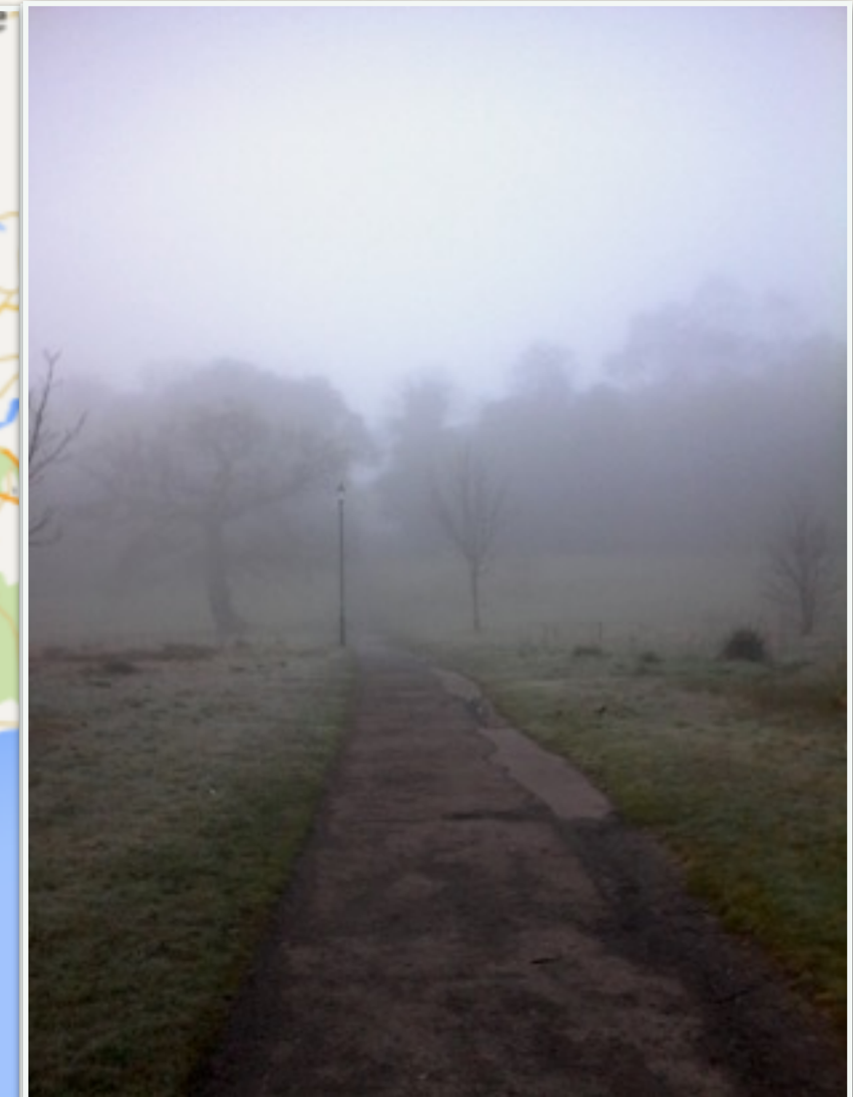

Abelian Symmetry Breaking and Modified Gravity

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Collaborations with Gustavo Niz and Gianmassimo Tasinato

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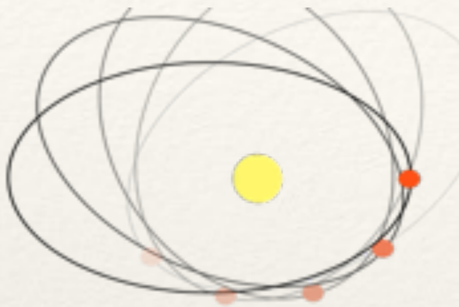
Coleg Gwyddoniaeth

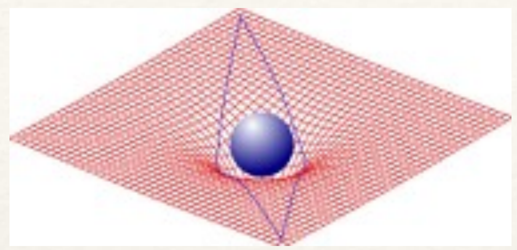
Swansea University
Prifysgol Abertawe

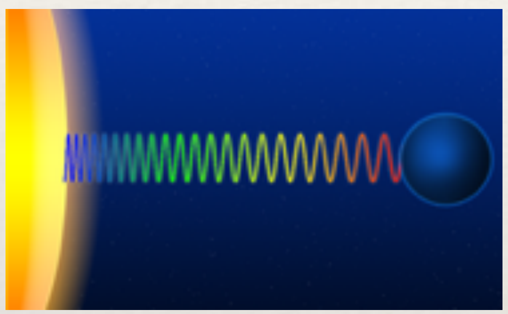
Outline

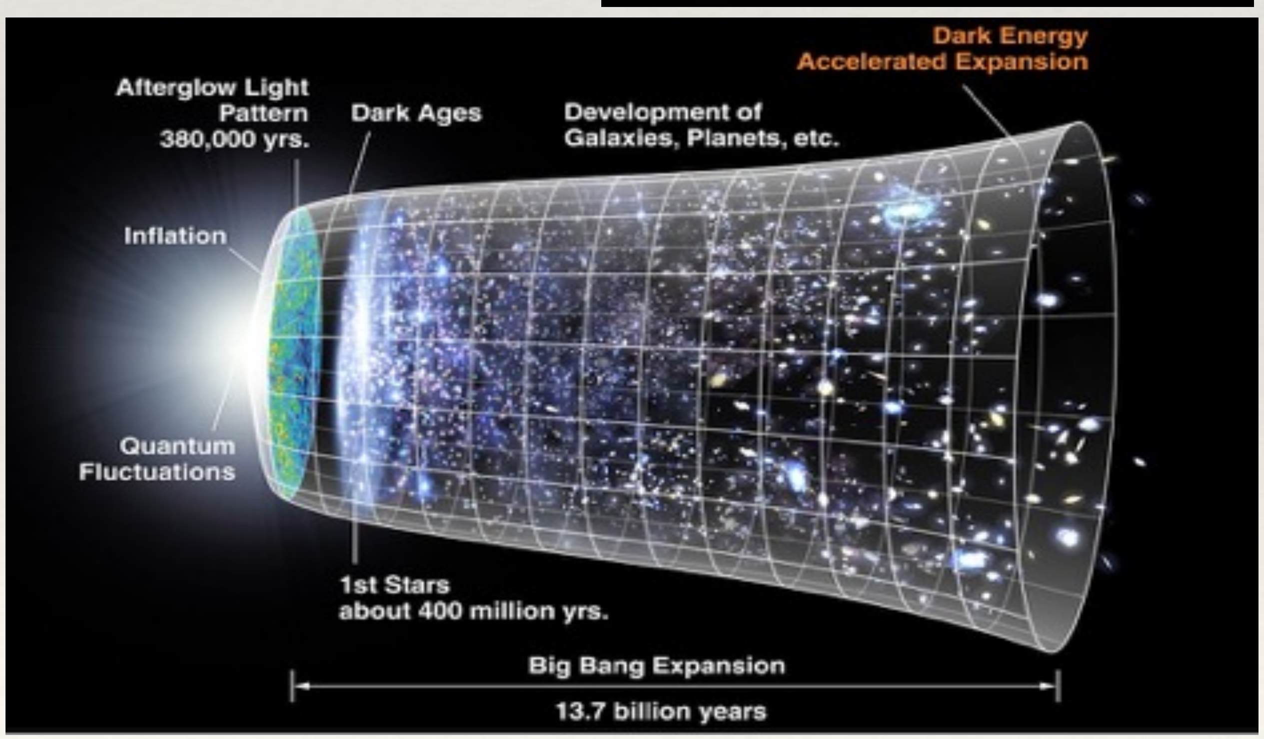
- Context
- Abelian Galileon-Higgs vortices
- Black holes in generalised Proca

