Perturbation theory for LSS: from the IR to the UV

Mathias Garny (CERN)

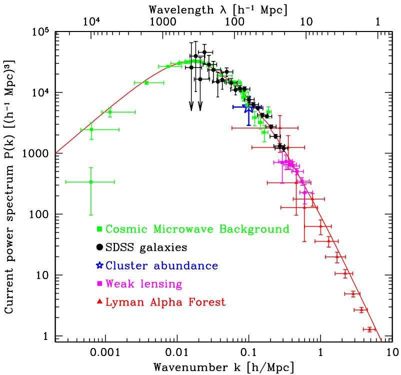
Cargese, 10.05.16

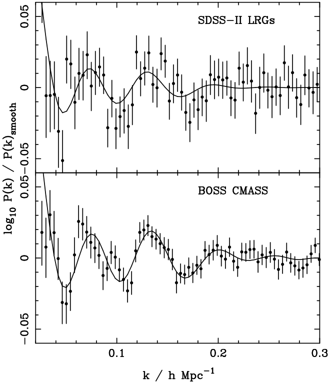
based on
1605.02149, 1512.05807 with D. Blas, M. Ivanov, S. Sibiryakov
1507.06665 with D. Blas, S. Flörchinger, N. Tetradis, U. Wiedemann
1508.06306 with T. Konstandin, R. Porto, L. Sagunski

The % Challenge

Power spectrum of density contrast $\delta(\mathbf{x}, z) = \rho(\mathbf{x}, z)/\bar{\rho}(z) - 1$

$$\langle \delta(\mathbf{k}, z)\delta(\mathbf{k}', z)\rangle = \delta^{(3)}(\mathbf{k} + \mathbf{k}')P(k, z)$$



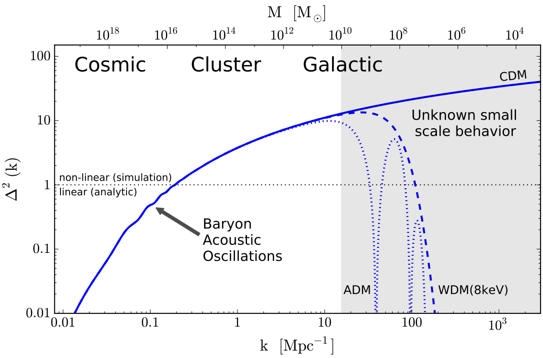


SDSS 2003 (Tegmark et al) 0310725

BOSS 1203.6594

Can we understand the weakly non-linear regime?

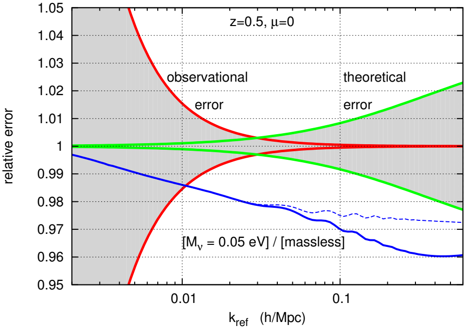
$$\Delta^2(k, z) = 4\pi k^3 P(k, z)$$



Kuhlen, Vogelsberger, Angulo 1209.5745

Theoretical error

Euclid forecast vs theoretical errors *Audren, Lesgourgues, Bird et. al. 1210.2194*



$$\sigma(M_\nu) \simeq \begin{cases} 25\text{meV} & \text{fiducial (2\%th. err. at } k = 0.4h/Mpc, z = 0.5) \\ 14\text{meV} & \text{th. err. } / = 10, k_{max} = 0.6h/Mpc \end{cases}$$

theoretical uncertainties from biased tracers, redshift-space distortions, relativistic effects, baryonic effects, non-linear clustering, ...

