# Nonlinear Gravitational Waves as Dark Energy in Warped Spacetimes

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Presentation based on:

R.Slagter, S.Pan: Found. of Phys. 2016 ( communicated by t'Hooft)
R.Slagter: Journ. of Mod. Phys. 2016
S.Pan, J.de Haro, A. Paliathanasir, R.Slagter: Mon.Not. Roy. Ast. Soc.2016
R.Slagter: Astrophys. Journ. 1986, 1983

# Overview

*I. Note on the Cauchy problem for GR*

*II. How to handle nonlinear gravitational waves*

*III. The Multiple-scale method*

*IV. Application to warped brane world models with U(1) scalar-gauge field(in the brane)*

*Spin-off:*

***a. Self-acceleration of FLRW possible without $\Lambda$?***

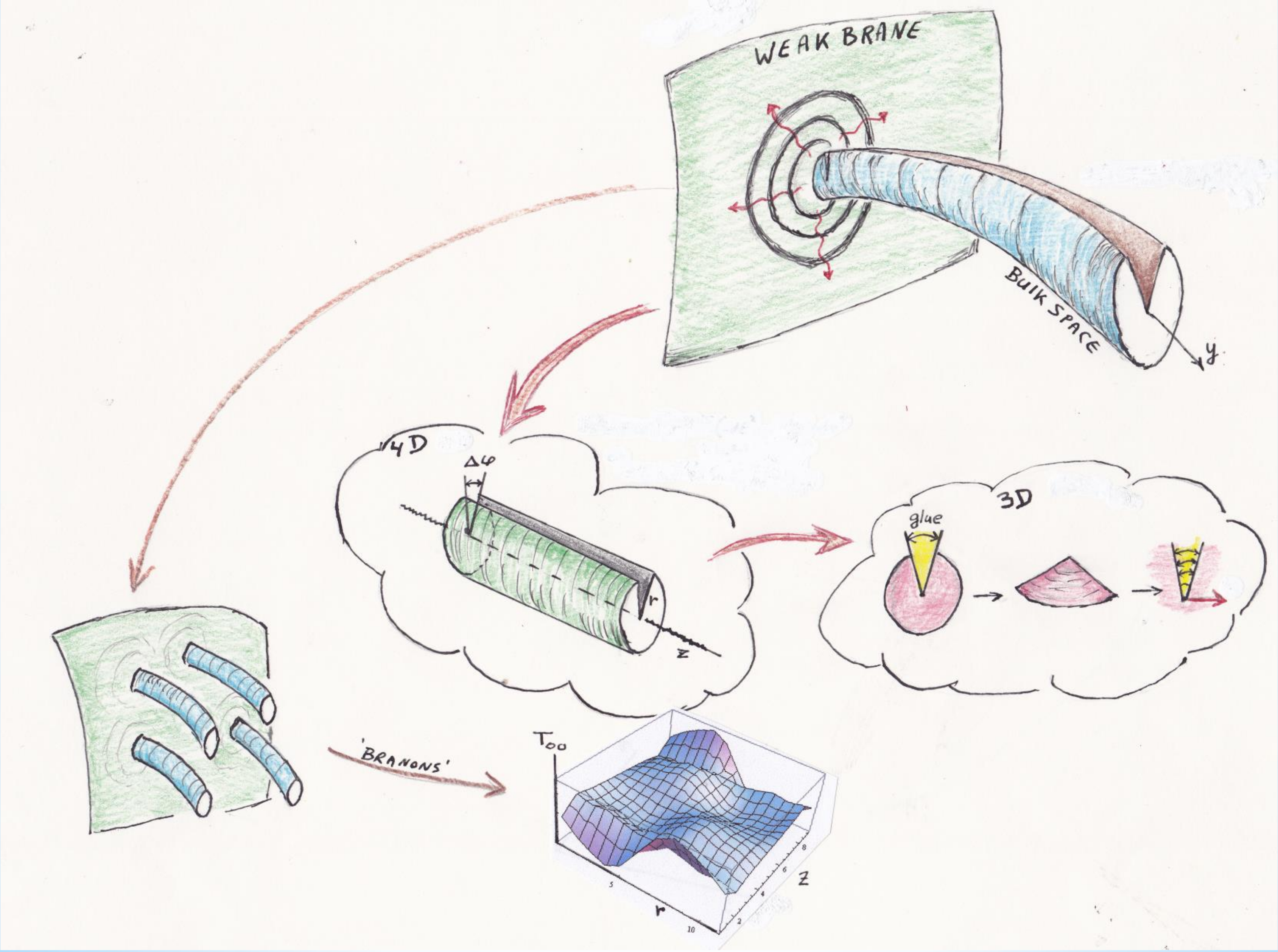
***[Slagter, Pan: Found of Phys, 2016]***

***b. Evidence via alignment of quasar polarization?***

***[Slagter: Journ Mod Phys,2016]***

**Overview article: R.Maartens: Liv.Rev. 2010 "Brane world models"**

# Artist impression of a cosmic string



# Some considerations on the Cauchy problem in GR

For linear problems: well understood
For nonlinear problems: we have
local Cauchy problem [well understood]
global Cauchy problem ( **strong cosmic censorship problem**)

***General***: Given a solution $u_0(x)$, does there exist a unique solution $u(t,x)$ of the PDE's with $u(0,x) = u_0(x)$

***Connection with practical physics:***

It is seldom that exact solutions of the Einstein eq. can be used: one needs numerical solutions or analytic approx ( expansion in a small parameter)

► The Einstein eq are essentially global hyperbolic eq:

► **A spacelike hypersurface S is called Cauchy surface, if each inextendible causal curve hits it precisely once**

► **Cauchy surfaces are the correct places to give data for the Cauchy problem**

► $n_\mu$ unit normal vector on S; define: $g_{\mu\nu} = h_{\mu\nu} + n_\mu n_\nu$ , $k_{\mu\nu} = \nabla_\gamma n_\delta h^\gamma_\mu h^\delta_\nu$
they constitute the initial data for the Einstein eq.

► There are **constraints:**

$$R - k_{ab}k^{ab} + \left(h^{ab}k_{ab}\right)^2 = \kappa T_{00} \qquad \nabla^b k_{ab} - \nabla_a\left(h^{bc}k_{bc}\right) = -\frac{1}{2}\kappa T_{0a}$$

# Considerations on gravitational waves in GR

Weak field approx:

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}({h_{\mu\nu}}^2) \qquad |h_{\mu\nu}| \ll 1$$

Einstein equations:

$$\partial^\gamma \partial_\mu h^{\mu\sigma} + \partial^\sigma \partial_\mu h^{\mu\gamma} - \partial^\gamma \partial^\sigma h - \partial^\mu \partial_\mu h^{\gamma\sigma} - \eta^{\gamma\sigma}(\partial_\beta \partial_\mu h^{\beta\mu} + \partial^\mu \partial_\mu h) = 16\pi G T^{\gamma\sigma}$$

Note that $h_{\mu\nu}$ is invariant under a coordinate transformation $x_\mu \to x_\mu + \xi_\mu$
so $h_{\mu\nu} \to h_{\mu\nu} - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)$
One usually choose the Lorentz-gauge: $\partial_\mu \left( h^{\mu\sigma} - \frac{1}{2}\eta^{\mu\sigma} h \right) = 0$ and additional gauge freedom $\partial^\mu \partial_\mu \xi^\sigma = 0$.

However: in high-energy situations ( or high curvature):

weak field approximation **not suitable**
**The background metric itself may respond to the strong gravitational field**

**And: weak-field approx runs into divergences when pushed to second order!**
**[long time ago:Trautman, Fock]**

# The multiple-scale analysis

**[ Or: high-frequency or "two-timing" method]**

**It is known that:**

► In d>4 the equations of (stringy) gravity are fully **non-linear** ( the usual Einstein equations are quasi-linear).

► **Non-uniformity** can appear in a regular perturbation expansion by interaction between consecutive orders of the perturbation scheme: **secular** terms can appear.

► **Discontinuities** of the 2-th order derivatives of the metric across a (d-1) dim submanifold

► **dispersion** of gravitational waves or shocks

**Now:**

►The **MS method** is a powerful method [ Choquet-Bruhat, 1969] to solve this Cauchy problem: the unknowns and their first derivatives cannot be given arbitrarily on a (d-1) dim submanifold: the Cauchy data must satisfy constraints

►Find an asymptotic solution of order p in an expansion parameter ω.

►This is also an approximate solution for appropriate boundedness conditions

►One obtains from lower to higher order: "gauge"-conditions on the fieldvariables
"back-reaction" on the background metric
propagation equations
.....

►Make additional restrictions on the expansion to maintain boundedness.

First of all: We need a physical meaningful expansion parameter:

for example: $$\frac{wavelength\ perturbation}{typical\ background\ dimension}$$

or: $$\frac{typical\ dimension\ of\ the\ extra\ dimension}{background\ dimension}$$

**So:** wave-like solutons of the non-linear hyperbolic system are characterized by different scales:

Regions with smooth variation of the solution: **background**
Regions with strong variation: **waves**

**Some results:**

Isaacson-1967; Choquet-Bruhat-Taub-1973;Choquet-Bruhat-1969,1976,1988
Slagter-Ap.J 1986
For sringy gravity ( Gauss-Bonnet term): Choquet-Bruhat-1988

## Simple example 1: Duffing's equation

$$\frac{d^2y}{dt^2} + \alpha y + \varepsilon y^3 = 0 \qquad \varepsilon \ small$$

When t is of order $\frac{1}{\epsilon}$ one gets serious problems: **Secular terms** in all orders, so violation of boundedness.

