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PARTIAL MASSLESSNESS BEYOND DE SITTER

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Institut d'Astrophysique de Paris

IESC Cargèse, May 2016

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Partial masslessness beyond de Sitter

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Based on work with

- Laura Bernard
- Cédric Deffayet
- Fawad Hassan
- Kurt Hinterbichler
- Angnis Schmidt-May

Outline of Talk

- Introduction & Motivation
- Bimetric attempts
- Covariant constraint approach
- Summary & Outlook

Introduction & Motivation

- History started 1939 with Fierz & Pauli
- Progress halted 1972 by no-go from Boulware & Deser
- Resparked interest in 2000s; A-HGS, CNPT ...
- Conjectured resolution in 2010 by dRGT
- Proved & extended by HR in 2011

M. Fierz & W. Pauli

Introduction & Motivation

- History started 1939 with Fierz & Pauli
- Progress halted 1972 by no-go from Boulwa
Deser

For more details see review by
A. Schmidt-May & M. von Strauss, J. Phys. A

- Conjectured resolution in 2010 by dRGT
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M. Fierz & W. Pauli

Introduction & Motivation

I) - Understanding gravity & particularly cosmology and the Dark sectors since they are modifications of GR

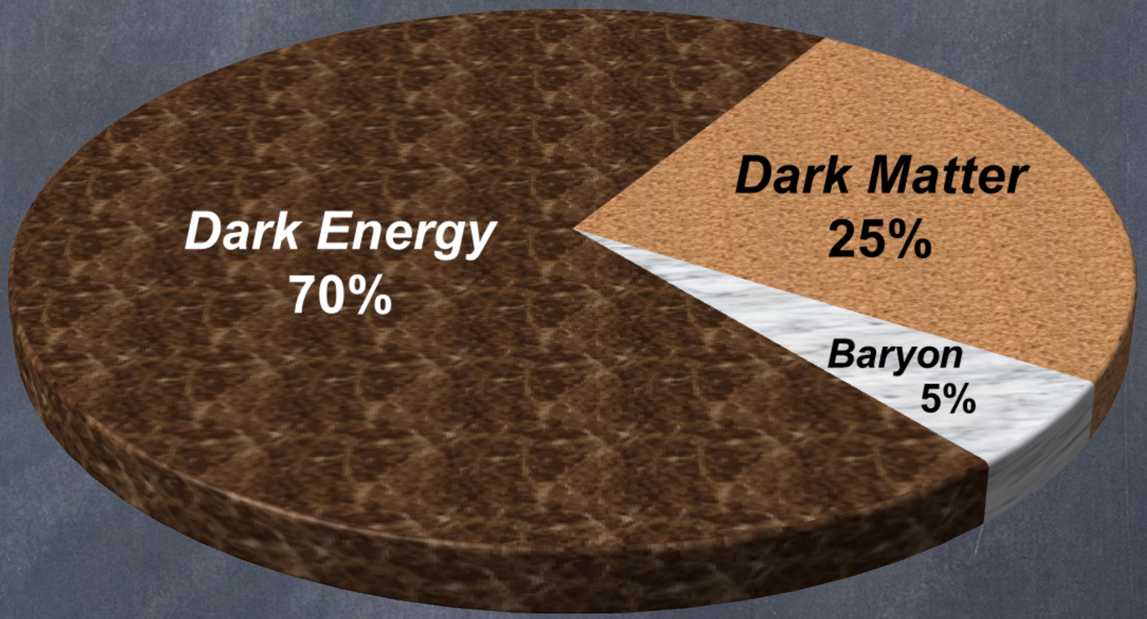
II) - Natural & direct generalisations of standard field theory (including GR), i.e. not ad hoc modifications. Therefore promising for understanding gravity at a deeper level. Immediately lead to two alternatives:

  - Either not realised in nature, but then we must understand why that is so

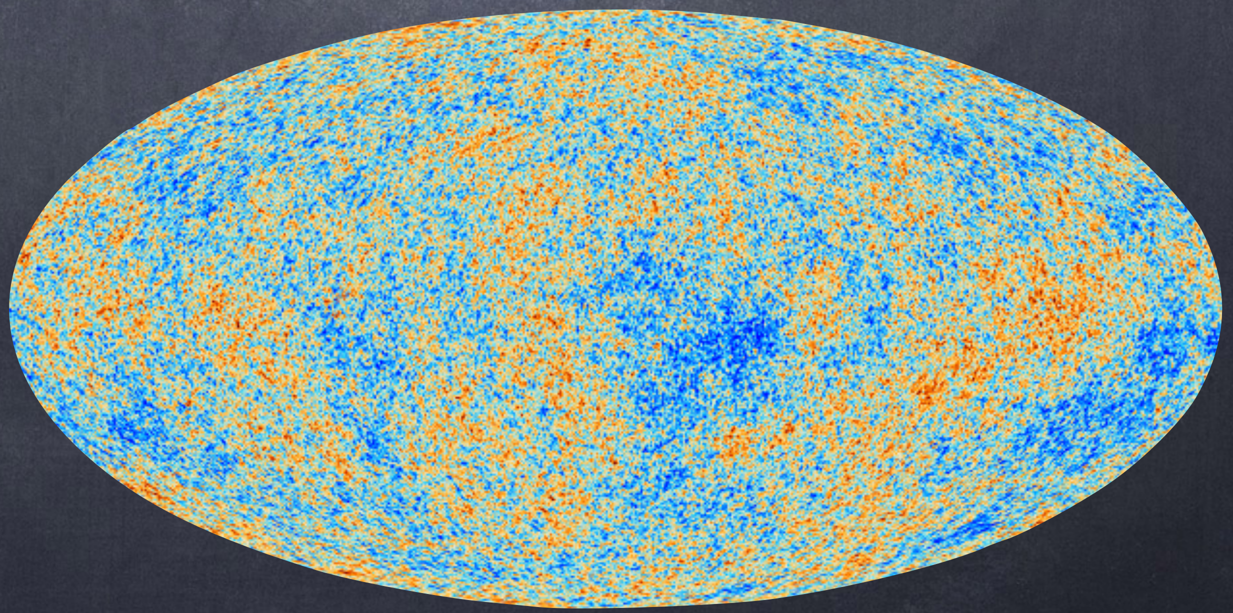
  - Or are realised in nature and therefore extremely important and almost certainly will increase our understanding of the Dark sectors -> pointing back to motivation I

Motivation I: Cosmology & Dark sectors

GR & the SM are quite adequate to explain observations thus far ✓



- Provided we accept the inclusion of, only indirectly inferred, Dark sources which totally dominate the energy budget
- And don't think too seriously about the cosmological constant problem(s) (CCP(s))
- Resolution of the CCP(s) seem to require new understanding of GR, QFT or both
- QFT very robust framework so modification of gravity away from GR appears to be more promising
- But also GR is a quite robust theory/model so modifications must make sense theoretically



Motivation II: Field theory

- Lower spin fields well understood and many do exist in nature. For the bosonic sector
  - Spin-0: Massive (Higgs) & massless (?)

$$(\nabla^2 - m^2)\phi = 0$$

  - Spin-1: Massive (Gauge bosons) & massless (photon)

$$\left(\nabla^2 - m^2 - \Lambda\right) A_\mu = 0\,, \qquad \nabla^\mu A_\mu = 0$$

  - Spin-2: Massless (Graviton ?) and massive (?)

$$\left(\nabla^2 - m^2 + \frac{2\Lambda}{3}\right) h_{\mu\nu} = 0\,, \qquad \nabla^\mu h_{\mu\nu} = 0\,, \qquad h = 0$$

Motivation II: Field theory

- Any spin-2 theory beg for non-linear completion, just as massless spin-2 theory beg for GR completion
- We must therefore consider a non-linear completion and the corresponding spin-2 particles either exist in nature or they do not