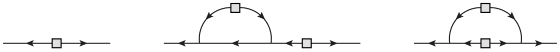
Perturbative solution

$$\delta(\mathbf{k}, z) = \sum_n \delta^{(n)}(\mathbf{k}, z)$$
$$= \sum_n \int_{k_1,\ldots,k_n} \delta_D(\mathbf{k} - \sum \mathbf{k}_i) F_n(\mathbf{k}_1, \ldots, \mathbf{k}_n; z)\, \delta(\mathbf{k}_1, z_{ini}) \cdots \delta(\mathbf{k}_n, z_{ini})$$

Power spectrum

$$P(k, z) = \overbrace{P_{11}(k)}^{P_{lin}} + \overbrace{(2P_{13} + P_{22})}^{P_{1-loop}} + (2P_{15} + 2P_{24} + P_{33}) + \ldots$$

where $P_{nm} = \langle \delta^{(n)} \delta^{(m)} \rangle$



e.g. $P_{22} = 2 \int_q d^3q\, F_2(\mathbf{q}, \mathbf{k} - \mathbf{q}; z)^2\, P_{ini}(q) P_{ini}(|\mathbf{k} - \mathbf{q}|)$

Fluid description

- Moments of Vlasov eq.

$$\begin{aligned} \frac{\partial \delta(\mathbf{x},\tau)}{\partial \tau} + \nabla \cdot \{(1+\delta(\mathbf{x},\tau))\mathbf{u}(\mathbf{x},\tau)\} &= 0 \quad \text{(continuity)} \\ \frac{\partial \mathbf{u}(\mathbf{x},\tau)}{\partial \tau} + \mathcal{H}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} &= -\nabla\Phi - \frac{1}{\rho}\nabla_j(\sigma_{ij}\rho) \quad \text{(Euler)} \\ \nabla^2\Phi(\mathbf{x},\tau) &= \frac{3}{2}\Omega_m\mathcal{H}^2\delta(\mathbf{x},\tau) \quad \text{(Poisson)} \end{aligned}$$

- stress tensor

$$\sigma_{ij} = \frac{1}{\rho} m \int d^3p \, \frac{p_i p_j}{a^2 m^2} f(\mathbf{x},\mathbf{p},t) - u_i u_j$$

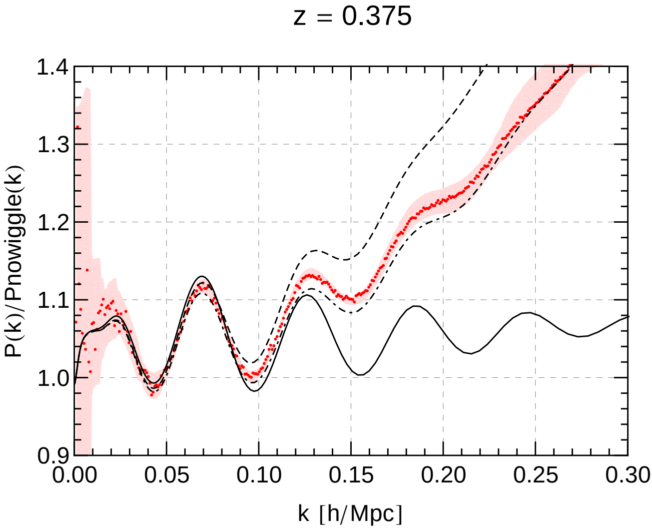
- Standard Perturbation Theory (SPT)

$\rightarrow$ Pressureless perfect fluid: $\sigma_{ij} = 0$

$\Rightarrow$ recursion relation for kernels $F_n(\mathbf{k}_1,\ldots,\mathbf{k}_n;z) \approx D_+(z)^n F_n^s(\mathbf{k}_1,\ldots,\mathbf{k}_n)$

$$F_1^s = 1, \qquad F_2^s(\mathbf{p},\mathbf{q}) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{p}\cdot\mathbf{q}}{pq}\left(\frac{p}{q}+\frac{q}{p}\right) + \frac{2}{7}\frac{(\mathbf{p}\cdot\mathbf{q})^2}{p^2q^2}$$

