Gravitational Baryogenesis

Elena Arbuzova

Dubna State University & Novosibirsk State University

based on common works with A.D. Dolgov

Hot Topics in Modern Cosmology

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Outline

- Introduction
- Gravitational Baryogenesis with bosons
- Gravitational Baryogenesis with fermions
- Conclusions

Standard Cosmological Model (SCM)

The Standard Model of Cosmology:

- the Standard Model of particle physics $\Rightarrow$ matter content
- General Relativity $\Rightarrow$ gravitational interaction
- the inflationary paradigm $\Rightarrow$ to fix a few problems of the scenario

SCM explains a huge amount of observational data:

- Hubble's law
- the primordial abundance of light elements
- the cosmic microwave background

Some Puzzles

Matter Inventory of the Universe:

- **Baryonic matter** (mainly protons and neutrons): **5%**
- **Dark matter** (weakly interactive particles not belonging to the MSM of particle physics): **25%**
- **Dark energy** (uniformly distributed substance with an unusual equation of state $\mathbf{P} \approx -\varrho$): **70%**

Matter dominance in the Universe:

- The amount of antimatter is very small and it can be explained as the result of high energy collisions in space.
- The existence of large regions of antimatter in our neighborhood would produce high energy radiation as a consequence of matter-antimatter annihilation, which is not observed.

A satisfactory model of our Universe should be able to explain the origin of the matter-antimatter asymmetry, which cannot be created within the MSM of particle physics.

Baryogenesis: the generation of the asymmetry between baryons and antibaryons

Different scenarios of baryogenesis are based, as a rule, on three well known *Sakharov principles (1967)*:

1. Non-conservation of baryonic number.
2. Breaking of symmetry between particles and antiparticles.
3. Deviation from thermal equilibrium.

NB. None of them is strictly necessary.

There are some interesting scenarios of baryogenesis for which one or several of the above conditions are not fulfilled.

A very popular scenario is the so called spontaneous baryogenesis (SBG).

- This mechanism does not demand an explicit C and CP violation and can proceed in thermal equilibrium.
- It is usually most efficient in thermal equilibrium.

Spontaneous Baryogenesis (SBG)

Cosmological baryon asymmetry can be created by SBG in thermal equilibrium:

- A. Cohen, D. Kaplan, Phys. Lett. B 199, 251 (1987); Nucl.Phys. B308 (1988) 913. A.Cohen, D.Kaplan, A. Nelson, Phys.Lett. B263 (1991) 86-92

Reviews:

- A.D.Dolgov, Phys. Repts 222 (1992) No. 6; V.A. Rubakov, M.E. Shaposhnikov, Usp. Fiz. Nauk, 166 (1996) 493; A.D. Dolgov, Surveys in High Energy Physics, 13 (1998) 83.

The term "spontaneous" is related to spontaneous breaking of underlying symmetry of the theory.

- Unbroken phase: the theory is invariant with respect to the global $U(1)$-symmetry, which ensures conservation of total baryonic number.
- Spontaneous symmetry breaking: the Lagrangian density acquires the term

$$\mathcal{L}_{\mathbf{SB}} = (\partial_\mu \theta) \mathbf{J}^\mu_{\mathbf{B}}$$

θ is the (pseudo) Goldstone field and $\mathbf{J}^\mu_{\mathbf{B}}$ is the baryonic current of matter fields.

NB: Due to the SSB this current is not conserved.

SBG: $\mathcal{L}_{\mathbf{SB}} = (\partial_\mu \theta) \mathbf{J}^\mu_{\mathbf{B}}$

Spatially homogeneous field $\theta = \theta(\mathbf{t})$:

$$\mathcal{L}_{\mathbf{SB}} = \dot{\theta}\, \mathbf{n_B}\,, \quad \mathbf{n_B} \equiv \mathbf{J^0_B}$$

- $\mathbf{n_B}$ is the baryonic number density, so it is tempting to identify $\dot{\theta}$ with the chemical potential, μ_B, of the corresponding system.

