Ghost Condensation and Horizon Entropy

Shinji Mukohyama
(YITP, Kyoto)

Ref. arXiv: 1602.06511 w/ S.Jazayeri, R.Saitou and Y.Watanabe

Based also on other works with
N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, T. Furukawa, K. Ichiki, K. Izumi, M. Luty, N. Sugiyama, J. Thaler, T. Wiseman, M. Zaldarriaga

Can we change gravity in IR?

- **Change Theory?**
  Massive gravity **Fierz-Pauli 1939**
  DGP model **Dvali-Gabadadze-Porrati 2000**

- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

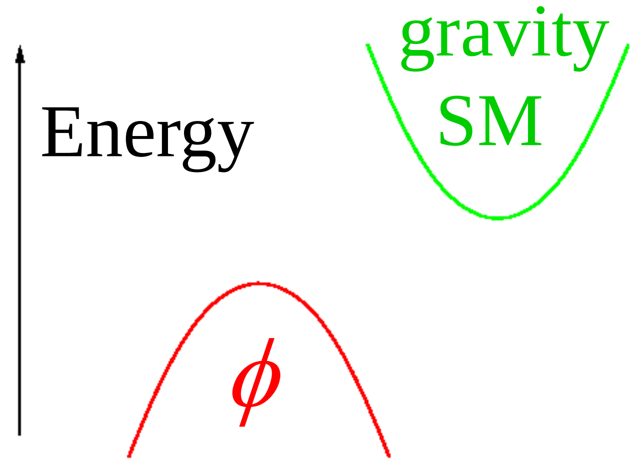
Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

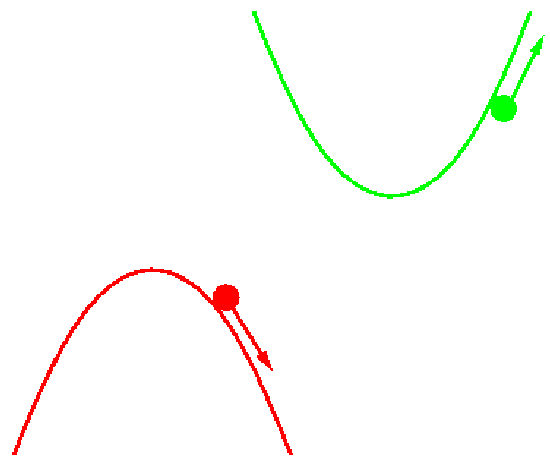
Suppose

$$L = L_{grav} + L_{SM} + L_\phi$$

$$L_\phi = -(\partial\phi)^2 + \cdots$$



Any coupling $\Rightarrow$ instability



Vacuum $\Rightarrow$

In analogy with Higgs potential, can this be stabilized?

$$L_\phi = M^4\left[-(\partial\phi)^2 + (\partial\phi)^4 \cdots\right] \quad ?$$

Clearly $(\partial\phi)^4 \cdots$ are higher dim ops. Really we should consider

$$L_\phi = P\left((\partial\phi)^2\right) + \tilde{P}\left((\partial\phi)^2\right)Q(\Box\phi) + \cdots$$

(Shift symmetry is assumed.)

Naively no sensible EFT.

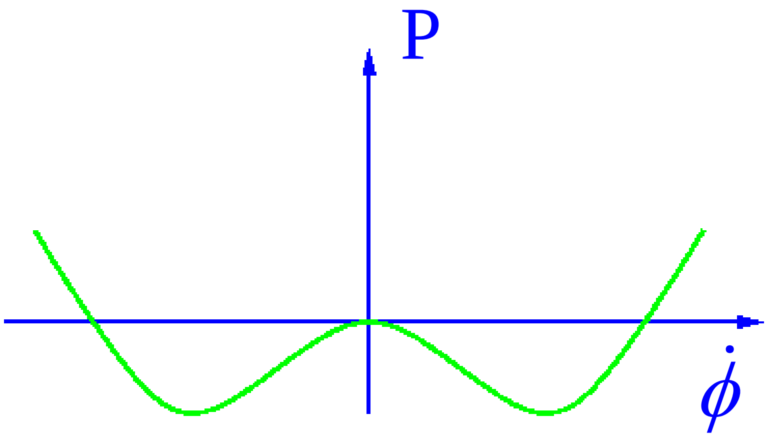
<u>NOT THE CASE.</u>

There is a sensible EFT + derivative expansion.