Linear FP

Constraints in linear FP theory: The FP equations

$$\delta E_{\mu\nu} = \mathcal{E}_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} - \Lambda \left(h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h \right) + \frac{m^2}{2} \left(h_{\mu\nu} - g_{\mu\nu} h \right) \approx 0$$

$$\left(\mathcal{E}_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} = -\frac{1}{2} \left[\delta^\rho_\mu \delta^\sigma_\nu \nabla^2 + g^{\rho\sigma} \nabla_\mu \nabla_\nu - \delta^\rho_\mu \nabla^\sigma \nabla_\nu - \delta^\rho_\nu \nabla^\sigma \nabla_\mu - g_{\mu\nu} g^{\rho\sigma} \nabla^2 + g_{\mu\nu} \nabla^\rho \nabla^\sigma \right] h_{\rho\sigma} \right)$$

Trace:

$$g^{\mu\nu} \delta E_{\mu\nu} = \nabla^2 h - \nabla^\mu \nabla^\nu h_{\mu\nu} + \left(\Lambda - \frac{3m^2}{2} \right) h \approx 0$$

Divergence:

$$\nabla^\mu \delta E_{\mu\nu} = \frac{m^2}{2} \left(\nabla^\mu h_{\mu\nu} - \nabla_\nu h \right) \approx 0$$

Double divergence:

$$\nabla^\mu \nabla^\nu \delta E_{\mu\nu} = \frac{m^2}{2} \left(\nabla^\mu \nabla^\nu h_{\mu\nu} - \nabla^2 h \right) \approx 0$$

Linear FP

The linear combination

$$2\nabla^{\mu}\nabla^{\nu}\delta E_{\mu\nu} + m^2 g^{\mu\nu}\delta E_{\mu\nu} = \frac{m^2}{2}\left(2\Lambda - 3m^2\right)\, h \approx 0$$

constitutes a scalar constraint. Together with divergence constraints the theory can be written

$$\left(\nabla^2 - m^2 + \frac{2\Lambda}{3}\right) h_{\mu\nu} = 0\,, \qquad \nabla^{\mu} h_{\mu\nu} = 0\,, \qquad h = 0$$

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Unitarity implies Higuchi bound
(cf Breitenlohner-Freedman bound in AdS)

$$3m^2 \geq 2\Lambda$$

Linear FP

The linear combination

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What about when $2\Lambda=3m^2$?

Linear FP

The linear combination

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$$\left(\nabla^2 - m^2 + \frac{2\Lambda}{3}\right) h_{\mu\nu} = 0\,, \qquad \nabla^{\mu} h_{\mu\nu} = 0\,, \qquad h = 0$$

What about when $2\Lambda = 3m^2$?

$$\longrightarrow \qquad \left[\nabla^{\mu}\nabla^{\nu} + \frac{m^2}{2} g^{\mu\nu}\right]\delta E_{\mu\nu} = 0$$

Linear FP

Implies the linear gauge symmetry

$$\Delta h_{\mu\nu} = \left[\nabla_\mu \nabla_\nu + \frac{m^2}{2} g_{\mu\nu}\right] \xi(x)$$

Action is trivially invariant since it can be written

$$S[h] \sim \int \mathrm{d}^4 x \sqrt{g}\, h^{\mu\nu} \delta E_{\mu\nu}$$

From group theory: coincides with existence of "short" UIRs in de Sitter

Further motivation

We now have an example of a theory where

$$\Lambda \sim m^2$$

is protected by a symmetry. Similarly

$$m^2 \sim 0$$

may be thought of as "technically natural" due to enhancement of diffeomorphism symmetry.
Furthermore dS favoured by unitarity

Further motivation

We now have an example of a theory where

$$\Lambda \sim m^2$$

is protected by a symmetry. Similarly

$$m^2 \sim 0$$

may be thought of as "technically natural" due to enhancement of diffeomorphism symmetry. Furthermore dS favoured by unitarity

- Small positive Λ may be regarded as technically natural!
- But spin-2 theories require nonlinear completion!

Early attempts I: Adding PM vertices

Zinoviev

Most obvious approach: Keep dS background fixed but add higher order PM interactions and constrain possible gauge symmetry

- Unique structure of cubic vertices constructed
- Only works for D=4 unless higher derivative terms are considered
- Apparent obstruction for quartic vertices

Early attempts II: Conformal gravity

Deser, Joung, Waldron

Conformal gravity action

$$S \sim \int \mathrm{d}^4x \sqrt{g}\, W^2 \sim \int \mathrm{d}^4x \sqrt{g} \left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 \right)$$

and equations

$$B_{\mu\nu} = 0$$

with

$$B_{\mu\nu} = -\nabla^2 P_{\mu\nu} + \nabla^\rho \nabla_{(\mu} P_{\nu)\rho} + W_{\rho\mu\nu\sigma} P^{\rho\sigma}\,, \qquad P_{\mu\nu} = R_{\mu\nu} - \frac{1}{6} g_{\mu\nu} R$$

Bach tensor

Schouten tensor

Early attempts II: Conformal gravity

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Conformal gravity action

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Under conformal transformation

$$\delta g_{\mu\nu} = 2\phi g_{\mu\nu} \qquad \delta P_{\mu\nu} = -\nabla_\mu \partial_\nu \phi$$

$$\longrightarrow \quad P_{\mu\nu} - \frac{m^2}{4} g_{\mu\nu} \longrightarrow P_{\mu\nu} - \frac{m^2}{4} g_{\mu\nu} - \left(\nabla_\mu \nabla_\nu + \frac{m^2}{2} g_{\mu\nu} \right) \phi$$

transforms exactly as a PM field!

Early attempts II: Conformal gravity

Deser, Joung, Waldron

- Correct structure linearly; PM field + massless spin-2

Maldacena

- Relative ghost between PM and massless field
- Not consistent PM theory nonlinearly
- Conjectured that PM can only propagate on Einstein backgrounds

Outline of Talk

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Basic details

$$\alpha \equiv \frac{m_f}{m_g}$$

Theory is defined by the covariant action

$$S[g,f] = m_g^2 \int \mathrm{d}^4x \left[\sqrt{|g|}\,R(g) + \alpha^2\sqrt{|f|}\,R(f) - 2m^2\sqrt{|g|}\,V(S;\beta_n)\right]$$

with the interactions governed by

$$\sqrt{|g|}\,V(S;\beta_n) = \sqrt{|g|}\sum_{n=0}^{4}\beta_n e_n(S) = \sqrt{|g|}\,\beta_0 + \sqrt{|g|}\sum_{n=1}^{3}\beta_n e_n(S) + \sqrt{|f|}\,\beta_4 = \sqrt{|f|}\,V(S^{-1};\beta_{4-n})$$

in terms of the square-root matrix

$$S = \sqrt{g^{-1}f}, \qquad S^\rho{}_\sigma S^\sigma{}_\nu = g^{\rho\mu}f_{\mu\nu}$$

along with the "elementary symmetric polynomials"

$$e_n(S) = S^{\mu_1}{}_{[\mu_1}\cdots S^{\mu_n}{}_{\mu_n]} = \tfrac{1}{n!}\delta^{\mu_1\cdots\mu_n}_{\nu_1\cdots\nu_n}S^{\nu_1}{}_{\mu_1}\cdots S^{\nu_n}{}_{\mu_n} = \tfrac{1}{n!(4-n)!}\epsilon^{\mu_1\cdots\mu_n\lambda_{n+1}\cdots\lambda_4}\epsilon_{\nu_1\cdots\nu_n\lambda_{n+1}\cdots\lambda_4}S^{\nu_1}{}_{\mu_1}\cdots S^{\nu_n}{}_{\mu_n}$$

or even more explicitly

$$e_0(S) = 1\,, \quad e_1(S) = \mathrm{Tr}(S)\,, \quad e_2(S) = \tfrac{1}{2}(\mathrm{Tr}(S)^2 - \mathrm{Tr}(S^2))\,, \quad \ldots \quad e_4(S) = \det(S)$$

