Gravitational Origin of Dark Matter

mostly based on arXiv 1604.08564

Angnis Schmidt-May

ETH Zürich

Hot Topics in Modern Cosmology
May 9, 2016
Cargèse

In collaboration with …

1604.08564

Eugeny Babichev
Luca Marzola
Martti Raidal
Federico Urban
Hardi Veermäe
Mikael von Strauss

Others:

Yashar Akrami
Oliver Baldacchino
Laura Bernard
Cédric Deffayet
Jonas Enander
Fawad Hassan
Mikica Kocic
Frank Könnig
Edvard Mörtsell
Rachel Rosen
Adam Solomon

review: ASM, Mikael von Strauss; 1512.00021

Contents

Motivation

Philosophy

Approach I

1. invent a model to explain observations
   ➡ many possibilities

2. check if model is consistent, fits into larger framework, has motivations besides cosmology, etc.

Philosophy

Approach 1

1. invent a model to explain observations
   → many possibilities
2. check if model is consistent, fits into larger framework, has motivations besides cosmology, etc.

Approach 2

1. construct a consistent model guided by a fundamental question
   → few possibilities
2. check if it can explain (part of) an observational phenomenon

Philosophy

Approach 1

1. invent a model to explain observations
   → many possibilities

2. check if model is consistent, fits into larger framework, has motivations besides cosmology, etc.

Approach 2

1. construct a consistent model guided by a fundamental question
   → few possibilities

2. check if it can explain (part of) an observational phenomenon

Consistent Field Theories

Standard Model of Particle Physics
& General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

Spin 2: graviton $g_{\mu\nu}$

Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

massive & massless

Spin 2: graviton $g_{\mu\nu}$

MASSLESS !

How do we describe massive spin-2 fields ?

Massless + Massive Spin-2 Fields

General Relativity

$$S_{\rm EH}[g] = M_{\rm P}^2 \int {\rm d}^4x \sqrt{g}\, \Big(R(g) - 2\Lambda \Big)$$

Einstein's equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$

Maximally symmetric solutions: $\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$

Linear perturbations of Einstein's equations, $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta g_{\rho\sigma} = 0 \qquad \bar{\mathcal{E}} \sim \nabla\nabla + \Lambda$$

➡ equation for a massless spin-2 field with 2 degrees of freedom,

tensor analogue of $\Box \phi = 0$

General Relativity

=

nonlinear theory

of massless spin-2

Linear Massive Gravity

Fierz & Pauli (1939)

Equation for a massive spin-2 field:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma}\delta g_{\rho\sigma} + \frac{m_{\rm FP}^2}{2}\left(\delta g_{\mu\nu} - \mathbf{a}\,\bar{g}_{\mu\nu}\delta g\right) = 0$$

tensor analogue of $\Box\phi - m^2\phi = 0$

Linear Massive Gravity

Fierz & Pauli (1939)

Equation for a massive spin-2 field:

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}\delta g_{\rho\sigma} + \frac{m_{\rm FP}^2}{2}\left(\delta g_{\mu\nu} - \mathbf{a}\,\bar{g}_{\mu\nu}\delta g\right) = 0$$

tensor analogue of $\Box\phi - m^2\phi = 0$

- propagates 5 degrees of freedom for $\mathbf{a} = \mathbf{1}$
- for $\mathbf{a} \neq \mathbf{1}$ there is an additional scalar mode which gives rise to a ghost instability

Ghosts

Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t \phi)^2 \cdots \quad \text{healthy}$$
$$\mathcal{L} = -(\partial_t \phi)^2 \cdots \quad \text{ghost}$$

- consequences: classical instability, negative probabilities at quantum level

  ➡ must be avoided!

Ghosts

Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t \phi)^2 \cdots \quad \text{healthy}$$

$$\mathcal{L} = -(\partial_t \phi)^2 \cdots \quad \text{ghost}$$

- consequences: classical instability, negative probabilities at quantum level

  → must be avoided!

See however Yashar's talk!

Ghosts

Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t \phi)^2 \cdots \quad \text{healthy}$$
$$\mathcal{L} = -(\partial_t \phi)^2 \cdots \quad \text{ghost}$$

- consequences: classical instability, negative probabilities at quantum level

→ must be avoided!

See however Yashar's talk!

