.

Hot topics in Modern Cosmology, Cargese

The simplest two-field Q-balls

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Inhomogeneous (and time-dependent) field configuration

motivation:

- DM candidate (soliton), to avoid restrictions on WIMP,
- clumping of DM, Axion/Bose star,
- baryogenesis, cogenesis,

main issues:

- stability
- energy (mass) spectrum
- long-range forces (massless fields)

Stability   *Charge* or Topology

Static solutions in theories with $V \geq 0 \rightarrow$ problem with Derrick theorem

(nonlinear kinetic term, gauge fields)

For pure scalar field theory scaling arguments restrict number of space-time dimensions $D < 3$

Stationary (but not static!) solution for $U(1)$-invariant scalar field theory:

$$\Phi = e^{i\omega t} f(r)$$

in ordinary $(3+1)$ space-time

(we also turn off gravity) only r dependence $\rightarrow$ Q-ball

Energy and Charge indeed static!

Charge (not electric!) $\to$ global $U(1)$ symmetry,

G. Rosen, J. Math. Phys. **9** (1968) 996

or Q-balls, S.R. Coleman, Nucl. Phys. B **262** (1985) 263 [Erratum **269** (1986) 744]

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi^* \Phi)$$

Single complex scalar field – Eq. of motion can be studied by method of classical mechanics (overshoot-undershoot method, where r corresponds to time)

thin-wall, like a snowball

Eq. is nonlinear, how to check (numerical) result?

$$\boxed{\mathrm{d}E/\mathrm{d}Q = \omega}$$

here

$$E = \int d^3x (\partial_0 \phi^* \partial_0 \phi + \partial_i \phi^* \partial_i \phi + V),$$
$$\text{and } Q = \mathrm{i} \int d^3x (\partial_0 \phi^* \phi - \phi^* \partial_0 \phi)$$

generalisation with additional real (massive) mediator scalar field →

NONTOPOLOGICAL SOLITONS

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + \frac{1}{2}\partial_\mu \Psi \partial^\mu \Psi - V(\Psi) - \lambda(\Phi^*\Phi)\Psi^2$$

R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739

Time-dependent background: how to investigate stability?

D.L.T. Anderson, G.H. Derrick, J. Math. Phys. **11** (1970) 1336

$$\delta\Phi = e^{i\omega t + |\gamma| t}(\psi_1(r) + \psi_2^*(r))$$

$$\begin{pmatrix} \hat{O}_{11} & \hat{O}_{12} \\ \hat{O}_{21} & \hat{O}_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} (\omega-\gamma)^2\psi_1 \\ (\omega+\gamma)^2\psi_2 \end{pmatrix}$$

Are the stable solutions in any other theory?