For simplicity

$$L_\phi = P\left((\partial\phi)^2\right)$$

in flat background.



Eq of motion

$$\partial_\mu\left[P' \cdot \partial^\mu \phi\right] = 0$$

Clearly any $\partial_\mu \phi =$ constant is a solution!
Suppose $\partial_\mu \phi$ =constant + timelike.
Go to frame where

$$\dot{\phi} = c, \partial_i \phi = 0 \Rightarrow \phi = ct$$

Solution for any c!

Look at small fluctuations

$$\phi = ct + \pi$$

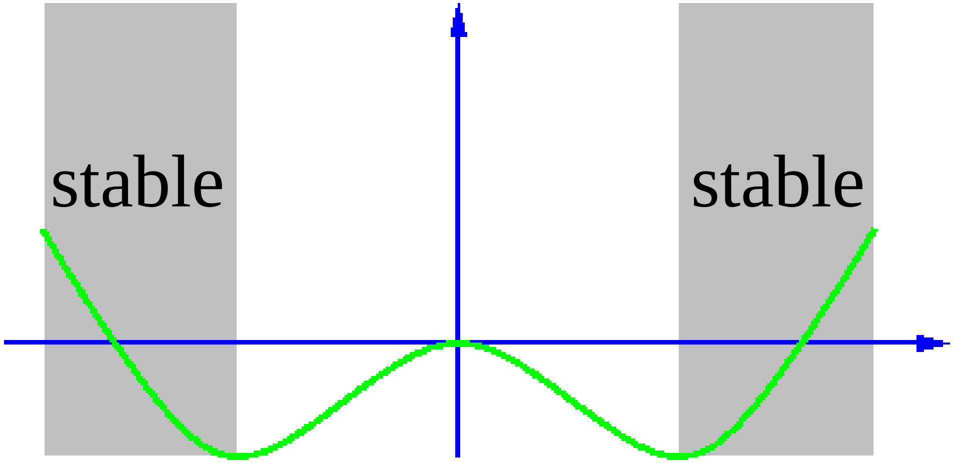
$$S = \int d^4x \left[\left(P'(c^2) + 2c^2 P''(c^2) \right) \dot{\pi}^2 - P'(c^2) \cdot (\nabla \pi)^2 \right]$$

So for $P'(c^2) + 2c^2 P''(c^2) > 0$

$$P'(c^2) > 0$$

NORMAL sign kinetic terms.

STABLE



In this language, we can have a good EFT because the background sits at some value c.

$\Longrightarrow$ doesn't sample entire function $P\left((\partial\phi)^2\right)$.
Taylor expansion of $P\left((\partial\phi)^2\right)$ around $(\partial\phi)^2 = c^2$ makes perfect sense.

Small fluctuations controlled by small # of parameters at low energies.

So far, possible backgrounds are labeled by continuous parameter c.

Situation changes in presence of gravity, in expanding universe.

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

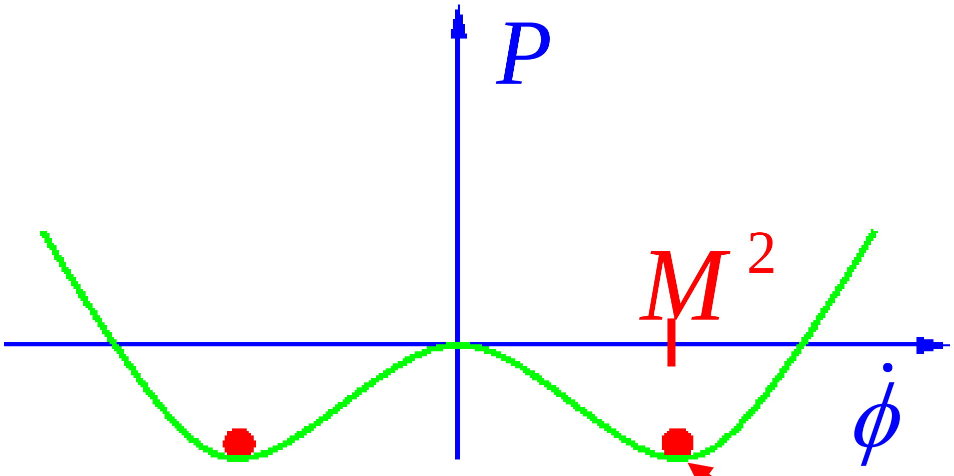
$$S = \int d^4x\sqrt{-g}\,P(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi)$$

$$T_{\mu\nu} \sim \; Pg_{\mu\nu} + P'\partial_\mu\phi\partial_\nu\phi$$

E.O.M.

