Gravitational Slip in Modified Gravity Theories

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HOT TOPICS IN MODERN COSMOLOGY
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WORK IN PROGRESS WITH
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Outline

- Motivations & scientific context
- Assumptions
- Gravitational slip → modified tensors
  - Overview: Bigravity; Einstein-Aether; Horndeski
- **Modified tensors → gravitational slip?**
- Conclusions

Motivation & Scientific context

- Accelerated expansion of the Universe: beyond reasonable doubt.
- ΛCDM or Modification of Gravity?
  - Λ? too small...
  - Beyond: New degree(s) of freedom
- Observations of background and perturbations: Can be used to discriminate and rule out models.

Why is this relevant?

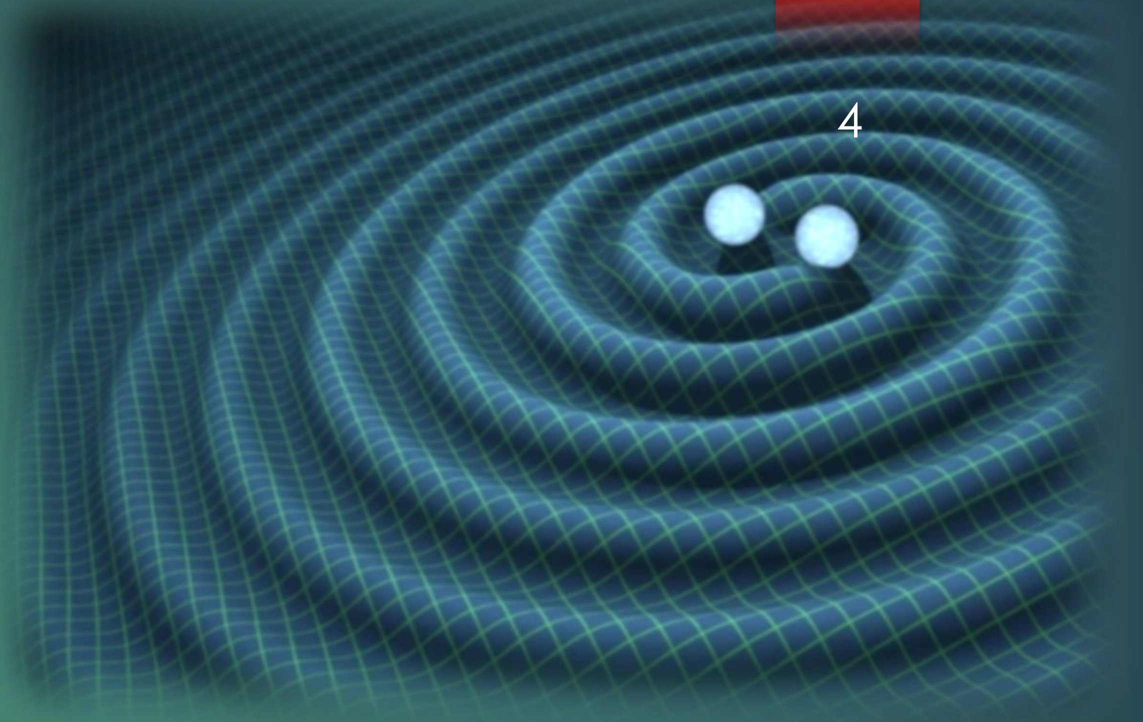
- Combining probes of Gravitational Waves and Large Scale Structure: New handle for constraining theories

- Both Gravitational Slip and Gravitational Waves impact on B-modes:
  - GW's: impact on $l \approx 100$
  - c_T shifts pick position of primordial B-modes
  - Lensing effects mostly around $z \sim 1$, $l \gtrsim 100$

- Friction term in GW's is degenerate with r (but background information can remove degeneracy)

Amendola, Ballesteros and Pettorino (2014)
Raveri, Baccigalupi, Silvestri and Zhou (2014)

Why is this relevant?



- Exciting times:
  - Detection of Gravitational waves
    - Ligo:

$m_{gw} \leq 1.2 \times 10^{-22}$ eV arrival time of different frequecies

$c_{gw} \lesssim 1.7$ time delay between detectors

(model-dependent bounds on c_{gw} much more stringent)

Blas et al 2016

Courtesy Caltech/MIT/LIGO Laboratory

Why is this relevant?