Status of SPT



black=linear, NLO(dashed), NNLO(dotdashed), red=Horizon Run 2

Diego Blas, MG, Thomas Konstandin 1309.3308

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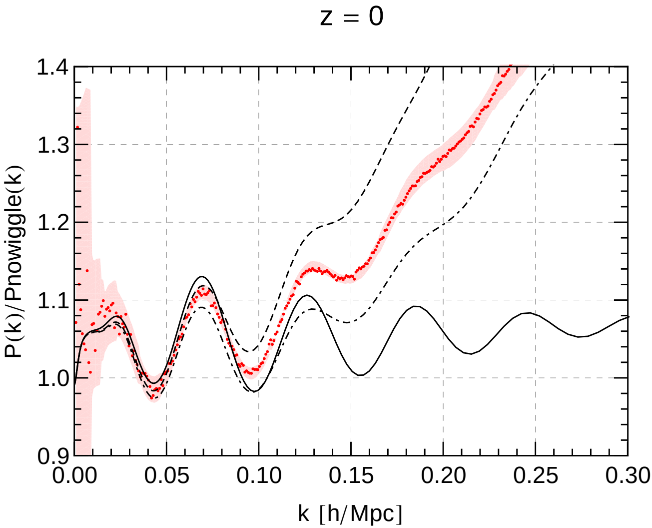

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Limitations at BAO scale

- Broadband shape at $z = 0$
  $\leftrightarrow$ impact of small scales (UV modes)

- Damping and phase-shift of BAO (broadening+shift in corr. fctn.)
  $\leftrightarrow$ impact of bulk flow (IR modes)

Scales

- $k \ll 0.1h/\mathrm{Mpc}$ bulk flows
- $k \sim 0.1 - 0.3h/\mathrm{Mpc}$ BAO
- $k \sim 0.3h/\mathrm{Mpc}$ non-linear
- $k \sim 1h/\mathrm{Mpc} \sim$ virial

IR and UV modes affect BAO scales via non-linear mode-coupling, need dedicated treatment for each regime.

Scales

- $k \ll 0.1h/\text{Mpc}$ bulk flows
- k_S
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- k_m
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- Resum IR modes $k < k_S$, coarse-grain (integrate out) UV modes $k > k_m$

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- Resum IR modes $k < k_S$, coarse-grain (integrate out) UV modes $k > k_m$
- Exact result independent of matching scales k_S, k_m
- Residual dependence in pert. calculation $\leftrightarrow$ theoretical error

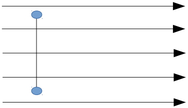
Outline

Impact of

- IR modes (large-scale bulk flows, broadening/shift of BAO peak)
- UV modes (small-scale pert., broadband shape)

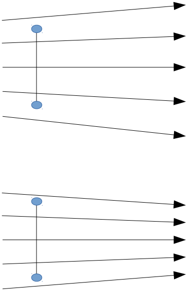
on modes at the BAO scale.

Bulk flows



- Overall displacement has no effect (Galilean inv./eq. principle)

Bulk flows



- Overall displacement has no effect (Galilean inv./eq. principle)
- Residual effect $\Rightarrow$ BAO broadening
- Rather well described in Zel'dovich app., LPT (other issues, $\rightarrow L_3$)

Bulk flows in SPT

- SPT kernels: IR divergence

$$F_3^s(k_1,k_2,k_3) \quad \to \quad \frac{k_1\cdot(k_2+k_3)}{3k_1^2}F_2^s(k_2,k_3) \qquad \text{for } k_1\to 0$$

$$F_n^s(\alpha k_1,\ldots,\alpha k_{n-1},k_n) \quad \to \quad \sim\frac{1}{\alpha^{n-1}} \qquad \text{for } \alpha\to 0$$

- SPT loop corr. to equal-time power spectrum

$$P_{nm}(k)=\langle\delta^{(n)}\delta^{(m)}\rangle\sim\frac{1}{\alpha^{n+m-2}}$$

- IR div. cancel when summing all diagrams at given order

$$\sum_{n+m=2L+2}P_{nm}(k)\sim\alpha^0$$

- Related to underlying Galilean symmetry/eq. principle *Jain, Bertschinger 95; Scoccimarro, Friemann 95; Creminelli, Norena, Simonovic, Vernizzi 13; Blas, MG, Konstandin 13; Carrasco, Foreman, Green, Senatore 13; Sugiyama, Spergel 13; Kehagias, Riotto 13; Peloso, Pietroni 13*

Time-sliced Perturbation Theory (TSPT)

- Goal: compute/resum *physical* effects from bulk flows
- Idea: formulation where cancellations of bulk effects are *manifest*