Modifications of General Relativity tend to be haunted by ghosts.
Modifying gravity is EXTREMELY difficult!

Fierz-Pauli theory is linear.
General Relativity is nonlinear.

Can we write down a
nonlinear mass term ?

Nonlinear Mass Term

... should not contain derivatives nor loose indices.

But if we try to contract the indices of the metric, we get: $g^{\mu\nu} g_{\mu\nu} = 4$

This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu} f_{\mu\nu} = \mathrm{Tr}\left(g^{-1} f\right) \qquad f^{\mu\nu} g_{\mu\nu} = \mathrm{Tr}\left(f^{-1} g\right)$$

Nonlinear Mass Term

... should not contain derivatives nor loose indices.

But if we try to contract the indices of the metric, we get: $g^{\mu\nu} g_{\mu\nu} = 4$

This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu} f_{\mu\nu} = \mathrm{Tr}\,(g^{-1} f) \qquad\qquad f^{\mu\nu} g_{\mu\nu} = \mathrm{Tr}\,(f^{-1} g)$$

➡ Massive gravity action is of the form

$$S_{\mathrm{MG}}[g] \;=\; S_{\mathrm{EH}}[g] - \int \mathrm{d}^4x\, V(g,f)$$

kinetic term   mass term

Nonlinear Mass Term

... should [...] derivatives nor loose indices.

But if we try to contr[...] $^{\mu\nu} g_{\mu\nu} = 4$

This is not a mas[...]

Simplest way o[...]

g^{μ}[...]

➜ Massive gravity

What determines $f_{\mu\nu}$?

Can we make it dynamical ?

$$S_{\rm MG}[g] = S_{\rm EH}[g] - \int {\rm d}^4x\, V(g,f)$$

kinetic term   mass term

Bimetric Theory

Nonlinear bimetric action:

$$S_{\mathrm{b}}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, \Big(R(g) - 2\Lambda\Big) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, \Big(R(f) - 2\tilde{\Lambda}\Big) - \int \mathrm{d}^4x\, V(g,f)$$

- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks nice …

… but unfortunately
the general theory
again has ghosts!

The Nonlinear Ghost

Can we extend the Fierz-Pauli mass term
by nonlinear interactions ?

$$\frac{m_{\rm FP}^2}{2}\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta g\right) + \mathbf{c_1}\delta g_\mu{}^\rho \delta g_{\rho\nu} + \mathbf{c_2}\delta g \delta g_{\mu\nu} + \dots$$

Can we choose coefficients $\mathbf{c_i}$ such that the ghost remains absent ?

The Nonlinear Ghost

Can we extend the Fierz-Pauli mass term
by nonlinear interactions ?

$$\frac{m_{\mathrm{FP}}^2}{2}\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta g\right) + \mathbf{c_1}\delta g_\mu{}^\rho \delta g_{\rho\nu} + \mathbf{c_2}\delta g \delta g_{\mu\nu} + \dots$$

Can we choose coefficients $\mathbf{c_i}$ such that the ghost remains absent ?

Boulware & Deser (1972): Beyond linear order this is impossible!

No consistent nonlinear massive gravity / bimetric theory ?

“

Massive gravity stinks.
If you want to modify gravity,
try something else...

Quote from lecture notes by Kurt Hinterbichler, 2010
(now turned into a very nice review!)

The Ghost-Free Theory

Development

Creminelli, Nicolis, Papucci, Trincherini (2005):
attempt to construct ghost-free candidate theory; fails only because of unfortunate sign mistake

de Rham, Gabadadze, Tolley (2010):
construction of candidate theory for massive gravity in flat reference frame; ghost-free in "decoupling limit"

Hassan, Rosen, ASM, von Strauss (2011/12):
proof of absence of ghost in fully nonlinear theory

Hassan & Rosen (2011):
generalisation to ghost-free bimetric theory

Interaction Potential

de Rham, Gabadadze, Tolley (2010);
Hassan & Rosen (2011)

$$V(g,f) = m^4 \sqrt{g} \sum_{n=1}^{3} \beta_n \, e_n\left(\sqrt{g^{-1}f}\right)$$

- arbitrary spin-2 mass scale m
- 3 interaction parameters β_n
- elementary symmetric polynomials $e_n(S)$
- square-root matrix S defined through $S^2 = g^{-1}f$

 - free bimetric theory

Hassan & Rosen (2011)

$$
\begin{aligned}
S_{\rm b}[g,f] \;&=\; m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
&+\; m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \;-\; \int \mathrm{d}^4x\, V(g,f)
\end{aligned}
$$

$$
V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n\left(\sqrt{f^{-1}g}\right)
$$

$$
e_1(S) = \mathrm{Tr}[S] \qquad e_2(S) = \tfrac{1}{2}\left((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\right)
$$

$$
e_3(S) = \tfrac{1}{6}\left((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\right)
$$

What is the physical content of ghost-free bimetric theory ?