- Exciting times:
  - Future probes of the Universe's largest scales: wide, deep and unprecendented precision.
- Standing out: Model-independent observable:

Gravitational Slip: $$\frac{\Phi}{\Psi} \equiv \eta$$

Weak Lensing

$$-\int_0^{r_s} dr\, \frac{r_s - r}{r\, r_s} \Delta_\Omega(\Phi + \Psi)$$

Redshift space-distortions

$$\frac{1}{\mathcal{H}} \partial_r(\boldsymbol{V}.\boldsymbol{n}) \approx \frac{1}{\mathcal{H}^2} \partial_r^2 \Psi$$

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Motta et al 2013
Amendola et al 2013

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~10% accuracy with Euclid

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Assumptions

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- First order in linear perturbations on FRW

$$ds^2 = -(1+2\mathbf{\Psi})dt^2 + a^2 2B_{,i}\, dt dx^i + a^2\left(1 - 2\mathbf{\Phi}\delta_{ij} + 2E_{,ij} + h_{ij}\right)dx^i dx^j$$

- Universe filled with dust (neglecting neutrinos and radiation).
- Matter moves on geodesics of $g_{\mu\nu}$ in the Jordan frame.

Assumptions

- Modification of gravity: addition of one extra
- Tensor: Massive Bigravity
- Vector: Einstein-Aether
- Scalar: Horndeski

Gravitational Slip & Gravitational Waves

- Slip equation

Gravitational Slip: property of geometry
Anisotropic Stress: property of matter

$$\Phi - \Psi = \sigma(t)\Pi(t,k) + \pi_m$$

- General modification of Gravity waves:

$$h''_{ij} + (2+\nu)\mathcal{H}\, h'_{ij} + c_T^2 k^2 h_{ij} + a^2\mu^2 h_{ij} = a^2\Gamma\gamma_{ij}$$

Spatial traceless part of $\delta^{(1)}G_{\mu\nu}$

Saltas et al 2014

Massive Bigravity

$$U(g,f) = M_P^2 \int d^4x \sqrt{-g} \sum_{n=0}^{4} \beta_i e_i$$

e_i , symmetric polynomials of $\sqrt{g^{-1}f}$

- Matter minimally coupled to g
- 1 massless + 1 massive graviton: 2+5 dofs, of which 1 longitudinal mode

Perturbations:

$$ds_g^2 = a^2 \Big\{ -(1+2\Psi_g)dt^2 + 2B_{g,i}\, dt dx^i + (1 - 2\Phi_g \delta_{ij} + 2\boldsymbol{E}_{g,ij} + h_{g_{ij}}) dx^i dx^j \Big\}$$

$$ds_f^2 = b^2 \Big\{ -c^2(1+2\Psi_f)dt^2 + 2B_{f,i}\, dt dx^i + (1 - 2\Phi_f \delta_{ij} + 2\boldsymbol{E}_{f,ij} + h_{f_{ij}}) dx^i dx^j \Big\}$$

- Scalar dofs $\{E_g, E_f\}$

Massive Bigravity

Slip equation:

$$\Phi - \Psi = a^2 r \bar{Z}(E_g - E_f)$$

Gravitational waves:

$$h_g'' + 2\mathcal{H} h_g' + k^2 h_g + a^2 r \bar{Z} h_g = -a^2 r \bar{Z} h_f$$

$$\sigma = a^2 r \bar{Z}, \qquad \bar{Z} = \bar{Z}(\beta_i, r)$$

$$\Pi = E_g - E_f$$

$$\nu = 0 \qquad \mu^2 = r\bar{Z}$$

$$c_T^2 = 1 \qquad \Gamma = -r\bar{Z}$$

Einstein-Aether

$$\mathcal{L}_v = \sum_{n=1}^{3} \mathcal{L}_n[\beta_n, (\nabla u)^2] + \lambda(u^\alpha u_\alpha + 1)$$

$$\delta u^i = \partial^i \delta u + \delta \bar{u}^i, \qquad \partial_i \delta \bar{u}^i = 0$$

- Dimensionless parameters β_i
- Non-vanishing vacuum violates LS: picks out a preferred frame, the Aether

Perturbations:

$$\Theta \equiv \partial_i V^i$$

$$\delta u^i \equiv \frac{1}{a} V^i$$

Einstein-Aether

Slip equation:

$$\Phi - \Psi = (\beta_1 + \beta_3)\frac{(a^2\Theta)'}{(k\ a)^2}$$

$$\sigma = (\beta_1 + \beta_3)$$

$$\Pi = \frac{(a^2\Theta)'}{(k\ a)^2}$$

Gravitational waves:

$$h_g'' + 2\mathcal{H}h_g' + \frac{k^2}{(1-\beta_1-\beta_3)}h_g = 0$$

$$\nu = 0 \qquad \mu^2 = 0$$

$$c_T^2 = (1-\beta_1-\beta_3)^{-1} \qquad \Gamma = 0$$

Horndeski

$$\mathcal{L} = \sum_{n=2}^{5} \mathcal{L}_n(\phi, X)$$

$$X = -\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$$

Perturbations:

$$v_X \equiv -\frac{\delta\phi}{\phi'}$$

- Run rate of M_P: α_M
- Braiding: α_B
- Kineticity: α_K
- Tensor speed excess: α_T
- *GR* : $\alpha_M = \alpha_B = \alpha_K = \alpha_T = 0$