The identification of $\dot{\theta}$ with chemical potential is questionable and depends upon the representation chosen for the fermionic fields:

- A.D. Dolgov, K. Freese, Phys.Rev. D **51** (1995) 2693-2702; A.D. Dolgov, K. Freese, R. Rangarajan, M. Srednicki, Phys.Rev. D **56** (1997) 6155, E.V. Arbuzova, A.D. Dolgov, V.A. Novikov, Phys. Rev. D **94** (2016) 123501.

Still the scenario is operative and presents a beautiful possibility to create an excess of particles over antiparticles in the universe.

Gravitational Baryogenesis (GBG)

Subsequently the idea of gravitational baryogenesis was put forward:

- H. Davoudiasl, R. Kitano, G. D. Kribs, H. Murayama, P. J. Steinhardt, Phys. Rev. Lett. **93** (2004) 201301, hep-ph/0403019.

The scenario of SBG was modified by the introduction of the coupling of the baryonic current to the derivative of the curvature scalar **R**:

$$\mathcal{L}_{GBG} = \frac{1}{M^2}(\partial_\mu R) J^\mu_B$$

where **M** is a constant parameter with the dimension of mass.

The addition of the curvature dependent term to the Hilbert-Einstein Lagrangian of GR leads to higher order gravitational equations of motion which are strongly unstable with respect to small perturbations.

- EA, A.D. Dolgov, *"Intrinsic problems of the gravitational baryogenesis"*, Phys.Lett. B769 (2017) 171-175, arXiv:1612.06206. *"Instability of gravitational baryogenesis with fermions"*, arXiv:1702.07477.

GBG with scalars

The action of the scalar model has the form:

$$A = \int d^4x \sqrt{-g} \left[\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} (\partial_\mu R) J^\mu - g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi^* + U(\phi, \phi^*) \right] - A_m$$

- where $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass, A_m is the matter action.

The baryonic number is carried by scalar field ϕ with potential $\mathbf{U}(\phi, \phi^*)$.

In contrast to scalar electrodynamics, the baryonic current of scalars is not uniquely defined.

Electrodynamics: the form of the electric current is dictated by the conditions of gauge invariance and current conservation $\Longrightarrow$ sea-gull term $\sim \mathbf{e}^2 \mathbf{A}_\mu |\phi|^2$.

The baryonic current of scalars is considerably less restricted.

Two extreme possibilities:

- the sea-gull term is absent $\Longrightarrow$ the current is not conserved
- the sea-gull term $\sim (\partial_\mu \mathbf{R}) \, |\phi|^2$ is included $\Longrightarrow$ the current conservation.

In both cases no baryon asymmetry can be generated without additional interactions.

GBG with scalars: non-invariant potential $U(\phi)$

If the potential $\mathbf{U}(\phi)$ is not invariant w.r.t. the $U(1)$-rotation $\phi \to \mathbf{e}^{\mathbf{i}\boldsymbol{\beta}}\phi \Longrightarrow$ the baryonic current defined in the usual way is not conserved.

$$\mathbf{J}_{1\mu} = \mathbf{iq}(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*)$$

- Here **q** is the baryonic number of ϕ

The equation of motion for field ϕ is:

$$\mathbf{D}^2\phi + \frac{\partial \mathbf{U}}{\partial \phi^*} = -\frac{\mathbf{iq}}{\mathbf{M}^2}(2\mathbf{D}_\mu \mathbf{R}\, \mathbf{D}^\mu \phi + \phi \mathbf{D}^2 \mathbf{R})$$

