\phi)\right) \exp\left(\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$$f(R) = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 2\kappa^2 V(\phi)\right) \exp\left(2\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$V(\phi)$ should be at least C^1.

Analogues of large-field (chaotic) inflation: $F(R) \approx R^2 A(R)$ for $R \to \infty$ with $A(R)$ being a slowly varying function of R, namely

$$|A'(R)| \ll \frac{A(R)}{R}\ , \ |A''(R)| \ll \frac{A(R)}{R^2}\ .$$

In particular,

$$f(R) \approx \frac{R^2}{6m^2 \ln^2(R/m^2)}$$

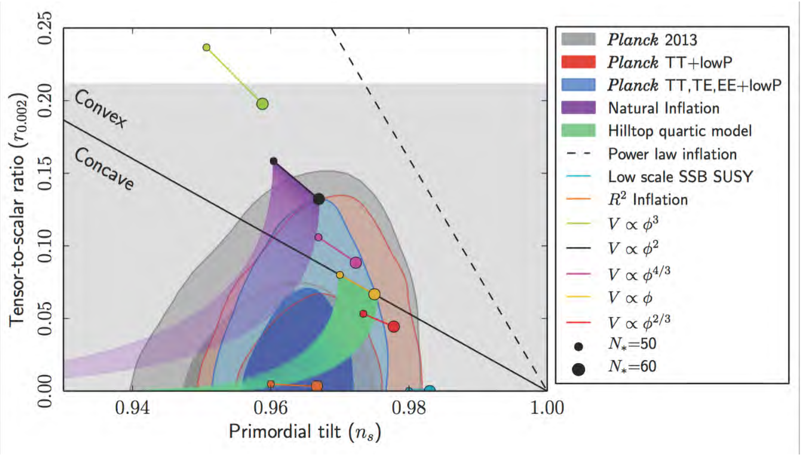
for $R \gg m^2$ to have the same n_s, r as for $V = m^2\phi^2/2$.

Analogues of small-field (new) inflation, $R \approx R_1$:

$$F'(R_1) = \frac{2F(R_1)}{R_1}\ , \ F''(R_1) \approx \frac{2F(R_1)}{R_1^2}\ .$$

Thus, all inflationary models in $f(R)$ gravity are close to the simplest one over some range of R.

Comparison with some simple models



The simplest models producing the observed scalar slope

I. In the Einstein gravity:

$$V(\phi) = \frac{m^2\phi^2}{2}$$

$$m \approx 1.8 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 2 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{8}{N} \approx 0.15$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

Almost excluded by data.

II. In the modified $f(R)$ gravity:

$$f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 3 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0,\ |\xi| \gg 1$, including the Brout-Englert-Higgs inflationary model.

Note similar predictions for inflaton masses and essentially the same prediction for H_{dS}.

Smooth potential reconstruction from scalar power spectrum in GR

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_\zeta(k(t(\phi))), \quad C = \frac{12\pi^2}{\kappa^6}$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^4}{12\pi^2}\int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int dN\sqrt{\frac{d\ln V}{dN}}$$

An ambiguity in the form of $V(\phi)$ because of an integration constant in the first equation. Information about $P_g(k)$ helps to remove this ambiguity.

In particular, if primordial GW are not discovered in the order $n_s - 1$:

$$r \ll 8|n_s - 1| \approx 0.3 \ ,$$

then $\left(\frac{V'}{V}\right)^2 \ll \left|\frac{V''}{V}\right|, \ |n_g| = \frac{r}{8} \ll |n_s - 1|, \ |n_g|N \ll 1$.

This is possible only if $V = V_0 + \delta V, \ |\delta V| \ll V_0$ – a plateau-like potential. Then

$$\delta V(N) = \frac{\kappa^4 V_0^2}{12\pi^2} \int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int \frac{dN}{\sqrt{V_0}} \sqrt{\frac{d(\delta V(N))}{dN}}$$

Here, integration constants renormalize V_0 and shift ϕ. Thus, the unambiguous determination of the form of $V(\phi)$ without knowledge of $P_g(k)$ becomes possible.

In particular, if $n_s - 1 = -\frac{2}{N} \approx -0.04$ for all $N \equiv \ln \frac{k_f}{k} = 1 - 60$ and $r \ll 8|n_s - 1|$, then

$$V(\phi) = V_0 \left(1 - \exp(-\alpha\kappa\phi)\right)$$

with $\alpha\kappa\phi \gg 1$ but α not very small, and

$$r = \frac{8}{\alpha^2 N^2}$$

More generally, if $n_s - 1 = -\frac{\beta}{N}$, $\beta > 1$, then

$$P_\zeta \propto N^\beta, \quad r \propto N^{-\beta}$$

$\beta < 2 : \delta V \propto -\phi^{-\frac{2(\beta-1)}{2-\beta}}$, $\phi \to \infty$ - large-field inflation.
$\beta > 2 : \delta V \propto -\phi^{\frac{2(\beta-1)}{\beta-2}}$, $\phi \to 0$ - small-field inflation.

Permitted 1σ interval for β: $(1.5, 2.4)$. The value $\beta = 2$ is aesthetically preferred.