Basic details

This lead to the equations of motion

$$E_{\mu\nu} \equiv \mathcal{G}_{\mu\nu}(g) + m^2 V_{\mu\nu}(g,f) = 0$$

$$\tilde{E}_{\mu\nu} \equiv \mathcal{G}_{\mu\nu}(f) + m^2 \tilde{V}_{\mu\nu}(g,f) = 0$$

with e.g.

$$V_{\mu\nu} = g_{\mu\rho}\Big[\beta_0 \delta^\rho_\nu - \beta_1\left(S^\rho{}_\nu - e_1\delta^\rho_\nu\right) + \beta_2\left([S^2]^\rho{}_\nu - e_1 S^\rho{}_\nu + e_2\delta^\rho_\nu\right)$$
$$- \beta_3\left([S^3]^\rho{}_\nu - e_1[S^2]^\rho{}_\nu + e_2 S^\rho{}_\nu - e_3\delta^\rho_\nu\right)\Big]$$

along with Bianchi constraints

$$\sqrt{|g|}\, g^{\mu\rho}\nabla_\rho V_{\mu\nu} = -\sqrt{|f|}\, f^{\mu\rho}\tilde{\nabla}_\rho \tilde{V}_{\mu\nu} = 0$$

and the identities

$$\sqrt{|g|}\, g^{\rho\mu} V_{\mu\nu} + \sqrt{|f|}\, f^{\rho\mu}\tilde{V}_{\mu\nu} - \sqrt{|g|}\, V\delta^\rho_\nu = 0$$

Basic details

The massive gravity limit (g massive) can be defined by

$$\alpha = \frac{m_f}{m_g} \to \infty \quad \text{and} \quad \Lambda_f = \frac{\beta_4 m^2}{\alpha^2}, m_g, m \ \text{all fixed}$$

This gives the limiting equations

$$\mathcal{G}_{\mu\nu} + m^2 V_{\mu\nu} = 0\,, \qquad \tilde{\mathcal{G}}_{\mu\nu} + \Lambda_f f_{\mu\nu} = 0$$

Works for solutions of the form

$$g_{\mu\nu} = g_{\mu\nu} + \mathcal{O}(\alpha^{-2}) \qquad f_{\mu\nu} = f^{\mathrm{E}}_{\mu\nu} + \mathcal{O}(\alpha^{-2})$$

Also perturbations behave like massive gravity if we take

$$g_{\mu\nu} \to g_{\mu\nu} + \frac{\delta g_{\mu\nu}}{m_g} \qquad f^{\mathrm{E}}_{\mu\nu} \to f^{\mathrm{E}}_{\mu\nu} + \frac{\delta f^{\mathrm{E}}_{\mu\nu}}{\alpha m_g}$$

Basic details

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Works for solu...

$$g_{\mu\nu} \ldots f^{\mathrm{E}}_{\mu\nu} + O(\alpha^{-2})$$

Also perturbat... ...ake

$$g_{\mu\nu} \to g_{\mu\nu} + \frac{\delta g_{\mu\nu}}{m_g} \qquad f^{\mathrm{E}}_{\mu\nu} \to f^{\mathrm{E}}_{\mu\nu} + \frac{\delta f^{\mathrm{E}}_{\mu\nu}}{\alpha m_g}$$

Recall talks by
Shinji Mukohyama
&
Angnis Schmidt-May

Proportional solutions & Mass spectrum

A conformal ansatz $f_{\mu\nu} = c^2 g_{\mu\nu}$ reduce the equations to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \begin{pmatrix} \Lambda_g \\ \Lambda_f \end{pmatrix} g_{\mu\nu} = 0$$

Consistency requires $\Lambda_g = \Lambda_f$:

$$\alpha^2 \beta_3 c^4 + (3\alpha^2\beta_2 - \beta_4)c^3 + 3(\alpha^2\beta_1 - \beta_3)c^2 + \left(\alpha^2\beta_0 - 3\beta_2\right) c - \beta_1 = 0$$

Generically determines $c = c(\alpha, \beta_n)$.

Proportional solutions & Mass spectrum

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$$\alpha^2 \beta_3 c^4 + (3\alpha^2\beta_2 - \beta_4) c^3 + 3(\alpha^2\beta_1 - \beta_3) c^2 + \left(\alpha^2\beta_0 - 3\beta_2\right) c - \beta_1 = 0$$

Generically determines $c = c(\alpha, \beta_n)$. Decoupled perturbations

$$\tilde{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta G_{\mu\nu} + \Lambda \delta G_{\mu\nu} = 0$$ massless

$$\tilde{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta M_{\mu\nu} + \Lambda \delta M_{\mu\nu} + \frac{\tilde{m}^2}{2} \left(\delta M_{\mu\nu} - g_{\mu\nu} \delta M\right) = 0$$ massive

with

$$\delta G_{\mu\nu} = \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu} \,, \qquad \delta M_{\mu\nu} = \frac{1}{2c} \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right)$$

Proportional solutions & Mass spectrum

The constant c is left undetermined by the PM values

$$\beta_1 = \beta_3 = 0\,, \qquad \alpha^4\beta_0 = 3\alpha^2\beta_2 = \beta_4$$

Higuchi bound saturated and nonlinear scaling symmetry realised

$$c \longrightarrow c + a\,, \qquad g_{\mu\nu} \longrightarrow \frac{1+(\alpha c)^2}{1+\alpha^2(c+a)^2} g_{\mu\nu}$$

Proportional solutions & Mass spectrum

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Higuchi bound saturated and nonlinear scaling symmetry realised

$$c \longrightarrow c + a\,, \qquad g_{\mu\nu} \longrightarrow \frac{1+(\alpha c)^2}{1+\alpha^2(c+a)^2}\, g_{\mu\nu}$$

- Covers all GR solutions
- Diagonalisable into mass eigenstates
- For PM values there is a dS preserving nonlinear scaling symmetry

Cosmological solutions

Bidiagonal solutions

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)$$

$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -X^2\mathrm{d}t^2 + Y^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)$$

Characterised by a modified Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3m_g^2} + \frac{m^2}{3}\left(\beta_0 + 3\beta_1\frac{Y}{a} + 3\beta_2\left(\frac{Y}{a}\right)^2 + \beta_3\left(\frac{Y}{a}\right)^3\right)$$

and the polynomial equation

$$\alpha^2\beta_3\left(\frac{Y}{a}\right)^4 + (3\alpha^2\beta_2 - \beta_4)\left(\frac{Y}{a}\right)^3 + 3(\alpha^2\beta_1 - \beta_3)\left(\frac{Y}{a}\right)^2 + \left(\frac{\alpha^2\rho}{m_g m^2} + \alpha^2\beta_0 - 3\beta_2\right)\frac{Y}{a} - \beta_1 = 0$$

Leaves $Y(t)/a(t)$ undetermined for the exact same parameters! Related by time-dependent PM trafo

Cosmological solutions

Bidiagonal solutions

Recall talk by
Adam Solomon

$g_{\mu\nu} \ldots {}^2)$

$f_{\mu\nu} \ldots \mathrm{d}\Omega^2)$

Characterised by the modified Friedmann equations

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3m_g^2} + \frac{m^2}{3}\left(\beta_0 + 3\beta_1\frac{Y}{a} + 3\beta_2\left(\frac{Y}{a}\right)^2 + \beta_3\left(\frac{Y}{a}\right)^3\right)$$

and the polynomial equation

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Leaves $Y(t)/a(t)$ undetermined for the exact same parameters! Related by time-dependent PM trafo

Direct approach towards nonlinear PM

Most direct approach to check for nonlinear PM: Mimic the linear FP analysis but for arbitrary backgrounds

- Requires linearised equations and constraints for arbitrary backgrounds
- Doable due to recent results but quite messy in practice

More on this approach later ...