GR & ΛCDM

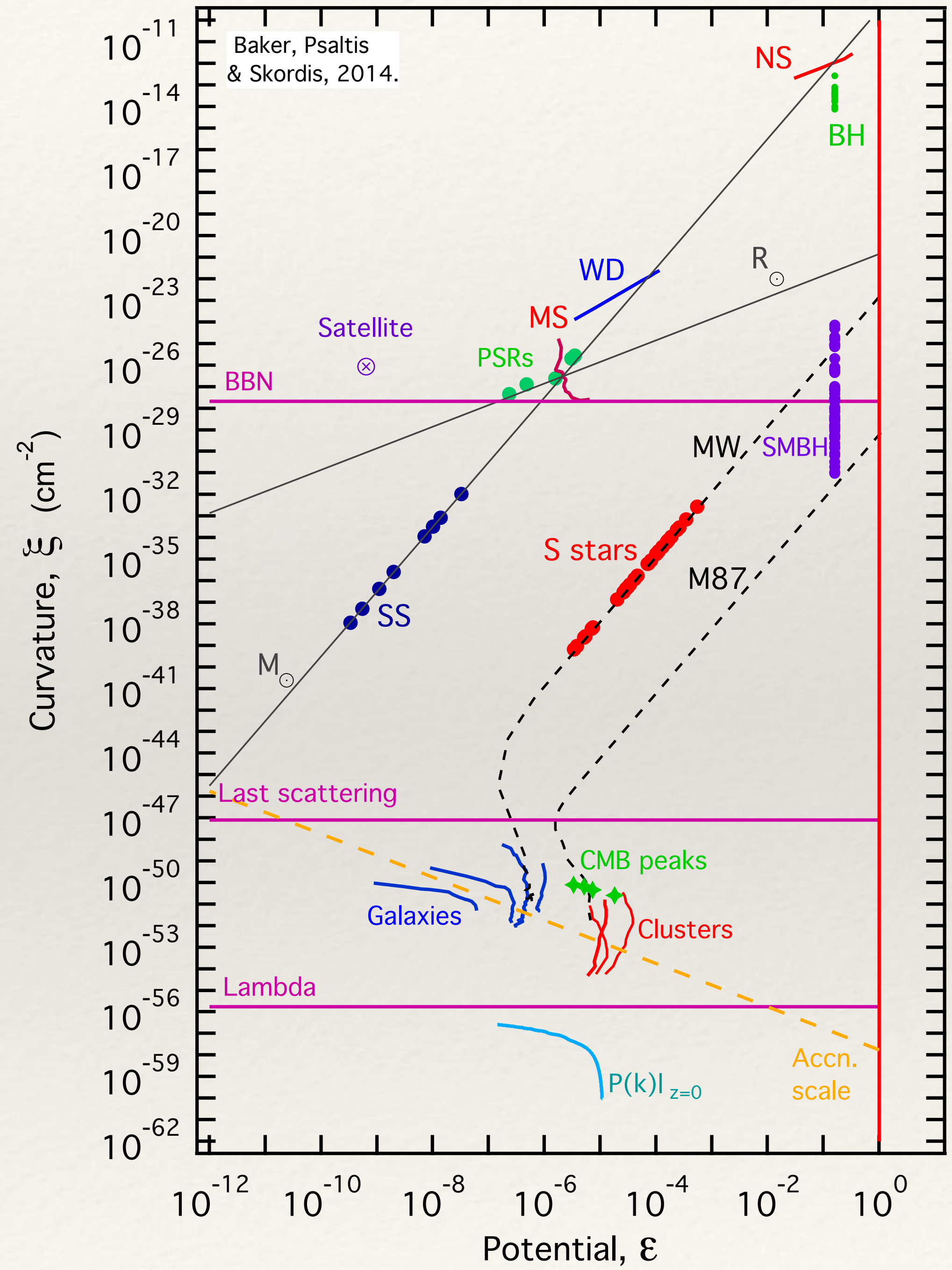








Dark Energy
Accelerated Expansion
Afterglow Light Pattern 380,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Inflation
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years



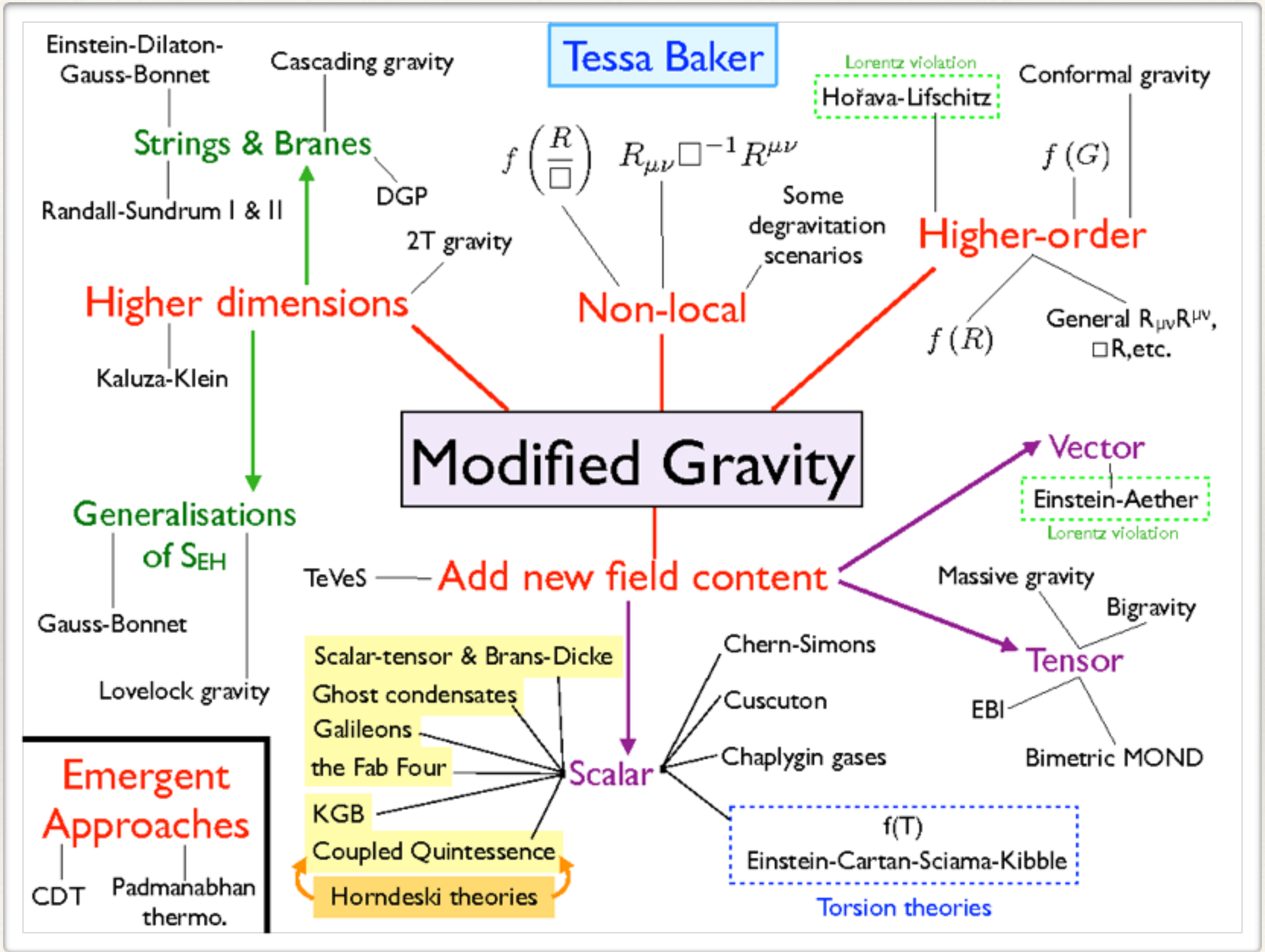
Baker, Psaltis & Skordis, 2014.
NS
BH
WD
R⊙
Satellite
MS
PSRs
BBN
MW
SMBH
S stars
M87
SS
M⊙
Last scattering
CMB peaks
Galaxies
Clusters
Lambda
Accn. scale
P(k)| z=0
Curvature, ξ (cm^{-2})
Potential, ε
10^{-11} 10^{-14} 10^{-17} 10^{-20} 10^{-23} 10^{-26} 10^{-29} 10^{-32} 10^{-35} 10^{-38} 10^{-41} 10^{-44} 10^{-47} 10^{-50} 10^{-53} 10^{-56} 10^{-59} 10^{-62}
10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0}

Lovelock's theorem

The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant.

In order to modify GR we must consider at least one of the following:

* New degrees of freedom,
* Higher order derivatives,
* Extra dimensions,
* Non-locality.


Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
Randall-Sundrum I & II
DGP
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
Add new field content
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
Emergent Approaches
CDT
Padmanabhan thermo.

Screening Mechanisms

Additional degrees of freedom must be screened:

- *Chameleon* The mass of the scalar mode becomes large in dense regions.