$$\partial_t[a^3P'\cdot\dot{\phi}] = 0 \;\Longrightarrow\; P'\dot{\phi} \to 0 \text{ as } a \to \infty$$

$\Longrightarrow$ $\dot{\phi} = 0$ (unstable ghost background) or $P'(\dot{\phi}^2) = 0$



In this language, we can have a good EFT because the background sits at the special value.

$\Rightarrow$ doesn't sample entire function $P\left((\partial\phi)^2\right)$.
Taylor expansion of $P\left((\partial\phi)^2\right)$ around $(\partial\phi)^2 = M^4$
makes perfect sense.

Small fluctuations controlled by small # of parameters at low energies.

Look at small perturbations $\phi = M^2 t + \pi$

$$\int d^4x \left[\left(P'(M^4) + M^4 P''(M^4) \right) \dot{\pi}^2 - P'(M^4)(\nabla \pi)^2 \right]$$

⇨ No spatial kinetic term for π!

Other terms $\tilde{P}\left((\partial\phi)^2\right) Q(\Box\phi) + \cdots$

do contain spatial kinetic terms but at least

$$(\nabla^2 \pi)^2 + \cdots$$

⇨ Low energy $S = \int d^4x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{M^2}(\nabla^2\pi)^2 + \cdots \right]$

$$\omega^2 \approx \frac{k^4}{M^2}$$

Positive definite Hamiltonian $\int d^3x \left[\frac{1}{2}\dot{\pi}^2 + \frac{\alpha}{M^2}(\nabla^2\pi)^2 + \cdots \right]$

⇨ STABLE!

Of course there are higher terms in effective theory.

$$S = \int d^4x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\nabla^2\pi)^2}{M^2} + \frac{\dot{\pi}(\nabla\pi)^2}{\tilde{M}^2} + \cdots \right]$$

Because of k^4, spatial fluctuations are less suppressed than we are used to…

To analyze low-energy theory, we must do proper analysis of scaling dimensions of operators!

$$E \to rE$$

$$dt \to r^{-1} dt$$

$$dx \to r^{-1/2} dx$$

$$\pi \to r^{1/4} \pi$$

Make invariant ➡ $\int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\nabla^2\pi)^2}{M^2} + \cdots \right]$

Leading operator in infrared $\int dt d^3x \frac{\dot{\pi}(\nabla\pi)^2}{\tilde{M}^2}$

scaling dimension 1/4. (Barely) irrelevant

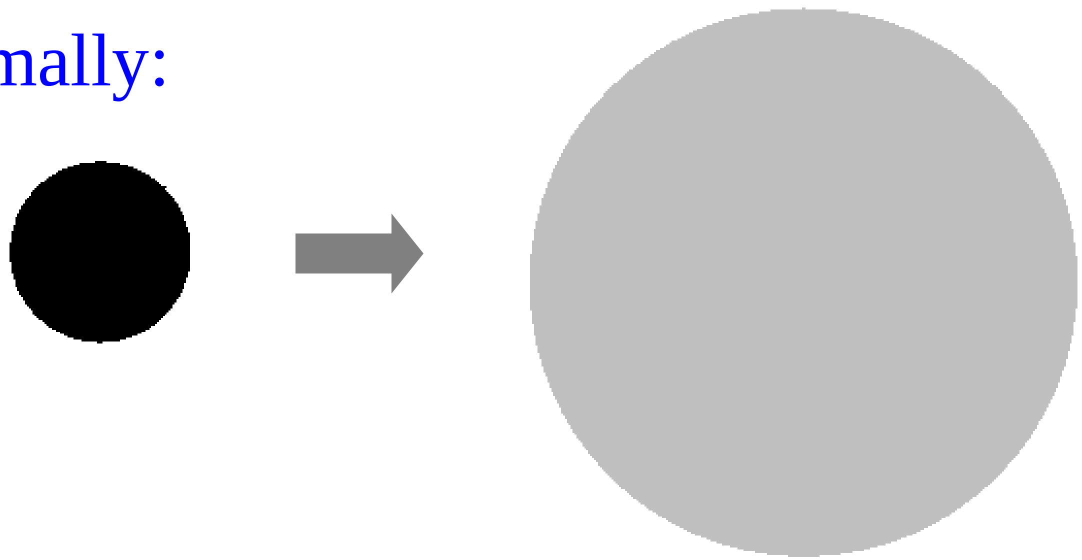
⇨ Good low-E effective theory.