Bellini & Sawicki, 2014

Horndeski

Slip equation:

$$\Phi - \Psi = \alpha_T \Phi + \mathcal{H}(\alpha_M - \alpha_T) v_X$$

Gravitational waves:

$$h_g'' + (2 + \alpha_M)\mathcal{H} h_g' + (1 + \alpha_T) k^2 h_g = 0$$

$$\sigma = \alpha_M - \alpha_T$$

$$\Pi = \frac{\alpha_T}{\sigma}\Phi + \mathcal{H} v_X$$

$$\nu = \alpha_M \qquad \mu^2 = 0$$

$$c_T^2 = 1 + \alpha_T \qquad \Gamma = 0$$

Modified Tensors → Gravitational Slip?

(In a modified gravity model, are there parameter choices which cancel gravitational slip? And is this configuration stable in time?)

Method

- Slip constraint:

$$\Phi - \Psi = C[\boldsymbol{X}] \equiv \boldsymbol{A}(t,k).\boldsymbol{X}(t,k) = 0$$

- Can we cancel away gravitational slip in a theory where gravitational waves are modified?
- Can the above constraint be mantained through cosmological evolution?

Method

- Fix a gauge
- Eliminate non-dynamical dofs:
- On the scalar sector: two coupled EoM:

$$\psi_1''=\psi_1''(\psi_1',\psi_1,\psi_2',\psi_2)$$

$$\psi_2''=\psi_2''(\psi_1',\psi_1,\psi_2',\psi_2)$$

Method

$$\boldsymbol{X}(t;k) = \{\psi_1, \psi_1', \psi_2, \psi_2'\}$$

4-dim phase space

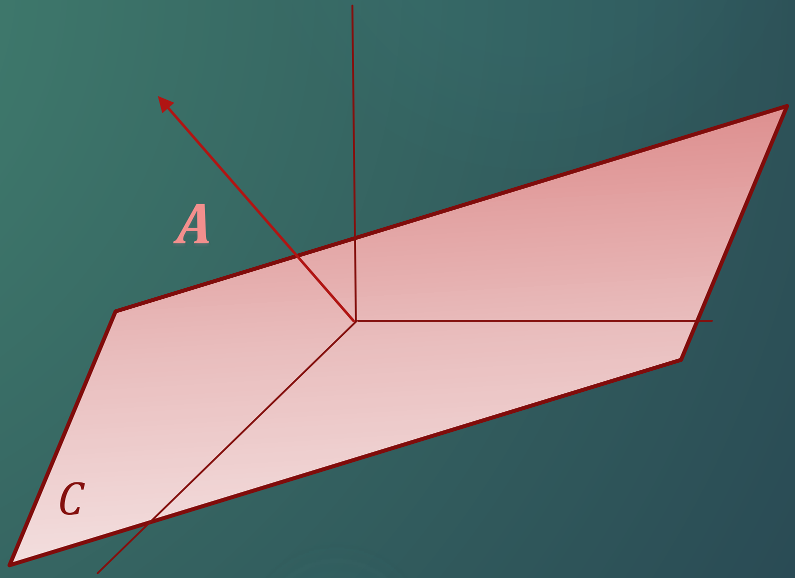
$$C = \boldsymbol{A}.\boldsymbol{X} = 0,$$

- The vector $\boldsymbol{A}$ in model-space defines a 3D hyperplane passing through the origin