  Khoury & Weltman, Phys. Rev. Lett. 93 (2004) 171104

$$\Box\phi + M^2(\rho)\phi = \frac{g}{M_{pl}}\rho$$

- *Symmetron* The mass of the scalar mode becomes large in dense regions.

  Hinterbichler & Khoury, Phys. Rev. Lett. 104, 231301

$$\Box\phi + m^2\phi = \frac{g(\rho)}{M_{pl}}\rho$$

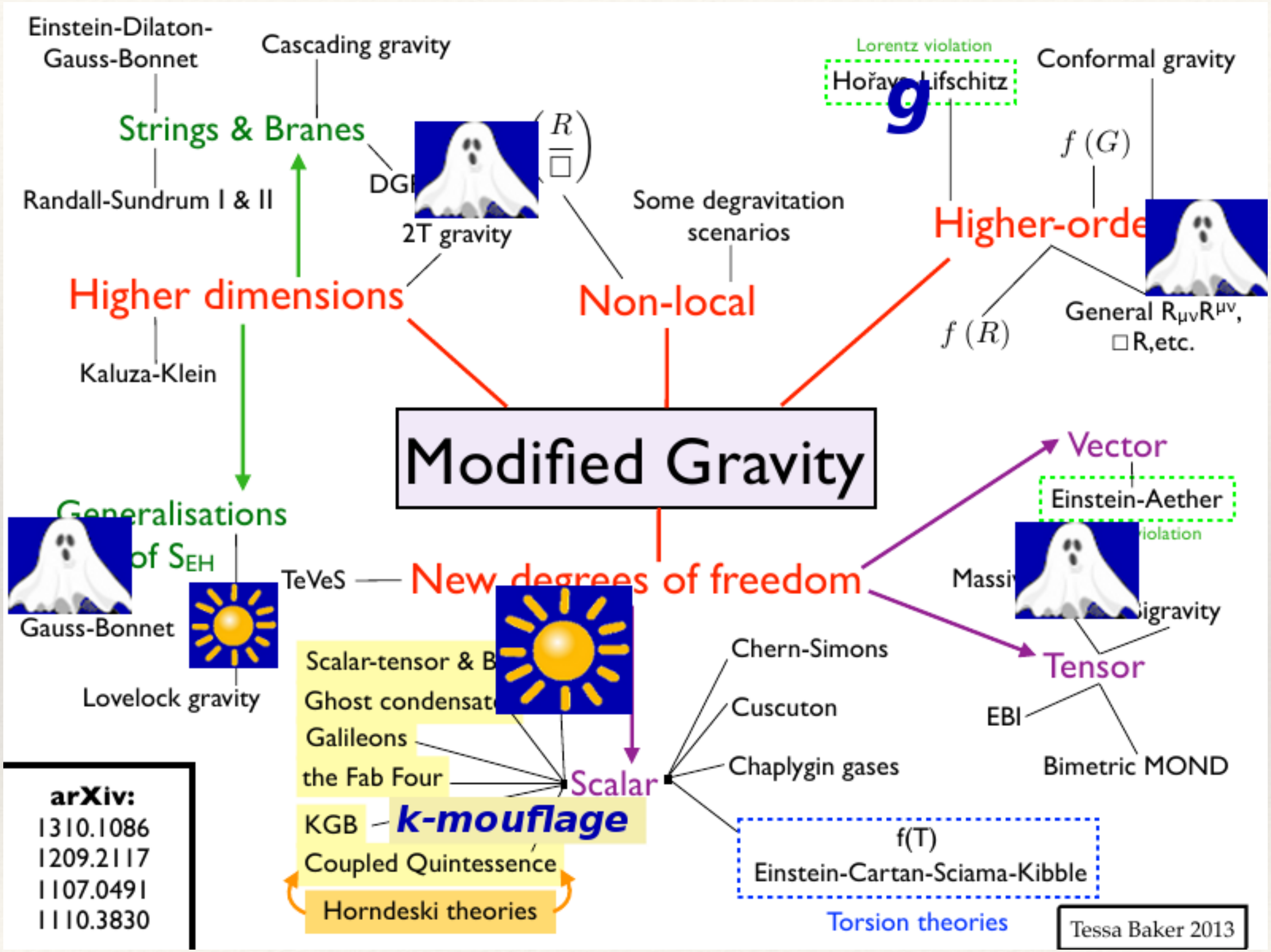
- *Vainshtein* NL derivative self-interactions become large in dense regions.

  Massive Gravity, DGP, Galileons...

$$K(\phi)\Box\phi + m^2\phi = \frac{g}{M_{pl}}\rho$$

Theoretical Consistency

* Higher order operators are negligible if $E/\Lambda \ll 1$. Background-dependent functions in front of kinetic terms, couplings and potentials may lead to a *strong coupling problem.*
* Try to avoid fine tunings — *technical naturalness.*
* Take care of the sign of the kinetic term — *ghost free.*



Reviews of the properties of MG models:

S. Tsujikawa, Lect.Notes Phys. 800:99-145, 2010

Clifton et al., Physics Reports 513, 1 (2012)

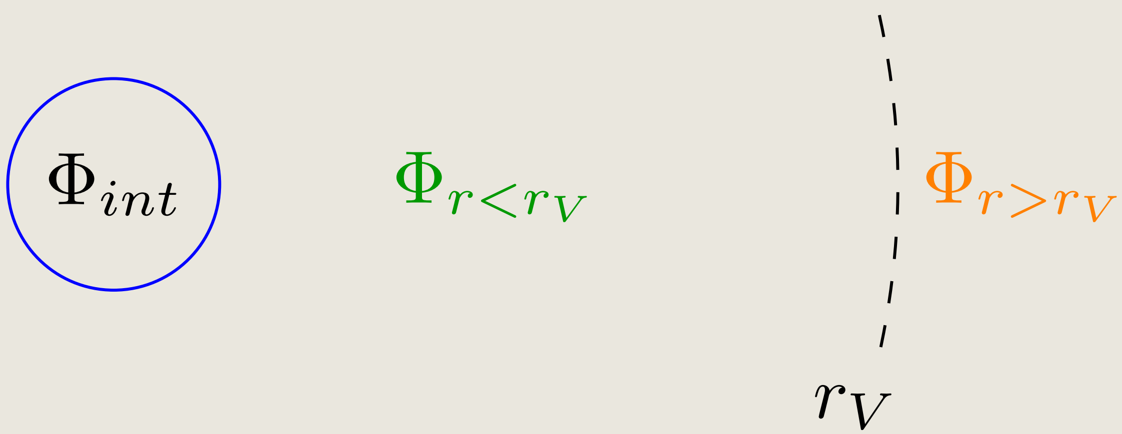
A. Barreira et al., arXiv:1504.01493

Some aspects of Galileons

- Scalar Galileons in strong gravity.
  JC, K. Koyama, G. Niz, G. Tasinato, JCAP10(2014)055
- Vortices in Higgs vector Galileons.
  JC, G. Tasinato, JHEP02(2016)063
- Black holes in generalised Proca action.
  JC, G. Niz, G. Tasinato, arXiv:1602.08697

(No)Scalarisation

Toy model



$$\Box\Phi + \frac{2\alpha}{\Lambda^3}[\ldots] = -4\pi\frac{\beta}{M_p^2}\Phi T$$