Proportional solutions

Hassan, ASM, von Strauss (2012)

$$\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu} \qquad \text{with} \quad c = \text{const.}$$

- gives two copies of Einstein's equations $(\alpha \equiv m_f/m_g)$:

$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_g(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$
$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_f(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$

- consistency condition: $\Lambda_g(\alpha, \beta_n, c) = \Lambda_f(\alpha, \beta_n, c)$ determines c

➜ Maximally symmetric backgrounds with $R_{\mu\nu}(\bar{g}) = \Lambda_g \bar{g}_{\mu\nu}$

Mass spectrum

Hassan, ASM, von Strauss (2012)

Perturbations around proportional backgrounds:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$$

Can be diagonalised into mass eigenstates:

$$\delta G_{\mu\nu} \propto \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu} \quad \text{massless (2 d.o.f.)}$$

$$\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu} \quad \text{massive (5 d.o.f.)}$$

Linearised equations:

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta G_{\rho\sigma} = 0$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta M_{\rho\sigma} + \frac{m_{\mathrm{FP}}^2}{2} \left(\delta M_{\mu\nu} - \bar{g}_{\mu\nu} \delta M \right) = 0$$

with Fierz-Pauli mass $m_{\mathrm{FP}} = m_{\mathrm{FP}}(\alpha, \beta_n, c)$

Ghost-free bimetric theory

=

nonlinear theory

of massless & massive spin-2

Deviations from General Relativity

What is the physical metric ?

How does matter couple
to the tensor fields ?

Matter coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

Only one metric can couple to matter!

$$S_{gf} = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \quad + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f)$$
$$- m^4 \int \mathrm{d}^4x \sqrt{g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1}f}\right)$$
$$+ \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g, \phi)$$

- only coupling that does not re-introduce the ghost
- $g_{\mu\nu}$ is gravitational metric

Matter coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

Only one metric can couple to matter!

$$S_{gf} = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \quad + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f)$$
$$- m^4 \int \mathrm{d}^4x \sqrt{g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1} f}\right)$$
$$+ \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g, \phi)$$

- only coupling that does not re-introduce the ghost
- $g_{\mu\nu}$ is gravitational metric

The gravitational metric is not massless !!

Physical Interpretation

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

Bimetric theory = General Relativity (GR) + corrections

Recall: $\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$

Assume that $\alpha = m_f/m_g$ is small (i.e. weak gravity!)

- the gravitational metric is almost massless
- the massive spin-2 field interacts only weakly with matter

Physical Interpretation

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

Bimetric theory = General Relativity (GR) + corrections

Recall: $\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$

Assume that $\alpha = m_f / m_g$ is small (i.e. weak gravity!)

- the gravitational metric is almost massless
- the massive spin-2 field interacts only weakly with matter

$\alpha \to 0$ is the General Relativity limit of bimetric theory

Large Spin-2 Mass

Babichev & Chrisostomi (2013)

For small α the Fierz-Pauli mass scales as: $m_{\mathrm{FP}} \sim \alpha^{-2}$

Thus it becomes large in the limit of small α.

- Interactions with a heavy field do not strongly affect the low-energy theory of the massless spin-2 mode, which therefore resembles GR.
- This can be verified explicitly in static point-source solutions and cosmological solutions of bimetric theory.

