Inflation with light Weyl ghost

A. Tokareva (INR, Moscow) in collaboration with M. Ivanov (EPFL, Lausanne), D. Gorbunov (INR, Moscow)

May, 10

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In both frameworks there is massive spin-2 ghost.

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  - Light ghost, $m < H_i$: amplitude of tensor perturbations damped by factor $m/\sqrt{2}H_i$. We studied the scalar perturbations for that case.

May the light ghost be safe?

- For $m \gtrsim 10^{10}$ GeV – the naturalness problem for the Higgs mass. Ghost impact on the ratio m_H/M_P becomes large (A.Salvio and A.Strumia, 1403.4226).

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- From short distance constraints: $m > 4 \cdot 10^{-12}$ GeV (S. J. Smullin, A. A. Geraci, D. M. Weld, A. Kapitulnik and J. Chiaverini, eConf C 040802, MOT004 (2004).)

Linearized action in the newtonian gauge at inflation

$$ds^2 = a(\eta)^2 \{-(1+2\Psi)\, d\eta^2 + 2\bar{\Psi}_i\, d\eta\, dx^i + [(1+2\Phi)\,\delta_{ij} + \bar{h}_{ij}]\, dx^i\, dx^j\}\,,\ \delta\phi = \chi.$$

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For de Sitter background – exact diagonalization.

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For de Sitter background – exact diagonalization. Vacuum initial conditions on field perturbations at $\eta \to \infty$:

$$\boxed{\Phi = \frac{H_0 q\eta}{2\pi\sqrt{6}M_P}\, e^{-iq\eta}, \qquad \chi = \frac{H_0}{2\pi}\, q\eta e^{-iq\eta}}$$

Superhorizon solutions

$$\Phi \sim a, \quad \chi \sim a, \quad \mathcal{R} = \Phi - \frac{H}{\dot{\phi}}\chi = -H_0/(2\pi\sqrt{2\epsilon}M_p)$$

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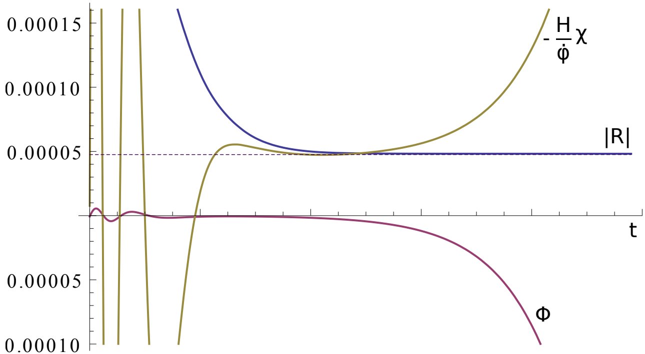

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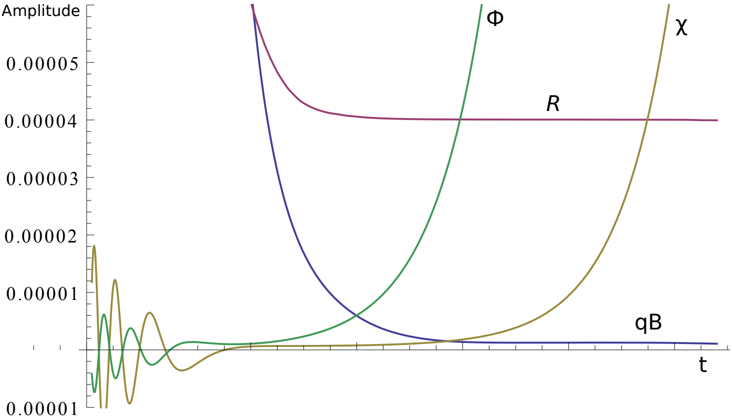
$$E = 0, \quad \mathcal{R} = -\frac{W_c}{2\epsilon} = -\frac{H_0}{2\pi\sqrt{2\epsilon}M_P}, \quad \boxed{B = \frac{\chi}{a\dot{\phi}} = \sqrt{\frac{\epsilon}{3}}\frac{\mathcal{R}}{q}}$$

Picture


Amplitude
0.00005
0.00004
0.00003
0.00002
0.00001
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Φ
χ
R
qB
t