Neglecting curvature and imposing

$$T = -M\left(\frac{4}{3}\pi R^3\right)^{-1}$$

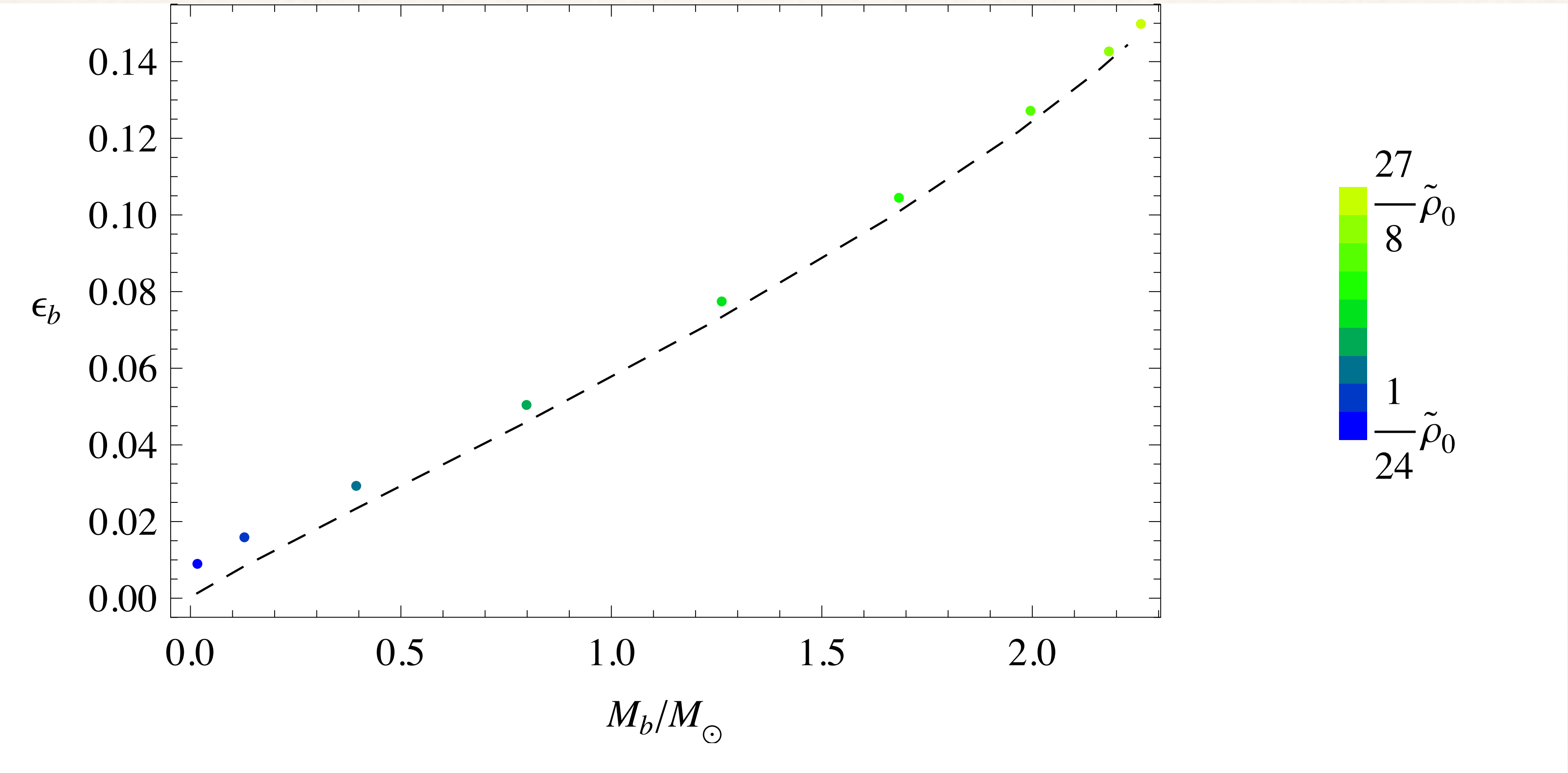
Match Φ and Φ' at R and r_V:

$$\phi_c - \phi_0 = \frac{\Lambda^3}{\alpha}\left[\frac{3}{16}r_V^2\left(\frac{R^{1/2}}{r_V^{1/2}} - 1\right) + \frac{3\beta R r_s}{320},\right]$$

The difference between the central and asymptotic values of the scalar field is always suppressed.

(No)Scalarisation

Binding energy of a polytropic compact star



Fixing an asymptotic value of the scalar field, deviations from GR inside a compact star get smaller as the density increases.

No hair? Scalar field solution breaks down near the maximum observed density of neutron stars.

Vector Galileons

* Use new fields to drive acceleration.
* Vectors have been less explored than scalars in DE models with derivative self-interactions.
* Can be relevant in cosmology.

  Tasinato, CQG31(2014)225004
* Can be obtained from spontaneous symmetry breaking

  Hull, Koyama, Tasinato, JHEP03(2015)154

Higgs Mechanism for Vector Galileons

Hull, Koyama, Tasinato, JHEP03(2015)154

- Gauge invariant action for a complex scalar field with higher order derivative couplings:

$$\mathcal{L} = -(\mathcal{D}_\mu\phi)(\mathcal{D}^\mu\phi)^* - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - V(\phi) + \underbrace{\mathcal{L}_{(8)} + \mathcal{L}_{(12)} + \mathcal{L}_{(16)}}$$

$$\mathcal{D}_\mu = \partial_\mu - igA_\mu$$

Gauge invariant operators of dimension 8, 12 and 16 that describe derivative self-interactions of the Higgs field.

$$V = -\mu^2\phi\phi^* + \frac{\lambda}{2}(\phi\phi^*)^2$$

$$v = \sqrt{\frac{\mu^2}{\lambda}}$$

Higgs Mechanism for Vector Galileons

Hull, Koyama, Tasinato, JHEP03(2015)154

- Demand that the Lagrangian is invariant under a U(1) gauge symmetry:

$$\phi \to \phi e^{i\xi}$$

$$A_\mu \to A_\mu + \frac{1}{g}\partial_\mu \xi$$

$$\Downarrow$$

$$\mathcal{D}_\mu \phi \to e^{i\xi}\mathcal{D}_\mu \phi$$

$$\mathcal{D}_\mu \mathcal{D}_\nu \phi \to e^{i\xi}\mathcal{D}_\mu \mathcal{D}_\nu \phi$$

Gauge invariant combinations:

$$L_{\mu\nu} \equiv \frac{1}{2}\left[(\mathcal{D}_\mu\phi)^*(\mathcal{D}_\nu\phi) + (\mathcal{D}_\nu\phi)^*(\mathcal{D}_\mu\phi)\right]$$

$$P_{\mu\nu} \equiv \frac{1}{2}\left[\phi^*\mathcal{D}_\mu\mathcal{D}_\nu\phi + \phi(\mathcal{D}_\mu\mathcal{D}_\nu\phi)^*\right]$$

$$Q_{\mu\nu} \equiv \frac{i}{2}\left[\phi(\mathcal{D}_\mu\mathcal{D}_\nu\phi)^* - \phi^*(\mathcal{D}_\mu\mathcal{D}_\nu\phi)\right]$$

$$\mathcal{L}_8 = \frac{1}{2!\Lambda^4}\varepsilon^{\alpha\beta\mu_1\mu_2}\varepsilon_{\alpha\beta\nu_1\nu_2}\left[\alpha_{(8)}L^{\nu_1}_{\mu_1}P^{\nu_2}_{\mu_2} + \beta_{(8)}L^{\nu_1}_{\mu_1}Q^{\nu_2}_{\mu_2}\right]$$

$$\mathcal{L}_{12} = \frac{1}{\Lambda^8}\varepsilon^{\alpha\mu_1\mu_2\mu_3}\varepsilon_{\alpha\nu_1\nu_2\nu_3}\left[\alpha_{(12)}L^{\nu_1}_{\mu_1}P^{\nu_2}_{\mu_2}P^{\nu_3}_{\mu_3} + \beta_{(12)}L^{\nu_1}_{\mu_1}Q^{\nu_2}_{\mu_2}Q^{\nu_3}_{\mu_3}\right]$$

$$\mathcal{L}_{16} = \frac{1}{\Lambda^{12}}\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\nu_1\nu_2\nu_3\nu_4}\left[\alpha_{(16)}L^{\nu_1}_{\mu_1}P^{\nu_2}_{\mu_2}P^{\nu_3}_{\mu_3}P^{\nu_4}_{\mu_4} + \beta_{(16)}L^{\nu_1}_{\mu_1}Q^{\nu_2}_{\mu_2}Q^{\nu_3}_{\mu_3}Q^{\nu_4}_{\mu_4}\right]$$

Higgs Mechanism for Vector Galileons

Hull, Koyama, Tasinato, JHEP03(2015)154

- Extract the physical content:

$$\phi = \varphi e^{ig\pi}, \qquad \hat{A}_\mu \equiv A_\mu - \partial_\mu \pi$$

Gauge invariants

- Covariant derivatives can be expressed in terms of these fields, e.g.

$$\mathcal{D}_\mu \phi = (\partial_\mu \varphi - ig\varphi \hat{A}_\mu) e^{ig\pi}$$

$$\Downarrow$$

- L, P and Q are symmetric:

$$L_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi + g^2 \varphi^2 \hat{A}_\mu \hat{A}_\nu$$
$$P_{\mu\nu} = \varphi \partial_\mu \partial_\nu \varphi - g^2 \varphi^2 \hat{A}_\mu \hat{A}_\nu$$
$$Q_{\mu\nu} = \frac{g}{2}[\partial_\mu(\varphi^2 \hat{A}_\nu) + \partial_\nu(\varphi^2 \hat{A}_\mu)]$$

$$\Downarrow$$

- $\mathcal{L}'s$ manifestly gauge invariant:

$$\mathcal{L}_8 = -\frac{g\beta_{(8)}}{\Lambda^4}(\partial_\mu \varphi \partial^\nu \varphi + g^2 \varphi^2 \hat{A}_\mu \hat{A}^\nu)\partial_\rho(\varphi^2 \hat{A}^\sigma)(\delta^\mu_\nu \delta^\rho_\sigma - \delta^\rho_\nu \delta^\mu_\sigma)$$