\- Yes

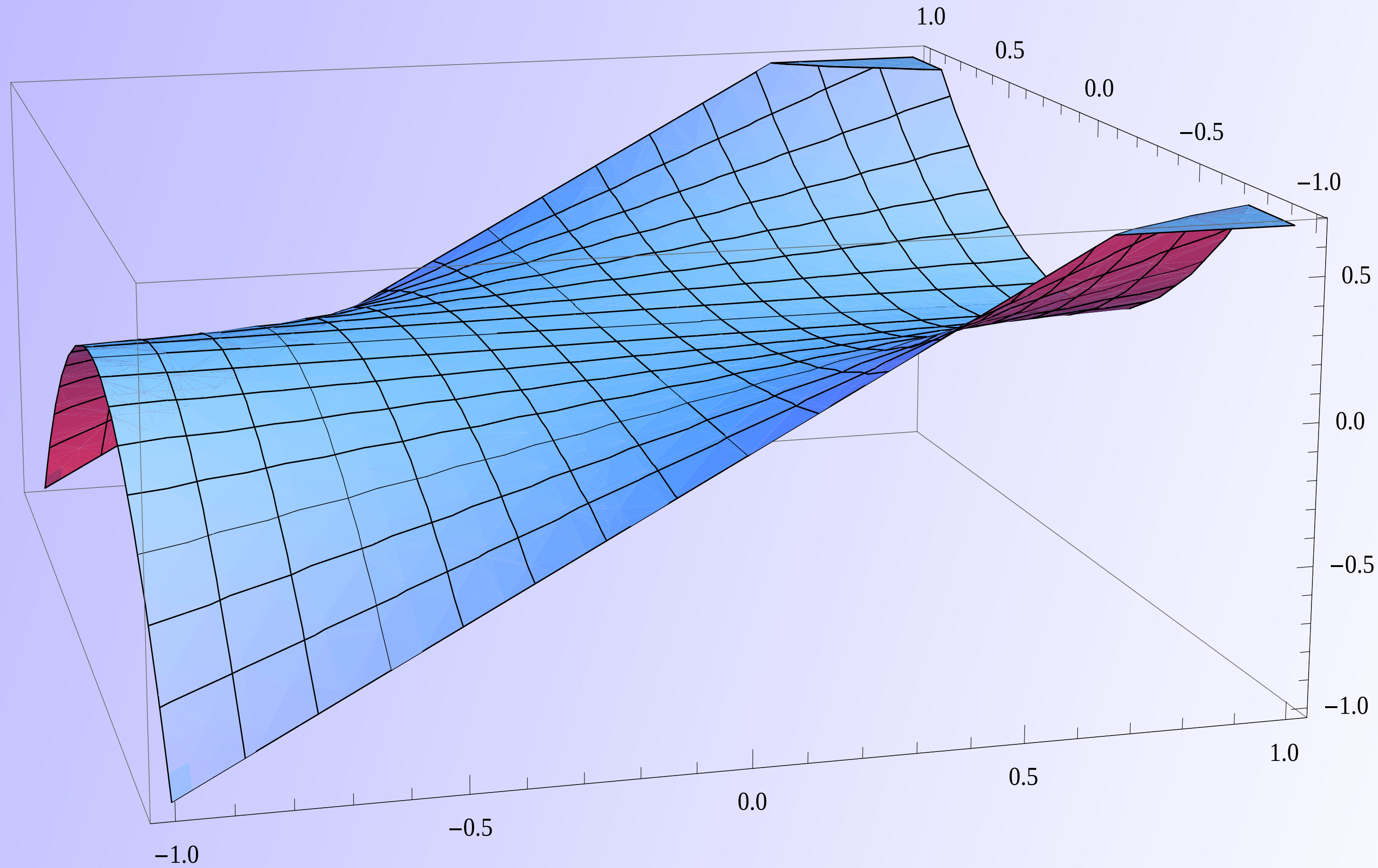
Q-criterion of stability: $\frac{dQ}{d\omega} < 0$

N.G. Vakhitov, A.A. Kolokolov, Radiophys. Quantum Electron. **16** (1973) 783 -NSE

R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739

The simplest $U(1)$-invariant Lagrangian with massless fields (E.N., M. Smolyakov 1605.02056)

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + \frac{1}{2}\partial_\mu \tilde{\Psi} \partial^\mu \tilde{\Psi} - h(\Phi^* \Phi)\tilde{\Psi}$$



Stable (classically) vacuum solution $\tilde{\Psi} = \tilde{\Psi}_0 > 0, \Phi = 0$. Then after shift

$$\tilde{\Psi} = \tilde{\Psi}_0 + \Psi$$

we have (not lenearization)

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + \frac{1}{2}\partial_\mu \Psi \partial^\mu \Psi - m^2 \Phi^* \Phi - h(\Phi^* \Phi)\Psi$$

where $m = \sqrt{h\tilde{\Psi}_0}$

This is Wick-Cutkosky model studied in Bethe-Salpeter formalism

Negative potential – critical bubble?

i.e. unstable static solution

For more solutions let us try

$$\Phi = \mathrm{e}^{\mathrm{i}\omega t} f(r)$$

$$\Psi = \phi(r)$$

and rederive $\mathrm{d}E/\mathrm{d}Q = \omega$.

The system of ODE:

$$(m^2 - \omega^2) f - \Delta f + h\phi f = 0,$$
$$-\Delta\phi + hf^2 = 0.$$

The Schrödinger–Poisson system.

