Minimal Theory of Massive gravity

Shinji Mukohyama
(YITP Kyoto)

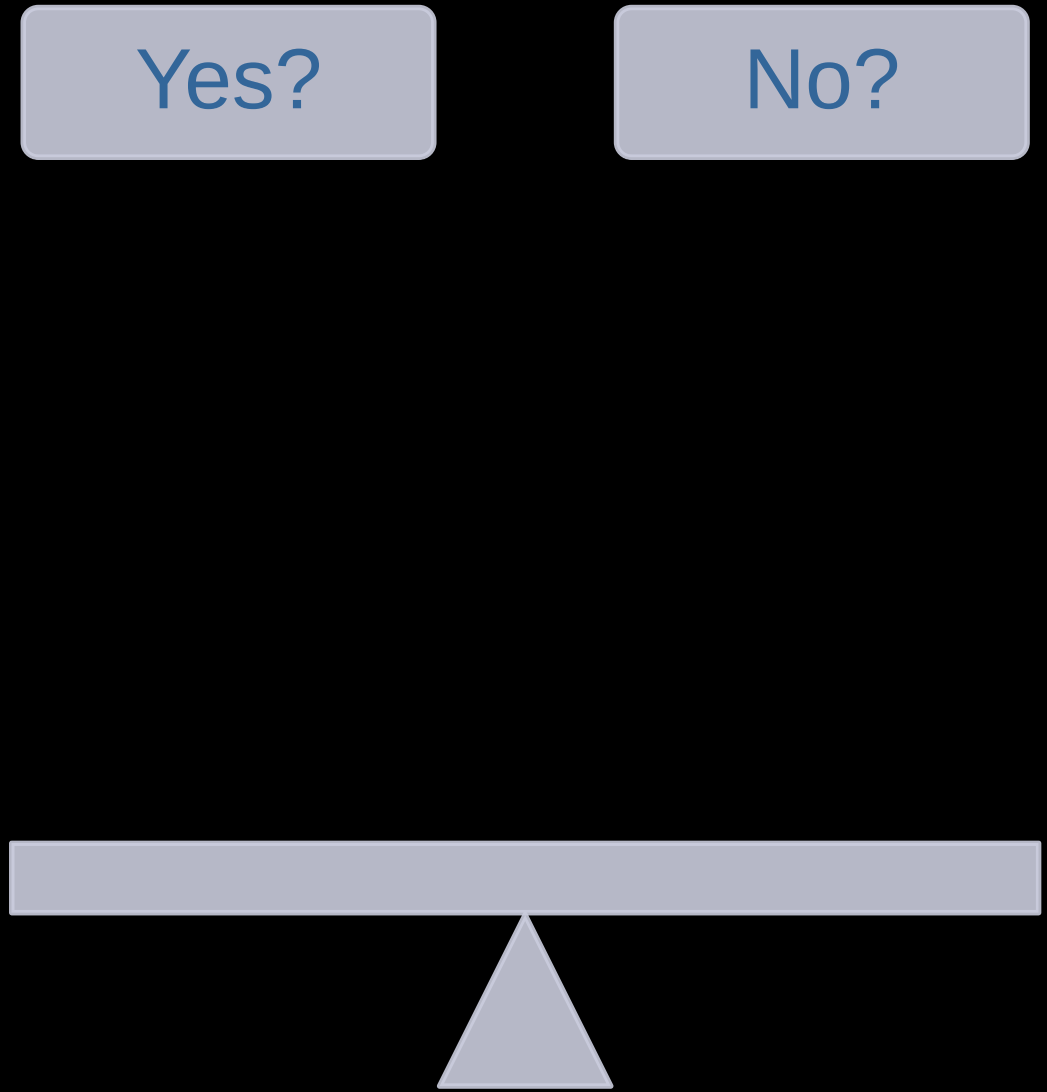
Ref. arXiv: 1506.01594 & 1512.04008 with Antonio DeFelice

Based also on other works with
Antonio DeFelice, Garrett Goon, Emir Gumrukcuoglu, Lavinia Heisenberg, Kurt Hinterbichler, David Langlois, Chunshan Lin, Ryo Namba, Atsushi Naruko, Takahiro Tanaka, Norihiro Tanahashi, Mark Trodden

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



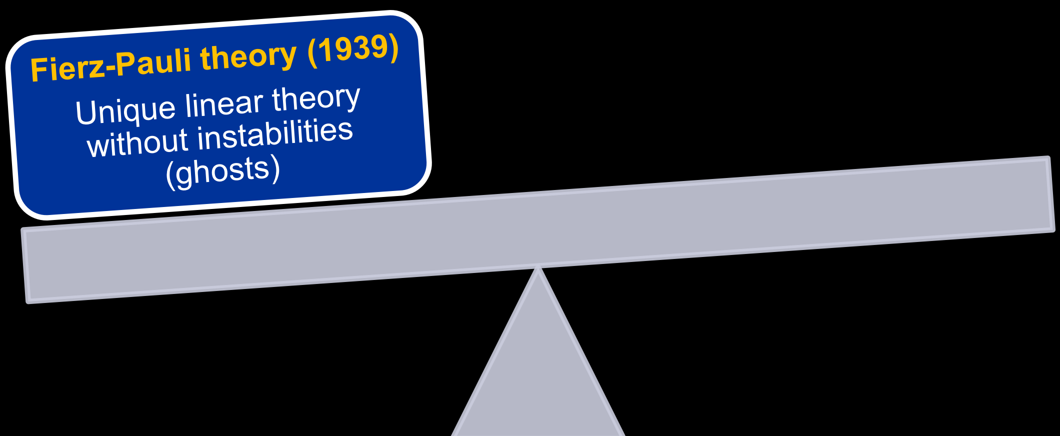
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