Higgs Mechanism for Vector Galileons

Hull, Koyama, Tasinato, JHEP03(2015)154

- Around the minimum of the potential: $\varphi = v + \frac{h}{\sqrt{2}}$

$$\mathcal{L}_{tot} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \boxed{-(g\,v)^2\hat{A}^2 - \frac{3g^3\beta_{(8)}v^4}{2\Lambda^4}\hat{A}_\mu\hat{A}^\mu\partial_\rho\hat{A}^\rho}$$
$$\boxed{+\ gauge\ field\ interactions + new\ interactions\ between\ \hat{A}_\mu\ and\ h + \ldots}$$

Same as the first two Lagrangians in generalised Proca action

L. Heisenberg, JCAP 1405 (2014) 015

The remaining two Lagrangians come from $\mathcal{L}_{(12)}$ and $\mathcal{L}_{(16)}$

Effects yet to be analysed. However they're suppressed by appropriate powers of $m_A = g\,v$

Vortices

- A simple way to study effects induced by vector Galileon interactions.

Vortices in the Abelian Higgs model

$$\mathcal{L}_{AH}[\Phi, A_a] = (D_a\Phi)^\dagger D^a\Phi - \frac{1}{4}F_{ab}F^{ab} - \frac{\lambda}{4}\left(\Phi^\dagger\Phi - \eta^2\right)^2$$

Introduce gauge inv. quantities, X and Â

$$\Phi(x^\alpha) = \eta X(x^\alpha)e^{i\chi(x^\alpha)}, \qquad A_a(x^\alpha) = \frac{1}{e}\left[\hat{A}_a(x^\alpha) - \partial_a\chi(x^\alpha)\right]$$

χ drops out from the Lagrangian

$$\mathcal{L}_{AH}[X, \hat{A}_a] = \eta^2\partial_a X\partial^a X + \eta^2 X^2 \hat{A}_a\hat{A}^a - \frac{1}{4e^2}F_{ab}F^{ab} - \frac{\lambda\eta^4}{4}\left(X^2 - 1\right)^2$$

massive

Vortices

Vortices in the Abelian Higgs model

The physical degrees of freedom obey

$$\Box X - X\hat{A}_a\hat{A}^a + \frac{\lambda\eta^2}{2}\left(X^2-1\right)X = 0$$
$$\partial_a F^{ab} + 2e^2\eta^2X^2\hat{A}^b = 0$$

Static solution: *Nielsen-Olesen vortex*, characterised by a winding number.

$$\left.\begin{array}{c}\text{At infinity:}\\ X \to 1\\ D_a\Phi \to 0 \Rightarrow A_a \to -\dfrac{N}{e}\partial_a\theta\\ \text{Furthermore:}\\ \chi = N\theta\end{array}\right\}\ \text{Flux} = 2\pi N$$

At *r=0*:

$$A_a(r\to 0) = 0, \quad X(r\to 0) = 0$$

It turns out to be consistent to set $A_r = 0$ everywhere.

Vortices

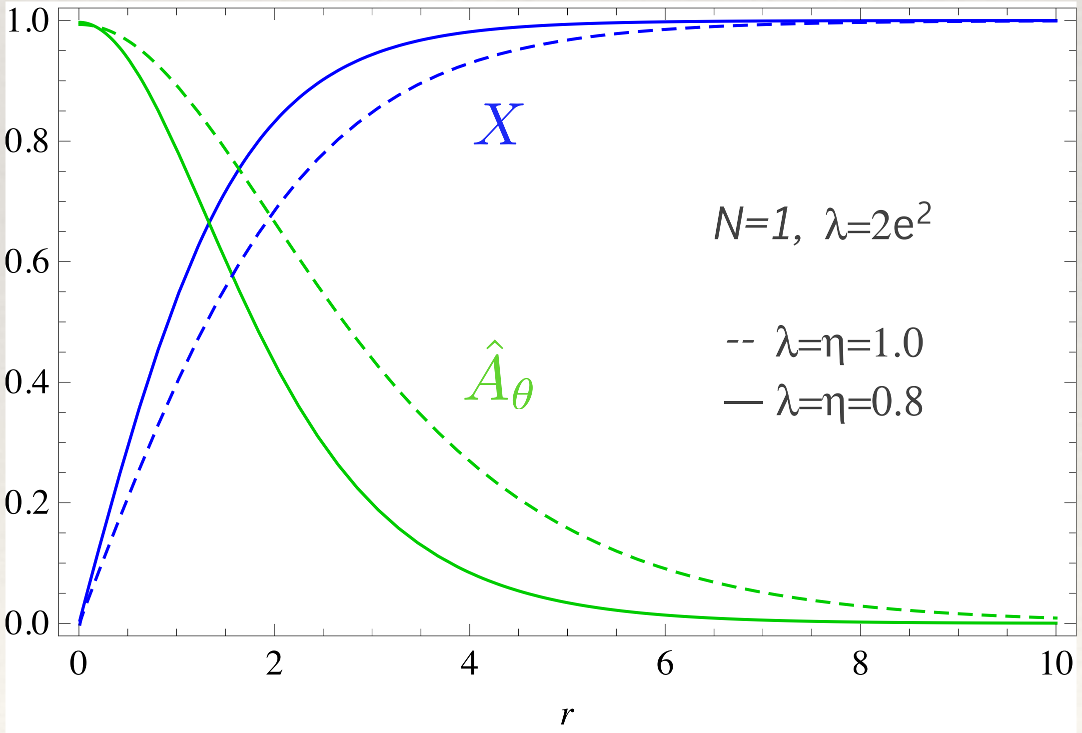
Vortices in the Abelian Higgs model

The solution is uniquely determined by:

- Rotational symmetry $$\hat{A}_i\, dx^i = \left[-\epsilon_{ij}\, \frac{x_j}{r}\, \hat{A}_\theta(r) + \frac{x_i}{r}\, \cancel{\hat{A}_r(r)}\right] dx^i$$
- Invariance under *reflection accompanied by complex conjugation of* Φ $$A_1(r,\, \theta) = -A_1(r, -\theta)$$ $$A_2(r,\, \theta) = A_2(r, -\theta)$$

$$r\eta^2 \lambda X \left(1 - X^2\right) - \frac{2\hat{A}_\theta^2 X}{r} + 2(X' + rX'') = 0$$

$$-2e^2 r \eta^2 \hat{A}_\theta X^2 - \hat{A}'_\theta + r\hat{A}''_\theta = 0$$



Vortices

Now with Galileons

Same gauge invariant fields and asymptotic as before,

$$\Phi(x^\alpha) = \eta X(x^\alpha) e^{i\chi(x^\alpha)}, \qquad A_a(x^\alpha) = \frac{1}{e}\left[\hat{A}_a(x^\alpha) - \partial_a \chi(x^\alpha)\right]$$

Consider only new contributions from vector field derivatives:

$$\mathcal{L}_{AHG} = \eta^2 \partial_a X \partial^a X + \eta^2 X^2 \hat{A}_a \hat{A}^a - \frac{1}{4e^2} F_{ab}F^{ab} - \frac{\lambda \eta^4}{4}\left(X^2 - 1\right)^2$$
$$+ \beta\eta^4 \left(\partial_a X \partial_b X + X^2 \hat{A}_a \hat{A}_b\right)\left[\eta^{ab}\partial^c (X^2 \hat{A}_c) - \partial^a (X^2 \hat{A}^b)\right]$$

$$r\eta^4 \lambda X\left(1 - X^2\right) - \frac{2\eta^2 \hat{A}_\theta^2 X}{r} + 2\eta^2 X' + 2r\eta^2 X''$$
$$+16\beta\eta^4 X^3 \hat{A}_r \left[\hat{A}_r^2 + \frac{2\hat{A}_\theta^2}{r^2} + \frac{\hat{A}_\theta^2 \hat{A}_r'}{r\hat{A}_r} - \frac{\hat{A}_\theta \hat{A}_\theta'}{r} - \frac{\hat{A}_r' X'}{2X\hat{A}_r} - \frac{X'^2}{2X^2} - \frac{X''}{2X}\right] = 0,$$

$$-2e^2 r\eta^2 \hat{A}_\theta X^2 - \hat{A}_\theta' + r\hat{A}_\theta'' + 12\beta e^2 \eta^4 X^4 r \hat{A}_\theta \left[\frac{\hat{A}_r}{r} + \hat{A}_r' + \frac{4\hat{A}_r X'}{3X}\right] = 0,$$

$$-r\eta^2 \hat{A}_r X^2 + 6\beta\eta^4 X^4 \left[\hat{A}_r^2 + \frac{\hat{A}_\theta^2}{r^2} - \frac{\hat{A}_\theta \hat{A}_\theta'}{r} - \frac{4\hat{A}_\theta^2 X'}{3Xr} + \frac{X'^2}{3X^2}\right] = 0.$$