Proof: Expand: $y(t) = \sum_0^\infty \varepsilon^n y_n(t)$ then:

$$y''_0 + y_0 = 0 \quad \rightarrow \quad y_0 = \cos(t)$$

$$y''_1 + y_1 = -y_0^3 \quad \rightarrow \quad y_1 = A\cos(t) + B\sin(t) + \frac{1}{32}\cos(3t) - \frac{3}{8} t \sin(t)$$

$y_1$ contains a secular term. However: **the solution is bounded for all t!**

**Solution: all secular terms sum up to zero. A hell of a job for many DE's!**

**Result:** $y(t) = cos[t\left(1 + \frac{3}{8}\varepsilon\right)]$

For the multiple scale method not necessary:

Substitute: $y(t) = Y_0(t, \tau) + \varepsilon Y_1(t, \tau) + \cdots.$ $\qquad \tau = \varepsilon t$

collecting powers of $\varepsilon$ : $\frac{\partial^2 Y_0}{\partial t^2} + Y_0 = 0$ $\frac{\partial^2 Y_1}{\partial t^2} + Y_1 = -Y_0^3 - 2\frac{\partial^2 Y_0}{\partial\tau\partial t}$

The general solution for $Y_0$: $\boldsymbol{Y_0(t,\tau) = A(\tau)e^{it} + A^*(\tau)e^{-it}}$

Substituting this solution in righthand side in Eq. for $Y_1$:

$$e^{it}\left[-3A^2A^* - 2i\frac{dA}{d\tau}\right] + e^{-it}\left[-3A(A^*)^2 + 2i\frac{dA^*}{d\tau}\right] - e^{3it}A^3 - e^{-3it}(A^*)^3$$

If we do not want secular terms in $Y_1$-equation to order ε, then the two terms in brackets must be zero! The solution is: $\boldsymbol{A(\tau) = R(0)e^{1\theta(0)+3iR(0)^2/2}}$
After substituting τ = ε t and proper boundary conditions, we obtain

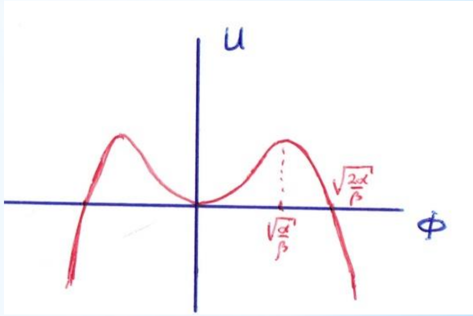
$$\boldsymbol{y(t) = \cos\left[t\left(1+\frac{3}{8}\varepsilon\right)\right] + o(\varepsilon)}, \quad \varepsilon\rightarrow 0,\ \varepsilon t = o(1),$$ [compare with sum sec.terms]

So: **one can keep track of the several orders of approximation**

## Example 2: Nonlinear Schrodinger equation

Consider:

$$\Phi_{tt} - \Phi_{xx} = \Phi(\beta\Phi^2 - \alpha)$$




$\Phi = 0$ is a stable solution. For $\alpha = 1, \beta = \frac{1}{6}$ and $\Phi$ small

Then an approx.: $U = 1 - cos\Phi$ (sin-Gordon eq)

How to handle the disturbances?

Multiple-Scale-method:

$$\Phi(x,t;\epsilon) = \sum_{k=1}^{\infty} \varepsilon^{kp}\,\Phi^{(k)}(x,t;\varepsilon)$$

We define "slow" variables": $X_k = \varepsilon^k x$ $\quad T_k = \epsilon^k t$

Then:

$$\Phi(t,x;\varepsilon) = \widetilde{\Phi}(x,t,X,T,X_1,T_1,\ldots\ldots)$$

And we can keep track of the several orders and can handle secular terms.

Lowest order:

$$(\partial_{xx} - \partial_{tt} - \alpha)\Phi^{(1)} = 0$$

[linearized eq.]

Next order:

$$(\partial_{xx} - \partial_{tt} - \alpha)\Phi^{(2)} + 2(\partial_{X_1 x} - \partial_{T_1 t})\Phi^{(1)} = 0$$

$$(\partial_{xx} - \partial_{tt} - \alpha)\Phi^{(3)} + (\partial_{X_1 X_1} - \partial_{T_1 T_1} + 2\partial_{X_2 x} - 2\partial_{T_2 t})\Phi^{(1)} + \beta\left(\Phi^{(1)}\right)^3 + 2(\partial_{X_1 x} - \partial_{T_1 t})\Phi^{(2)} = 0$$

# The multiple scale method formally

Let us consider the formal series of the relevant fields $\boldsymbol{F_i}$ in **x** on a manifold M, dependent on different scales $(\boldsymbol{x}, \xi, \chi, ..)$

$$F_i(\boldsymbol{x}, \xi, \chi..) = \sum_{n=0}^{\infty} \omega^{-n} F_i^{(n)}(\boldsymbol{x}, \xi, \chi..)$$

with $\xi = \omega\Theta(\boldsymbol{x})$, $\chi = \breve{\omega}\Pi(\boldsymbol{x})$, ... scalar ( phase functions) on M.

**Here:** $\frac{1}{\boldsymbol{\omega}}$ is the ratio of the **characteristic wavelength** of the perturbations to the dimension of the background; $\frac{1}{\breve{\boldsymbol{\omega}}}$ ratio **extra dim**. to background dim.

$F_i$ are the **metric components (**and the **scalar-gauge fields)**.

$$\frac{d\boldsymbol{g}_{\mu\nu}}{d\boldsymbol{x}^{\sigma}} = \boldsymbol{g}_{\mu\nu,\sigma} + \boldsymbol{\omega l}_{\sigma}\dot{\boldsymbol{g}}_{\mu\nu} + \breve{\boldsymbol{\omega}}\boldsymbol{m}_{\sigma}\breve{\boldsymbol{g}}_{\mu\nu} + .. \quad \text{with} \quad \boldsymbol{g}_{\mu\nu,\sigma} \equiv \frac{\partial \boldsymbol{g}_{\mu\nu}}{\partial \boldsymbol{x}^{\sigma}}, \quad \dot{\boldsymbol{g}}_{\mu\nu} \equiv \frac{\partial \boldsymbol{g}_{\mu\nu}}{\partial \xi} \ ...$$

If one substitutes the expansions into the fieldequations, one says that $F_i$ are asymptotic solutions, if in the replaced series

$$f_i(\boldsymbol{x}, \xi, \chi..) = \sum_{n=-m}^{\infty} \omega^{-n} f_i^{(n)}(\boldsymbol{x}, \xi, \chi..)$$

all $\boldsymbol{f_i^{(n)}}$=0

# The multiple scale method formally

Now

$F_i(x,\xi,\chi..) = \sum_{n=0}^{\infty} \omega^{-n} F_i^{(n)}(x,\xi,\chi..)$ is also an ***approximate wavelike***

solution of order p on W ⊂ $M$, if for all x∈ $W$

$|f_i(x,\xi,\chi..)| \leq C.\omega^{-p}$ for all $\xi, \chi, ...$ C const

**Nice example:** stringy gravity—Choquet-Bruhat; **J Math. Phys.**1988

► gravity+Gauss-Bonnet term in n-dim: field eq second order PDE's

► Cauchy problem is well described with constraints

►

# Why Warped 5D Space times?

•The explanation of the acceleration with a cosmological constant is rather **problematic:**

- ► **Coincidence-problem:** $\boldsymbol{\Omega_{\Lambda} \sim \Omega_{M}}$
- ► **Finetuning-problem:** $\boldsymbol{\rho_{\Lambda,obs} \sim 10^{-57} GeV^{4}}$ $\boldsymbol{\rho_{\Lambda,theor} \sim 1\, TeV^{4}}$
- ► **Ad hoc modifications:** of the **Friedmann** equation risky, specially when considering density perturbations: do it **covariantly**

► **Disturbances don't survive in 4D models** : at least some of them are needed for the observed large-scale structures

In **warped 5D model**: they do survive

**So modify GR : D-branes.**

**1. Dvali-Gabadadze- Porrati (DGP)**

**⇒2. Randall-Sundrum (RS)**

In general:

Gravity leakage at late-times initiates acceleration, due to weakening of gravity on the brane – **not** due to any negative pressure field.