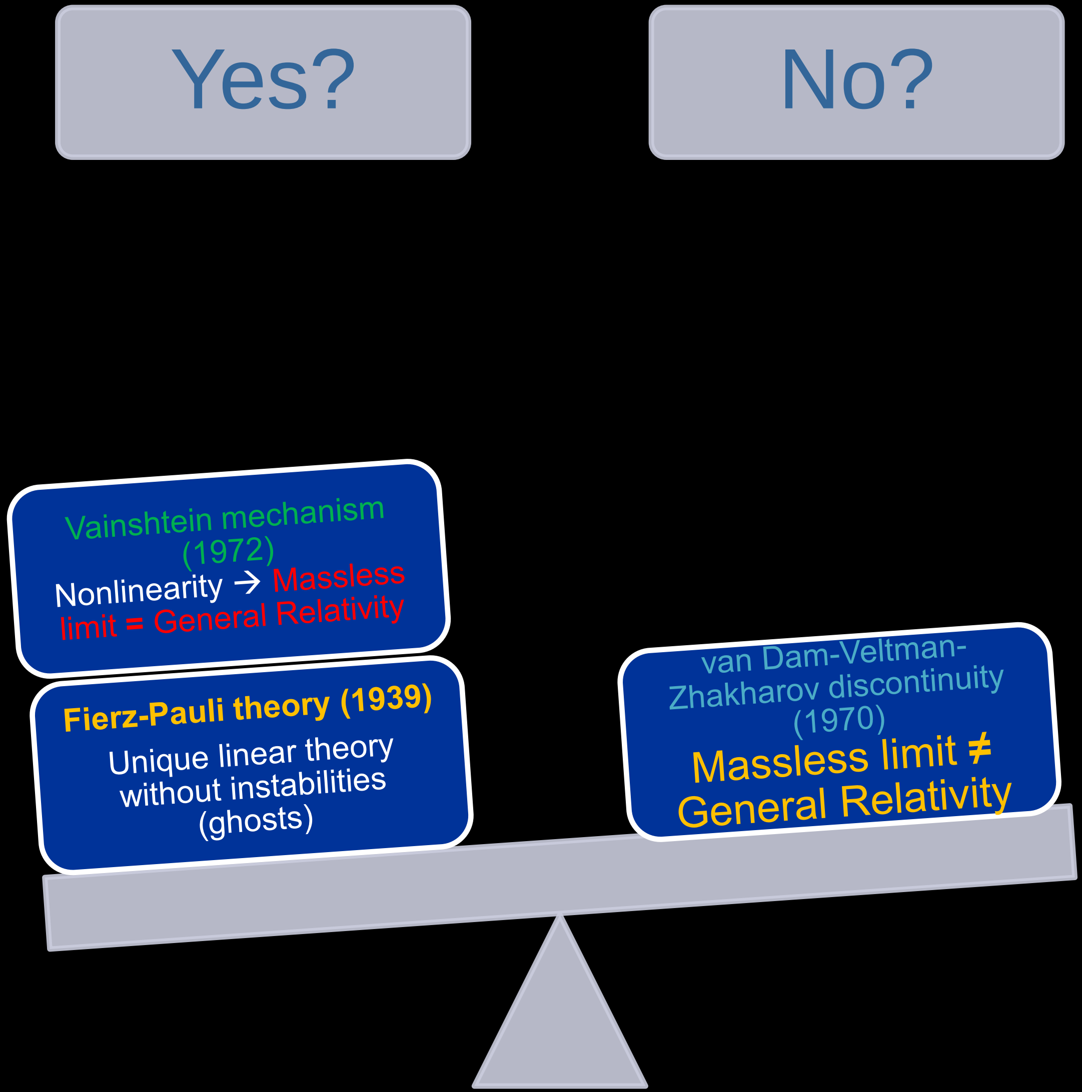
van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?	No?
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6^{th} d.o.f.@Nonlinear level → Instability (ghost)
Fierz-Pauli theory (1939) Unique linear theory without instabilities (ghosts)	van Dam-Veltman-Zhakharov discontinuity (1970) Massless limit ≠ General Relativity

Nonlinear massive gravity

de Rham, Gabadadze 2010
de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical (but technically natural)
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a (a=0,1,2,3)**
- **Poincare symmetry in the field space:**

$$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$$

→ $$f_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

fiducial metric

Pullback of Minkowski metric in field space to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x \sqrt{-g}\,(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$

$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f}\right)^\mu_{\ \nu}$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

$$[\mathcal{A}] \equiv Tr\mathcal{A}$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

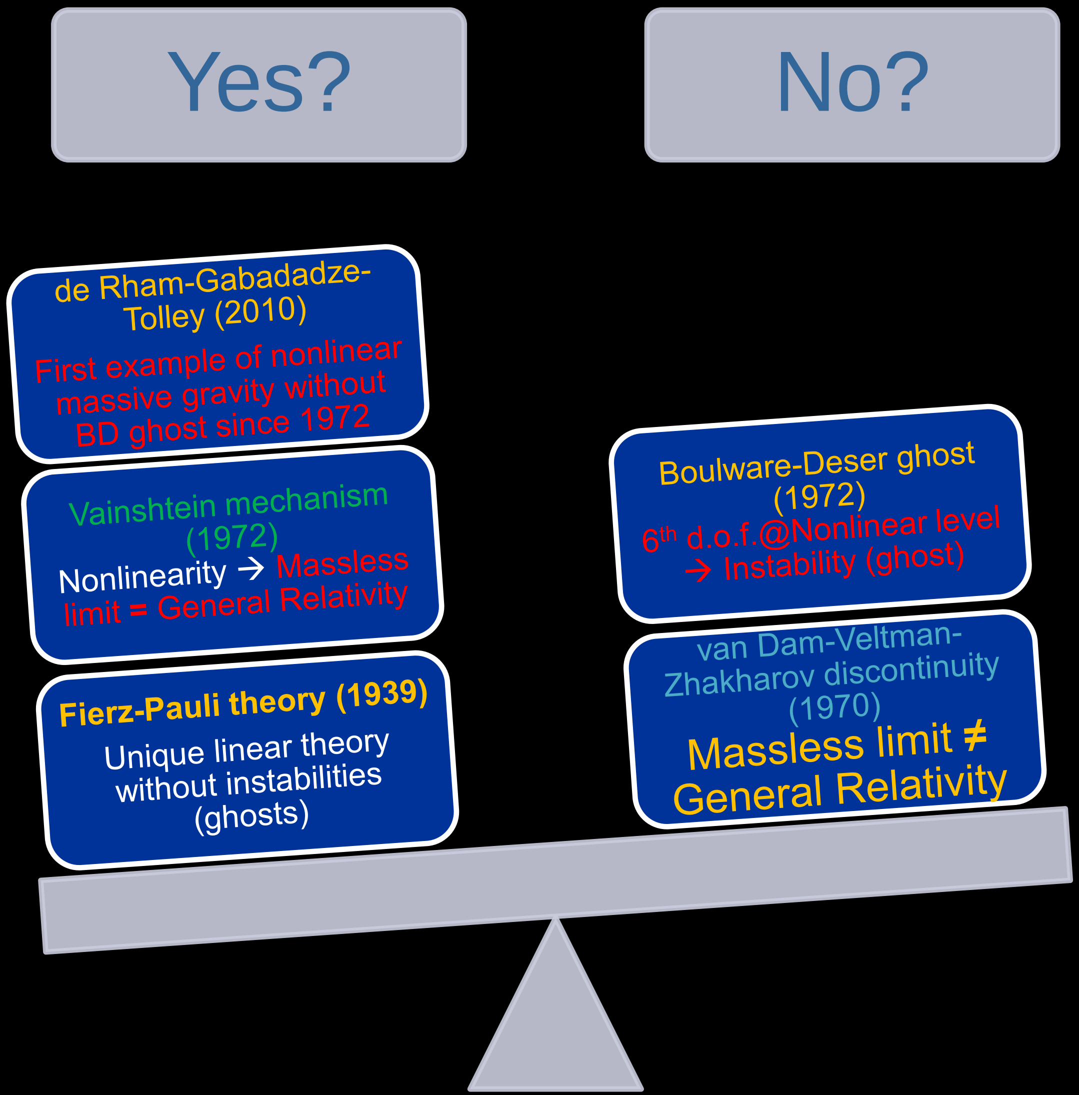
$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \quad \Rightarrow \quad \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