Ghost-free bimetric theory

=

General Relativity +
additional (heavy?) tensor field

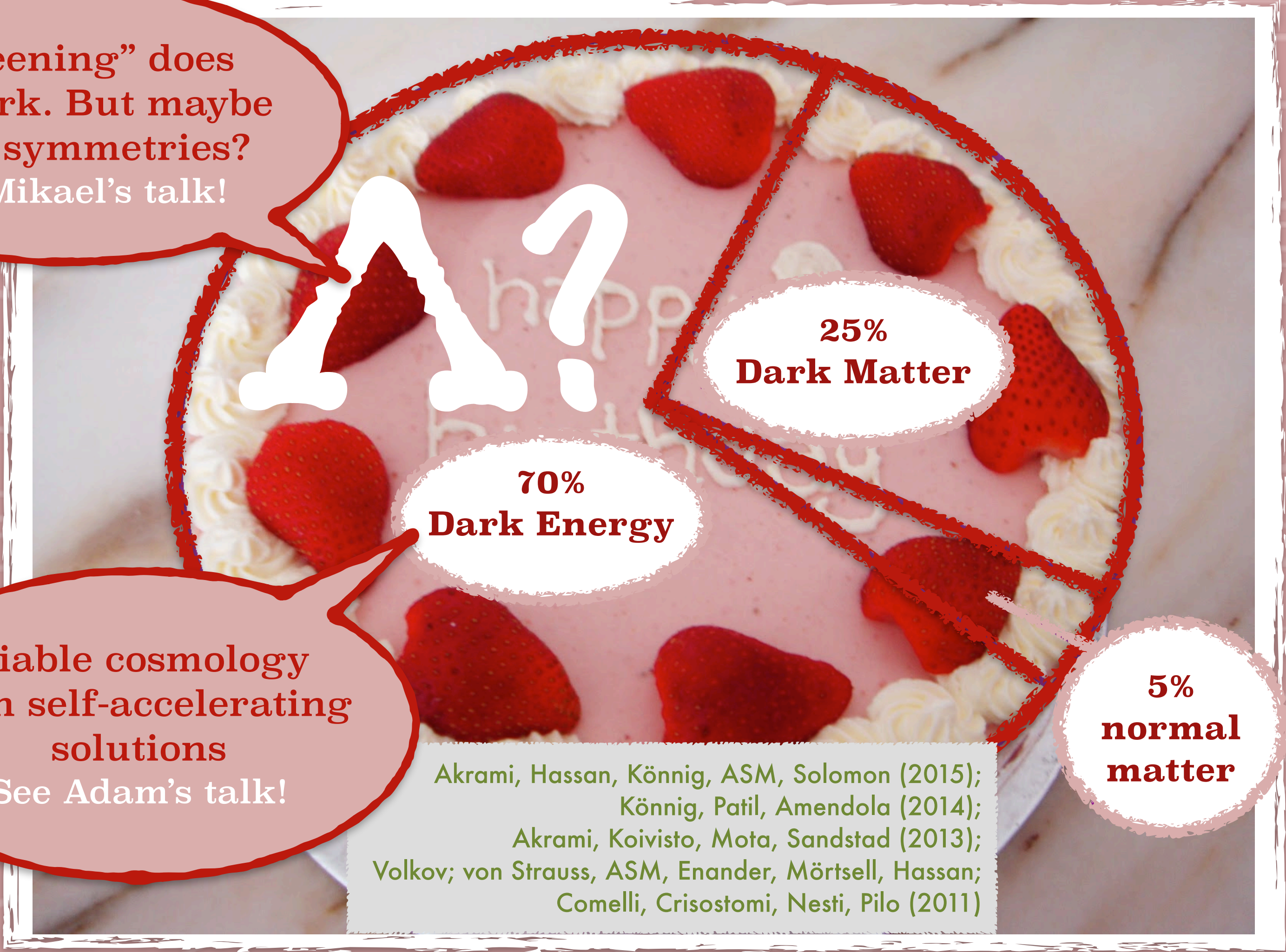
Cosmology


happy
birthday
25%
Dark Matter
70%
Dark Energy
5%
normal
matter


Λ?
25%
Dark Matter
70%
Dark Energy
5%
normal
matter

"Screening" does not work. But maybe extra symmetries?
See Mikael's talk!

“Screening” does not work. But maybe extra symmetries?
See Mikael’s talk!



Viable cosmology with self-accelerating solutions
See Adam’s talk!