**4D gravity is recovered at high energy via the lightest KK modes of the graviton**

**By-product: hierarchy problem solved [ for example in RS-1 model]**

# Present State of our Universe

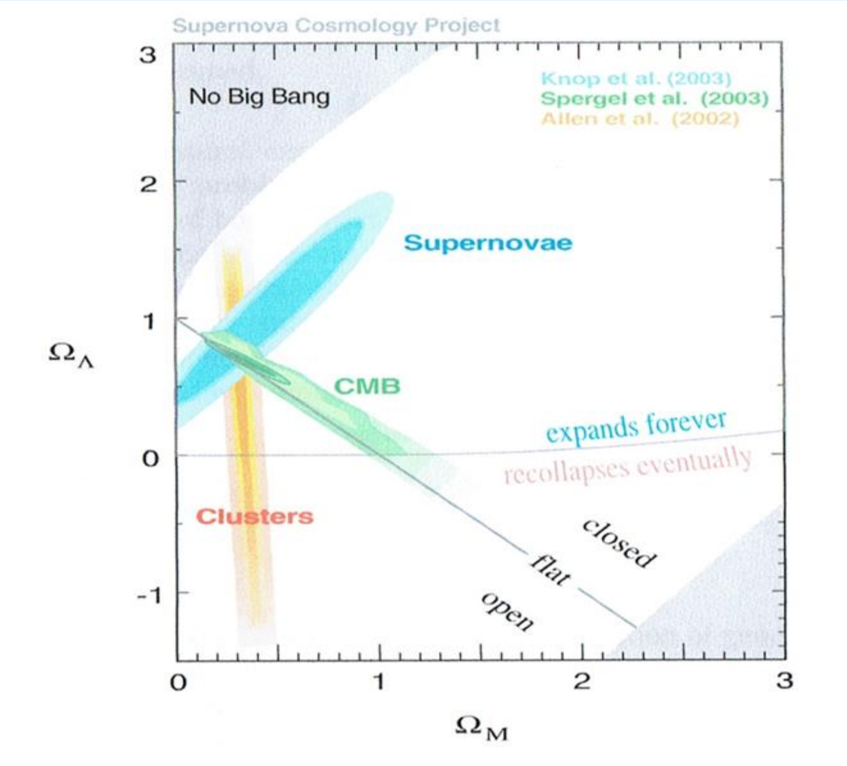
► The expansion of our universe is **accelerating.**

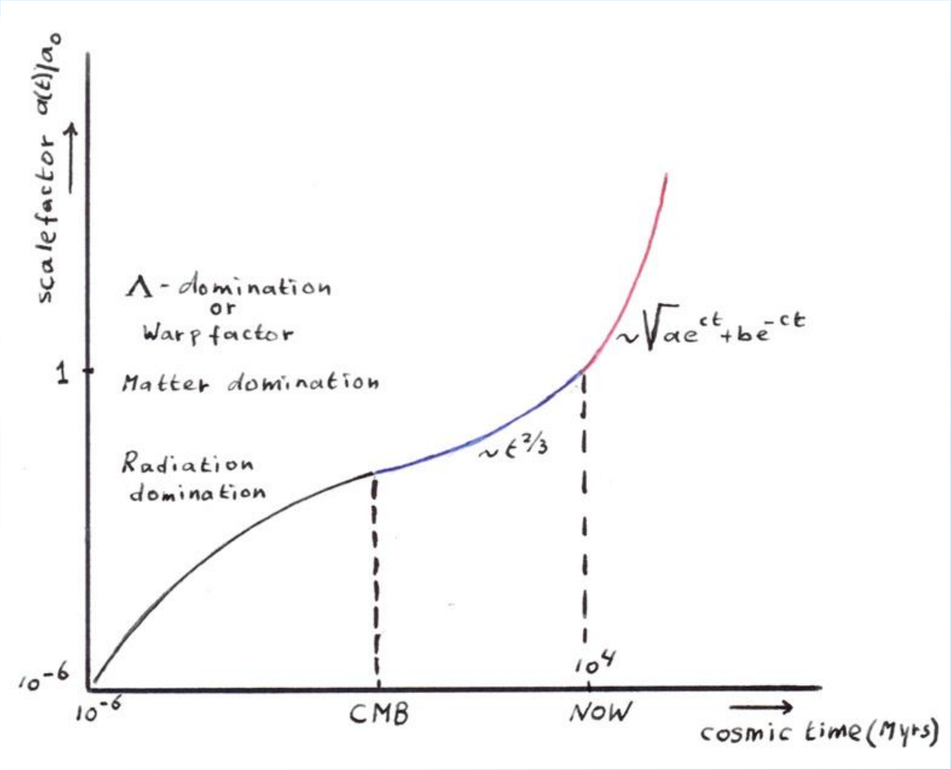
► One needs **dark energy** with an effectively negative pressure, $p < -\frac{1}{3}\rho$

**LCDM: w = -1** **[ Planck 2015: $w > -1$ ? ]**

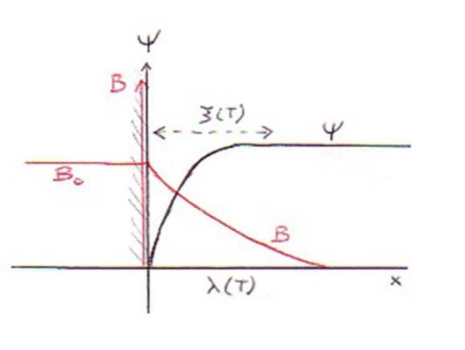
► We should live now in the cosmological constant dominated era (and approx. )

$\Omega_\Lambda = 0.73 \qquad \Omega_M = \Omega_{DM}\,(=0.23) + \Omega_B\,(=0.046)$



# Why Cosmological Cosmic Strings?



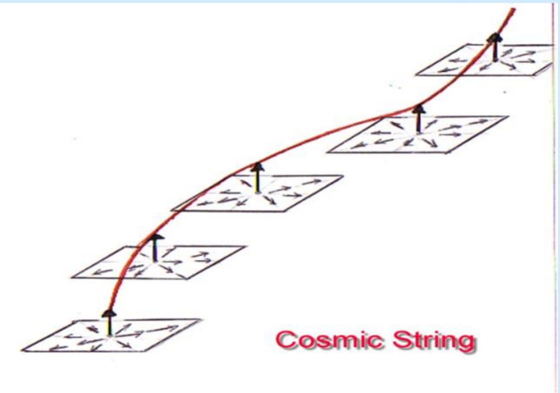

- The U(1) scalar gauge field has 1. lived up to his reputation! :
  2. triggered inflation
  3. GL theory of super-cond.
  4. Nielsen-Olesen vortex

- Gen Rel field eq:

$$G_{\mu\nu} = \kappa^2{}_4 T_{\mu\nu} \qquad D_\mu D^\mu \Phi - 2\frac{\partial U}{\partial \Phi^*} = 0 \qquad \nabla^\mu F_{\mu\nu} - \frac{1}{2} ie[\Phi(D_\nu\Phi)^* - \Phi^*(D_\nu\Phi)] = 0$$

**Cosmic string collection of points in false vacuum.

** Angle deficit Minkowski minus wedge



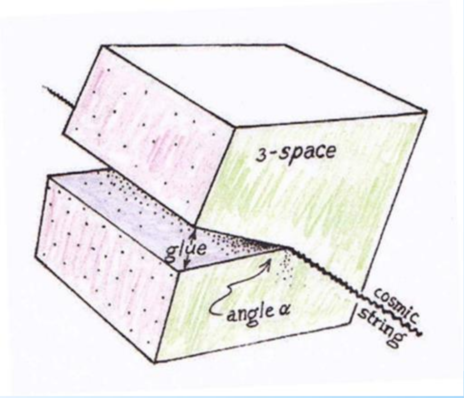

$$ds^2 = -e^{a_0}(dt^2 - dz^2) + dr^2 + e^{-2a_0}(k_2 r + a_2)^2 \, d\varphi^2$$

Question: What about cylindrical GW from CS in expanding universe? [ Gregory, 1989]

It turns out: C-energy ~ $\frac{r_{cs}}{R_H}$ extremely small

Expected disturbances fade away during expansion

[Importance of cyl symm grav waves was already noticed by Einstein-Rosen[1936]]



# Why Cosmological Cosmic Strings?

- U(1) CS can be embedded into a flat 4D FRW along the polar axis

- **However:** The approx spacetime becomes conical:[ not pleasant]

$$ds^2 = a(t)^2[-dt^2 + dr^2 + K(r)^2 dz^2 + (1 - 4\pi G\mu)^2 S(r)^2 d\varphi^2]$$

**and can be matched on the well known FLRW spacetime by suitable transformation**

$$ds^2 = a(t)^2 \left[-dt^2 + \frac{dR^2}{1 - kR^2} + R^2 d\theta^2 + (1 - 4\pi G\mu)^2 R^2 sin^2\theta d\varphi^2\right]$$

- Result: No contribution from the gravitation waves from the CS because

$$\frac{r_{CS}}{R_H} \sim \frac{\dot{a}}{a} \sim \mathbf{10^{-20}}$$

- Disturbances are damped rapidly by $(\frac{r_{CS}}{R_H})^2$

- Asymptotic conical ST ( angle deficit) is **problematic**. Also found in radiative cyl. Einstein-Rosen ST: C-energy related to angle deficit [just as mass is related to angle deficit for CS].