According to definition, the current divergence is:

$$\mathbf{D}_\mu \mathbf{J}_1^\mu = \frac{2\mathbf{q}^2}{\mathbf{M}^2}\left[\mathbf{D}_\mu \mathbf{R}\,(\phi^* \mathbf{D}^\mu \phi + \phi \mathbf{D}^\mu \phi^*) + |\phi|^2 \mathbf{D}^2 \mathbf{R}\right] + \mathbf{iq}\left(\phi \frac{\partial \mathbf{U}}{\partial \phi} - \phi^* \frac{\partial \mathbf{U}}{\partial \phi^*}\right)$$

- If $\mathbf{U} = \mathbf{U}(|\phi|)$, the last term disappears, but the current is non-conserved.
- this non-conservation does not lead to any cosmological baryon asymmetry
- it can produce or annihilate an equal number of baryons and antibaryons.

To create cosmological baryon asymmetry we need new types of interactions, e.g.

$$\mathbf{U}_4 = \lambda_4 \phi^4 + \lambda_4^* \phi^{*4}$$

Equations of Motion

The corresponding equations of motion for gravitational field

$$\frac{m_{Pl}^2}{16\pi}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)$$
$$-\frac{1}{M^2}(\left[R_{\mu\nu}-(D_\mu D_\nu-g_{\mu\nu}D^2)\right]D_\alpha J_1^\alpha$$
$$+\frac{1}{2}g_{\mu\nu}J_1^\alpha\, D_\alpha R-\frac{1}{2}(J_{1\nu}D_\mu R+J_{1\mu}D_\nu R))$$
$$-\frac{1}{2}(D_\mu\phi\, D_\nu\phi^*+D_\nu\phi\, D_\mu\phi^*)+\frac{1}{2}g_{\mu\nu}(D_\alpha\phi\, D^\alpha\phi^*-U(\phi))=\frac{1}{2}\,T_{\mu\nu}$$

- D_μ is the covariant derivative, $T_{\mu\nu}$ is the energy-momentum tensor of matter

Taking the trace with respect to $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ we obtain:

$$\frac{m_{Pl}^2}{16\pi}\,R+\frac{1}{M^2}\left[(R+3D^2)D_\alpha J_1^\alpha+J_1^\alpha\,D_\alpha R\right]-D_\alpha\phi\,D^\alpha\phi^*+2U(\phi)=-\frac{1}{2}\,T_\mu^\mu$$

EoM in FRW background: $ds^2 = dt^2 - a^2(t)d\mathbf{r}^2$

In the homogeneous case the equation for the curvature scalar:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2}\left[(R + 3\partial_t^2 + 9H\partial_t) D_\alpha J_1^\alpha + \dot{R}\, J_1^0\right] = -\frac{T^{(tot)}}{2}$$

- $\mathbf{J_1^0}$ is the baryonic number density of the ϕ-field, $\mathbf{H} = \dot{\mathbf{a}}/\mathbf{a}$ (Hubble par.)
- $\mathbf{T^{(tot)}}$ is the trace of the energy-momentum tensor of matter including contribution from the ϕ-field.

In the homogeneous and isotropic cosmological plasma

$$\mathbf{T^{(tot)}} = \varrho - 3\mathbf{P}\,,$$

where ϱ and $\mathbf{P}$ are the energy density and the pressure of the plasma.

Relativistic plasma:

- $\varrho = \pi^2 \mathbf{g}_* \mathbf{T}^4/30$ with $\mathbf{T}$ and $\mathbf{g}_*$ being respectively the plasma temperature and the number of particle species in the plasma.
- The Hubble parameter: $\mathbf{H}^2 = 8\pi\varrho/(3\mathbf{m}_{Pl}^2) \sim \mathbf{T}^4/\mathbf{m}_{Pl}^2$.