HEALTHY SECTOR IN ABSENCE OF GRAVITY

NOTE: Background breaks Lorentz invariance spontaneously – prefered frame where ϕ is spatially isotropic. No different from CMBR or any cosmological fluid.

What makes the ghost condensate special is that, unlike other cosmological fluid, IT DOES NOT DILUTE AS UNIVERSE EXPANDS.

Normally:

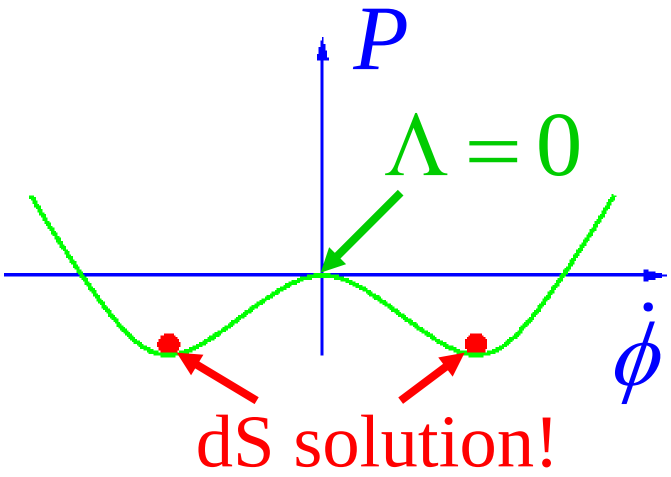


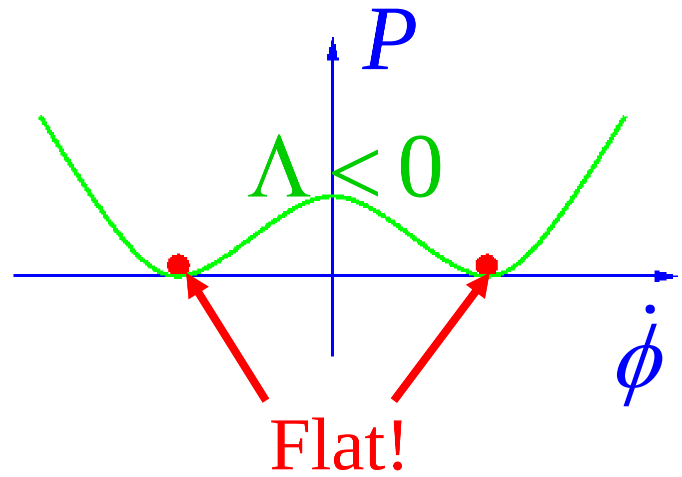
$\Rightarrow$ In deep future, flat or dS (maximal symmetry) EMPTY OF FLUID

NOT HERE! $\dot{\phi} = M^2$ even as universe expands.

$$T_{\mu\nu} = P(M^4) g_{\mu\nu} + P'(M^4) \partial_\mu \phi \partial_\nu \phi$$

Exactly that of c.c. $-P(M^4)$!





Present acceleration can be due to ghost condensate even with Λ=0 (in the symmetric phase). w=p/ρ=-1 exactly. BUT NOT A C.C.

PHYSICAL FLUID WITH PHYSICAL EXCITATIONS.

Systematic construction of Low-energy effective theory

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

Backgrounds characterized by

✧ $\left\langle \partial_\mu \phi \right\rangle \neq 0$ and timelike

✧Background metric is maximally symmetric, either Minkowski or dS.

⇨

$$L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00} - 2\dot{\pi} \right)^2 - \frac{\alpha_1}{M^2} \left(K + \vec{\nabla}^2 \pi \right)^2 - \frac{\alpha_2}{M^2} \left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi \right) \left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi \right) + \cdots \right\}$$

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

➡ Write down most general action invariant under this residual symmetry.