So: Surviving disturbances must be very small ( otherwise conflict with observ)

# Problems for Cosmic Strings from Observations

► density perturbations : $\frac{\delta\rho}{\rho} \sim G\mu = \eta^2 / M_p^{\ 2} \sim 10^{-6}$ for GUT scale

► in first instance correct with observations

► Now: inconsistencies with new CBM power spectrum COBE, WMAP

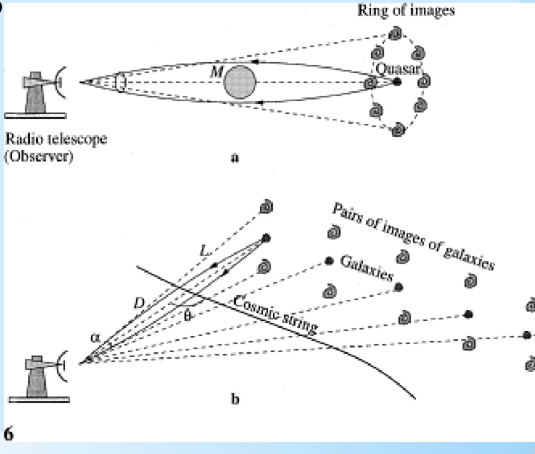
► They cannot provide a satisfactory explanation for the magnitude of the initial density perturbations

► How to handle **super-massive CS** with $G\mu >> 1$ [ phase transition at energy much larger than GUT ]

This is **interesting** for perturbation analysis

[The angle deficit will increase with the **energy scale** of symmetry breaking]

► where is the axially symmetric gravitational lensing-effect?




► Cosmological CS: late-time conical residu [unwanted]
[Gregory, 1989]

So **Exit CS** study??

# Rescue of CS

**reborn CS** → Go to warped 5D RS model

*** in the brane: unobservable angle deficit

*** asymptotically: no conical space time

[Slagter, 2012, IJMPD]

*** So no conflict with:

1. CMB-spectrum
2. Absence of axially symm. double images
3. The effective 4D spacetime of the CS in agreement with GUT;

→ CS can be produced in superstring theory

→ Super massive CS with Gμ >> 1 will be warped down to GUT scale( $10^{-7}$)

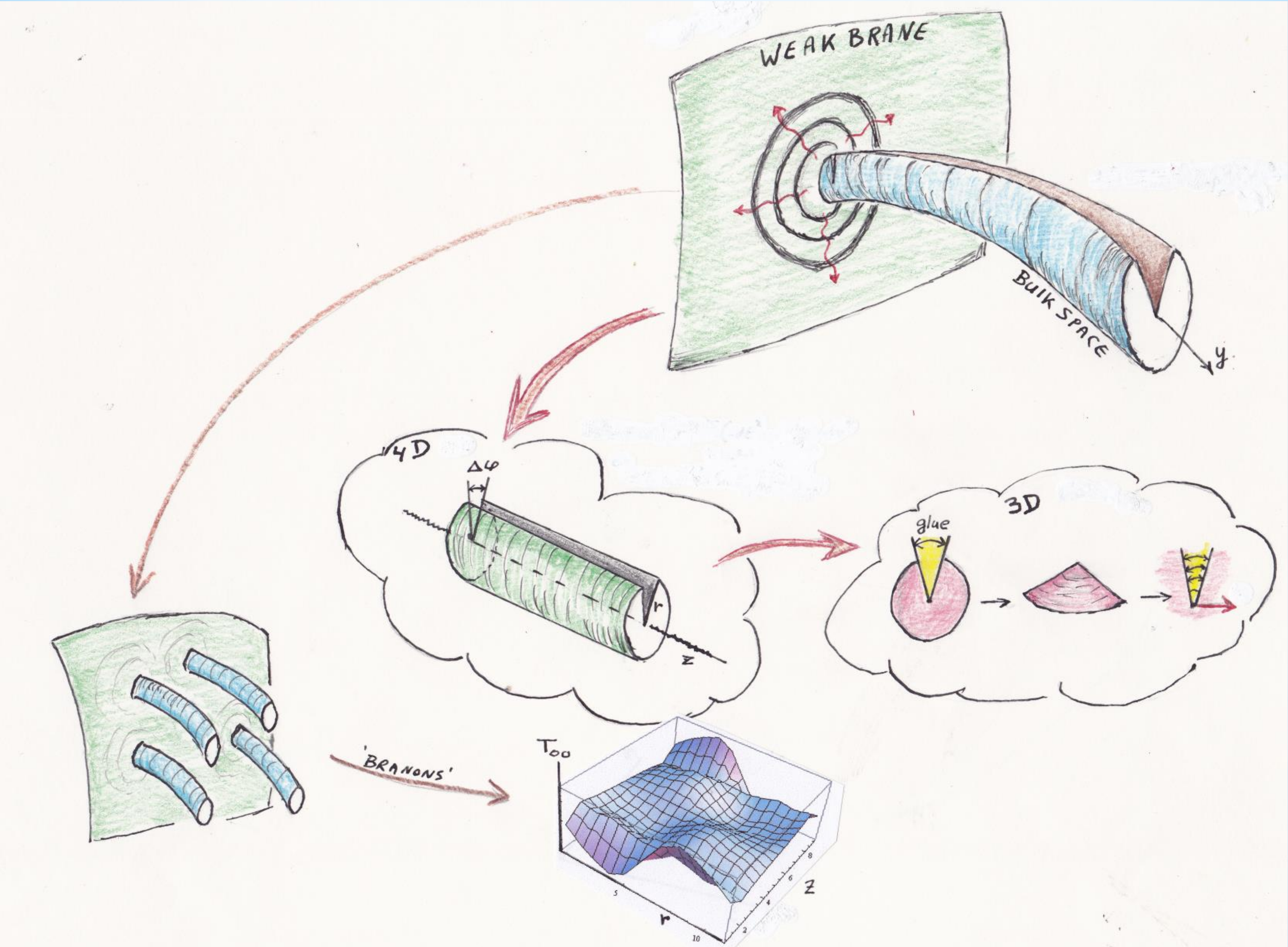
→ Disturbances in the spatial components of the stress-energy tensor cause cylindrical symmetric waves, amplified due to the presence of the bulk space with warp factor

►► Mass: $\mu = 2\pi F(y) \int_0^{\infty} e^{-A} K\sigma dr$ with F the WARPFACTOR

so: building up a huge mass in the bulk : KK-modes on brane

►► Test of RS type models against observational constraint possible !

Cern: KK-particles detectable?



WEAK BRANE
BULK SPACE
y
4D
Δφ
r
z
3D
glue
'BRANONS'
T00
r
z

# The warped 5D model with the U(1) scalar-gauge field

We consider the warped spacetime: $[{}^4g_{\mu\nu} = {}^5g_{\mu\nu} - n_\mu n_\nu]$ (n normal to brane)

$$ds^2 = \mathcal{W}(t,r,y)^2\left[e^{2(\gamma(t,r)-\psi(t,r))}\left(-dt^2 + dr^2\right) + e^{2\psi(t,r)}dz^2 + r^2e^{-2\psi(t,r)}d\varphi^2\right] + dy^2$$

With W the warpfactor. We reside on the BRANE y=0. Gravity can prop. in BULK

We consider: scalar-gauge field in brane: [empty BULK; only $\Lambda_5$ ]

$$\Phi = \eta X(t,r)e^{i\varphi}, \qquad A_\mu = \frac{1}{\varepsilon}[P(t,r) - 1]\nabla_\mu\varphi, \qquad V(\Phi) = \frac{1}{8}\beta(\Phi^2 - \eta^2)^2$$

From the 5D-eq:
[Slagter-Pan;2016]
Found of Phys

$$\mathcal{W} = \frac{e^{\sqrt{-\frac{1}{6}\Lambda_5}(y-y_0)}}{\alpha\sqrt{r}}\sqrt{(d_1e^{\alpha t} - d_2e^{-\alpha t})(d_3e^{\alpha r} - d_4e^{-\alpha r})}$$

The modified 4D effective Einstein equations:

$${}^4G_{\mu\nu} = -\Lambda_{eff}\,{}^4g_{\mu\nu} + \kappa_4^2\,{}^4T_{\mu\nu} + \kappa_5^4\,S_{\mu\nu} - \mathcal{E}_{\mu\nu}$$

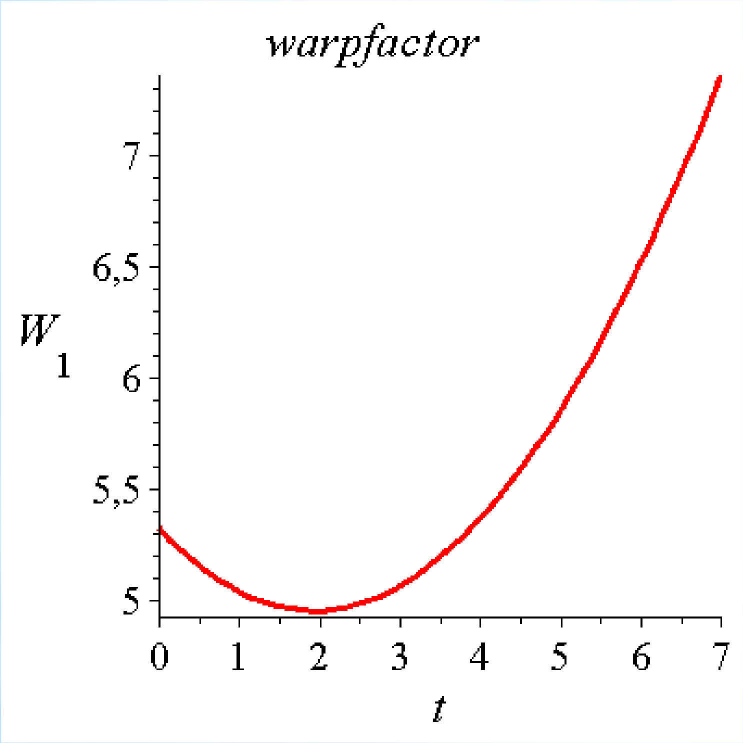
S is the quadratic term in the energy-momentum tensor [from extrinsic curv. terms in proj. Einstein tensor]