Setting $\hat{A}_r = 0$ is inconsistent.

$$\mathcal{L}_6 \supset \frac{4\beta\eta^4}{r^3} X^2 \hat{A}_r \left(2\hat{A}_\theta{}^2 X^2 - \ldots\right)$$

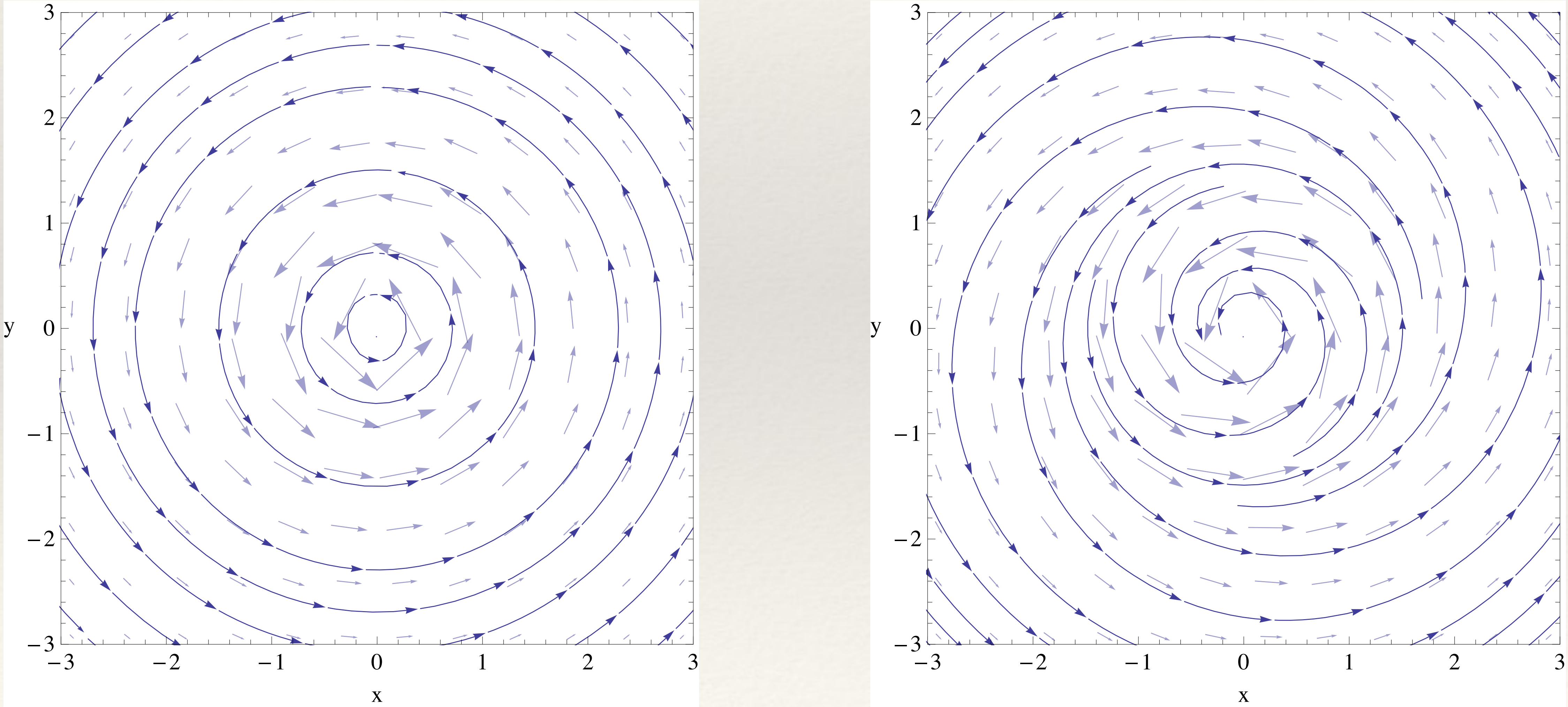
Vortices

Now with Galileons

$$\hat{A}_r = \frac{r}{12\beta\eta^2 X^2}\left[1 - \sqrt{1 - \left(\frac{12\beta\eta^2 X^2}{r}\right)^2 \left[\frac{\hat{A}_\theta}{r^2}(\hat{A}_\theta - r\hat{A}'_\theta) + \frac{X'}{3X}\left(\frac{X'}{X} - \frac{4\hat{A}_\theta^2}{r}\right)\right]}\right]$$

- Only $\hat{A}_\theta$ and X are "dynamical"

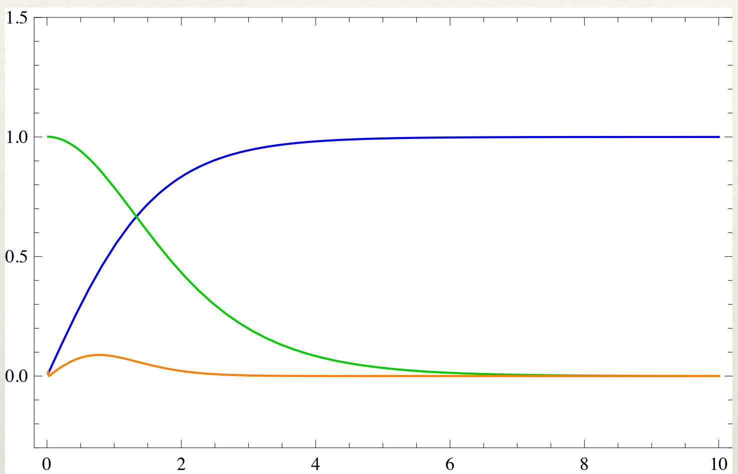
$\hat{A}$ for a N-O and a Galileon vortex