(➡ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\begin{cases} (h_{00})^2 & \textbf{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

Action for π

$$\xi^0 = \pi \quad \begin{cases} h_{00} \to h_{00} - 2\partial_0\pi \\ K_{ij} \to K_{ij} + \partial_i\partial_j\pi \end{cases}$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2\pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i\vec{\nabla}^j\pi\right)\left(K_{ij} + \vec{\nabla}_i\vec{\nabla}_j\pi\right) + \cdots\right\}$$

$$E \to rE$$

$$dt \to r^{-1} dt$$

$$dx \to r^{-1/2} dx$$

$$\pi \to r^{1/4} \pi$$

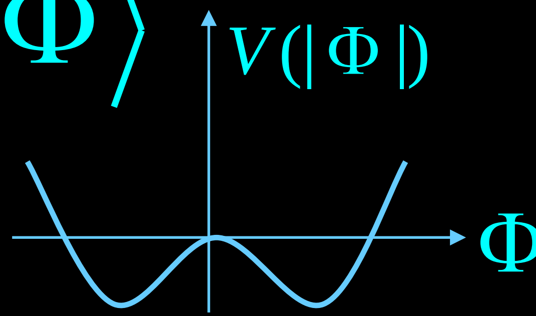
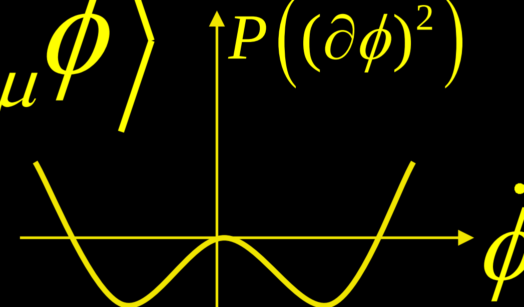
Make invariant $\Rightarrow \int dt\, d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots \right]$

Leading nonlinear operator in infrared $\int dt\, d^3x \frac{\dot{\pi}(\nabla\pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

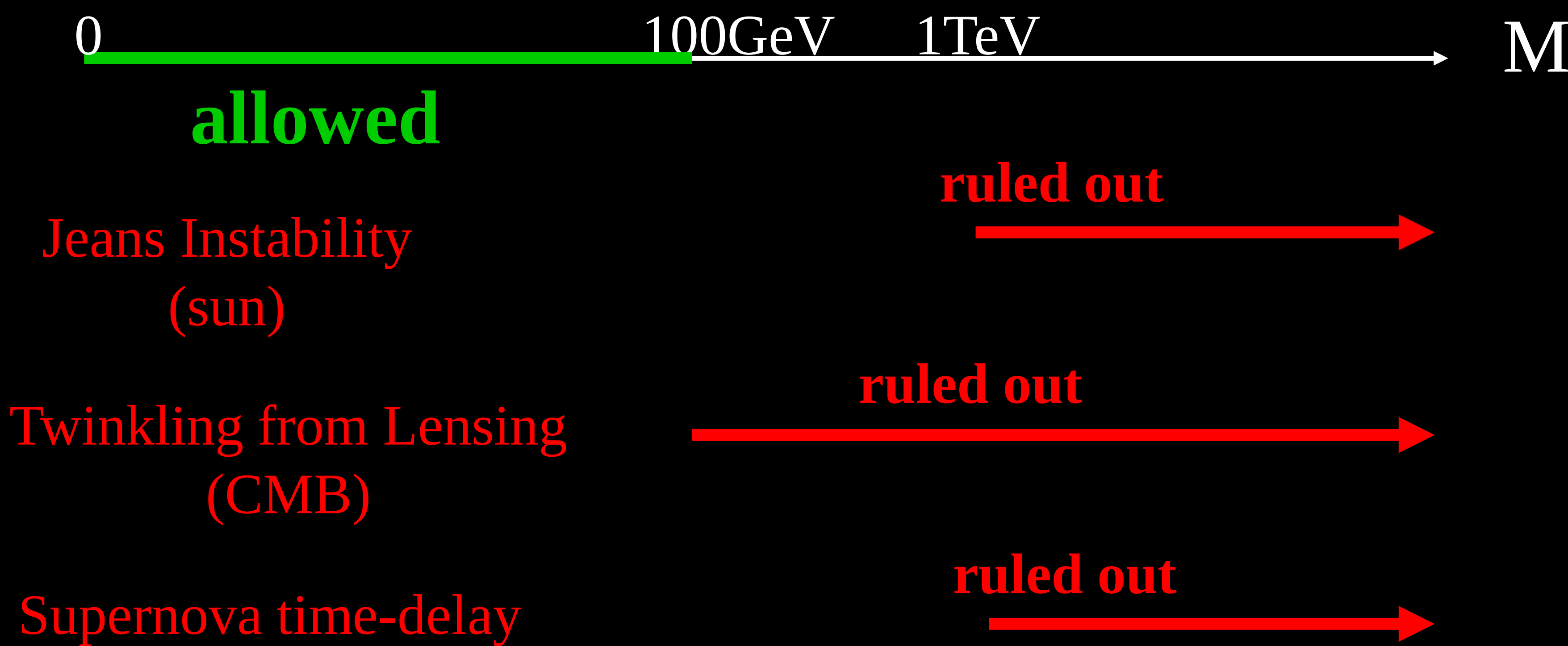
$\Rightarrow$ Good low-E effective theory