Perturbative approach & Conformal gravity

Equations are of the form

$$\mathcal{G}_{\mu\nu}(g) + V_{\mu\nu}(g,f) = 0$$

$$\mathcal{G}_{\mu\nu}(f) + \tilde{V}_{\mu\nu}(g,f) = 0$$

Perturbative ansätze, eg

$$f_{\mu\nu}[g] = a^2 g_{\mu\nu} + \frac{b}{m^2} P_{\mu\nu} + \frac{c_1}{m^4} P^2_{\mu\nu} + \frac{c_2}{m^4}\left[\tfrac{1}{3} e_2(P) g_{\mu\nu} - P P_{\mu\nu}\right] + \mathcal{O}\left(\frac{P^3}{m^6}\right)$$

Result in single higher derivative equation (infinite expansion)

$$\Lambda\, g_{\mu\nu} + a\, R_{\mu\nu} + b\, g_{\mu\nu} R + \mathcal{O}\left(R^2\right) = 0$$

Perturbative approach & Conformal gravity

Single higher derivative equation (infinite expansion)

$$\Lambda\, g_{\mu\nu} + a\, R_{\mu\nu} + b\, g_{\mu\nu} R + \mathcal{O}\left(R^2\right) = 0$$

Perturbative approach & Conformal gravity

Single higher derivative equation (infinite expansion)

$$\Lambda g_{\mu\nu} + a R_{\mu\nu} + b g_{\mu\nu} R + \mathcal{O}\left(R^2\right) = 0$$

$$B_{\mu\nu} = 0$$

For the PM parameter values we get Weyl invariance to lowest order

$$g_{\mu\nu} \longrightarrow \phi g_{\mu\nu} + \mathcal{O}(R)$$

Perturbative approach & Conformal gravity

Single higher derivative equation (infinite expansion)

$$\Lambda g_{\mu\nu} + a R_{\mu\nu} + b g_{\mu\nu} R + \mathcal{O}\left(R^2\right) = 0$$

$$B_{\mu\nu} = 0$$

For the PM parameter values we get Weyl invariance to lowest order

$$g_{\mu\nu} \longrightarrow \phi g_{\mu\nu} + \mathcal{O}(R)$$

Bootstrapping within this formulation reveals symmetry up to 6th order

$$\Delta g_{\mu\nu} = \phi g_{\mu\nu} + \frac{a}{2}\left(P_{\mu\nu}\phi + \nabla_\mu \partial_\nu \phi\right) + \mathcal{O}(R^2, R^3)$$

$$\Delta f_{\mu\nu} = \phi f_{\mu\nu} + \frac{a}{2}\left(\tilde{P}_{\mu\nu}\phi + \tilde{\nabla}_\mu \partial_\nu \phi\right) + \mathcal{O}(\tilde{R}^2, \tilde{R}^3)$$

Perturbative approach & Conformal gravity

- Connection to conformal gravity at lowest order in derivatives
- Propagates 6 modes instead of 7 on Einstein spacetimes
- Perturbative gauge invariance up to at least 6th order in derivatives

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The most direct approach

Perturbed equations for general backgrounds

$$\delta E_{\mu\nu} = \delta\mathcal{G}_{\mu\nu} + m^2 \delta V_{\mu\nu} = 0$$

$$\delta\tilde{E}_{\mu\nu} = \delta\tilde{\mathcal{G}}_{\mu\nu} + \frac{m^2}{\alpha^2}\delta\tilde{V}_{\mu\nu} = 0$$

Construct generalised traces

$$\Phi^g_k \equiv [S^k]^\nu{}_\rho g^{\rho\mu}\delta E_{\mu\nu}$$

$$\Psi^g_k \equiv [S^k]^\nu{}_\rho f^{\rho\mu}\delta\tilde{E}_{\mu\nu}$$

And generalised divergences

$$\Psi^g_k \equiv [S^k]^\nu{}_\rho \nabla^\rho \nabla^\mu \delta E_{\mu\nu}$$

$$\Psi^f_k \equiv [S^k]^\nu{}_\rho \tilde{\nabla}^\rho \tilde{\nabla}^\mu \delta\tilde{E}_{\mu\nu}$$

The most direct approach

Now build the most general linear combination

$$\mathcal{C} \equiv \sum_{k=0}^{3} \left(u_k^g \Phi_k^g + v_k^g \Psi_k^g + u_k^f \Phi_k^f + v_k^f \Psi_k^f \right)$$

and find scalar coefficients $\{u^{g,f}, v^{g,f}\}$ such that

$$\mathcal{C} \sim 0$$

Equal mod lower order derivatives

The most direct approach

Now build the most general linear combination

$$\mathcal{C} \equiv \sum_{k=0}^{3} \left(u_k^g \Phi_k^g + v_k^g \Psi_k^g + u_k^f \Phi_k^f + v_k^f \Psi_k^f \right)$$

and find scalar coefficients $\{u^{g,f}, v^{g,f}\}$ such that

$$\mathcal{C} \sim 0$$

If this is possible the scalar constraint is given by

$$\mathcal{C} = 0$$

If this vanishes identically we have a gauge symmetry

The most direct approach

Why not checked earlier?

- Constraint only recently found
- Quite cumbersome expression to work with
- Not manifestly covariant in general

Less ambitious approach: Restricted background and MG limit

To make headway:

- Focus on MG limit. Only one equation
- Consider simple model with $\beta_3 = 0$
  Constraint is manifestly covariant
- Restricted class of backgrounds but more general then Einstein spacetimes

Less ambitious approach: Restricted background and MG limit

The field equations we consider are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + m^2 \left[\beta_0 g_{\mu\nu} - \beta_1 \left(S_{\mu\nu} - e_1 g_{\mu\nu} \right) + \beta_2 \left(S_{\mu\rho} S^{\rho}{}_{\nu} - e_1 S_{\mu\nu} + e_2 g_{\mu\nu} \right) \right] = 0$$

or equivalently

$$m^{-2} R^{\mu}{}_{\nu} = \left(\beta_0 + \frac{1}{2} e_1 \beta_1 \right) \delta^{\mu}_{\nu} + \left(\beta_1 + \beta_2 e_1 \right) S^{\mu}{}_{\nu} - \beta_2 (S^2)^{\mu}{}_{\nu}$$

Less ambitious approach: Restricted background and MG limit

The field equations we consider are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + m^2 \left[\beta_0 g_{\mu\nu} - \beta_1 \left(S_{\mu\nu} - e_1 g_{\mu\nu}\right) + \beta_2 \left(S_{\mu\rho} S^{\rho}{}_{\nu} - e_1 S_{\mu\nu} + e_2 g_{\mu\nu}\right)\right] = 0$$

or equivalently

$$m^{-2} R^{\mu}{}_{\nu} = \left(\beta_0 + \frac{1}{2} e_1 \beta_1\right) \delta^{\mu}_{\nu} + \left(\beta_1 + \beta_2 e_1\right) S^{\mu}{}_{\nu} - \beta_2 (S^2)^{\mu}{}_{\nu}$$

Covariant constraint

$$\mathcal{C} \equiv (S^{-1})^{\nu}{}_{\rho} \nabla^{\rho} \nabla^{\mu} \delta E_{\mu\nu} + \frac{m^2 \beta_1}{2} g^{\mu\nu} \delta E_{\mu\nu} + m^2 \beta_2 S^{\mu\nu} \delta E_{\mu\nu} \approx 0$$

If $\mathcal{C} = 0$ off-shell $\longrightarrow$ linear gauge symmetry

$$\Delta h_{\mu\nu} = \left[(S^{-1})_{\mu}{}^{\rho} \nabla_{\rho} \partial_{\nu} + (S^{-1})_{\nu}{}^{\rho} \nabla_{\rho} \partial_{\mu} + m^2 \beta_1 g_{\mu\nu} + 2 m^2 \beta_2 S_{\mu\nu}\right] \xi(x)$$