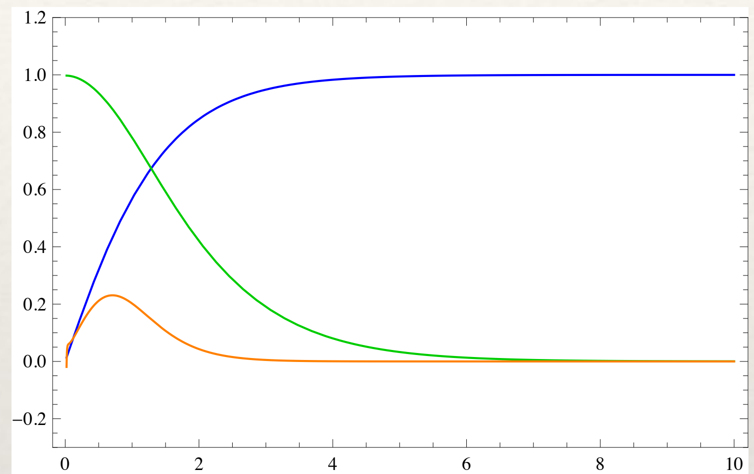
Vortices

Now with Galileons

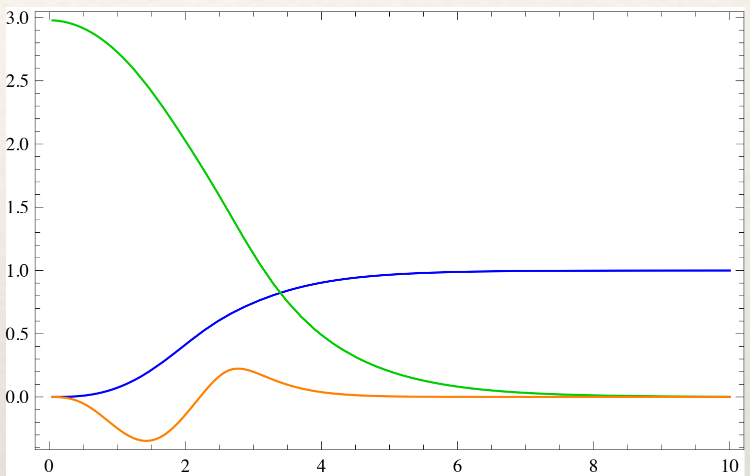
X, $\hat{A}_\theta$ and $\hat{A}_r$



$\beta = 0.10$

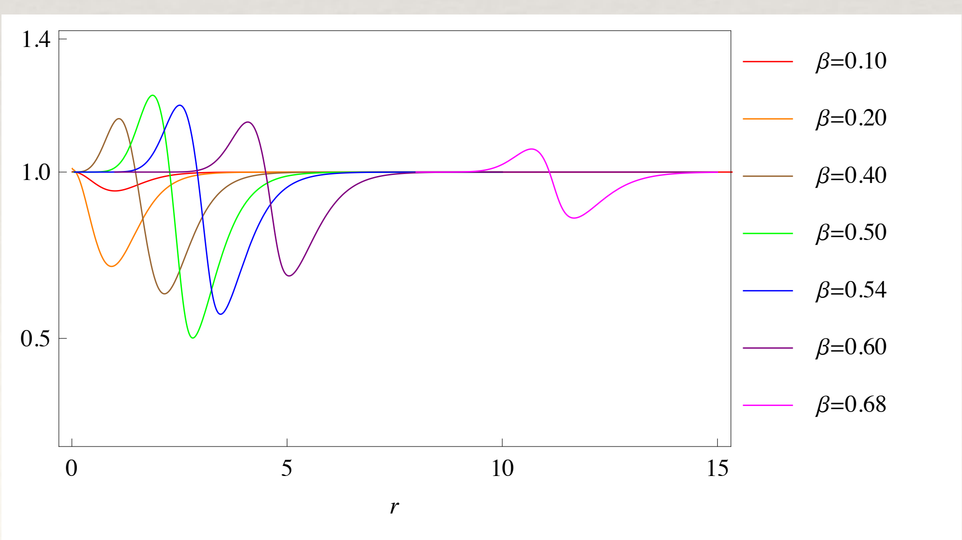


$\beta = 0.20$



$\beta = 0.50$

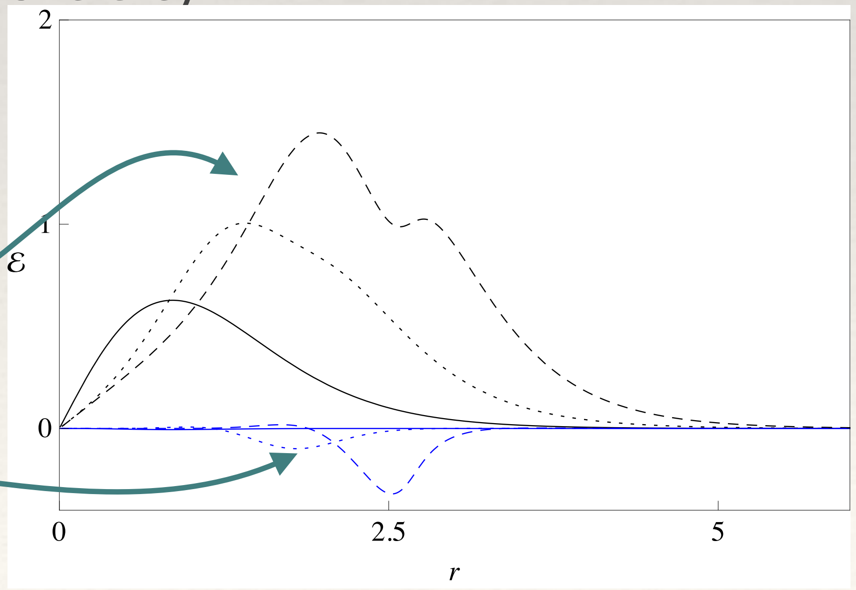
- Given β, $\hat{A}_r$ constraints the allowed values for the vorticity



Non-linear contributions

Total energy

Energy from L_6

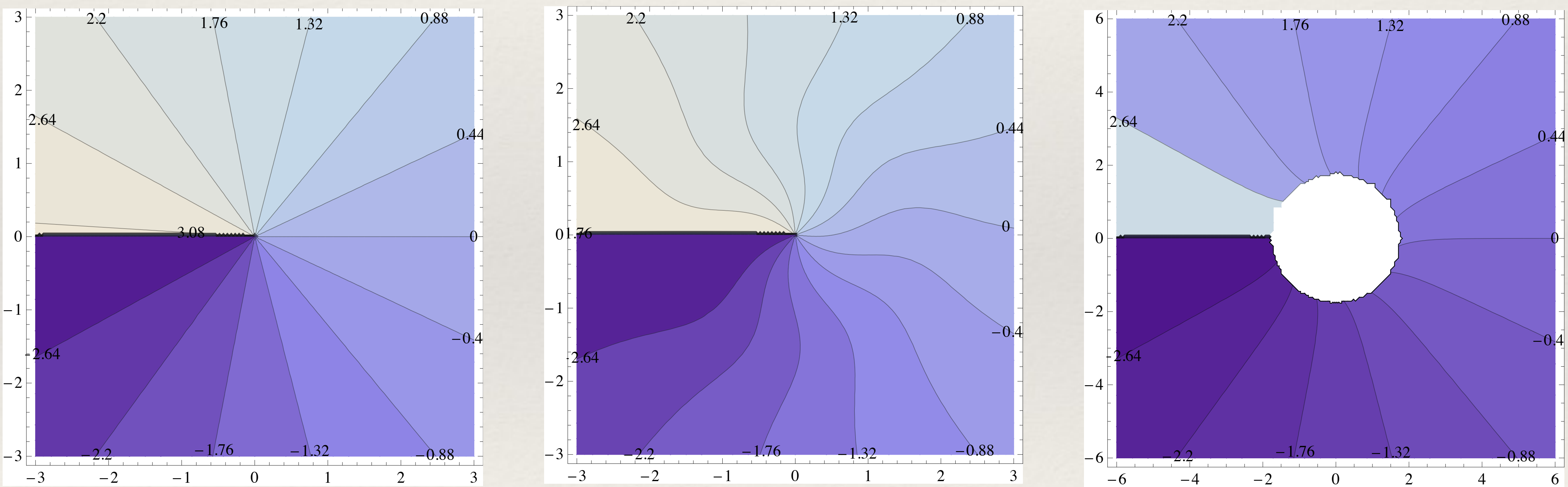


Vortices

Visual trick

$$\hat{A}_r = e\,A_r + \partial_r \chi$$

- The non-trivial profile of $\hat{A}_r$ can be attributed either to A_r or to the phase
- Gauge choice: $A_r = 0,\ \chi = N\theta + \tilde{\chi}(r)$



Lines of constant phase for N-O, β=0.4 and β=0.5

Vortices

Minimal (and weak) coupling to gravity

- Is the geometry of the vortex reflected on the spacetime?

$$G_{ab} = 8\pi G(T^{(AH)}_{ab} + T^{(6)}_{ab})$$

$\downarrow$

Contains r-θ component

$\downarrow$

$$ds^2 = e^{2(\gamma(r)-\Psi(r))}(dt^2 - dr^2) - e^{2\Psi}dz^2 - \alpha(r)^2 e^{-2\Psi}d\theta^2 - \beta\,\omega(r)drd\theta$$

- Further assumption: small β limit $\Rightarrow \hat{A}_r \sim \mathcal{O}(\beta\eta^2)$
- Only $T^{(AH)}$ contributes at the lowest order, however it does contain Galileon effects.
- Asymptotically: $X \approx 1 - x_0 \dfrac{e^{-\eta\sqrt{\lambda}r}}{\sqrt{r}}, \quad \hat{A}_\theta \approx a_0\sqrt{r}e^{-\sqrt{2}e\eta r}$ N-O

$$\Rightarrow \hat{A}_r \approx \frac{b_0}{r}e^{-2\sqrt{2}e\eta r}, \qquad \omega \approx -\frac{1}{r^{3/2}}e^{-2\sqrt{2}er\eta}$$ New fields

Vortices

Minimal (and weak) coupling to gravity

- Diff. invariance

$$d\theta \to d\theta - \frac{\beta\,\omega}{2\,\alpha^2}\,e^{2\psi}\,dr$$

This redefinition effectively eats up the Galileon contribution that would source an off-diagonal component of the EMT.

The coordinate system adapts to the vortex, and the derivative interactions modulate the radial dependence of A_θ and X.

The Galileon effects can be seen as a further contribution to the angular deficit of the cosmic string. However, in the end it is only seen in the curvature invariants.

Some aspects of Galileons

- Scalar Galileons in strong gravity.