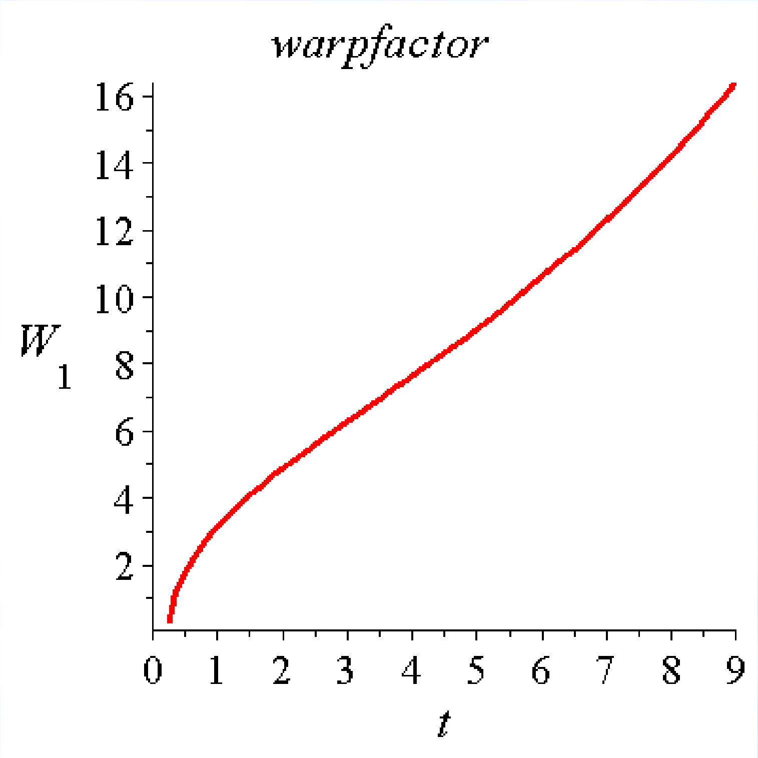
$\mathcal{E}$ is part of the 5D Weyl tensor C and carries inf.of grav.field outside the brane

$$\mathcal{E}_{\mu\nu} = {}^5C_{\alpha\gamma\beta\delta}n^\gamma n^\delta\,{}^4g_\mu^\alpha\,{}^4g_\nu^\beta$$

$\Lambda_{eff} = 0$ (RS-finetuning)

# * Exact solutions




Slagter-Pan;2016--Found of Phys

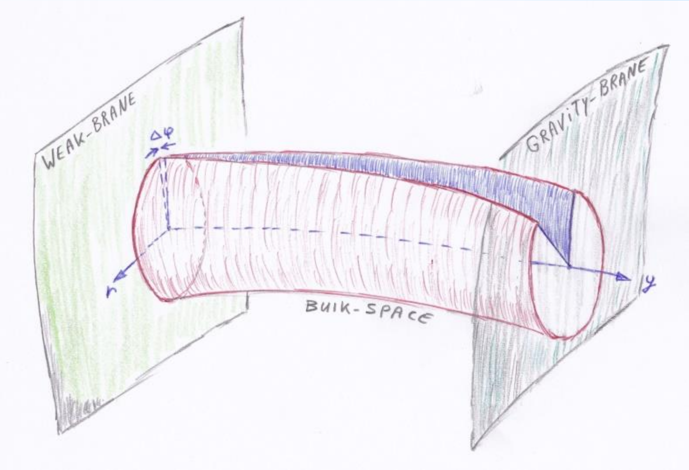
10.1007_s10701-016-0002-2.pdf

# The warped 5D model with the U(1) scalar-gauge field

The scalar-gauge field equations:

$$D^{\mu}D_{\mu}\Phi = 2\frac{dV}{d\Phi^{*}} \qquad {}^{4}\nabla^{\mu}F_{\nu\mu} = \frac{1}{2}i\varepsilon(\Phi(D_{\nu}\Phi)^{*} - \Phi^{*}D_{\nu}\Phi)$$

With $D_\mu \Phi = {}^4\nabla_\mu \Phi + i\epsilon A_{\mu\Phi}$.




►The scalar gauge field can build-up a **huge mass** per unit length (or **angle-deficit**) by the warpfactor W: **$G\mu \sim 1$**

►**Can induce massive KK-modes felt on the brane.**
[while manifestation on **brane** will be warped down to GUT scale consistent with observation]

►**Disturbances** can cause cyl. symm waves amplified by the warpfactor and could survive natural damping due to the expansion of the universe.

►Could possible explane "**self-acceleration**" **[ dark energy]** with $\Lambda_{eff}$=0 !

# The nonlinear wave approximation in 5D GenRel

We expand:

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(x) + \frac{1}{\omega} h_{\mu\nu}(x,\xi,\chi,..) + \frac{1}{\omega^2} k_{\mu\nu}(x,\xi,\chi,..) + \cdots$$

$$A_\mu = \bar{A}_\mu(x) + \frac{1}{\omega} B_\mu(x,\xi,\chi,..) + \frac{1}{\omega^2} C_\mu(x,\xi,\chi,..) + \cdots$$

$$\Phi = \bar{\Phi}(x) + \frac{1}{\omega} \Psi(x,\xi,\chi,..) + \frac{1}{\omega^2} \Xi(x,\xi,\chi,..) + \cdots$$

We define

$$\frac{dg_{\mu\nu}}{dx^\sigma} = g_{\mu\nu,\sigma} + \omega l_\sigma \dot{g}_{\mu\nu} + \breve{\omega} k_\upsilon \breve{g}_{\mu\nu} + .. \qquad g_{\mu\nu,\sigma} = \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \qquad \dot{g}_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial \xi}$$

The rapid variations occur in the directions of $l_\mu$ , $k_\mu$ transversal to the sub-minifolds of constant phase .

For the time being: only $l_\mu = \frac{\partial \Theta}{\partial x^\mu}$ [ now $\Theta = t - r$]

The perturbations can be **φ-dep.**

We write:

$$\Gamma^{\alpha}_{\mu\nu} = \bar{\Gamma}^{\alpha}_{\mu\nu} + \Gamma^{\alpha(0)}_{\mu\nu} + \frac{1}{\omega}\Gamma^{\alpha(1)}_{\mu\nu} + \ \dots$$

$$R^{\sigma}_{\mu\tau\nu} = \omega\, R^{\sigma(-1)}_{\mu\tau\nu} + \bar{R}^{\sigma}_{\mu\tau\nu} + R^{\sigma(0)}_{\mu\tau\nu} + \frac{1}{\omega}\, R^{\sigma(1)}_{\mu\tau\nu} + \cdots$$

with $$\Gamma^{\sigma(0)}_{\mu\nu} = \tfrac{1}{2}\,\bar{g}^{\beta\sigma}\left(l_\mu \dot{h}_{\beta\nu} + l_\nu \dot{h}_{\beta\mu} - l_\beta \dot{h}_{\mu\nu}\right)$$

$$\Gamma^{\alpha(1)}_{\mu\nu} = \tfrac{1}{2}\left(h^{\sigma}_{\mu:\nu} + h^{\sigma}_{\nu:\mu} - h^{:\sigma}_{\mu\nu} - \ l_\nu \dot{k}^{\sigma}_{\mu} + l_\mu \dot{k}^{\sigma}_{\nu} - l^{\sigma}\dot{k}_{\mu\nu}\right) - h^{\sigma}_{\rho}\Gamma^{\rho(0)}_{\mu\nu}$$

We substitute the expansions into the fieldequations and subsequently put zero the various powers of ω

From the $\omega^{-1}$ **Einstein**: $\,^4G^{(-1)}_{\mu\nu} = -\mathcal{E}^{(-1)}_{\mu\nu}$ ("gauge" cond)

**Scalar**: $l^\mu l_\mu \ddot{\Psi} = 0$ **[note: this is the Eikonal eq., or $\ddot{\Psi}$ ]**

**gaugefield**: $l^\mu \ddot{B}_\mu = 0$

Normally one imposes a priori gauge-conditions: $l^\mu\left(\ddot{h}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\ddot{h}\right) = 0$

The contribution of $\mathcal{E}^{(-1)}_{\mu\nu}$ changes the conditions on $h_{\mu\nu}$

**Further**: we take $l^\mu l_\mu = 0$ (Eikonal cond)

$l^\mu l_\mu \neq 0$ means that $h_{\mu\nu}$ arises from a coord transformation.