Less ambitious approach: Restricted background and MG limit

Generally difficult due to derivative terms

$$\mathcal{C} = m^2\beta_2\Big[\Big(A^{\beta\lambda} + \tilde{A}^{\beta\lambda}\Big)\,\delta g_{\beta\lambda} + B^{\beta\lambda}_{\rho}\,\nabla^{\rho}\delta g_{\beta\lambda}\Big]$$

$$\begin{aligned}
\tilde{A}^{\beta\lambda} = e_1\tilde{N}^{\beta\lambda} + [S^{-1}]^{\nu}{}_{\gamma}\Big(& S^{\beta}_{\rho}\nabla^{\lambda}S^{\rho}_{\nu}\nabla^{\gamma}e_1 - S^{\beta}_{\rho}\nabla_{\nu}S^{\rho\lambda}\nabla^{\gamma}e_1 + S^{\lambda}_{\rho}\nabla^{\gamma}S^{\beta}_{\mu}\nabla_{\nu}S^{\rho\mu} + S^{\beta}_{\mu}\nabla^{\gamma}S^{\lambda}_{\rho}\nabla_{\nu}S^{\rho\mu} \\
& + S^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu\rho}\nabla_{\nu}S^{\beta}_{\rho} + S^{\mu\rho}\nabla^{\gamma}S^{\lambda}_{\mu}\nabla_{\nu}S^{\beta}_{\rho} - 2S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu\lambda}\nabla_{\nu}e_1 - S^{\rho}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\beta}S^{\lambda}_{\rho} - S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu}_{\rho}\nabla^{\lambda}S^{\rho}_{\nu} \\
& - S^{\mu}_{\rho}\nabla^{\gamma}S^{\beta}_{\mu}\nabla^{\lambda}S^{\rho}_{\nu} - S^{\beta}_{\rho}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\lambda}S^{\rho}_{\mu} + S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\lambda}e_1 + S^{\mu}_{\rho}\nabla^{\gamma}S_{\mu\nu}\nabla^{\rho}S^{\beta\lambda} - S^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\rho}S^{\beta}_{\rho} \\
& - S^{\lambda}_{\mu}\nabla^{\gamma}S^{\beta}_{\rho}\nabla^{\mu}S^{\rho}_{\nu} - S^{\beta}_{\rho}\nabla^{\gamma}S^{\lambda}_{\mu}\nabla^{\mu}S^{\rho}_{\nu} - S^{\lambda}_{\mu}\nabla^{\gamma}S^{\rho}_{\nu}\nabla^{\mu}S^{\beta}_{\rho} + 2S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu\lambda}\nabla^{\rho}S_{\rho\nu} + 2S^{\beta}_{\rho}\nabla^{\gamma}S_{\mu\nu}\nabla^{\mu}S^{\rho\lambda} \\
& + S^{\lambda}_{\rho}S^{\beta}_{\mu}\nabla^{\gamma}\nabla_{\nu}S^{\rho\mu} + [S^2]^{\lambda\rho}\nabla^{\gamma}\nabla_{\nu}S^{\beta}_{\rho} - [S^2]^{\beta\lambda}\nabla^{\gamma}\nabla_{\nu}e_1 - [S^2]^{\beta}_{\rho}\nabla^{\gamma}\nabla^{\lambda}S^{\rho}_{\nu} - S^{\lambda}_{\mu}S^{\beta}_{\rho}\nabla^{\gamma}\nabla^{\mu}S^{\rho}_{\nu} \\
& + [S^2]^{\beta\lambda}\nabla^{\gamma}\nabla^{\rho}S_{\rho\nu}\Big) + \nabla^{\beta}S^{\lambda}_{\gamma}\nabla^{\gamma}e_1 - \nabla_{\gamma}S^{\beta\lambda}\nabla^{\gamma}e_1 - \nabla^{\mu}S^{\rho}_{\mu}\nabla^{\beta}S^{\lambda}_{\rho} - \nabla^{\mu}S^{\beta}_{\rho}\nabla^{\lambda}S^{\rho}_{\mu} + \nabla^{\mu}S^{\beta}_{\mu}\nabla^{\lambda}e_1 \\
& + \nabla_{\mu}S^{\mu}_{\rho}\nabla^{\rho}S^{\beta\lambda} - \nabla^{\mu}S^{\lambda}_{\mu}\nabla^{\rho}S^{\beta}_{\rho} - \nabla^{\rho}S^{\lambda}_{\mu}\nabla^{\mu}S^{\beta}_{\rho} + 2\nabla_{\mu}S^{\beta}_{\rho}\nabla^{\mu}S^{\rho\lambda} - S^{\beta}_{\rho}\nabla^{\lambda}\nabla^{\mu}S^{\rho}_{\mu} + S^{\beta}_{\gamma}\nabla^{\gamma}\nabla^{\lambda}e_1 \\
& - S^{\lambda}_{\gamma}\nabla^{\gamma}\nabla^{\rho}S^{\beta}_{\rho} + S^{\beta}_{\rho}\nabla^{\gamma}\nabla_{\gamma}S^{\rho\lambda}\,.
\end{aligned}$$

$$\begin{aligned}
B^{\beta\lambda}_{\rho} = e_1M^{\beta\lambda}_{\rho} + [S^{-1}]^{\nu}{}_{\gamma}\Big(& \delta^{\gamma}_{\rho}S^{\lambda}_{\delta}S^{\beta}_{\mu}\nabla_{\nu}S^{\delta\mu} + \delta^{\gamma}_{\rho}[S^2]^{\lambda\mu}\nabla_{\nu}S^{\beta}_{\mu} - \delta^{\gamma}_{\rho}[S^2]^{\beta\lambda}\nabla_{\nu}e_1 - \delta^{\gamma}_{\rho}[S^2]^{\beta}_{\mu}\nabla^{\lambda}S^{\mu}_{\nu} \\
& - \delta^{\gamma}_{\rho}S^{\lambda}_{\mu}S^{\beta}_{\delta}\nabla^{\mu}S^{\delta}_{\nu} + \delta^{\gamma}_{\rho}[S^2]^{\beta\lambda}\nabla^{\mu}S_{\mu\nu} + S^{\beta\lambda}S^{\mu}_{\rho}\nabla^{\gamma}S_{\mu\nu} + [S^2]^{\beta\lambda}\nabla^{\gamma}S_{\rho\nu} - \delta^{\beta}_{\rho}[S^2]^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu} - S^{\beta}_{\rho}S^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\Big) \\
& - S^{\beta}_{\delta}\nabla^{\lambda}S^{\delta}_{\rho} + S^{\beta}_{\rho}\nabla^{\lambda}e_1 - S^{\lambda}_{\mu}\nabla^{\mu}S^{\beta}_{\rho} + 2S^{\beta}_{\delta}\nabla_{\rho}S^{\delta\lambda} + \delta^{\beta}_{\rho}S^{\lambda}_{\gamma}\nabla^{\gamma}e_1 - S^{\beta\lambda}\nabla_{\rho}e_1 + S^{\beta\lambda}\nabla_{\mu}S^{\mu}_{\rho} \\
& - \delta^{\beta}_{\rho}S^{\lambda}_{\delta}\nabla^{\mu}S^{\delta}_{\mu} - S^{\beta}_{\rho}\nabla^{\mu}S^{\lambda}_{\mu}\,.
\end{aligned}$$