  JC, K. Koyama, G. Niz, G. Tasinato, JCAP10(2014)055

- Vortices in Higgs vector Galileons.

  JC, G. Tasinato, JHEP02(2016)063

- **Black holes in generalised Proca action.**

  JC, G. Niz, G. Tasinato, arXiv:1602.08697

Generalised Proca

L. Heisenberg, JCAP 1405 (2014) 015

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \sum_{n=2}^{5}\beta_n\mathcal{L}_n$$

$$\mathcal{L}_2 = G_2(X) \qquad X = -\frac{1}{2}A_\mu A^\mu$$

$$\mathcal{L}_3 = G_3(X)(D_\mu A^\mu)$$

$$\mathcal{L}_4 = G_4(X)R + G_{4,X}\left[(D_\mu A^\mu)^2 + c_2 D_\rho A_\sigma D^\sigma A^\rho - (1+c_2)D_\rho A_\sigma D^\sigma A^\rho\right]$$

$$\mathcal{L}_5 = \ldots$$

- G's are arbitrary functions,
- The non-minimal couplings to gravity keep the eqs. of motion 2nd order,
- 3 degrees of freedom (+ gravity).

Black holes in gen. Proca

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_{pl}^2}{2}R - \frac{1}{4}F^2 - \Lambda + \beta\left[\boxed{(D_\mu A^\mu)^2 - D_\mu A_\nu D^\nu A^\mu} - \frac{1}{2}A^2 R\right]\right\}$$

- GR + Maxwell + c.c. + particular choice $G_4 = X$ and $c_2 = 0$
- c_2 would lead to a redefinition of the coupling to F.

Total derivative in flat space

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{4}F^2 - \Lambda + \beta\, G_{\mu\nu}A^\mu A^\nu\right]$$

New contributions excited only by the coupling to gravity

Black holes in gen. Proca

$$\frac{M_{pl}^2}{2}G_{\mu\nu} = \frac{1}{2}\left[F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F^2\right] - \beta\left[\frac{1}{2}g_{\mu\nu}(D_\alpha A^\alpha)^2 - 2A_{(\mu}D_{\nu)}D^\alpha A_\alpha + g_{\mu\nu}A_\alpha D^\alpha D^\beta A_\beta\right.$$
$$+\frac{1}{2}g_{\mu\nu}D_\alpha A_\beta D^\beta A^\alpha - 2D^\alpha A_{(\mu}D_{\nu)}A_\alpha + D_\alpha\left(A_{(\nu}D_{\mu)}A^\alpha + A_{(\mu}D^\alpha A_{\nu)} - A^\alpha D_{(\mu}A_{\nu)}\right)$$
$$\left.-\frac{1}{2}\left(A^2 G_{\mu\nu} + A_\mu A_\nu R - D_\mu D_\nu A^2 + g_{\mu\nu}\Box A^2\right)\right]$$

$$D^\mu F_{\mu\nu} = -2\beta G_{\mu\nu}A^\mu$$

Key to avoid Bekenstein's "no Proca-hair" theorem

$$D^\mu F_{\mu\nu} = m^2 A_\nu$$

- A_t and A_i cannot be turned on simultaneously without violating time-reversal symmetry.
- There are only purely electrical and purely magnetic cases, for which one can show that certain integral identity is violate, concluding that the vector field profiles are not supported.

Black holes in gen. Proca

Asymptotically flat configurations without c.c.

- The coupling to $G_{\mu\nu}$ can switch off the terms that would violate time-reversal symmetry.

$$ds^2 = -f(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2,$$
$$A_\mu = (A_0(r), \pi(r), 0, 0).$$

$\downarrow$ Plug in the field equations

$$R = \frac{(2f - rf')(2f' + rf'')}{2r(f + rf')^2}$$

$\downarrow$

Asymptotically, if $f \sim r^n$, $R \sim r^{-2}$

Black holes in gen. Proca

Asymptotically flat configurations without c.c.

- Asymptotic flatness can be recovered for non-trivial profiles only if

$$\beta = \frac{1}{4}$$

Plug back to the exact field equations

$$f = h = 1 - \frac{2M}{r}$$

$$A_0 = \frac{Q}{r} + P$$

$$\pi = \frac{\sqrt{Q^2 + 2\,P\,Q\,r + 2\,M\,P^2\,r}}{r - 2M}$$

- Unique solution
- Asymptotic flatness not imposed
- The new integration constant, P, controls the asymptotic profile of π:

$$\pi \sim \frac{1}{r} \text{ if } P = 0$$

$$\pi \sim \frac{1}{\sqrt{r}} \text{ otherwise}$$

Black holes in gen. Proca

Asymptotically flat configurations without c.c.

* Abelian symmetry breaking terms completely screen the geometry from the vector fields. (Similar to stealthy Schwarzschild configurations in scalar-tensor gravity, Babichev, Charmousis JHEP1408(2014))
* $G_{\mu\nu} = 0$ avoids Bekenstein's theorem.
* All curvature invariants and the EMT are well behaved for r > 0.
* π does not contribute to $F_{\mu\nu}$. No violation of no-hair conjecture.
* However, an object coupled to A_μ can probe the longitudinal profile.
* Stable under spherical symmetric, but time dependent, perturbations provided that the charges are small. More general perturbations not tested yet.

Black holes in gen. Proca

Inclusion of c.c.

- Interestingly, we can impose asymptotic flatness again with

$$\beta = \frac{1}{4}$$

$$\begin{aligned}
f &= 1 - \frac{2M}{r} + \frac{4r^2\Lambda_P}{3} + \frac{4}{5}r^4\Lambda_P^2\,, \\
h &= f\,\left(1+2r^2\Lambda_P\right)^{-2}, \\
A_0 &= \frac{Q}{r} + P\left(1+\frac{2}{3}r^2\Lambda_P\right), \\
\pi &= \sqrt{\frac{A_0^2}{fh} - \frac{P^2(1+2r^2\Lambda_P) - 8M_{pl}^2 r^2\Lambda_P}{h}} \\
&= \frac{1+2r^2\Lambda_P}{\sqrt{f}}\left[\frac{\left(3Q + rP\left(3+2r^2\Lambda_P\right)\right)^2}{9r^2 f} - P^2(1+2r^2\Lambda_P) + 8M_{pl}^2 r^2\Lambda_P\right]^{1/2}
\end{aligned}$$

$$\Lambda_P \equiv \frac{\Lambda}{P^2 - 4M_{pl}^2}$$

$$P^2 \neq 4M_{pl}^2$$

Black holes in gen. Proca

Inclusion of c.c.

- The electric charge is still screened from the geometry.
- A_0 and π do not vanish at infinity.
- If Q = P = 0 the electric field is turned off, but the longitudinal profile is still sourced by the cosmological constant.
- Additional essential singularity if $\Lambda_P < 0$

$$R = -\frac{8\Lambda_P(15M - 5r + 4\Lambda_P^2 r^5)}{5r(1+2\Lambda_P r^2)^3}$$ (Vanishes at infinity)

- $G_{rr}=0$, this avoids Bekenstein's no-go arguments.
- $f = 1 - \frac{2M}{r} + \frac{4r^2\Lambda_P}{3} + \frac{4}{5}r^4\Lambda_P^2 \implies$ 1 real zero $\implies$ Only 1 horizon
- The horizon covers the sing. if $\sqrt{-2\Lambda_P}M > 4/15$

Black holes in gen. Proca

Generalisations, conclusions, ...

* Without cosmological constant it is easy to include slow rotation.
  * The metric is the slow rotation limit of Kerr (not Kerr-Newman).
  * The time and angular components of the vector field are the slow rotation limit of Kerr-Newman.

* Also without Λ, generalisation to arbitrary dimensions is straightforward

$$ds^2 = -\,f(r)\,dt^2 + h(r)^{-1}\,dr^2 + r^2\,d\Omega^2_{(d-2)}\,,$$
$$A_\mu = (A_0(r),\,\pi(r),\,0,\,\ldots\,,0)$$

$$\beta = \frac{d-3}{2\,d-4}$$

$$f = h = 1 - \frac{2M_d}{r^{d-3}}$$

$$A_0 = \frac{Q_d}{r^{d-3}} + P_d$$

$$\pi = \frac{\sqrt{Q_d^2 + 2\,M_d\,P_d^2\,r^{d-3} + 2\,P_d\,Q_d\,r^{d-3}}}{r^{d-3} - 2M_d}$$

Different asymptotic? (fix different β)

More general Lagrangians

Complete study of stability

Astrophysical consequences