$\rightarrow$ disentangle time-evolution from statistical averaging

Blas, MG, Ivanov, Sibiryakov 1605.02149, 1512.05807

Time-sliced Perturbation Theory (TSPT)

- Time-evolution of probability distribution functional instead of fields

$$Z[J;\eta] = \int \mathcal{D}\delta\, \mathcal{P}[\delta;\eta] e^{\int_k \delta(k) J(k)}$$

- Corr. fctns. by taking J-derivatives

$$\langle \delta(k_1)\delta(k_2)\rangle = \frac{\partial^2 Z}{\partial J(k_1)\partial J(k_2)}\Big|_{J=0} = P(k_1)\delta_D(k_1+k_2)$$

- Time-evolution of $\mathcal{P}$: Liouville equation

$$\partial_\eta \mathcal{P} + \int_k \frac{\partial}{\partial \delta(k)} \dot{\delta}(k)\, \mathcal{P}[\delta;\eta] = 0$$

- First step: time-evolution of $\mathcal{P}$ to time $\eta = \ln(a)$
- Second step: statistical averaging in terms of fields $\delta(k)$ at time η

$\Rightarrow$ manifestly Galilean invariant

Time-sliced Perturbation Theory (TSPT)

- Perturbative evaluation in terms of $\delta(k)$ at time η

$$\mathcal{P}[\delta;\eta] = \mathcal{N}^{-1} e^{-\Gamma[\delta;\eta]}$$

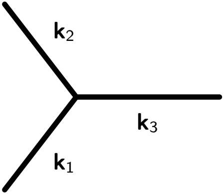
$$\Gamma[\delta;\eta] = \sum_n \int_{k_i} \Gamma_n(k_1,\ldots,k_n;\eta)\delta(k_1)\cdots\delta(k_n)$$

- Liouville eq. (+ICs) gives recursion for Γ_n
- EdS: time-dependence $\Gamma_n = D_+(z)^{-2}\,\bar{\Gamma}_n$

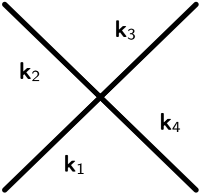
$$\begin{aligned}
\bar{\Gamma}_2(k_1,k_2) &= \frac{\delta_D(k_1+k_2)}{P_{lin}(k_1)} \\
\bar{\Gamma}_3(k_1,k_2,k_3) &= -\frac{(k_1+k_2)^2 k_1\cdot k_2}{2k_1^2k_2^2}\bar{\Gamma}_2(k_1+k_2,k_3) + \ \mathit{perm.}
\end{aligned}$$

Time-sliced Perturbation Theory (TSPT)

$$= P_{lin}(k,z) = D_+(z)^2 P_{lin}(k)$$



$$= -D_+^{-2}(z)\frac{\bar{\Gamma}_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{3!}$$



$$= -D_+^{-2}(z)\frac{\bar{\Gamma}_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)}{4!}$$

Time-sliced Perturbation Theory (TSPT)

- IR safe (finite for $k_1 \to 0$)

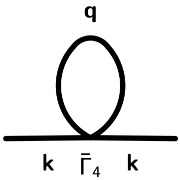
$$\bar{\Gamma}_3(k_1, k_2, k_3) \to \left(\underbrace{-\frac{k_1 \cdot k_2}{k_1^2 P_{lin}(k_3)} - \frac{k_1 \cdot k_3}{k_1^2 P_{lin}(k_2)}}_{1/k_1 \text{ terms cancel}} - \frac{k_1^2 k_2 \cdot k_3}{2k_2^2 k_3^2 P_{lin}(k_1)} \right) \delta_D(k_1+k_2+k_3)$$

- IR safe including sub-leading terms

$$\bar{\Gamma}_n(\alpha k_1, \ldots, \alpha k_{n-2}, k_{n-1}, k_n) \to\sim \alpha^0 \qquad \text{for } \alpha \to 0$$

Time-sliced Perturbation Theory (TSPT)