Massive gravity: history

Simple question: Can graviton have mass?

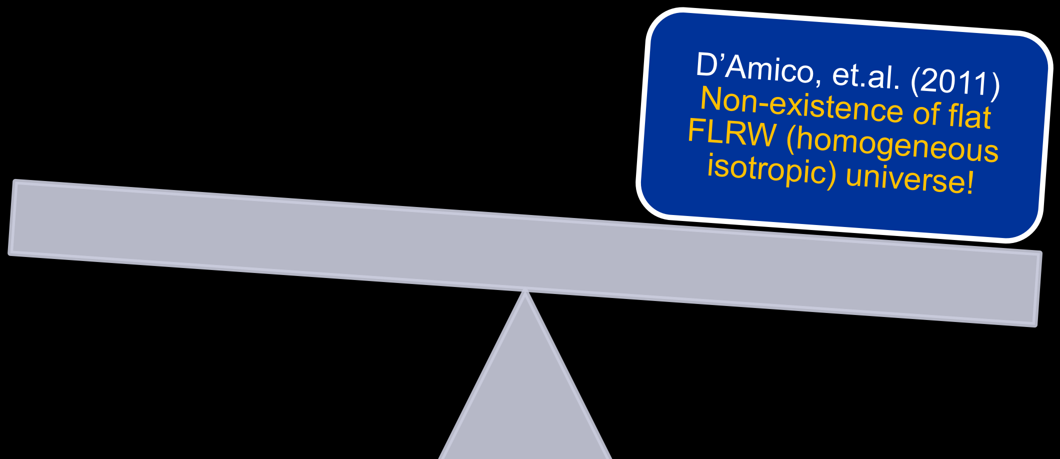
May lead to acceleration without dark energy



Cosmological solutions in nonlinear massive gravity

Good?

Bad?



Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Consistent Theory found in 2010 but

No Viable Cosmology?

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\upsilon}$ spontaneously breaks diffeo.
- Both $g_{\mu\upsilon}$ and $f_{\mu\upsilon}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\upsilon}$
- No closed FLRW chart
- Open FLRW ansatz

$$
\begin{aligned}
\phi^0 &= f(t)\sqrt{1+|K|(x^2+y^2+z^2)},\\
\phi^1 &= \sqrt{|K|}f(t)x,\\
\phi^2 &= \sqrt{|K|}f(t)y,\\
\phi^3 &= \sqrt{|K|}f(t)z.
\end{aligned}
$$

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2\,dt^2 + |K|\,(f(t))^2\,\Omega_{ij}(x^k)dx^i dx^j$$

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2 dt^2 + a(t)^2\Omega_{ij}dx^i dx^j\,,\\
\Omega_{ij}dx^i dx^j &= dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}\,,
\end{aligned}
$$

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for ϕ^a (a=0,1,2,3)

$$(\dot{a} - \sqrt{|K|}N)\left[\left(3 - \frac{2\sqrt{|K|}f}{a}\right) + \alpha_3\left(3 - \frac{\sqrt{|K|}f}{a}\right)\left(1 - \frac{\sqrt{|K|}f}{a}\right) + \alpha_4\left(1 - \frac{\sqrt{|K|}f}{a}\right)^2\right] = 0$$