Less ambitious approach: Restricted background and MG limit

Generally difficult due to derivative terms

$$\mathcal{C} = m^2\beta_2\left[\left(A^{\beta\lambda} + \cancel{\tilde{A}^{\beta\lambda}}\right)\delta g_{\beta\lambda} + B^{\beta\lambda}_{\rho}\,\cancel{\nabla^{\rho}}\delta g_{\beta\lambda}\right]$$

$$\begin{aligned}
\tilde{A}^{\beta\lambda} = {}& e_1\tilde{N}^{\beta\lambda} + [S^{-1}]^{\nu}{}_{\gamma}\Big(S^{\beta}_{\rho}\nabla^{\lambda}S^{\rho}_{\nu}\nabla^{\gamma}e_1 - S^{\beta}_{\rho}\nabla_{\nu}S^{\rho\lambda}\nabla^{\gamma}e_1 + S^{\lambda}_{\rho}\nabla^{\gamma}S^{\beta}_{\mu}\nabla_{\nu}S^{\rho\mu} + S^{\beta}_{\mu}\nabla^{\gamma}S^{\lambda}_{\rho}\nabla_{\nu}S^{\rho\mu} \\
& + S^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu\rho}\nabla_{\nu}S^{\beta}_{\rho} + S^{\mu\rho}\nabla^{\gamma}S^{\lambda}_{\mu}\nabla_{\nu}S^{\beta}_{\rho} - 2S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu\lambda}\nabla_{\nu}e_1 - S^{\rho}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\beta}S^{\lambda}_{\rho} - S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu}_{\rho}\nabla^{\lambda}S^{\rho}_{\nu} \\
& - S^{\mu}_{\rho}\nabla^{\gamma}S^{\beta}_{\mu}\nabla^{\lambda}S^{\rho}_{\nu} - S^{\beta}_{\rho}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\lambda}S^{\rho}_{\mu} + S^{\beta}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\lambda}e_1 + S^{\mu}_{\rho}\nabla^{\gamma}S_{\mu\nu}\nabla^{\rho}S^{\beta\lambda} - S^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\nabla^{\rho}S^{\beta}_{\rho} \\
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& + S^{\lambda}_{\rho}S^{\beta}_{\mu}\nabla^{\gamma}\nabla_{\nu}S^{\rho\mu} + [S^2]^{\lambda\rho}\nabla^{\gamma}\nabla_{\nu}S^{\beta}_{\rho} - [S^2]^{\beta\lambda}\nabla^{\gamma}\nabla_{\nu}e_1 - [S^2]^{\beta}_{\rho}\nabla^{\gamma}\nabla^{\lambda}S^{\rho}_{\nu} - S^{\lambda}_{\mu}S^{\beta}_{\rho}\nabla^{\gamma}\nabla^{\mu}S^{\rho}_{\nu} \\
& + [S^2]^{\beta\lambda}\nabla^{\gamma}\nabla^{\rho}S_{\rho\nu}\Big) + \nabla^{\beta}S^{\lambda}_{\gamma}\nabla^{\gamma}e_1 - \nabla_{\gamma}S^{\beta\lambda}\nabla^{\gamma}e_1 - \nabla^{\mu}S^{\rho}_{\mu}\nabla^{\beta}S^{\lambda}_{\rho} - \nabla^{\mu}S^{\beta}_{\rho}\nabla^{\lambda}S^{\rho}_{\mu} + \nabla^{\mu}S^{\beta}_{\mu}\nabla^{\lambda}e_1 \\
& + \nabla_{\mu}S^{\mu}_{\rho}\nabla^{\rho}S^{\beta\lambda} - \nabla^{\mu}S^{\lambda}_{\mu}\nabla^{\rho}S^{\beta}_{\rho} - \nabla^{\rho}S^{\lambda}_{\mu}\nabla^{\mu}S^{\beta}_{\rho} + 2\nabla_{\mu}S^{\beta}_{\rho}\nabla^{\mu}S^{\rho\lambda} - S^{\beta}_{\rho}\nabla^{\lambda}\nabla^{\mu}S^{\rho}_{\mu} + S^{\beta}_{\gamma}\nabla^{\gamma}\nabla^{\lambda}e_1 \\
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$$\begin{aligned}
B^{\beta\lambda}_{\rho} = {}& e_1M^{\beta\lambda}_{\rho} + [S^{-1}]^{\nu}{}_{\gamma}\Big(\delta^{\gamma}_{\rho}S^{\lambda}_{\delta}S^{\beta}_{\mu}\nabla_{\nu}S^{\delta\mu} + \delta^{\gamma}_{\rho}[S^2]^{\lambda\mu}\nabla_{\nu}S^{\beta}_{\mu} - \delta^{\gamma}_{\rho}[S^2]^{\beta\lambda}\nabla_{\nu}e_1 - \delta^{\gamma}_{\rho}[S^2]^{\beta}_{\mu}\nabla^{\lambda}S^{\mu}_{\nu} \\
& - \delta^{\gamma}_{\rho}S^{\lambda}_{\mu}S^{\beta}_{\delta}\nabla^{\mu}S^{\delta}_{\nu} + \delta^{\gamma}_{\rho}[S^2]^{\beta\lambda}\nabla^{\mu}S_{\mu\nu} + S^{\beta\lambda}S^{\mu}_{\rho}\nabla^{\gamma}S_{\mu\nu} + [S^2]^{\beta\lambda}\nabla^{\gamma}S_{\rho\nu} - \delta^{\beta}_{\rho}[S^2]^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu} - S^{\beta}_{\rho}S^{\lambda}_{\mu}\nabla^{\gamma}S^{\mu}_{\nu}\Big) \\
& - S^{\beta}_{\delta}\nabla^{\lambda}S^{\delta}_{\rho} + S^{\beta}_{\rho}\nabla^{\lambda}e_1 - S^{\lambda}_{\mu}\nabla^{\mu}S^{\beta}_{\rho} + 2S^{\beta}_{\delta}\nabla_{\rho}S^{\delta\lambda} + \delta^{\beta}_{\rho}S^{\lambda}_{\gamma}\nabla^{\gamma}e_1 - S^{\beta\lambda}\nabla_{\rho}e_1 + S^{\beta\lambda}\nabla_{\mu}S^{\mu}_{\rho} \\
& - \delta^{\beta}_{\rho}S^{\lambda}_{\delta}\nabla^{\mu}S^{\delta}_{\mu} - S^{\beta}_{\rho}\nabla^{\mu}S^{\lambda}_{\mu}\,.
\end{aligned}$$

But solvable for backgrounds with $\nabla_{\rho}S_{\mu\nu} = 0$

Less ambitious approach: Restricted background and MG limit

Generally difficult due to derivative terms

$$\mathcal{C} = m^2\beta_2\Big[\Big(A^{\beta\lambda} + \tilde{A}^{\beta\lambda}\Big)\,\delta g_{\beta\lambda} + B^{\beta\lambda}_{\rho}\nabla^{\rho}\delta g_{\beta\lambda}\Big]$$

$$A^{\beta\lambda} = m^2\Big[\beta_0\left(e_1\, S^{\beta\lambda} - 2[S^2]^{\beta\lambda}\right) + \beta_2\left(e_1^2[S^2]^{\beta\lambda} - 2\,e_2\,[S^2]^{\beta\lambda} - e_1[S^3]^{\beta\lambda}\right)\Big]$$

Only need to solve two algebraic equations

$$A^{\beta\lambda} = 0$$

$$m^{-2}R^{\mu}_{\ \nu} = \left(\beta_0 + \frac{1}{2}e_1\beta_1\right)\delta^{\mu}_{\nu} + (\beta_1 + \beta_2 e_1)\,S^{\mu}_{\ \nu} - \beta_2(S^2)^{\mu}_{\ \nu}$$