$$C' = \boldsymbol{A}'.\boldsymbol{X} + \boldsymbol{A}.\boldsymbol{X}' = 0$$

EoM: $\mathbf{X}' = \boldsymbol{M}.\mathbf{X}$

$$C' = \boldsymbol{B}.\boldsymbol{X} = 0$$



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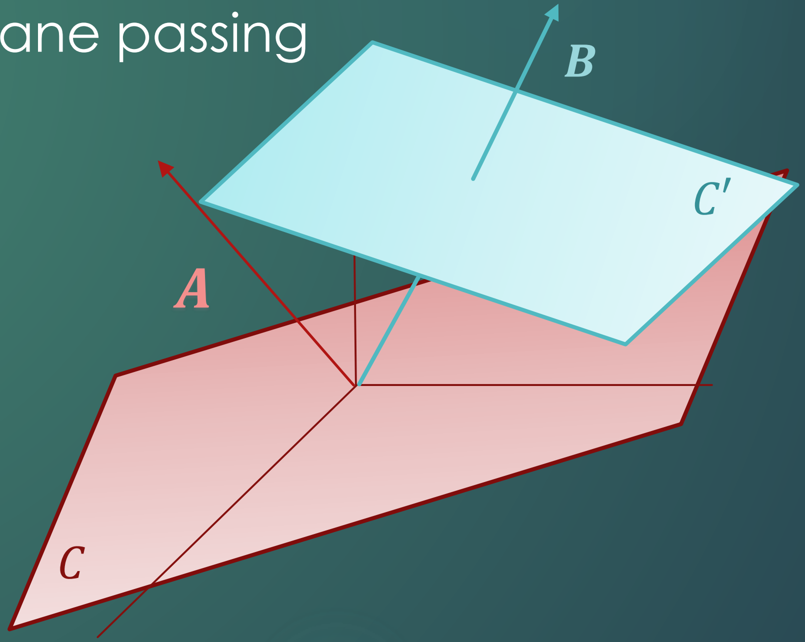
EoM: $\mathbf{X}' = \boldsymbol{M}.\mathbf{X}$

$$C' = \boldsymbol{B}.\boldsymbol{X} = 0$$

- $\boldsymbol{B}$ generically defines a LI 3D hyperplane that contains the origin



Method

- In 4D: at most 4 linearly-independent hyperplanes.

- Cases:
  - a) $C' = \lambda_0\, C$ phase space is a 3D subspace
  - b) $C'' = \lambda_0\, C + \lambda_1 C'$ phase space is a 2D subspace
  - c) $C''' = \lambda_0\, C + \lambda_1 C' + \lambda_2 C''$ phase space is a 1D subspace

Method

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- If $\{C, C', C'', C'''\}$ are all linearly-independent, the only intersection is the origin: no perturbations

Method

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Increasing fine-tuning

- If $\{C, C', C'', C'''\}$ are all linearly-independent, the only intersection is the origin: no perturbations

$\mathcal{E}x$: Cancelling Gravitational Slip in Massive Bigravity

$$C = a^2 r\bar{Z}\left(E_g - E_f\right) = \mathcal{Z}\,\Delta E = \boldsymbol{V_0}.\boldsymbol{X}$$

$$\boldsymbol{V_0} = \{\mathcal{Z}, -\mathcal{Z}, 0, 0\}, \qquad \boldsymbol{X} = \{E_g, E_f, E_g', E_f'\}$$

$$C = 0$$

Model-Space | Phase-Space

$$\mathcal{Z} = 0$$

$$\Delta E = E_g - E_f = 0$$

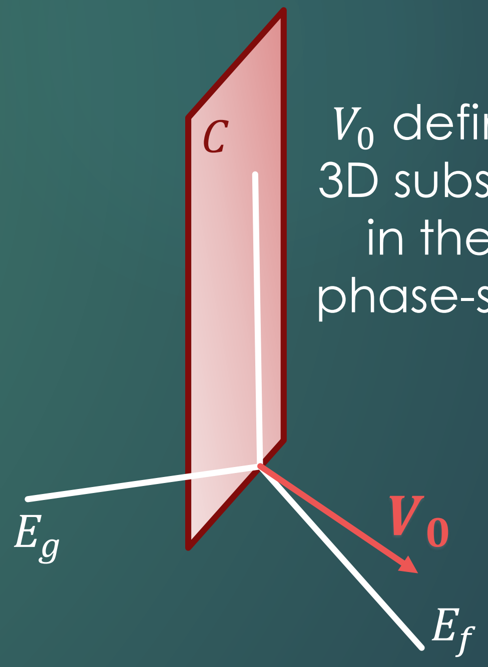
GW not modified

$$h_g'' + 2\mathcal{H}h_g' + k^2 h_g + \mathcal{Z}h_g = -\mathcal{Z}h_f$$

$\mathcal{Z} = 0 \rightarrow c = 1$, de Sitter

Trivially: All higher derivatives hyperplanes

$C \approx C' \approx C'' \approx \cdots = 0$



V_0 defines a 3D subspace in the 4D phase-space

$\mathcal{E}x$: Cancelling Gravitational Slip in Massive Bigravity

$$C' = \mathcal{Z}'\,\Delta E + \mathcal{Z}\,\Delta E' = \boldsymbol{V_1}.\boldsymbol{X}$$