- The first sol $\dot{a} = \sqrt{|K|}N$ implies $g_{\mu\nu}$ is Minkowski

  → we consider other solutions

$$f = \frac{a}{\sqrt{|K|}}X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

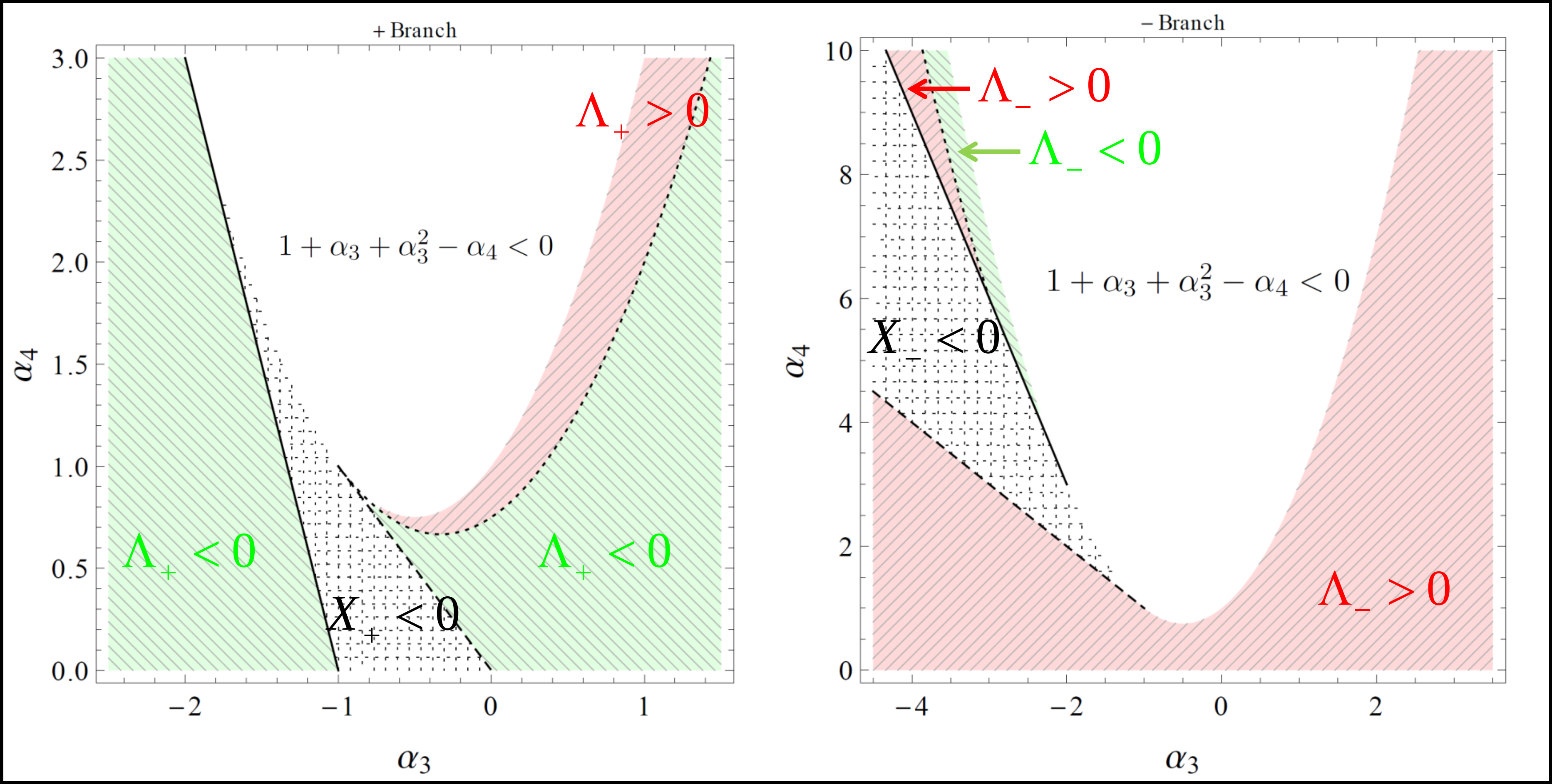
- Latter solutions do not exist if K=0

- Metric EOM → self-acceleration

$$3H^2 + \frac{3K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3 + \alpha_4)^2}\left[(1 + \alpha_3)(2 + \alpha_3 + 2\alpha_3^2 - 3\alpha_4) \pm 2(1 + \alpha_3 + \alpha_3^2 - \alpha_4)^{3/2}\right]$$

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

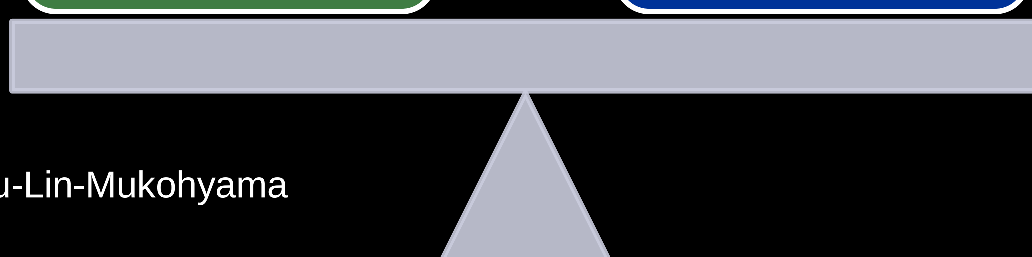
Cosmological solutions in nonlinear massive gravity

Good?

Bad?