$P^{1-\mathrm{loop}}(\eta; k) =$



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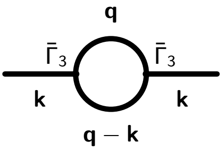
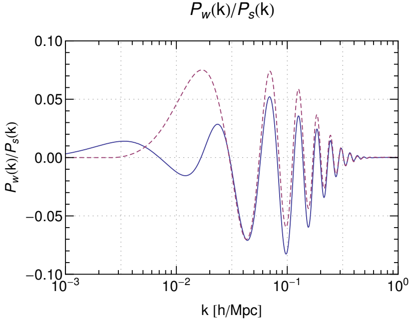


Each diagram is IR safe by itself, no large cancellations

Time-sliced Perturbation Theory (TSPT)

- Decomposition in smooth and oscillating (wiggly) component

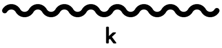
$$P_{lin}(k) = \underbrace{P_s(k)}_{\text{smooth}} + \underbrace{P_w(k)}_{\text{BAO}}$$



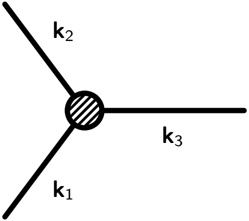
- $P_w(k) \sim \sin(kr_s)e^{-(k/k_{Silk})^2}$, $r_s = 110$ Mpc$/h$, $1/k_{Silk} = 5$Mpc$/h$

$\Rightarrow$ decomposition $\Gamma_n = \Gamma_n^s + \Gamma_n^w$

Time-sliced Perturbation Theory (TSPT)

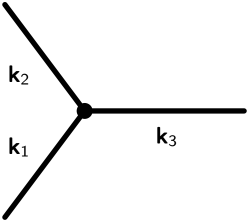


$= P_w(k)\,,$



$= -\frac{1}{3!}\Gamma_3^w(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$

$= P_s(k)\,,$



$= -\frac{1}{3!}\Gamma_3^s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$

Bulk flows in TSPT

- Physical IR enhancement for $1/r_s \ll k_1 \ll k_2, k_3$

$$\Gamma_3^w(k_1, k_2, k_3) \quad \to \quad \frac{k_1 \cdot k_2}{k_1^2} \left(\frac{P_w(k_2 + k_1) - P_w(k_2)}{P_s(k_2)^2} \right) \delta_D(k_1 + k_2 + k_3)$$

$$= \quad \Bigg[\underbrace{\frac{k_1 \cdot k_2}{k_1^2} \left(e^{k_1 \cdot \nabla} - 1 \right)}_{\equiv \mathcal{D}_{k_1}} P_w(k_2) \Bigg] \frac{1}{P_s(k_2)^2} \delta_D(k_1 + k_2 + k_3)$$

$$\mathcal{D}_{k_1} e^{ikr_s} \quad \sim \quad \frac{k_1 \cdot k}{k_1^2} (e^{ik_1 \mu r_s} - 1) e^{ikr_s} \sim \begin{cases} \mathcal{O}(k/k_1) & 1/r_s \ll k_1 \ll k \\ \mathcal{O}(kr_s) & k_1 \ll 1/r_s \ll k \end{cases}$$

$$\sim \quad \mathcal{O}(1/\varepsilon)$$

$$\varepsilon \equiv \max\left[k_1/k,\ 1/(kr_s)\right]$$

$$\Gamma_n^w(k_1, \ldots, k_n) \quad \to \quad (-1)^{n-2} \mathcal{D}_{k_1} \mathcal{D}_{k_2} \cdot \mathcal{D}_{k_{n-2}} P_w(k_n) \frac{1}{P_s(k_n)^2} \delta_D(\sum k_i)$$

$$\sim \quad \mathcal{O}(1/\varepsilon^{n-2})$$

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- k_S
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Bulk flows in TSPT

- Separate into soft and hard loops, separation scale k_S
- Expansion parameters

$$\sigma_S^2 = \int_{q<k_S} d^3q P(q)$$
$$\sigma_h^2 = \int_{q>k_S} d^3q P(q)$$
$$\varepsilon \equiv \max\left[k_S/k,\, 1/(kr_s)\right]$$