Equation for $\mathbf{R}$: the fourth order

The covariant divergence of the current in the homogeneous case:

$$\mathbf{D_\alpha J_1^\alpha} = \frac{\mathbf{2q^2}}{\mathbf{M^2}} \left[\dot{\mathbf{R}} \left(\phi^* \dot{\phi} + \phi \dot{\phi}^* \right) + \left(\ddot{\mathbf{R}} + \mathbf{3H\dot{R}} \right) \phi^* \phi \right] + iq \left(\phi \frac{\partial U}{\partial \phi} - \phi^* \frac{\partial U}{\partial \phi^*} \right)$$

The expectation values of the products of the quantum operators ϕ, ϕ^*, and their derivatives after the thermal averaging:

$$\langle \phi^* \phi \rangle = \frac{\mathbf{T^2}}{\mathbf{12}}, \quad \langle \phi^* \dot{\phi} + \dot{\phi}^* \phi \rangle = \mathbf{0}.$$

Equation of motion for the classical field $\mathbf{R}$ in the cosmological plasma:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{q^2}{6M^4} \left(R + 3\partial_t^2 + 9H\partial_t \right) \left[\left(\ddot{\mathbf{R}} + 3H\dot{R} \right) T^2 \right] + \frac{1}{M^2} \dot{R} \langle J_1^0 \rangle = -\frac{T^{(tot)}}{2}$$

- $\langle J_1^0 \rangle$ is the thermal average value of the baryonic number density of ϕ.
- This term can be neglected, since it is small initially and subdominant later.

NB: We obtained the 4th order differential equation for $\mathbf{R}$.

Exponential Solutions

Keeping only the linear in **R** terms and neglecting higher powers of **R**, such as $\mathbf{R^2}$ or **HR**, we obtain the linear fourth order equation:

$$\frac{d^4R}{dt^4} + \mu^4 R = -\frac{1}{2}\, T^{(tot)}, \quad \mu^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2}\,.$$

The homogeneous part of this equation has exponential solutions:

$$R \sim e^{\lambda t}, \quad \lambda = |\mu| e^{i\pi/4 + i\pi n/2}, \quad n = 0, 1, 2, 3$$

- There are two solutions with positive real parts of λ.

Curvature scalar is exponentially unstable w.r.t. small perturbations, so **R** should rise exponentially fast with time and quickly oscillate around this rising function.

The characteristic rate of the perturbation explosion is much larger than the rate of the universe expansion, if:

$$(\mathrm{Re}\,\lambda)^4 > H^4 = \left(\frac{8\pi\varrho}{3m_{Pl}^2}\right)^2 = \frac{16\pi^6 g_*^2}{2025}\frac{T^8}{m_{Pl}^4}$$

- $\varrho = \pi^2 g_* T^4/30$ is energy density of the primeval plasma at temperature T
- $g_* \sim 10 - 100$ is the number of relativistic degrees of freedom in the plasma.

This condition is fulfilled if

$$\frac{2025}{2^9\pi^7 q^2 g_*^2}\frac{m_{Pl}^6 M^4}{T^{10}} > 1 \quad \text{or} \quad T \lesssim m_{Pl}^{3/5} M^{2/5}$$

Let us stress that at these temperatures the instability is quickly developed and the standard cosmology would be destroyed.

To preserve the successful BBN results $\Longrightarrow$ impose the condition that the development of the instability was longer than the Hubble time at the BBN epoch at $T \sim 1$ MeV $\Longrightarrow$ M should be extremely small: $M < 10^{-32}$ MeV $\Longrightarrow$
A tiny M leads to a huge strength of coupling

$$\mathcal{L}_{GBG} = \frac{1}{M^2}(\partial_\mu R) J_B^\mu$$

It surely would lead to pronounced effects in stellar physics.