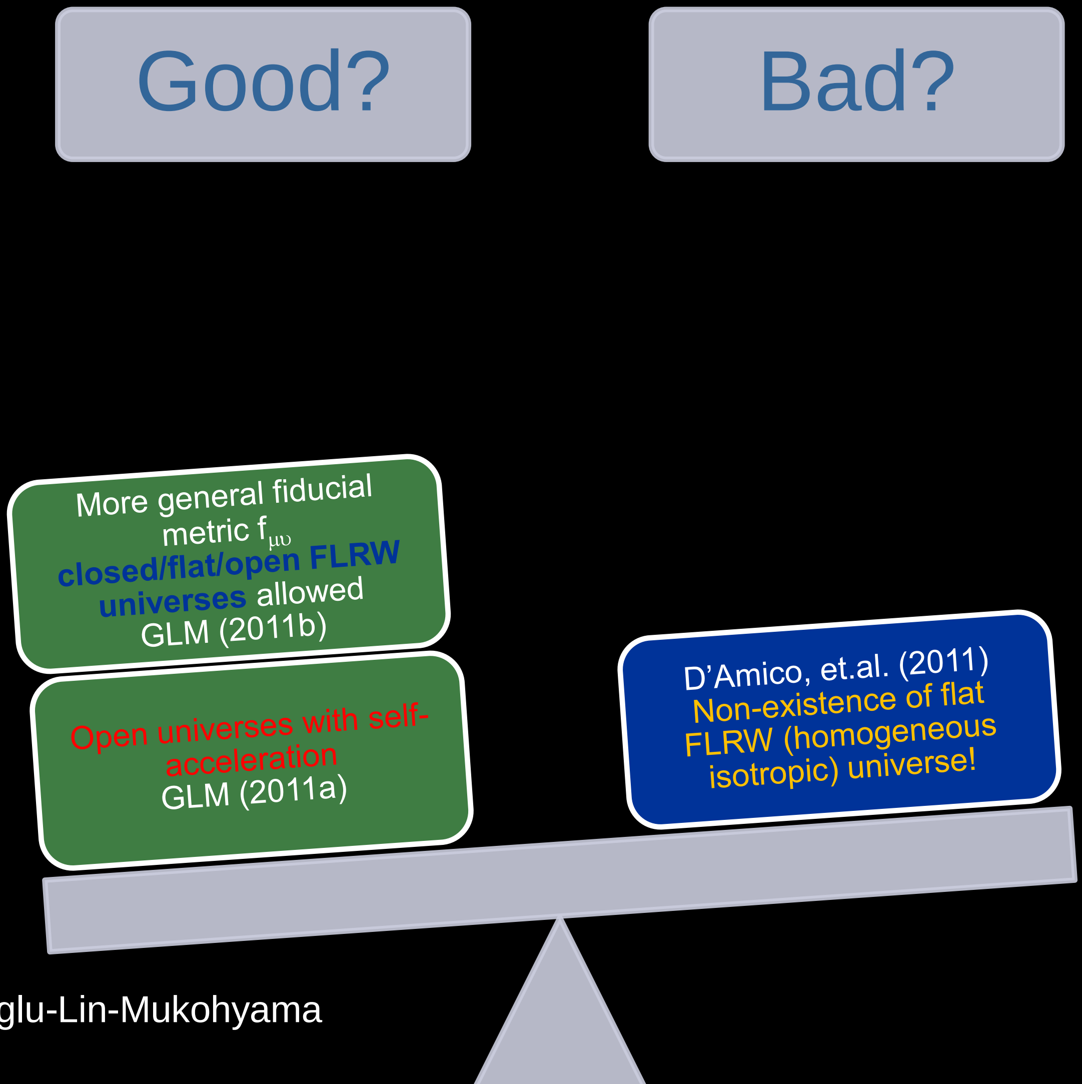
Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!



GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama

Summary of Introduction + α

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

New backgrounds or Extended theories

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic
- (ii) Extended quasidilaton (De Felice&Mukohyama 2013), Bimetric theory (Hassan, Rosen 2011; DeFelice, Nakamura, Tanaka 2013; DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Rubakov 2004; Dubovsky 2004; Blas, Comelli, Pilo 2009; Comelli, Nesti, Pilo 2012; Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (de Rham, Heisenberg, Ribeiro 2014; Gumrukcuoglu, Heisenberg, Mukohyama 2014, 2015), New quasidilaton (Mukohyama 2014), ...
- They provide stable cosmology.

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]
+ De Felice, arXiv: 1303.4154 [hep-th]

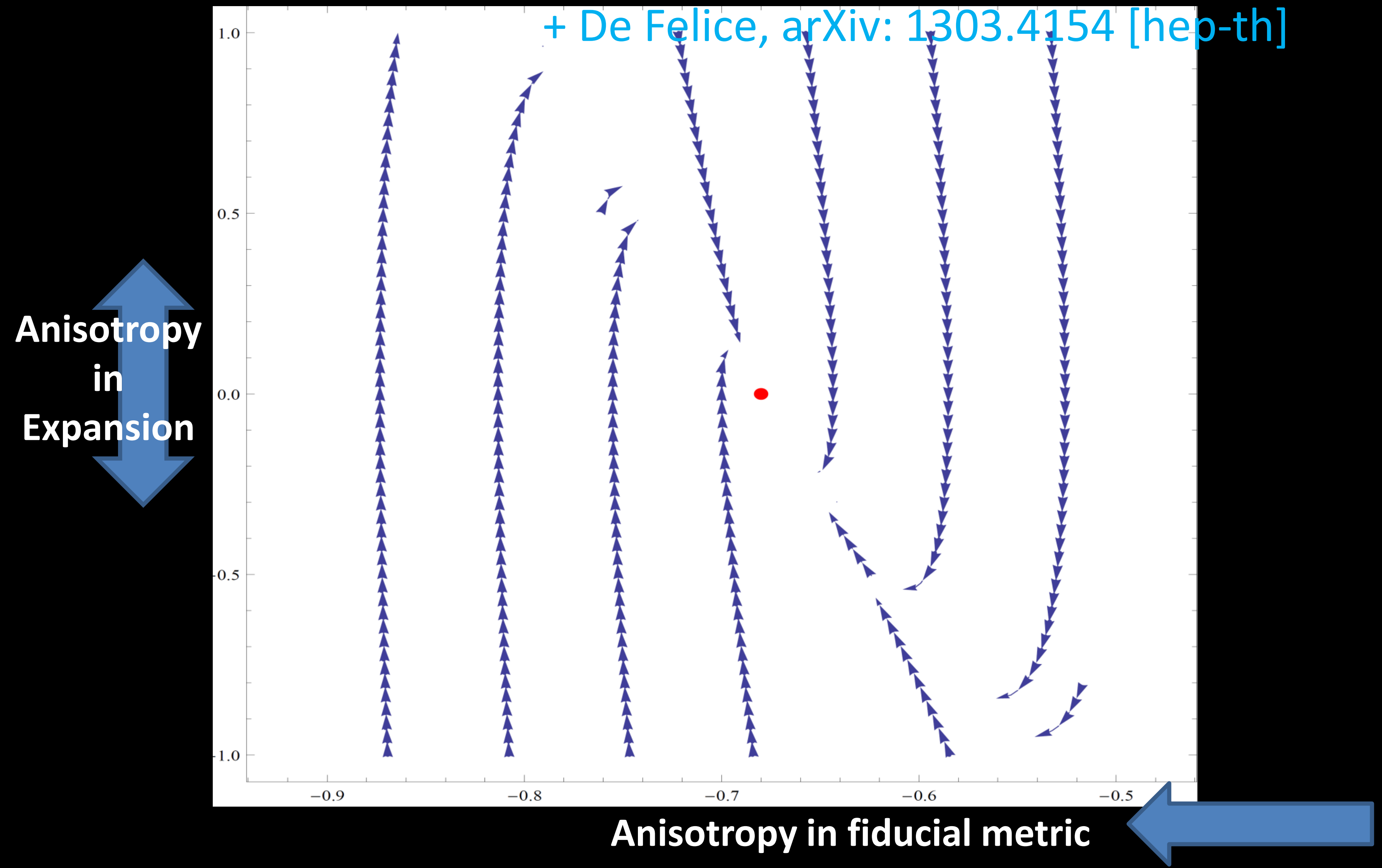
- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.
→ Anisotropic FLRW universe!
statistical anisotropy expected
(suppressed by small m_g^2)