Akrami, Hassan, König, ASM, Solomon (2015);
König, Patil, Amendola (2014);
Akrami, Koivisto, Mota, Sandstad (2013);
Volkov; von Strauss, ASM, Enander, Mörtsell, Hassan;
Comelli, Crisostomi, Nesti, Pilo (2011)

“Screening” does not work. But maybe extra symmetries?
See Mikael’s talk!
Λ?
What if it has spin-2 ?
25%
Dark Matter
70%
Dark Energy
5%
normal matter
Viable cosmology with self-accelerating solutions
See Adam’s talk!
Akrami, Hassan, Könnig, ASM, Solomon (2015);
Könnig, Patil, Amendola (2014);
Akrami, Koivisto, Mota, Sandstad (2013);
Volkov; von Strauss, ASM, Enander, Mörtsell, Hassan;
Comelli, Crisostomi, Nesti, Pilo (2011)

Spin-2
Dark Matter

Dark Matter

Aoki, Mukohyama (2016); Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic action for mass eigenstates:

$$S_{(2)} = \frac{1}{2} \int \mathrm{d}^4x \Big[\delta G_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} \delta G_{\rho\sigma} + \delta M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} \delta M_{\rho\sigma} - \frac{m_{\rm FP}^2}{2} (\delta M^{\mu\nu} \delta M_{\mu\nu} - \delta M^2) - \frac{1}{m_{\rm Pl}} \Big(\delta G^{\mu\nu} - \alpha\, \delta M^{\mu\nu} \Big) T_{\mu\nu} \Big]$$

Dark Matter

Aoki, Mukohyama (2016); Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic action for mass eigenstates:

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}^4x \Big[\delta G_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta G_{\rho\sigma} + \delta M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta M_{\rho\sigma} - \frac{m_{\rm FP}^2}{2}(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2) - \frac{1}{m_{\rm Pl}}\Big(\delta G^{\mu\nu} - \alpha\,\delta M^{\mu\nu}\Big)T_{\mu\nu}\Big]$$

In the General Relativity (GR) limit of bimetric theory, $\alpha \to 0$:

massive spin-2 field decouples from matter, interacts only with gravity.

A large spin-2 mass further suppresses deviations from GR.

Dark Matter

Aoki, Mukohyama (2016);
Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic action for mass eigenstates:

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}^4x \Big[\delta G_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta G_{\rho\sigma} + \delta M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta M_{\rho\sigma} - \frac{m_{\mathrm{FP}}^2}{2}(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2) - \frac{1}{m_{\mathrm{Pl}}}\Big(\delta G^{\mu\nu} - \alpha\,\delta M^{\mu\nu}\Big)T_{\mu\nu}\Big]$$

In the General Relativity (GR) limit of bimetric theory, $\alpha \to 0$:

massive spin-2 field decouples from matter, interacts only with gravity.

A large spin-2 mass further suppresses deviations from GR.

Basically we get: GR + Standard Model + spin-2 dark matter candidate

Consistency checks

Our spin-2 dark matter is part of gravity (!) and...

- ... gravitates just like baryonic matter
- ... does not decay into gravitons and its decay rate into Standard Model fields is sufficiently small: $\Gamma(\delta M \to XX) \sim \frac{\alpha^2 m_{\mathrm{FP}}^3}{m_{\mathrm{Pl}}^2}$

➡ automatically stable

- ... has interactions with baryonic matter which are naturally suppressed by the Planck scale
- ... can be produced thermally for a mass of 1 - 10^8 TeV

Detection

- not observable in current indirect and direct detection experiments

- massive spin-2 field may gravitate differently in curved backgrounds

  → non-standard behaviour of dark matter around massive objects ?

- dark matter self-interactions: could be observable in cluster collisions and in power spectrum

- correlations with gravitational waves → Ask Shinji!

Lessons learned (and to be learned..)

Ghost-free bimetric theory...

- is one of the few known consistent modifications of General Relativity
- describes nonlinear interactions of massless and massive spin-2 fields
- can be interpreted as gravity in the presence of an extra spin-2 field
- contains an interesting dark matter candidate whose coupling to baryonic matter is suppressed by the Planck scale

Ghost-free bimetric theory...

- is one of the few known consistent modifications of General Relativity
- describes nonlinear interactions of massless and massive spin-2 fields
- can be interpreted as gravity in the presence of an extra spin-2 field
- contains an interesting dark matter candidate whose coupling to baryonic matter is suppressed by the Planck scale

And what is next ?

- develop better understanding of phenomenology
- can we detect this ??

Thank you for you attention!