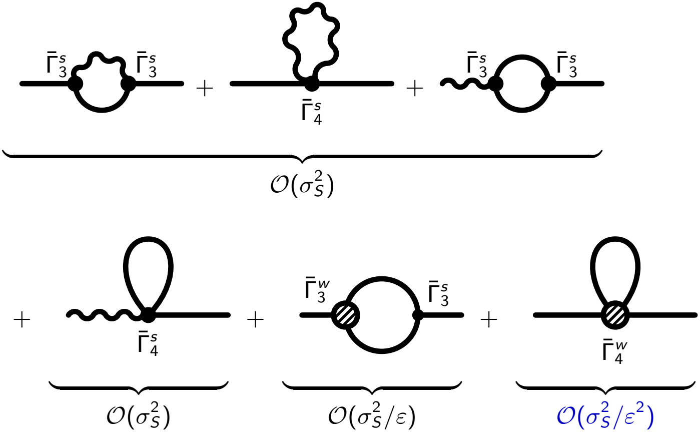
- Exact result independent of $k_S \Rightarrow$ residual dependence gives estimate of th. error
- For $1/r_s \sim 0.01h/\mathrm{Mpc} \ll k_S \lesssim k_{Silk} \sim 0.2h/\mathrm{Mpc} \Rightarrow$

$$\sigma_S^2 \sim 0.1 - 0.2$$
$$\sigma_S^2/\varepsilon^2 \sim \mathcal{O}(1)$$

Bulk flows in TSPT

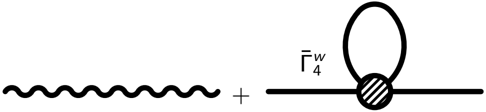
Power spectrum, one soft loop ($q < k_S$)

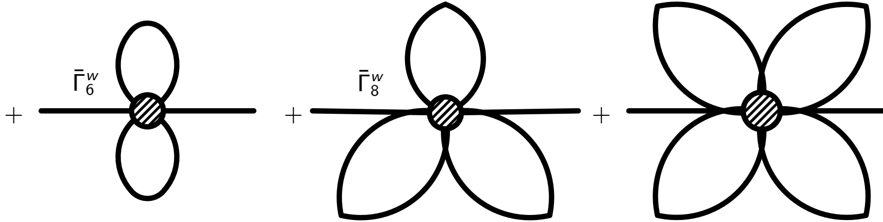
$P_w^{1-loop}(\eta; k) =$



IR resummation in TSPT

$P_w^{IR\ res,LO}(\eta;k) =$

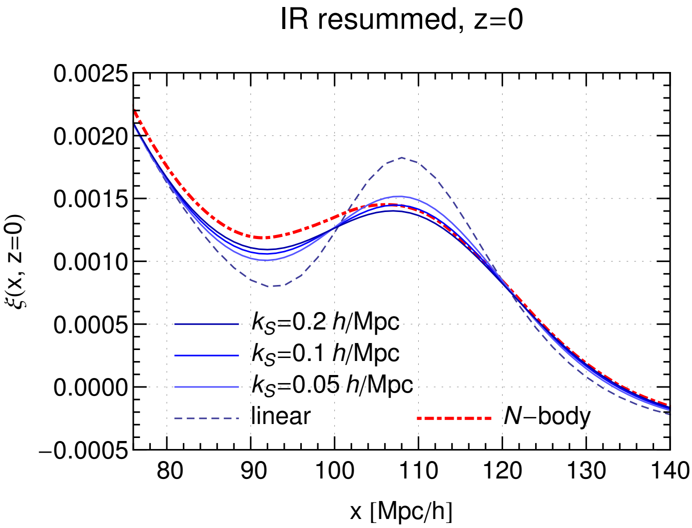




$$= \exp\left(-\frac{1}{2}\int_{q<k_S} d^3q P_s(\eta;q)\mathcal{D}_q\mathcal{D}_{-q}\right) P_w(\eta;k)$$

Resum all contributions of order $(\sigma_S^2/\varepsilon^2)^{L_s}$, $L_s = 1, 2, 3, \ldots$