Robust prediction

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

Bounds on symmetry breaking scale M

Arkani-Hamed, Cheng, Luty and Mukohyama and Wiseman, JHEP 0701:036,2007



So far, there is no conflict with experiments and observations if M < 100GeV.

Ghost condensation

- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100\text{GeV}$.

Holography and GSL

- Do holographic dual descriptions always exist? PROBABLY NO. e.g.) A de Sitter space is only meta-stable and a unitary holographic dual is not known.
- How about ghost condensate?
- Let's look for violation of GSL in ghost condensate, since violation of GSL would indicate absence of holographic dual. (GSL is expected to be dual to ordinary 2^{nd} law.)
- Three proposals: (i) semi-classical heat flow; (ii) analogue of Penrose process; (iii) negative energy.
- **The generalized 2^{nd} law holds in the presence of ghost condensate. (Mukohyama 2009, 2010)**

Ghost inflation and de Sitter entropy bound

S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- **Black holes & cosmology** in gravity theories are **as important as Hydrogen atoms & blackbody radiation** in quantum mechanics
- Provide **non-trivial tests** for theories of gravity e.g. black-hole entropy in string theory
- **Does the theory of ghost condensation pass those tests?**
- **Ghost condensation is known to be consistent with BH thermodynamics (Mukohyama 2009, 2010)**
- **How about de Sitter thermodynamics?**

de Sitter thermodynamics

- de Sitter (dS) spacetime is one of the three spacetimes with maximal symmetry
- dS horizon has temperature $T_H = H/(2\pi)$
- In quantum gravity, a dS space is probably unstable (e.g. KKLT, Susskind, …). So, let's consider a dS space as a part of inflation
- Friedmann equation → 1st law with entropy $S = A/(4G_N) = \pi/(G_N H^2)$ (This is in contrast with analogue gravity systems.)

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Slow roll inflation (non-eternal)

$$\dot{H} = -4\pi G_{\mathrm{N}} \dot{\phi}^2$$

$$S = \pi/(G_{\mathrm{N}} H^2) \qquad dN = H dt$$

$$\frac{\delta\rho}{\rho} \sim \frac{\delta a}{a} \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}}$$

$$\frac{dS}{dN} = \frac{8\pi^2\dot{\phi}^2}{H^4} \sim \left(\frac{\delta\rho}{\rho}\right)^{-2}$$

$|\delta\rho/\rho| \lesssim 1$ for non-eternal inflation

$$N_{\mathrm{tot}} \lesssim S_{\mathrm{end}} - S_{\mathrm{beginning}} < S_{\mathrm{end}}$$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Eternal inflation

$$\delta\rho/\rho \gtrsim 1 \quad \Rightarrow \quad \Delta N \gtrsim \Delta S.$$

- Fluctuation generated during eternal epoch would collapse to form BH → unobservable!