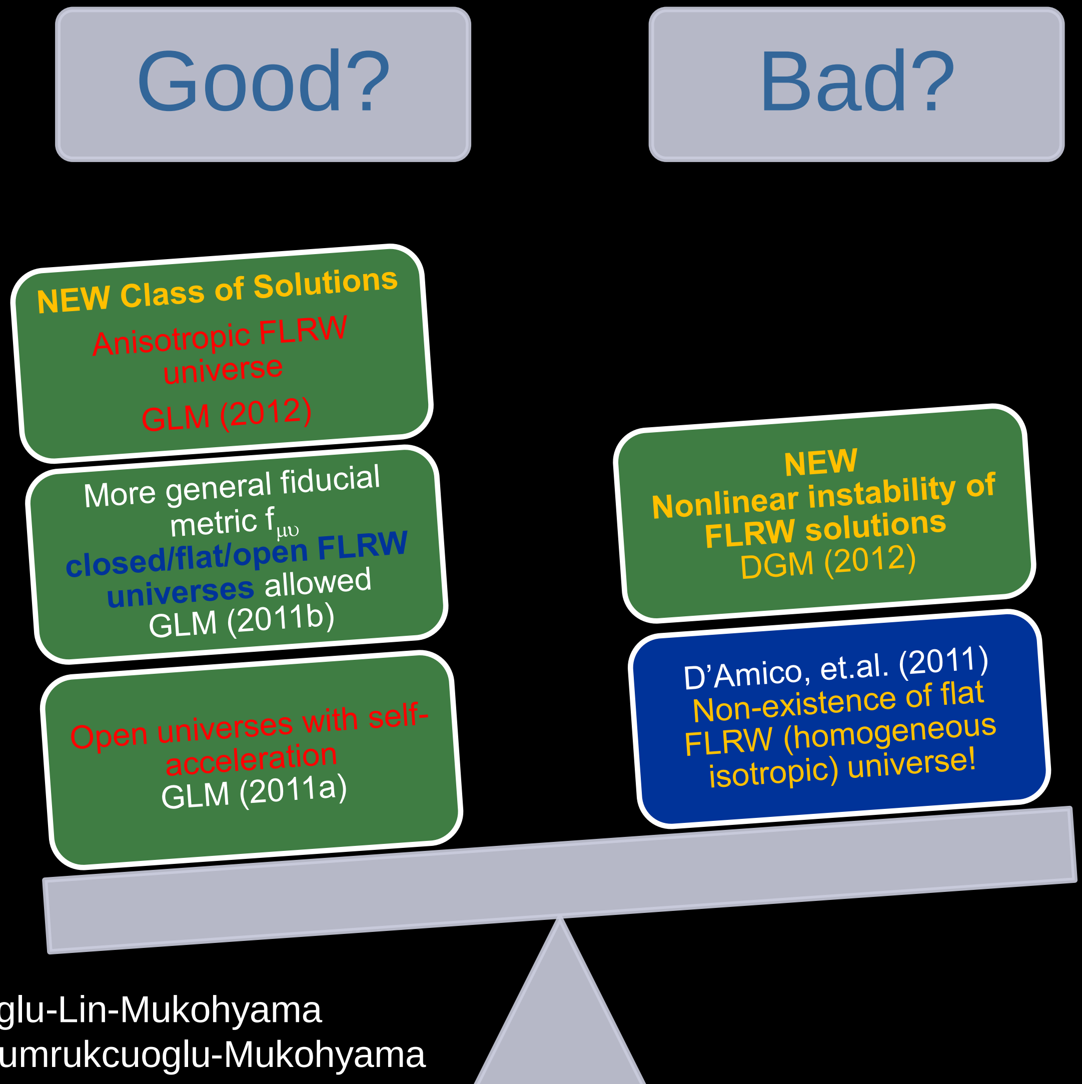
New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]

+ De Felice, arXiv: 1303.4154 [hep-th]