$\boldsymbol{V_1} = \{\mathcal{Z}', -\mathcal{Z}', \mathcal{Z}, -\mathcal{Z}\}$, $\{\boldsymbol{V_0}, \boldsymbol{V_1}\}$ ***LI*:**

$$C' \approx C \approx 0$$

Model-Space	Phase-Space
$\mathcal{Z} = 0$	$\Delta E' = E'_g - E'_f = 0$
Intersection in a 3D subsapce	Intersection in a 2D subsapce

GW not modified

$\mathcal{Z} = 0 \rightarrow c = 1$, de Sitter

Trivially: All higher derivatives hyperplanes

$$C \approx C' \approx C'' \approx \cdots = 0$$

$\mathcal{Ex}$: Cancelling Gravitational Slip in Massive Bigravity

$$C'' = \mathcal{Z}''\Delta E + 2\,\mathcal{Z}'\Delta E' + \mathcal{Z}\,\Delta E'' \qquad + \quad EoM$$

$$C'' = c_0\,\Delta E + c_1\,\Delta E' + c_3\,E_g' + c_4\,E_g = \boldsymbol{V_2}.\boldsymbol{X}$$

$$\{\boldsymbol{V_0}, \boldsymbol{V_1}, \boldsymbol{V_2}\}\ \boldsymbol{LI}$$

$$C'' \approx C' \approx C \approx 0$$

Intersection in a 2D subsapce

Model-Space

$$c_3 = c_4 = 0 \rightarrow de\ Sitter$$

Intersection in a 1D subsapce

Phase-Space

$$E_g' = -\frac{c_4}{c_3}\,E_g$$

$\mathcal{E}x$: Cancelling Gravitational Slip in Massive Bigravity

$$C''' = d_0\, \Delta E + d_1\, \Delta E' + d_2 \left(E_g' + \frac{c_4}{c_3}\, E_g \right) + d_3 E_g = \boldsymbol{V_3}.\boldsymbol{X} = 0$$

$$\{\boldsymbol{V_0}, \boldsymbol{V_1}, \boldsymbol{V_2}, \boldsymbol{V_3}\}\ \boldsymbol{LI}$$

$$C''' \approx C'' \approx C' \approx C \approx 0$$

Model-Space ⟵ ⟶ Phase-Space

Intersection in a 1D subsapce

LD :

$d_3 = 0 \to de\ Sitter$

L I :

Intersection is the origin:

no perturbations

Conclusion:

It is not possible to cancel away Gravitational Slip in Massive Bigravity if Gravitational Waves are modified.

Modified GW → Graviational Slip

$\mathcal{E}x$: Cancelling Gravitational Slip in Horndeski

$$C = 0$$

Model-Space ← → Phase-Space

No Modified GW

Reduce phase-space: 3D

$$C' \approx C \approx 0$$

Model-Space ← → Phase-Space

No Modified GW

Reduce phase-space: 2D

$$C'' \approx C' \approx C$$

$\mathcal{E}x$: Cancelling Gravitational Slip in Horndeski

$$C'' \approx C' \approx C$$

Model-Space

Phase-Space

Two set of solutions:

1- Analytically solved:

a) No modified MG

b) No evolution of δ_m

2- Numerically solved,

a)Scalar field decouples

b)Gravity is switched off

L I :

Intersection is 1D subspace

C''' ?

Remarks and conclusions

- Modifications of Gravitational waves seem to imply the presence of Gravitational slip
- This can be used to rule out theories: e.g. B-modes of CMB polarization, direct detection of GWs and LSS.
- For more dof‘s extension of procedure is straightforward, but do „conclusions“ hold?
- Could it be promising to extend this to vector perturbations?

Thank you!

Beyond Horndeski

Effective action term $\alpha_H \, \delta N \, \delta R$:
no tensor contributions

Slip equation:

$$\Phi - \Psi = \alpha_T \Phi + \mathcal{H}(\alpha_M - \alpha_T) v_X - \alpha_H \, (\Psi + v_X')$$

$$\sigma = \alpha_M - \alpha_T$$

$$\Pi = \frac{\alpha_T}{\sigma} \Phi + \mathcal{H} v_X - \frac{\alpha_H}{\sigma} (\Psi + v_X')$$

Gravitational waves:

$$h_g'' + (2 + \alpha_M)\mathcal{H} h_g' + (1 + \alpha_T) k^2 h_g = 0$$

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Counter example