Together with the condition $\nabla_{\rho}S_{\mu\nu} = 0$

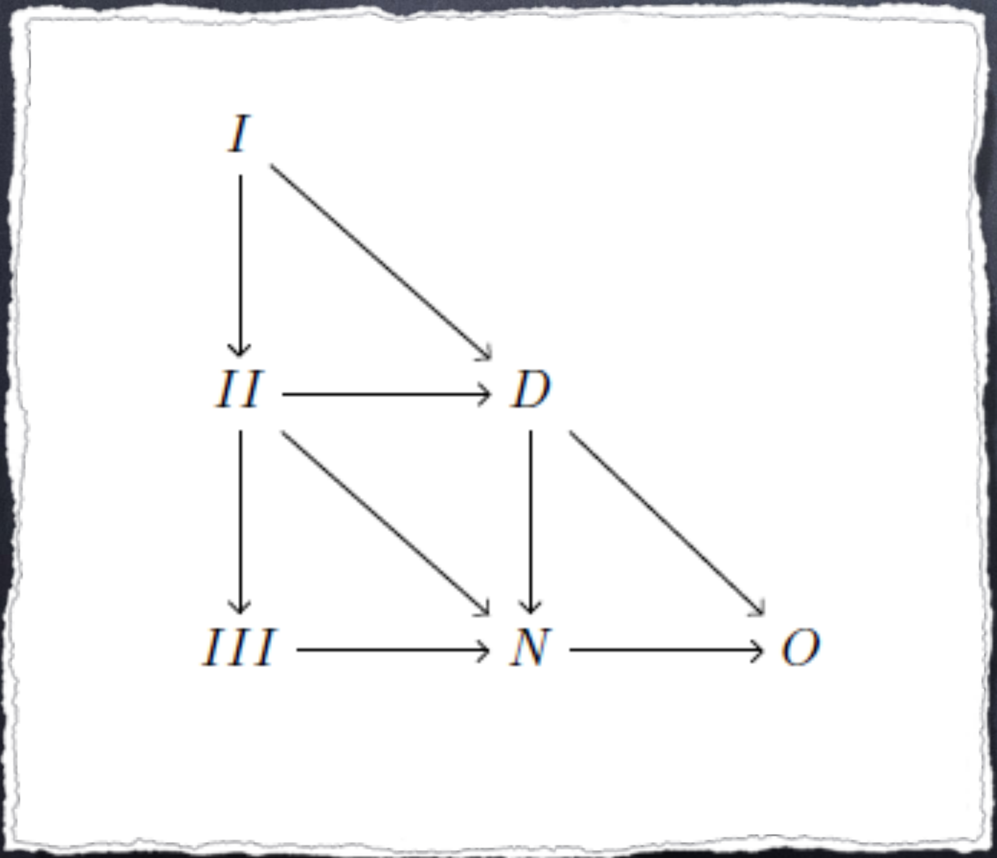
Ricci symmetric spacetimes

- Spacetimes with a CCT are very restricted
- Here it also implies Ricci symmetric $\nabla_\rho R_{\mu\nu} = 0$
- Restricted class of backgrounds but more general then Einstein spacetimes
- Heavily studied by differential geometers

Ricci symmetric spacetimes

Only three possible spacetimes, with known metrics

- Petrov type D: A 2x2 decomposable spacetime $W_{\mu\nu\rho\sigma} \neq 0\,, \quad B_{\mu\nu} \neq 0$
- Petrov type N: Restricted PP wave $W_{\mu\nu\rho\sigma} \neq 0\,, \quad B_{\mu\nu} = 0$
- Petrov type O: Includes Einstein static $W_{\mu\nu\rho\sigma} = 0\,, \quad B_{\mu\nu} = 0$



Petrov classification of spacetimes due to properties of Weyl tensor and their possible degeneration. Type I is most general, all others algebraically special and type O has vanishing Weyl tensor

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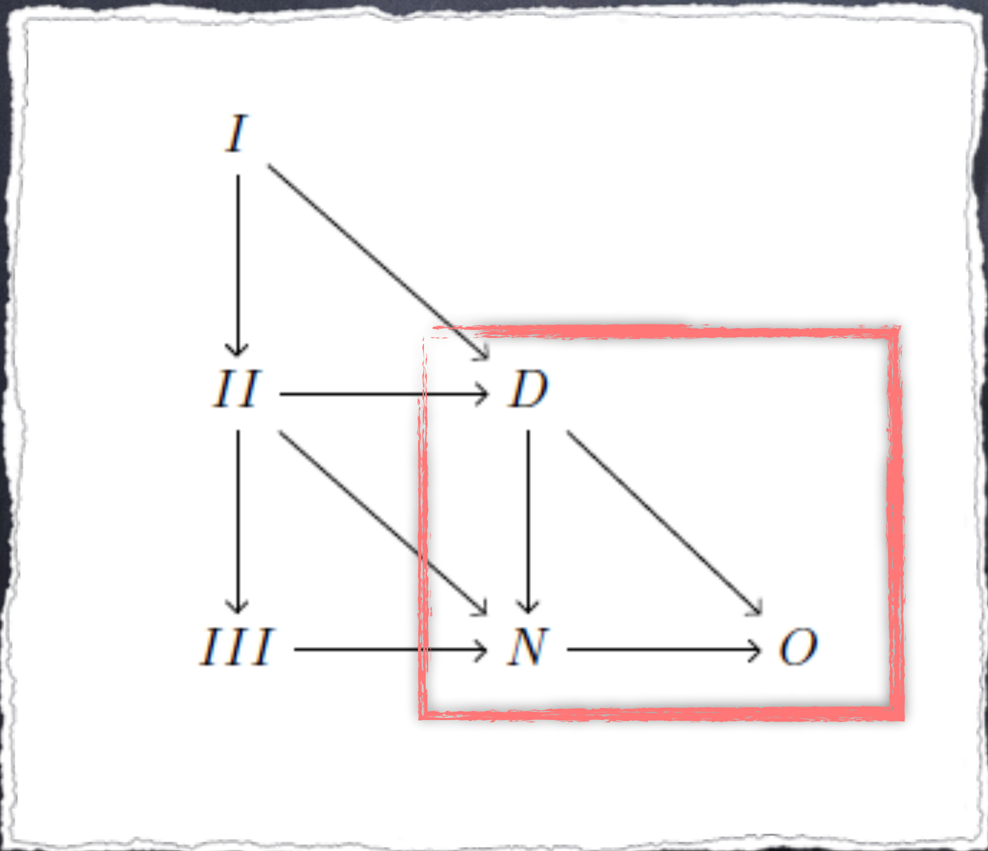

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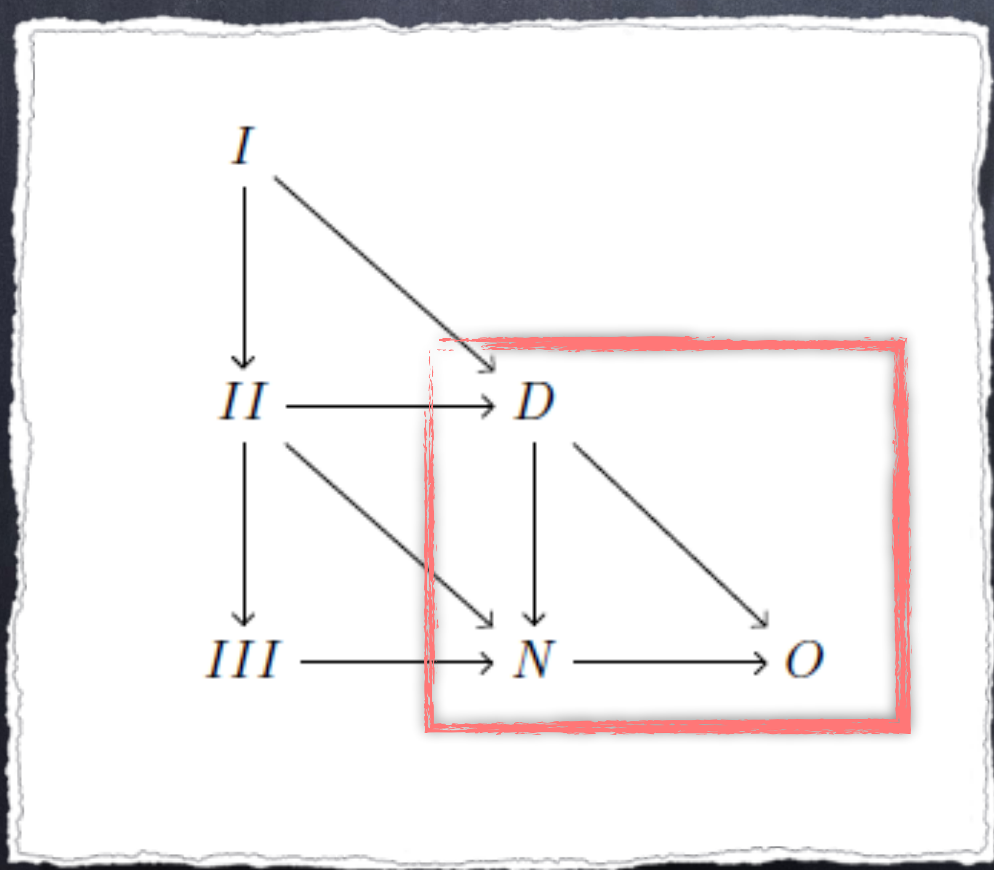