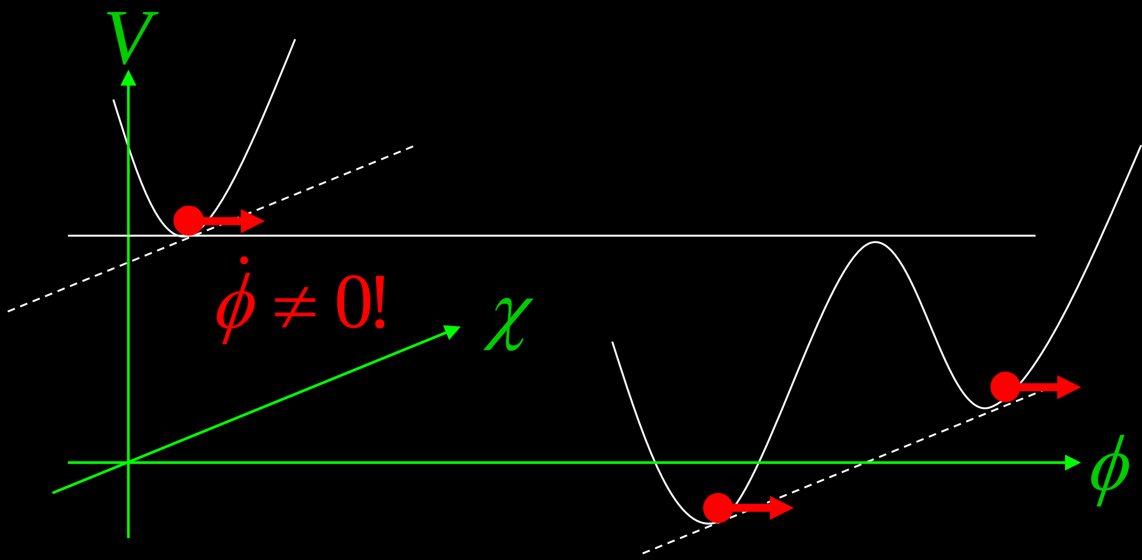
$$\Rightarrow \quad \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation

- Does ghost inflation satisfy the bound?

Ghost inflation

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004



Similar to hybrid inflation but **NOT SLOW ROLL**

Scale-invariant perturbations

cf. tilted ghost inflation, Senatore (2004)

$$\frac{\delta\rho}{\rho} \sim \frac{H\delta\pi}{\dot{\phi}} \sim \left(\frac{H}{M}\right)^{5/4}$$

scaling dim of π

$$\delta\pi \sim M \cdot (H/M)^{1/4} \qquad \dot{\phi} \sim M^2$$

[compare $\frac{H}{M_{Pl}\sqrt{\varepsilon}}$]

Prediction of Large non-Gauss.

Leading non-linear interaction $\beta \dfrac{\dot{\pi}(\nabla \pi)^2}{M^2}$

non-G of $\sim \beta \left(\dfrac{H}{M}\right)^{1/4}$ ← scaling dim of op.

$\sim \beta \left(\dfrac{\delta\rho}{\rho}\right)^{1/5}$

$$\int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2 \pi)^2}{M^2} + \cdots \right]$$

[Really "0.1" $\times(\delta\rho/\rho)^{1/5} \sim 10^{-2}$. VISIBLE.
In usual inflation, non-G ~ $(\delta\rho/\rho) \sim 10^{-5}$ too small.]

⇒ $f_{NL} \sim 82\ \beta\alpha^{-4/5}$, equilateral type

Planck 2015 constraint (equilateral type)

$f_{NL} = -4 \pm 43$ (68% CL statistical) ➔ $-0.6 \leq \beta\alpha^{-4/5} \leq 0.5$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Eternal inflation

$$\delta\rho/\rho \gtrsim 1 \Rightarrow \Delta N \gtrsim \Delta S.$$

- Fluctuation generated during eternal epoch would collapse to form BH → unobservable!

$$\Rightarrow \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation

- Does ghost inflation satisfy the bound?
The answer appears to be "no" since N_{tot} can be arbitrarily large. Swampland?

Lower bound on Λ?