Cosmological solutions in nonlinear massive gravity

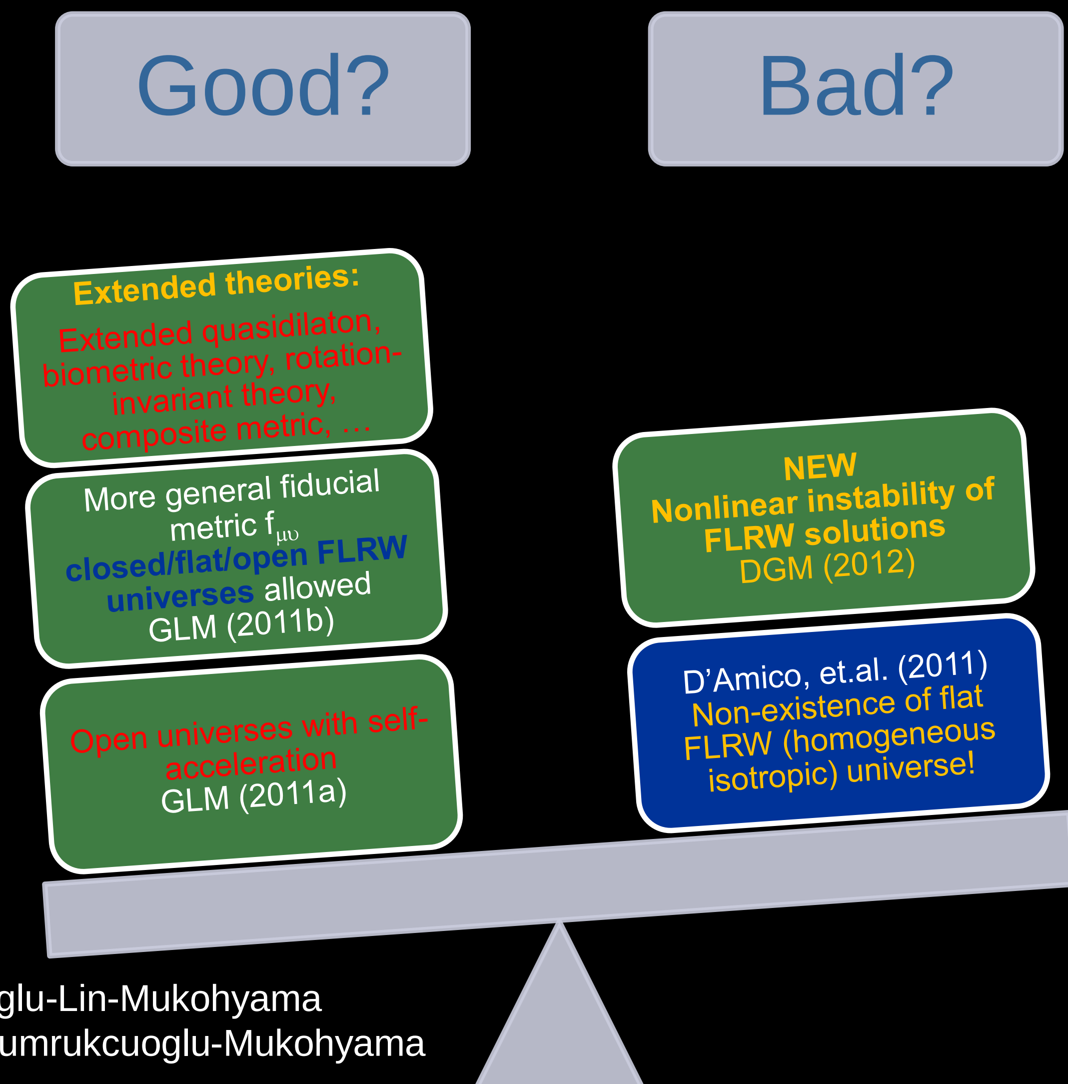


GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

New backgrounds or Extended theories

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic
- (ii) Extended quasidilaton (De Felice&Mukohyama 2013), Bimetric theory (Hassan, Rosen 2011; DeFelice, Nakamura, Tanaka 2013; DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Rubakov 2004; Dubovsky 2004; Blas, Comelli, Pilo 2009; Comelli, Nesti, Pilo 2012; Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (de Rham, Heisenberg, Ribeiro 2014; Gumrukcuoglu, Heisenberg, Mukohyama 2014, 2015), New quasidilaton (Mukohyama 2014; De Felice, Gumrukcuoglu, Heisenberg, Mukohyama, Tanahashi 2016), ...
- They provide stable cosmology.

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

More recent development

Minimal Theory of Massive Gravity

De Felice & Mukohyama, arXiv: 1506.01594
1512.04008

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Cosmology of MTMG

- Constraint $\mathcal{C}_0 \approx 0$ $\quad X \doteq \tilde{a}/a$

$$(c_3 + 2c_2X + c_1X^2)(\dot{X} + NHX - MH) = 0$$

- Self-accelerating branch

$$X = X_\pm \doteq \frac{-c_2 \pm \sqrt{c_2^2 - c_1c_3}}{c_1} \qquad \lambda = 0$$

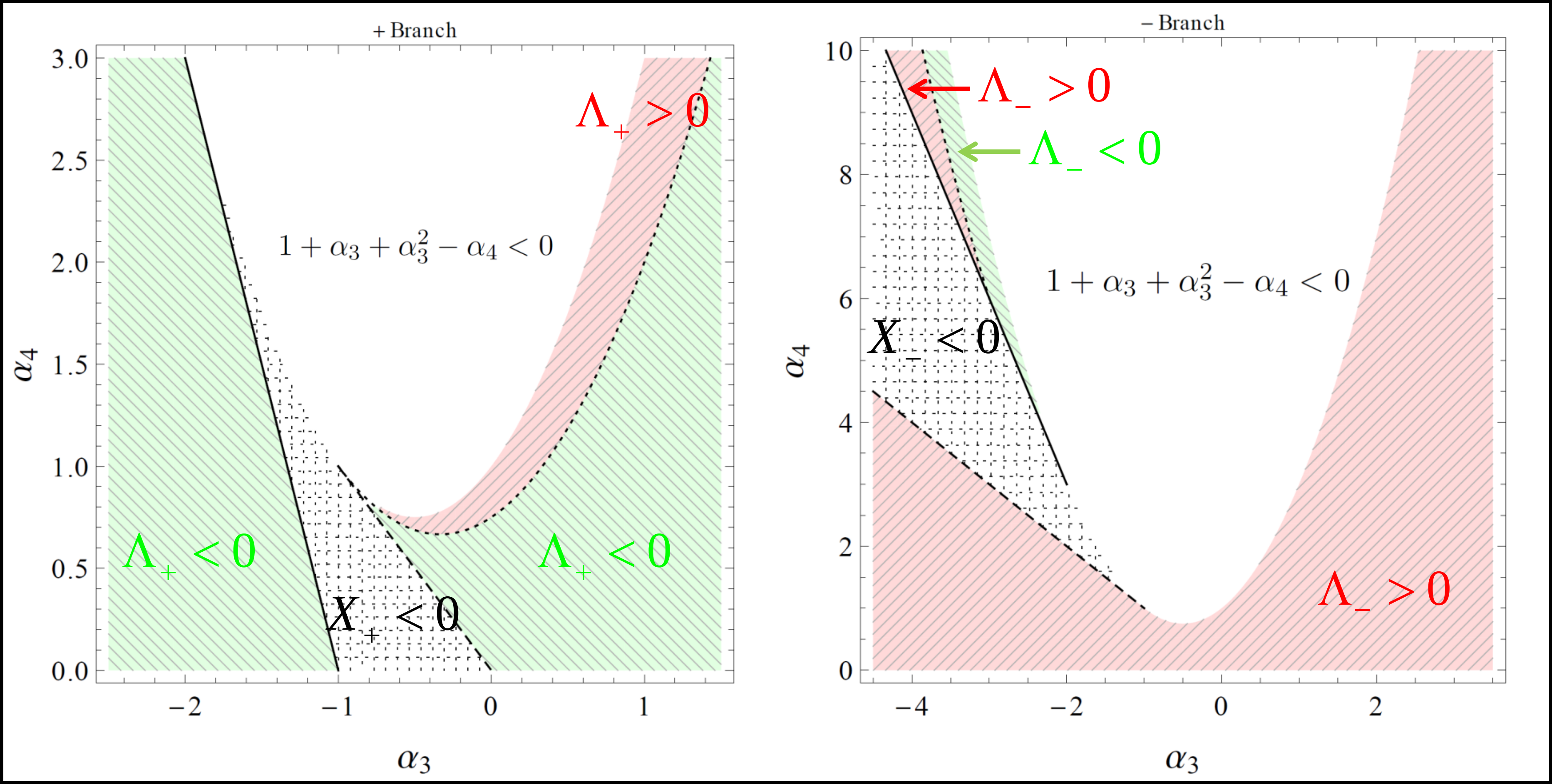
$$3M_{\rm P}^2H^2 = \frac{m^2M_{\rm P}^2}{2}\left(c_4 + 3c_3X + 3c_2X^2 + c_1X^3\right) + \rho$$