- Linear PM symmetry for these backgrounds in massive gravity model
- Does not work for general backgrounds in massive gravity
- Also works for equations more general than massive gravity ...

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Only three possible spacetimes, with known metrics

- Petrov type D: A 2x2 decomposable spacetime $W_{\mu\nu\rho\sigma} \neq 0\,, \quad B_{\mu\nu} \neq 0$
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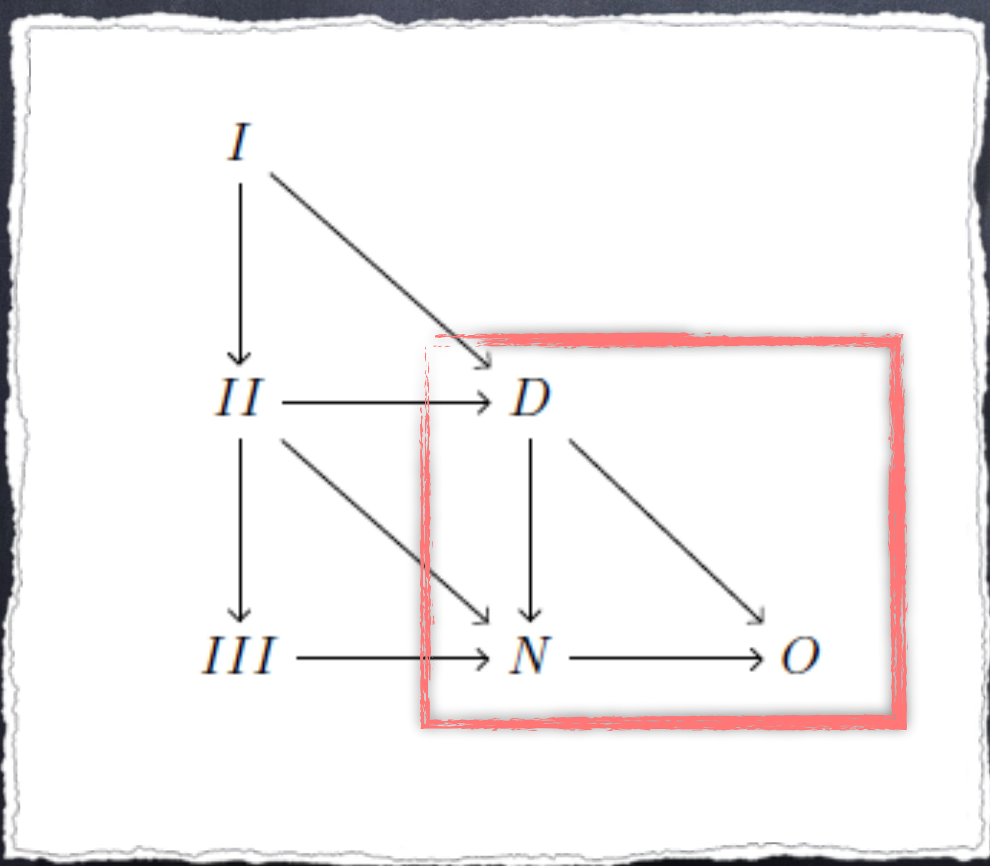

- Linear PM symmetry for these backgrounds in massive gravity model
- Does not work for general backgrounds in massive gravity
- Also works for equations more general than massive gravity ...

Ricci symmetric spacetimes

Reverse engineering; constructive approach

Consider general equations of the form

$$E_{\mu\nu} \equiv \mathcal{G}_{\mu\nu} + \Lambda g_{\mu\nu} + F_{\mu\nu}(g, R) = 0$$

For the Ricci symmetric spacetimes we have that

$$(R^2)_{\mu\nu} = r_1 R_{\mu\nu} + r_2 g_{\mu\nu}$$

Result in linearised equations of the form

$$\begin{aligned}\delta E_{\mu\nu} = & \, \delta\mathcal{G}_{\mu\nu} + a_1 h_{\mu\nu} + a_2 g_{\mu\nu} h + b_1 \left(R_\mu{}^\rho h_{\rho\nu} + R_\nu{}^\rho h_{\rho\mu} \right) \\ & + b_2 g_{\mu\nu} R^{\rho\sigma} h_{\rho\sigma} + b_3 R_{\mu\nu} h + b_4 R_{\mu\nu} R^{\rho\sigma} h_{\rho\sigma} + b_5 R_\mu{}^\rho R_\nu{}^\sigma h_{\rho\sigma}\end{aligned}$$

Ricci symmetric spacetimes

Reverse engineering; constructive approach

Write down most general linear constraint

$$\mathcal{C} \equiv \nabla^\mu \nabla^\nu \delta E_{\mu\nu} + c_1 R^\nu{}_\rho \nabla^\rho \nabla^\mu \delta E_{\mu\nu} + c_2 g^{\mu\nu} \delta E_{\mu\nu} + c_3 R^{\mu\nu} \delta E_{\mu\nu}$$

Demand that $\mathcal{C} = 0$

Linear gauge symmetry given by

$$\Delta h_{\mu\nu} = \left(\nabla_\mu \nabla_\nu + c_1 R_{\rho(\mu} \nabla^\rho \nabla_{\nu)} + c_2 g_{\mu\nu} + c_3 R_{\mu\nu} \right) \xi(x)$$

$$c_i = c_i(a_i, b_i)$$

Ricci symmetric spacetimes

- Linear PM symmetry exist for Ricci symmetric spacetimes in massive gravity, not only Einstein
- But also for more general equations, which still propagate at most 5 dof
- In general many solutions within each class of Ricci symmetric spacetimes
- Unique solution which works for all of these classes

Outline of Talk

- ~~Introduction & Motivation~~
- ~~Bimetric attempts~~
- ~~Covariant constraint approach~~
- Summary & Outlook

Summary & Outlook

- Bimetric theory is an extended theory of gravity motivated from first principles
- Interesting connections between different gravity theories
- Realises linear PM symmetry on Einstein backgrounds but also on Ricci symmetric backgrounds
- PM theory may provide framework for naturally small CC, protected by symmetries
- Promising indications of nonlinear realisation of PM, but also technical obstacles

Summary & Outlook

- Is there a fully nonlinear PM theory?
- If so, is it the proposed bimetric model?
- Or possibly an extension of this model, including e.g. higher spin fields?
- What about consistent matter couplings of such a PM field?

Thank you for your kind attention!

Mikael von Strauss
Institut d'Astrophysique de Paris

IESC Cargèse, May 2016

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