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Result:

$$\boxed{\mathcal{R} = const, \quad B = \sqrt{\frac{\epsilon}{3}\frac{\mathcal{R}}{q}} \left(\frac{t}{\sqrt{\gamma}}\right)^{-p/2} e^{\pm it/\sqrt{\gamma}}}$$

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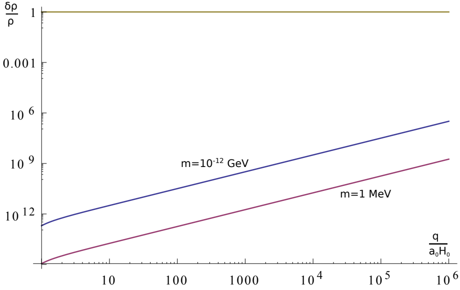

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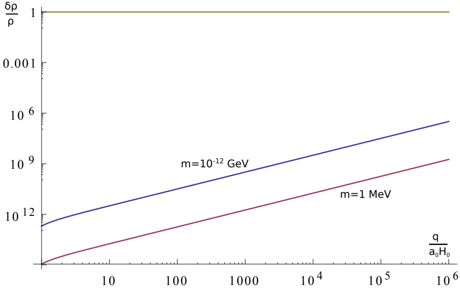
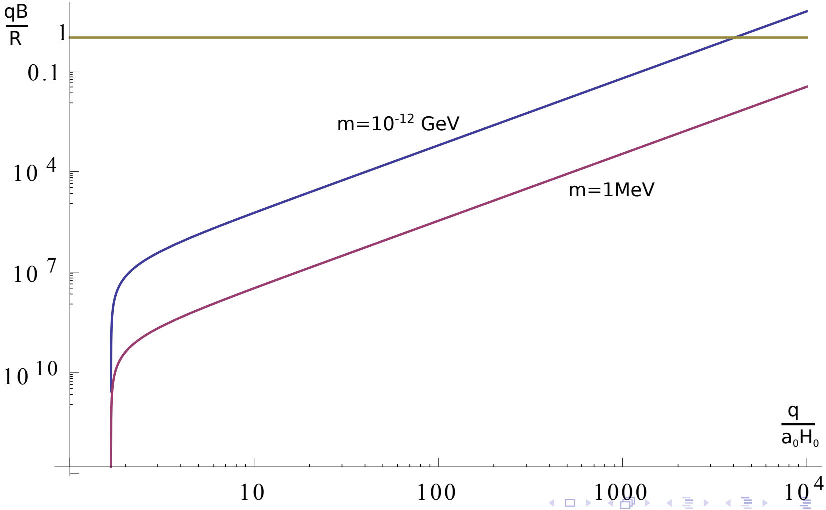
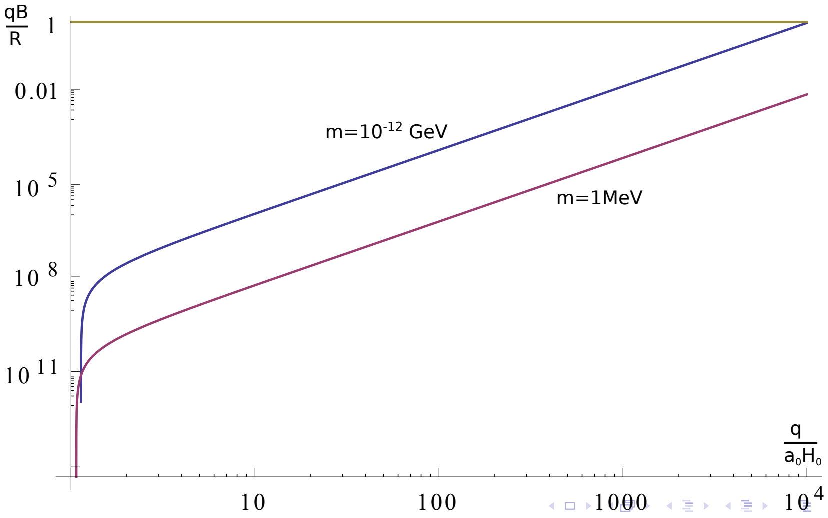


- The relative ghost impact to CMB is characterized by $g_{0i} = n_i qB/\mathcal{R}$ in comoving gauge.

$n_i g_{oi}$ spectrum at recombination



$n_i g_{oi}$ spectrum now



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- The cosmological ghost scalar perturbations do not affect the Planck data.

Thanks for your attention!