With boundary conditions

$$\partial_r f|_{r=0} = 0, \qquad \lim_{r\to\infty} f(r) = 0,$$
$$\partial_r \phi|_{r=0} = 0, \qquad \lim_{r\to\infty} \phi(r) = 0.$$

Main reparametrization

$$R = r\sqrt{m^2 - \omega^2}, \qquad F(R) = \frac{h}{m^2-\omega^2} f(r), \qquad G(R) = \frac{h}{m^2-\omega^2}\phi(r),$$

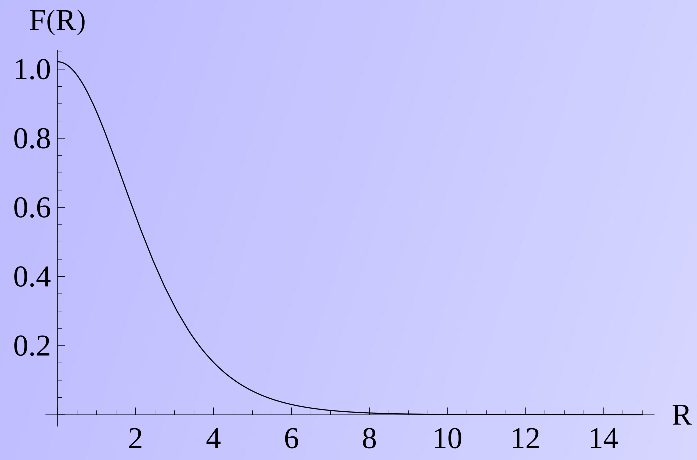
resulting in

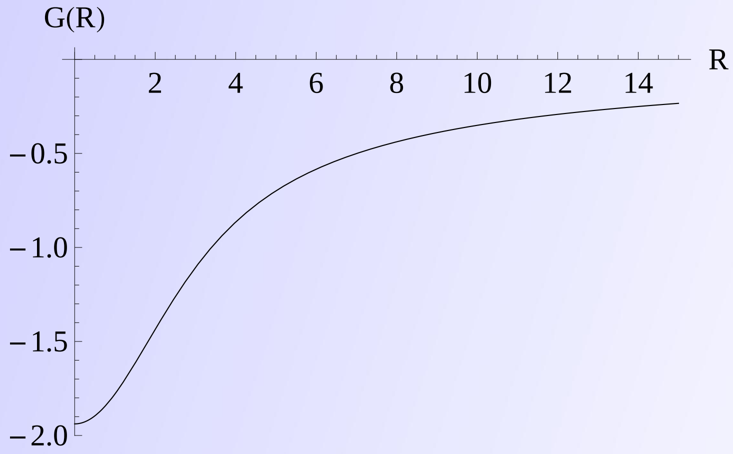
$$-\Delta_R F + F + FG = 0,$$
$$-\Delta_R G + F^2 = 0,$$

with the boundary conditions

$$\partial_R F|_{R=0} = 0, \qquad \lim_{R\to\infty} F(R) = 0, \qquad \partial_R G|_{R=0} = 0, \qquad \lim_{R\to\infty} G(R) = 0.$$

Exponentially localized complex field, $F \sim \mathrm{e}^{-\sqrt{m^2-\omega^2}}$ and real field with tail $\frac{1}{r}$