# The effective brane Einstein equations

The $\omega^{(0)}$- Einstein equations:

$$ {}^4\bar{G}_{\mu\nu} + {}^4G^{(0)}_{\mu\nu} = -\Lambda_{eff}\,{}^4\bar{g}_{\mu\nu} + \kappa_4^2({}^4\bar{T}_{\mu\nu} + {}^4T^{(0)}_{\mu\nu}) + \kappa_5^4\left(\bar{S}_{\mu\nu} + S^{(0)}_{\mu\nu}\right) - (\bar{\mathcal{E}}_{\mu\nu} + \bar{\mathcal{E}}^{(0)}_{\mu\nu}) $$

where the part of the Weyl tensor is:

$$ \mathcal{E}_{\mu\nu} = n^\gamma n^\delta\,{}^4g^\alpha_\mu\,{}^4g^\beta_\nu[{}^5R_{\alpha\gamma\beta\delta} - \frac{1}{3}({}^5g_{\alpha\gamma}\,{}^5R_{\delta\beta} - {}^5g_{\alpha\delta}\,{}^5R_{\gamma\beta} - {}^5g_{\beta\delta}\,{}^5R_{\gamma\alpha} + {}^5g_{\beta\delta}\,{}^5R_{\gamma\alpha}) + \frac{1}{12}({}^5g_{\alpha\gamma}\,{}^5g_{\delta\beta} - {}^5g_{\alpha\delta}\,{}^5g_{\gamma\beta})\,{}^5R] $$

Now we take only $h_{11}, h_{44}\quad h_{13}\quad h_{14}\quad h_{55}\ \neq\ 0$

One can also integrate the equations wrt to $\xi$ : propagation equations

**Then:** substitute back these equations: $(\Lambda_{eff} = 0\ (RS\ finetuning))$

$$ {}^4\bar{G}_{\mu\nu} = \kappa_4^2\,{}^4\bar{T}_{\mu\nu} + \kappa_5^4\bar{S}_{\mu\nu} - \bar{\mathcal{E}}_{\mu\nu} + \frac{1}{\tau}\int\left(\kappa_4^2T^{(0)}_{\mu\nu} + \kappa_5^4\,S^{(0)}_{\mu\nu} - {}^4G^{(0)}_{\mu\nu} - \mathcal{E}^{(0)}_{\mu\nu}\right)d\xi $$

one says: ► $-\int \mathcal{E}^{(0)}_{\mu\nu}\,d\xi$ is the KK-mode contribution of the perturbative 5D graviton

► can play the role of effective CC ( same sign)

► is an extra "back-reaction" term which contain $\dot{h}_{55}$

# The scalar-gauge field equations

Simplified case: $l_\mu = [1, -1, 0, 0, 0]$
Then: first order gauge field: $\boldsymbol{B_\mu = [B_0, B_0, 0, B, 0]}$

From the **gauge field** eq: : The $\bar{A}_\mu$ is as the unperturbed case.
The first order perturbations:

$$\partial_t \dot{\Psi} = \partial_r \dot{\Psi} + \left[\frac{\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}}}{\bar{\mathcal{W}}} + \frac{1}{2r}\right] \dot{\Psi}$$

$$\partial_t \dot{B} = \partial_r \dot{B} + \left[\partial_r \bar{\psi} - \partial_t \bar{\psi} - \frac{1}{2r}\right] \dot{B} + e^{2\bar{\psi}} \frac{(\partial_r \bar{P} - \partial_t \bar{P})}{2r^2 \bar{\mathcal{W}}^2 \varepsilon} \dot{h}_{44}$$

$$\partial_t \dot{B}_0 = \partial_t \dot{B}_0 - e^{2\bar{\gamma}} \frac{\partial_\varphi \dot{B}}{r^2} - \varepsilon e^{2\bar{\gamma} - 2\bar{\psi}} \bar{\mathcal{W}}^2 \bar{X} \Psi \sin \varphi + e^{2\bar{\psi}} \frac{(\partial_t \bar{P} - \partial_r \bar{P})}{2r^2 \bar{\mathcal{W}}^2 \varepsilon} \dot{h}_{14}$$

► We observe: **φ-dependent** parts arise, amplified by warpfactor!
► One needs: $\boldsymbol{l^\mu \bar{A}_\mu = 0}$ , otherwise real and imaginary parts interacts as propagation progresses.
► We omitted for time being $C_\mu$ and the $\boldsymbol{\kappa_5^4 \left(\bar{S}_{\mu\nu} + S_{\mu\nu}^{(0)}\right)}$ term
► Approximate wave solution no longer axially symmetric! [also found by Choquet B]

# The scalar background field equation

After integration we obtain for the background scalar field

$$\overline{D}^{\alpha}\overline{D}_{\alpha}\overline{\Phi} - \frac{1}{2}\beta\overline{\Phi}(\overline{\Phi}\overline{\Phi}^{*} - \eta^{2}) = \frac{1}{\tau}\int \left(h^{\mu\nu} l_{\mu} l_{\nu} \ddot{\Psi} + \bar{g}^{\mu\nu}\Gamma^{\alpha(0)}_{\mu\nu}\dot{\Psi}\right) d\xi$$

►There is a "backreaction" from the HF perturbations

# The background Einstein equations to order $\omega^{(0)}$

In our special model, we have decoupled background equations:

$$\partial_{tt}^2\bar{\mathcal{W}} = -\partial_{rr}^2\bar{\mathcal{W}} + \frac{2}{\bar{\mathcal{W}}}(\partial_t\bar{\mathcal{W}}^2 + \partial_r\bar{\mathcal{W}}^2) - \bar{\mathcal{W}}(\partial_t\bar{\psi}^2 + \partial_r\bar{\psi}^2) + \frac{\bar{\mathcal{W}}}{r}(\partial_r\bar{\gamma} - \partial_t\bar{\gamma})$$
$$+ 2(\partial_r\bar{\mathcal{W}} - \partial_t\bar{\mathcal{W}})(\partial_t\bar{\psi} - \partial_r\bar{\psi} + \partial_r\bar{\gamma} - \partial_t\bar{\gamma}) + 2\bar{\mathcal{W}}\partial_t\bar{\psi}\partial_r\bar{\psi} - 4\frac{\partial_t\bar{\mathcal{W}}\partial_r\bar{\mathcal{W}}}{\bar{\mathcal{W}}}$$
$$- 2\partial_{tr}\bar{\mathcal{W}} - \frac{3}{4}\kappa_4^2\left(e^{2\bar{\psi}}\frac{(\partial_t\bar{P} - \partial_r\bar{P})^2}{\bar{\mathcal{W}}r^2\epsilon^2} + \bar{\mathcal{W}}(\partial_t\bar{X} - \partial_r X)^2\right)$$

$$\partial_{tt}^2\bar{\psi} = \partial_{rr}^2\bar{\psi} + \frac{\partial_t\bar{\psi}}{r} + \frac{2}{\bar{\mathcal{W}}}(\partial_r\bar{\mathcal{W}}\partial_r\bar{\psi} - \partial_t\bar{\mathcal{W}}\partial_r\bar{\psi}) - \frac{\partial_r\bar{\mathcal{W}}}{r\bar{\mathcal{W}}} + \frac{3e^{2\bar{\psi}}}{4\bar{\mathcal{W}}^2r^2\epsilon^2}\kappa_4^2(\partial_t\bar{P}^2 - \partial_r\bar{P}^2 - \bar{\mathcal{W}}^2\varepsilon^2\bar{X}^2\bar{P}^2e^{2\bar{\gamma}-2\bar{\psi}})$$

$$\partial_t\bar{\gamma} = \partial_r\bar{\gamma}$$
$$+ \frac{1}{\partial_t\bar{\mathcal{W}} - \partial_r\bar{\mathcal{W}} - \frac{\bar{\mathcal{W}}}{2r}}\left[\frac{1}{2}\bar{\mathcal{W}}(\partial_t\bar{\psi} - \partial_r\bar{\psi})^2 + \frac{\partial_r\bar{\mathcal{W}}}{r} - \partial_{tr}\bar{\mathcal{W}} + \partial_{rr}\bar{\mathcal{W}} + \frac{2\partial_t\bar{\mathcal{W}}\partial_r\bar{\mathcal{W}}}{\bar{\mathcal{W}}}\right.$$
$$+ (\partial_r\bar{\mathcal{W}} - \partial_t\bar{\mathcal{W}})(\partial_r\bar{\psi} - \partial_t\bar{\psi}) - \frac{\partial_r\bar{\mathcal{W}}^2 + 3\partial_t\bar{\mathcal{W}}^2}{2\bar{\mathcal{W}}}$$
$$+ \kappa_4^2\frac{\bar{\mathcal{W}}}{16}\left(7\partial_t\bar{X}^2 + 5\partial_r\bar{X}^2 - 12\partial_t\bar{X}\partial_r\bar{X} + 5e^{2\bar{\gamma}}\frac{\bar{X}^2\bar{P}^2}{r^2} + 6e^{2\bar{\psi}}\frac{(\partial_r\bar{P} - \partial_t\bar{P})^2}{\bar{\mathcal{W}}^2r^2\epsilon^2}\right.$$
$$\left.\left. + \bar{\mathcal{W}}^2\varepsilon e^{2\bar{\gamma}-2\bar{\psi}}(\bar{X}^2 - \eta^2)^2\right)\right]$$