$\Lambda_{\rm eff}$ from graviton mass term (even with c_4=0)

Scalar/vector parts are the same as ΛCDM

Time-dependent mass for gravity waves

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

Cosmology of MTMG

- Constraint $\mathcal{C}_0 \approx 0$ $\qquad X \doteq \tilde{a}/a$

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0$$

- “Normal” branch

$$H = XH_f \qquad \lambda = \frac{4(H_f X - H)\,N}{m^2\,(c_1 X^2 + 2c_2\,X + c_3)\,M}$$

$$3M_{\rm P}^2 H^2 = \frac{m^2 M_{\rm P}^2}{2}\left(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3\right) + \rho$$

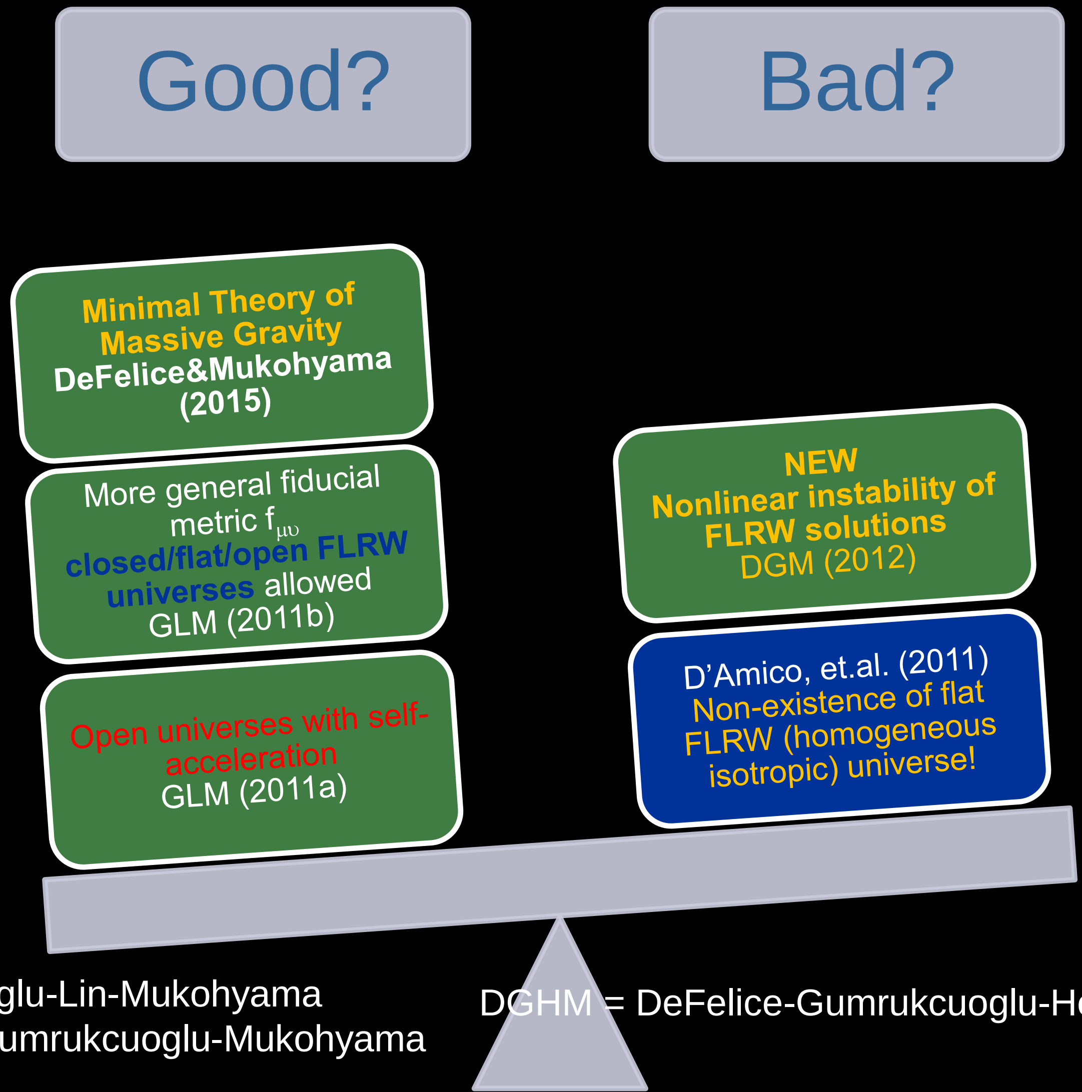
Dark component with **w ≠ -1 without extra dof**

Scalar part recovers GR in UV ($L \ll m^{-1}$) but deviates from GR in IR ($L \gg m^{-1}$)

Vector part is the same as GR

Time-dependent mass for gravity waves

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama

Summary

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have infinitely strong coupling and ghost instability
- Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
- Minimal theory of massive gravity with 2dof provides a nonlinear completion of dRGT self-accelerating cosmology