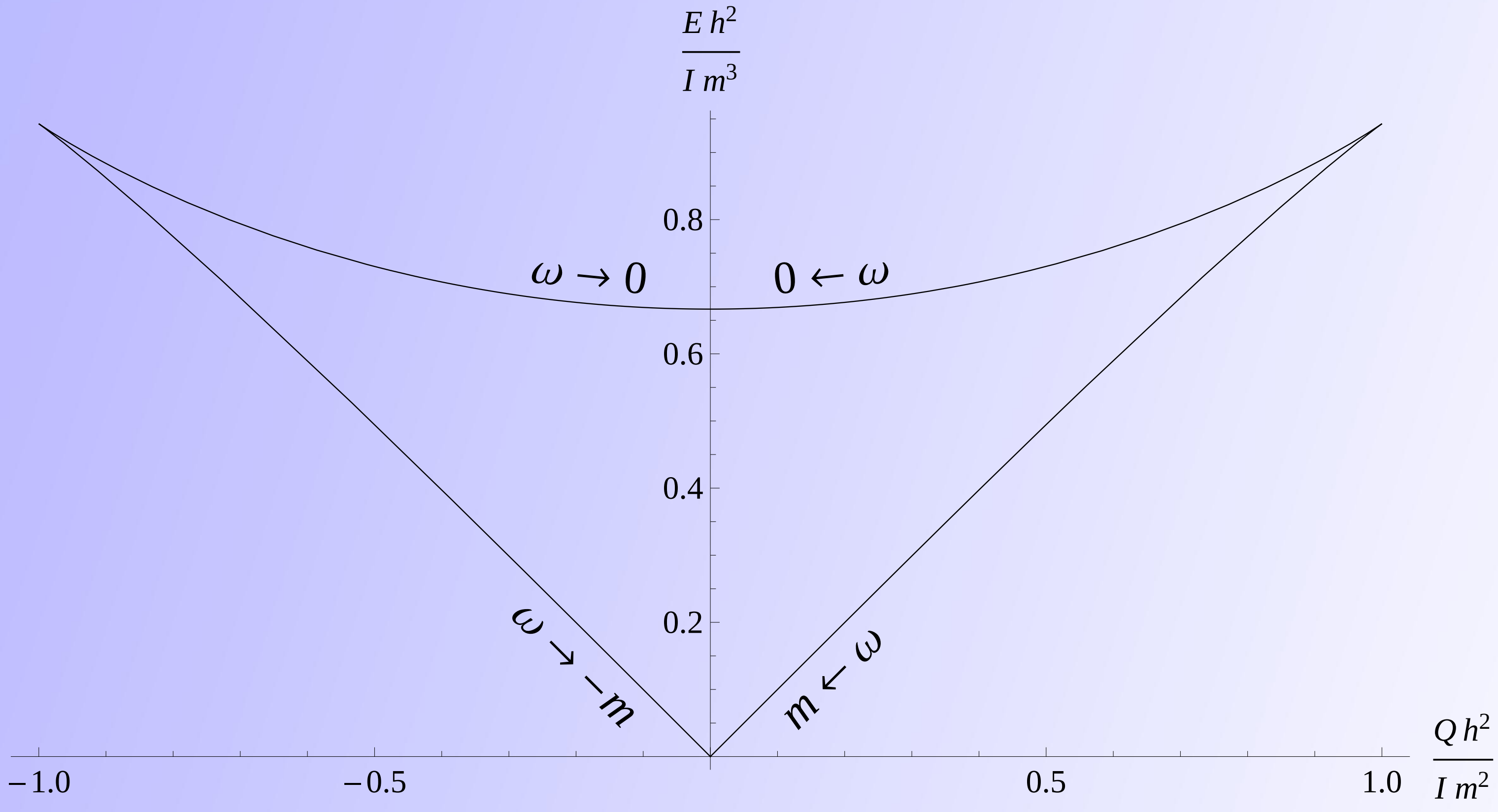




The only value needed for the $E(Q)$ is $I = 4\pi \int\limits_0^\infty F^2 R^2 dR$.

$Q = \frac{2\omega\sqrt{m^2-\omega^2}}{h^2} I$ and $E = \omega Q + \frac{2(m^2-\omega^2)^{\frac{3}{2}}}{3h^2} I$

Numerically, $I = 44.05$.



Expression for the lowest branch

$$E = \frac{\sqrt{2}\,I}{3\,h^2}\left(2m^2 + \sqrt{m^4 - \frac{Q^2h^4}{I^2}}\right)\sqrt{m^2 - \sqrt{m^4 - \frac{Q^2h^4}{I^2}}}$$

here $Q_{max} = -Q_{min} = \frac{Im^2}{h^2}$, and maximal energy $E_{max} = \frac{2\sqrt{2}\,Im^3}{3h^2}$.

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Conclusions

- Critical bubble and 2-particle bound state are connected by continuous family of solutions
- Interesting connection with the Schrödinger–Poisson systems, which appear, for example, when one considers the Newtonian limit for boson stars
- Order of magnitude coincidence with bound energy for Wick-Cutkosky solution even for $Q = 2$
- Reparametrization helps to consider model in 3 spatial dimensions
- Massless real field – we are working beyond thin-wall approximation

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THANK YOU!