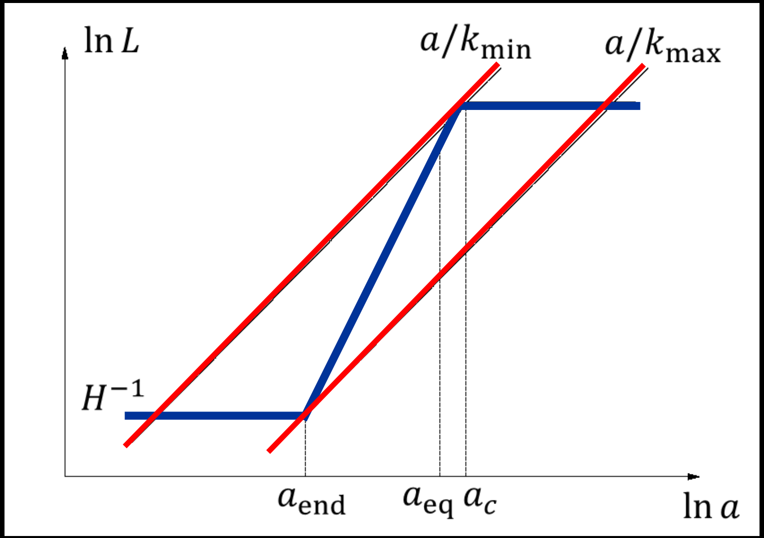
S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- Tiny Λ prevents earlier inflationary modes from being observed.

$$\frac{a_{\rm end}}{a_{\rm reh}} \sim \left(\frac{\rho_{\rm reh}}{\rho_{\rm inf}}\right)^{1/3} \qquad \frac{a_{\rm reh}}{a_{\rm eq}} \sim \left(\frac{s_{\rm eq}}{s_{\rm reh}}\right)^{1/3}$$

$\ddot{a}(t = t_c) = 0$ with

$$6M_{\rm Pl}^2 \frac{\ddot{a}}{a} = -\rho_{\rm m}^{\rm eq}\left(\frac{a_{\rm eq}}{a}\right)^3 + 2\rho_\Lambda$$



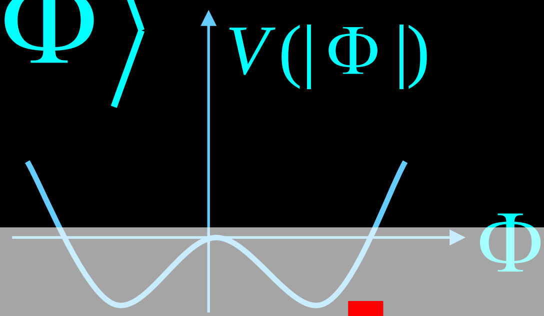
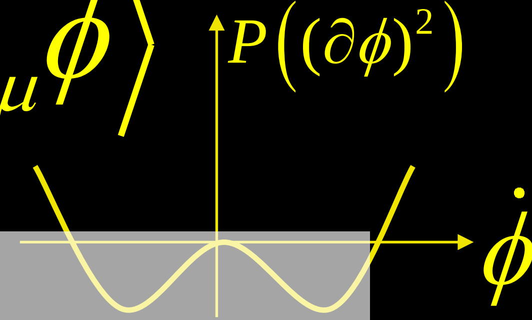
- $N_{\rm obs} \sim \ln(k_{\rm max}/k_{\rm min}) \lesssim S = \pi/(G_N H^2)$

$$\Rightarrow \quad \Omega_\Lambda \gtrsim \exp\left[-10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right] \qquad M \lesssim 100\,{\rm GeV}$$

- In our universe, $\Omega_\Lambda = O(1)$ and thus the bound is well satisfied.

Summary

- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100$GeV.
- It appears easy but is actually difficult to violate the generalized 2nd law by ghost condensate. (Mukohyama 2009, 2010)
- Ghost inflation predicts large non-Gaussianity that can be tested.
- de Sitter entropy bound appears to be violated but is actually satisfied by ghost inflation.

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

Thank you very much!

Cosmological Page time

S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- Hawking rad from BH → $S_{rad} = S_{ent}$ increases but S_{BH} ($\geqq S_{ent}$) decreases → semi-classical description should break down @ Page time, when S_{BH} ~ half of $S_{BH,init}$
- After inflation, we expect to see O(1) deviation from semi-classical description @ Page time, when $N_{obs} \sim S_{end}$
- For example, if Λ decays at $a=a_{decay}$ then

$$\frac{a_{\rm Page}}{a_{\rm decay}} \sim \left(\frac{M}{100\,{\rm GeV}}\right)^{-1}\left(\frac{a_{\rm decay}}{a_{\rm eq}}\right)^{2}\exp\left[10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right]$$