Let us omit the assumption $r \ll 8|n_s - 1| \approx 0.3$, but keep $\beta = 2$ ($P_\zeta = P_0 N^2$). Then:

$$V = V_0 \frac{N}{N + N_0} = V_0 \tanh^2 \frac{\kappa\phi}{2\sqrt{N_0}}$$

$$r = \frac{8N_0}{N(N + N_0)}$$

$r \sim 0.003$ for $N_0 \sim 1$. From the upper limit on r: $N_0 < 100$ for $N = 57$.

For the one-parametric $R + R^2$ inflationary model, $N_0 = 3/2$.

Where is the primordial GW contribution to CMB temperature anisotropy?

For $1 \ll \ell \lesssim 50$, the Sachs-Wolfe plateau occurs for the contribution from GW, too:

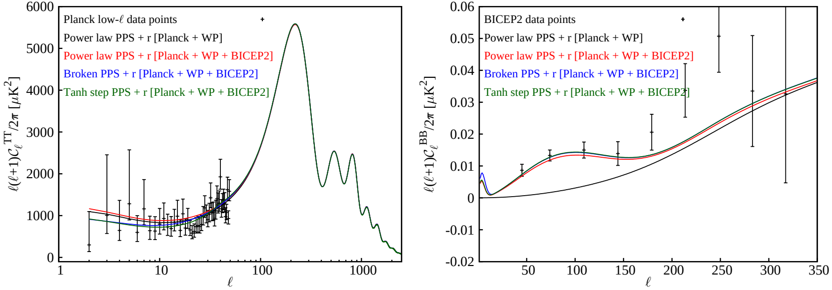
$$\ell(\ell+1)C_{\ell,g} = \frac{\pi}{36}\left(1 + \frac{48\pi^2}{385}\right)P_g$$

assuming $n_t = 1$ (A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985)). So,

$$C_\ell = C_{\ell,s} + C_{\ell,g} = (1 + 0.775r)C_{\ell,s}$$

.

For larger $\ell > 50$, $\ell(\ell+1)C_{\ell,s}$ grows and the first acoustic peak forms at $\ell \approx 200$, while $\ell(\ell+1)C_{\ell,g}$ decreases quickly. Thus, the presence of GW should lead to a step-like enhancement of $\ell(\ell+1)C_\ell$ for $\ell \lesssim 50$.


Planck low-ℓ data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
ℓ(ℓ+1)C_ℓ^TT/2π [μK^2]
ℓ
BICEP2 data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
ℓ(ℓ+1)C_ℓ^BB/2π [μK^2]
ℓ

The most critical argument against $r \sim 0.1$:
no sign of GW in the CMB temperature anisotropy power spectrum.

Instead of the $\sim 10\%$ increase of $\ell(\ell+1)C_\ell$ over the multipole range $2 \ll \ell < 50$, a $\sim 10\%$ depression is seen for $20 \lesssim \ell \lesssim 40$ (see e.g. Fig. 39 of arXiv:1303.5076). The feature exists even if $r \ll N^{-1}$ but the presence of $r \sim 0.1$ makes it larger.

More detailed analysis in D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1406, 061 (2014), arXiv:1403.7786 :
the power-law form of $P_\zeta(k)$ is excluded at more than 3σ CL.

Next step: studying of local features in the same range

The effect of at least the same order: an upward wiggle at $\ell \approx 40$ (the Archeops feature) and a downward one at $\ell \approx 22$.

Lesson: irrespective of the search for primordial GW from inflation, features in the anisotropy spectrum for $20 \lesssim \ell \lesssim 40$ confirmed by WMAP and Planck should be taken into account and studied seriously. Some new physics beyond one slow-rolling inflaton may show itself through them.

A more elaborated class of model suggested by previous studies of sharp features in the inflaton potential caused, e.g. by a fast phase transition occurred in another field coupled to the inflaton during inflation:
D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1408, 048 (2014); arXiv:1405.2012

In particular, the potential with a sudden change of its first derivative:

$$V(\phi) = \gamma\phi^2 + \lambda\phi^p(\phi - \phi_0)\,\theta(\phi - \phi_0)$$

which generalizes the exactly soluble model considered in A. A. Starobinsky, JETP Lett. **55**, 489 (1992) produces $-2\Delta \ln \mathcal{L} = -11.8$ compared to the best-fitted power law scalar spectrum, partly due to the better description of wiggles at both $\ell \approx 40$ and $\ell \approx 22$.

Conclusions

- Inflation is being transformed into a normal physical theory, based on some natural assumptions confirmed by observations and used to obtain new theoretical knowledge from them.
- First quantitative observational evidence for small quantities of the first order in the slow-roll parameters: $n_s(k) - 1$ and $r(k)$.
- The quantitative theoretical prediction of these quantities is based on gravity (space-time metric) quantization and requires very large space-time curvature in the past of our Universe with a characteristic length only five orders of magnitude larger than the Planck one.

- Using the measured value of $n_s - 1$ and assuming a scale-free scalar power spectrum leads to the prediction that the region $r > 10^{-3}$ is well expected. Under the same assumptions, r can be even larger and close to its present observational upper limit in two-parametric inflationary models having large, but not too large $N_0 \sim N$. However, this requires a moderate amount of parameter tuning.
- Regarding CMB temperature anisotropy, small features in the multipole range $20 \lesssim \ell \lesssim 40$ at the accuracy level $\sim 1\ \mu\mathrm{K}$ which mask the GW contribution to CMB temperature anisotropy have to be investigated and understood. They may reflect some fine structure of inflation (i.e. fast phase transitions in other quantum fields coupled to an inflaton during inflation).
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.