# The Einstein propagation equations to order $\omega^{(0)}$

We obtain the propagation equations by substituting back the integrated equations:

$$\partial_t \dot{h}_{11} = \partial_r \dot{h}_{11} + \frac{e^{2\bar{\gamma}}}{r^2}\left(\partial_r \bar{\psi} - \partial_t \bar{\psi} - \frac{1}{2r}\right)\dot{h}_{44} + \frac{1}{2}\left(\ddot{k}_{22} + \ddot{k}_{11}\right) - \ddot{k}_{12} + \frac{2}{\bar{\mathcal{W}}}\left(\partial_t \bar{\mathcal{W}} - \partial_r \bar{\mathcal{W}}\right.$$
$$\left. + \bar{\mathcal{W}}\left(\partial_r \bar{\psi} - \partial_t \bar{\psi} + \partial_t \bar{\gamma} - \partial_r \bar{\gamma}\right)\right)\dot{h}_{11} + \frac{1}{2} e^{2\bar{\gamma} - 2\bar{\psi}} \bar{\mathcal{W}}^2 \left(\frac{\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}}}{\bar{\mathcal{W}}} + \frac{1}{2r}\right)\dot{h}_{55}$$
$$+ \kappa_4^2 e^{2\bar{\gamma} - 2\bar{\psi}} \bar{\mathcal{W}}^2 \left(\partial_t \bar{X} - \partial_r X\right)\dot{\Psi} \cos\varphi$$

$$\partial_t \dot{h}_{44} = \partial_r \dot{h}_{44} + \left(2\partial_r \bar{\psi} - 2\partial_t \bar{\psi} - \frac{3}{2r} + \frac{\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}}}{\bar{\mathcal{W}}}\right)\dot{h}_{44} + \frac{\kappa_4^2}{\varepsilon}\left(\partial_t \bar{P} - \partial_r \bar{P}\right)\dot{B}$$
$$+ \frac{1}{2} r^2 e^{-2\bar{\psi}} \bar{\mathcal{W}}^2 \left(\partial_t \bar{\psi} - \partial_r \bar{\psi} + \frac{1}{2r}\right)\dot{h}_{55}$$

$$\partial_t \dot{h}_{55} = \partial_r \dot{h}_{55}$$

$$\partial_t \dot{h}_{13} = \partial_r \dot{h}_{13} + 2\left(\frac{\partial_t \bar{\mathcal{W}} - \partial_r \bar{\mathcal{W}}}{\bar{\mathcal{W}}} + \partial_t \bar{\psi} - \partial_r \bar{\psi}\right)\dot{h}_{13} + \ddot{k}_{13} - \ddot{k}_{23}$$

$$\partial_t \dot{h}_{14} = \partial_r \dot{h}_{14} + 2\left(\partial_t \bar{\psi} - \partial_r \bar{\psi} + \frac{1}{r} + \frac{\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}}}{\bar{\mathcal{W}}}\right)\dot{h}_{14} + \ddot{k}_{24} - \ddot{k}_{14}$$
$$+ 2\kappa_4^2 e^{2\bar{\gamma} - 2\bar{\psi}} \bar{\mathcal{W}}^2 \bar{X} \bar{P} \dot{\Psi} \sin\varphi + \partial_\varphi \left[\dot{h}_{11} + \frac{e^{2\bar{\gamma}}}{r^2}\dot{h}_{44} - e^{2\bar{\gamma} - 2\bar{\psi}} \bar{\mathcal{W}}^2 \dot{h}_{55}\right]$$

►These propagation equations are linear in the first order derivative.
Appearance of combinations of $\ddot{h}_{\mu\nu}$ and $\ddot{k}_{\mu\nu}$ terms:
distortion of the shape of the waves

►The equation for $\dot{h}_{55}$ is as expected: $\dot{\boldsymbol{h}}_{55} = \mathcal{M}_1(t,r).\ \mathcal{M}_2(\varphi, y, \xi)$ :
the brane part must be separable from the bulk part.

►There is an interaction between the HF perturbations from the bulk, the matterfields on the brane and the evolution of $\dot{h}_{ij}$

►The bulk contribution $\dot{h}_{55}$ is amplified by the warpfactor!

►It is a reflection of the massive KK-modes felt on the brane.

►The $\dot{h}_{55}$ contribution disappears when: $\left[\partial_r\bar{\psi} - \partial_t\bar{\psi} - \frac{1}{2r}\right]=0$ ( physically not very interesting: $\bar{\psi} = a\log(r) + b$; so a testparticle in this field is repelled from the cylinder

►Effectively a **dark-energy** term in Einstein equations
**However:** a more general solution must be investigated with $\kappa_5^4\left(\bar{S}_{\mu\nu} + S_{\mu\nu}^{(0)}\right)$
For example in $\bar{\psi}_{tt}$: terms at rhs:

$$\kappa_5^4 \int \left(\dot{\Psi}\dot{B}(\bar{X}_t - \bar{X}_r)(\bar{P}_t - \bar{P}_r)cos\varphi\right)d\xi$$

# Example of a solution

Consider the last eq. for $\dot{h}_{14}$ : For $\dot{h}_{14}$=0 (for the moment): integration to φ:

$$\dot{h}_{11} = e^{2\bar{\gamma}-2\bar{\psi}}\bar{\mathcal{W}}^2\left[\dot{h}_{55} - 2\kappa_4^2\bar{X}\bar{P}\int(\dot{\Psi}\sin\varphi)d\varphi\right]$$

Let us consider $\Psi = \widehat{\Psi}(t,r,\xi)cos\varphi$ Then we obtain:

$$\dot{h}_{11} = 2\kappa_4^2\bar{X}\bar{P}e^{2\bar{\gamma}-2\bar{\psi}}\bar{\mathcal{W}}^2\dot{\widehat{\Psi}}cos2\varphi$$

With two extremal values on $[0,\pi]$ and amplified by warpfactor.
$\gamma_t$ can also be written as:

$$\partial_t\,\bar{\gamma} = \frac{2r}{\bar{W}_1 + 2r\partial_r\bar{W}_1}\Big[\partial_r\bar{W}_1\partial_t\bar{\psi} + \partial_t\bar{W}_1\partial_r\bar{\psi} + \partial_{tr}\bar{W}_1 - \partial_r\bar{\gamma}\partial_t\bar{W}_1 + \bar{W}_1\partial_r\bar{\psi}\partial_t\bar{\psi}$$
$$- \frac{2}{\bar{W}_1}\partial_r\bar{W}_1\partial_t\bar{W}_1 + \frac{3}{4}\kappa_4^2\Big\{\bar{W}_1\partial_r\bar{X}\partial_t\bar{X} + \frac{e^{2\bar{\psi}}}{r^2e^2\bar{W}_1}\partial_r\bar{P}\partial_t\bar{P} + \frac{1}{\tau}\int\Big(\bar{W}_1\dot{\Psi}^2 + \frac{e^{2\bar{\psi}}}{r^2\bar{W}_1}\dot{B}^2\Big)d\xi\Big\}$$
$$- \frac{3}{16\tau}\int\Big(\bar{W}_1\dot{h}_{55}^2 - \frac{2e^{4\bar{\psi}}}{r^4\bar{W}_1^3}\dot{h}_{44}^2 - \frac{4e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1}(\dot{h}_{11}\dot{h}_{55})\Big].\qquad(20)$$

# Possible evidence of cosmic strings via alignment of quasar and BH polarizations?

There appeared two investigations on polarization vectors on BH and quasars:
D.Hutsemekers, et al, **Alignment of quasar polarizations with large-scale structures**
A.Taylor, et al, **Alignment of Radio Galaxies in deep radio imaging of ELAIS N1**

## Alignment of quasar polarizations with large-scale structures*

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**ABSTRACT**

We have measured the optical linear polarization of quasars belonging to Gpc-scale quasar groups at redshift $z \sim 1.3$. Out of 93 quasars observed, 19 are significantly polarized. We found that quasar polarization vectors are either parallel or perpendicular to the directions of the large-scale structures to which they belong. Statistical tests indicate that the probability that this effect can be attributed to