IR resummation in TSPT



Blas, MG, Ivanov, Sibiryakov 1605.02149, 1512.05807

IR resummation in TSPT

- LO agrees essentially with deriv. based on symmetry arguments

  Baldauf, Mirbabayi, Simonovic, Zaldarriaga, 1504.04366

- TSPT: systematic power counting
- Two types of NLO corrections:
  - NLO_h: Hard loop $\propto \sigma_h^2 \times (\sigma_S^2 \times 1/\varepsilon^2)^{L_s}$, $L_s = 1, 2, 3, \ldots$
  - NLO_s: Subleading soft $\propto \varepsilon \times (\sigma_S^2 \times 1/\varepsilon^2)^{L_s}$, $L_s = 1, 2, 3, \ldots$

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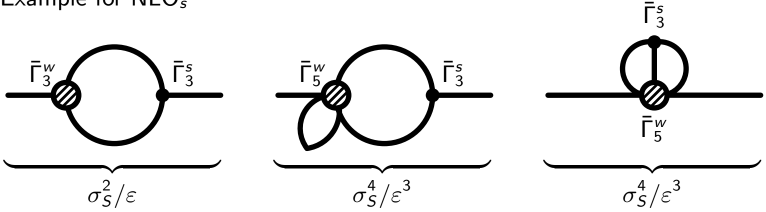
- Example for NLO$_s$



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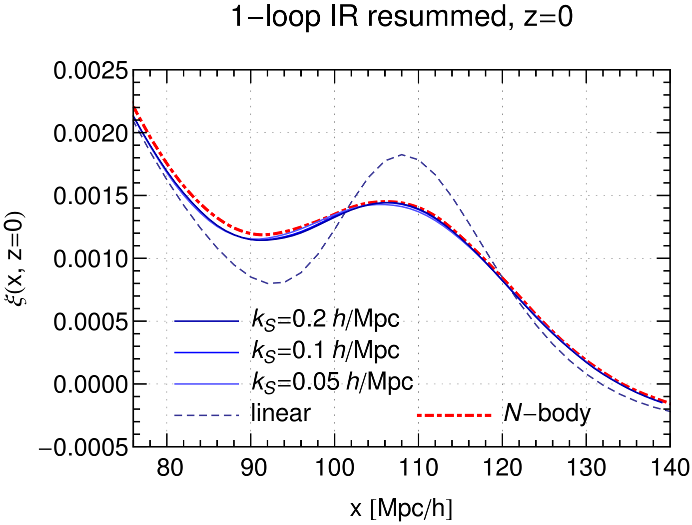
  - NLO$_s$: Subleading soft $\propto \varepsilon \times (\sigma_S^2 \times 1/\varepsilon^2)^{L_s}$, $L_s = 1, 2, 3, \ldots$
- At the end of the day, rather compact result

$$P_w^{IR\ res, LO+NLO} = P_s + (1 + g^2\mathcal{S})e^{-g^2\mathcal{S}}P_w + P^{1-loop}[P_s + e^{-g^2\mathcal{S}}P_w] \\ + g^4(\mathcal{S}^a + \varkappa\mathcal{S}^b)e^{-g^2\mathcal{S}}P_w$$

where $g \equiv D_+(z)$, $\mathcal{S} \equiv \frac{1}{2}\int_{q<k_S} d^3q P_s(\eta; q)\mathcal{D}_q\mathcal{D}_{-q}$

IR resummation in TSPT



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Shift of BAO peak

- Non-linear clustering $\Rightarrow$ shift in BAO peak position $\delta x/x \sim -0.4\%$

e.g. Crocce, Scoccimarro 0704.2783

- Potentially sensitive to modifications of gravity

e.g. Bellini, Zumalacarregui, 1505.03839

- TSPT: NLO$_s$ terms $\leftrightarrow$ odd number of derivatives

$$P_w^{IR\ res,NLO}(z,k) = D(z)^4 e^{-k^2 D(z)^2 \Sigma^2} \left(H(k) P_w(k) + S(z,k) \frac{dP_w(k)}{dk} \right) .$$

$$S(z,k) = s\,k + \left(\Sigma_{Silk}^2 + D(z)^2 \Sigma_b^2\right) k^3 \ ,$$

- Analytical result for BAO shift

k_S , h/Mpc	$\Delta x^{NLO}/x_{BAO}$		
	Full	App 1	App 2
0.05	-0.38%	-0.43%	-0.46%
0.1	-0.41%	-0.45%	-0.40%
0.2	-0.45%	-0.50%	-0.32%