## Alignments of Radio Galaxies in Deep Radio Imaging of ELAIS N1

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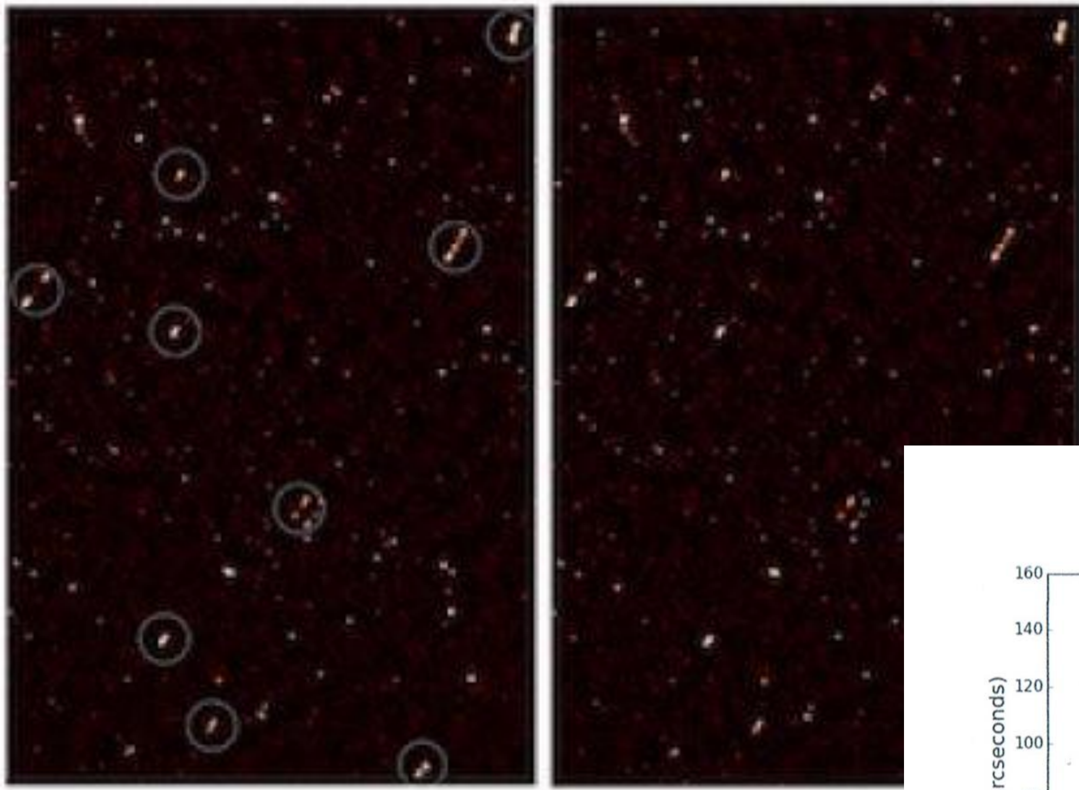
**ABSTRACT**

We present a study of the distribution of radio jet position angles of radio galaxies over an area of 1 square degree in the ELAIS N1 field. ELAIS N1 was observed with the Giant Metrewave Radio Telescope at 612 MHz to an rms noise level of 10 $\mu$Jy and angular resolution of $6'' \times 5''$. The image contains 65 resolved radio galaxy jets. The spatial distribution reveals a prominent alignment of jet position angles along a "filament" of about 1°. We examine the possibility that the apparent alignment arises from an underlying random distribution and find that the probability of chance



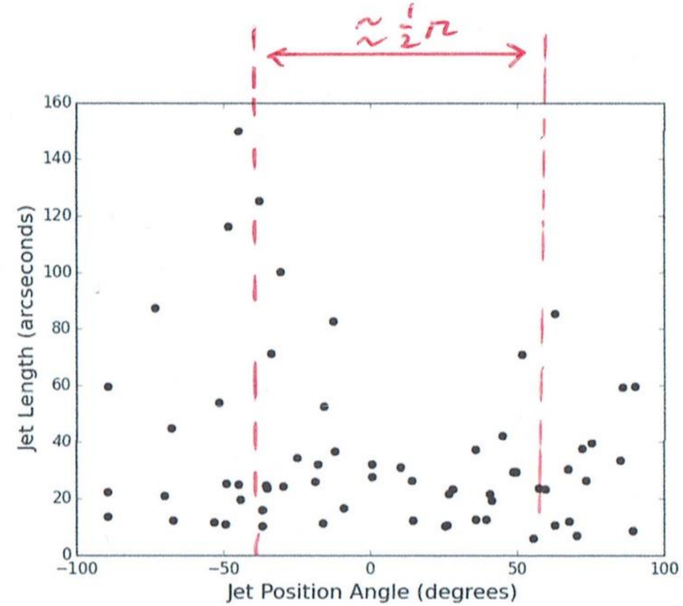
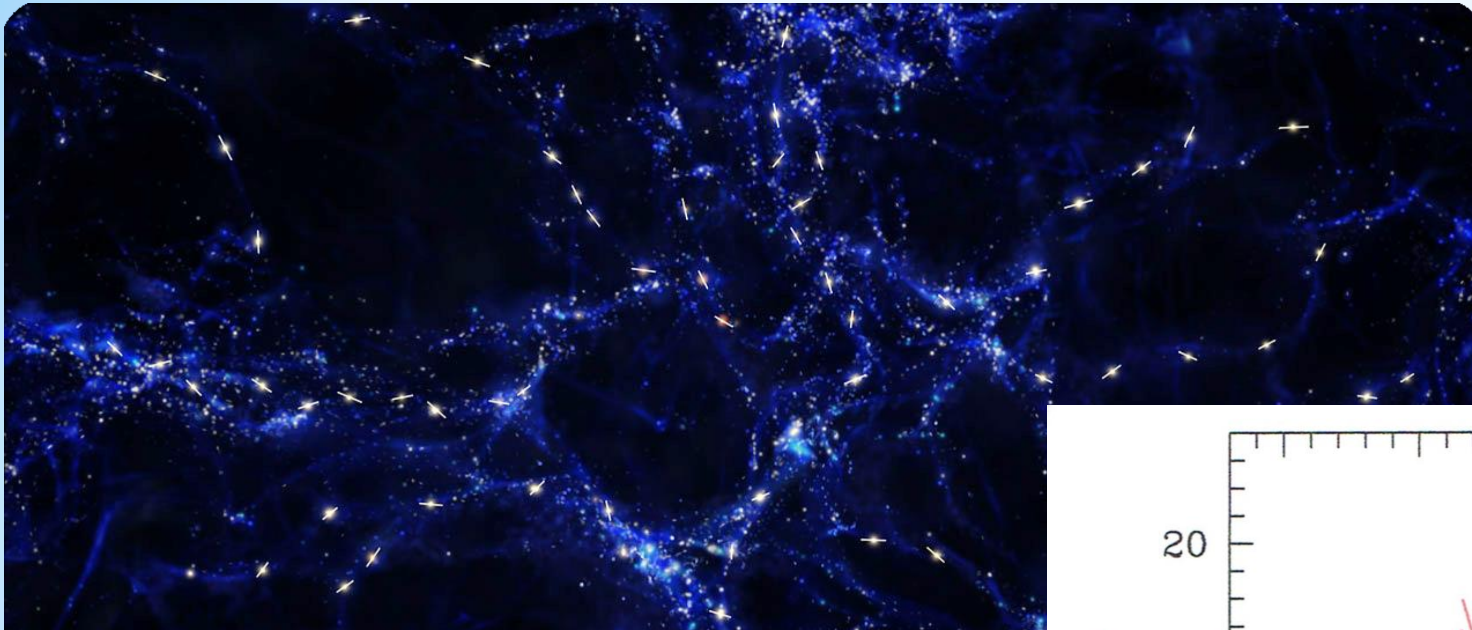



Figure 4. The length of the 64 radio jets plotted against jet position angle. The longest jets are preferentially present in the excess of object with polarisation angle ~ −40°.

Two preferred directions



*Artist's impression of the alignment of the spin axes of quasars to the l*
*Universe. These alignments are over billions of light-years, the larges*
*Credits : ESO/M. Kornmesser*

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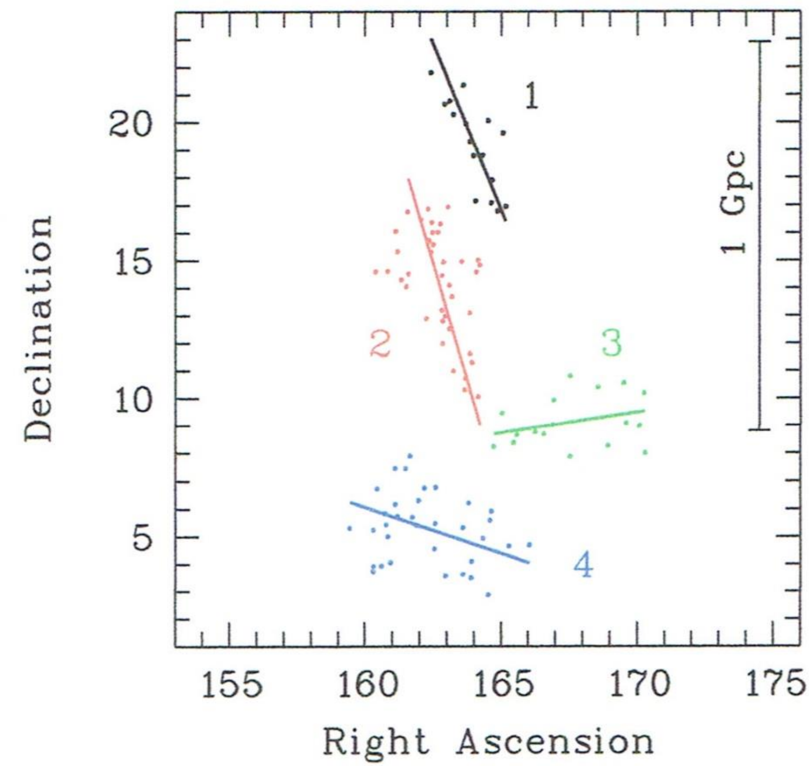


**Fig. 4.** The quasar groups and their orientations on the sky. Right ascensions and declinations are in degree. The superimposed lines illustrate the orientations of the four groups labelled 1, 2, 3, 4. The comoving distance scale at redshift $z = 1.3$ is indicated assuming a flat Universe with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_m = 0.27$.

Prospect: Wait for new data from Gaia! [500 000 pulsars?]

Then: next order results can be tested.