GBG with fermions

We start from the action in the form:

$$A = \int d^4x \sqrt{-g} \left[\frac{m_{Pl}^2}{16\pi} R - \mathcal{L}_m \right]$$

with

$$\begin{aligned} \mathcal{L}_m &= \frac{i}{2}(\bar{Q}\gamma^\mu \nabla_\mu Q - \nabla_\mu \bar{Q}\, \gamma^\mu Q) - m_Q \bar{Q}\, Q \\ &+ \frac{i}{2}(\bar{L}\gamma^\mu \nabla_\mu L - \nabla_\mu \bar{L}\gamma^\mu L) - m_L \bar{L}\, L \\ &+ \frac{g}{m_X^2}\left[(\bar{Q}\, Q^c)(\bar{Q}L) + (\bar{Q}^c Q)(\bar{L}Q)\right] + \frac{f}{m_0^2}(\partial_\mu R) J^\mu + \mathcal{L}_{other} \end{aligned}$$

- Q is the quark (or quark-like) field with non-zero baryonic number, L is another fermionic field (lepton)
- ∇_μ is the covariant derivative of Dirac fermion in tetrad formalism (see e.g. lectures I. L. Shapiro, arXiv:1611.02263)
- $J^\mu = \bar{Q}\gamma^\mu Q$ is the quark current with γ^μ being the curved space gamma-matrices; $\mathcal{L}_{other}$ describes all other forms of matter.

EoM for fermions

The four-fermion interaction between quarks and leptons

$$\frac{g}{m_X^2}\left[(\bar{Q}\,Q^c)(\bar{Q}L)+(\bar{Q}^cQ)(\bar{L}Q)\right]$$

is introduced to ensure the necessary non-conservation of the baryon number

- g is a dimensionless coupling constant.
- m_X is a constant parameter with dimension of mass, which may be of the order of $10^{14}-10^{15}$ GeV in grand unified theories.

Lagrangian leads to the following equations of motion for quarks:

$$i\gamma^\mu\nabla_\mu Q = m_Q Q - \frac{f}{m_0^2}(\partial_\mu R)\gamma^\mu Q - \frac{g}{m_X^2}\left[2Q^c(\bar{Q}L)+(\bar{Q}Q^c)L\right]$$

$$i\nabla_\mu\bar{Q}\,\gamma^\mu = -m_Q\bar{Q} + \frac{f}{m_0^2}(\partial_\mu R)\bar{Q}\gamma^\mu + \frac{g}{m_X^2}\left[2\bar{Q}^c(\bar{L}Q)+\bar{L}(\bar{Q}^cQ)\right]$$

and leptons:

$$i\gamma^\mu\nabla_\mu L = m_L L - \frac{g}{m_X^2}(\bar{Q}^cQ)Q$$

$$i\nabla_\mu\bar{L}\,\gamma^\mu = -m_L\bar{L} + \frac{g}{m_X^2}(\bar{Q}Q^c)\bar{Q}$$

Taking variation of action over metric, $\mathbf{g}^{\mu\nu}$, we obtain equations for gravitational field in the form:

$$\frac{m_{Pl}^2}{8\pi}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = T_{\mu\nu}^{m}$$

where energy-momentum tensor, $\mathbf{T}_{\mu\nu}^{m}$, is defined as:

$$T_{\mu\nu}^{m} = \frac{2}{\sqrt{-g}}\frac{\delta A_m}{\delta g^{\mu\nu}} \quad \text{with} \quad A_m = \int d^4x\sqrt{-g}\mathcal{L}_m$$

The gravitational EoM, obtained this way, can be written as:

$$\frac{m_{Pl}^2}{8\pi}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = -g_{\mu\nu}\mathcal{L}_m +$$
$$\frac{i}{4}\left[(\bar{Q}(\gamma_\mu\nabla_\nu + \gamma_\nu\nabla_\mu)Q - (\nabla_\nu\bar{Q}\,\gamma_\mu + \nabla_\mu\bar{Q}\,\gamma_\nu)Q\right] +$$
$$\frac{i}{4}\left[(\bar{L}(\gamma_\mu\nabla_\nu + \gamma_\nu\nabla_\mu)L - (\nabla_\nu\bar{L}\,\gamma_\mu + \nabla_\mu\bar{L}\,\gamma_\nu)L\right] -$$
$$\frac{2f}{m_0^2}\left[R_{\mu\nu} + g_{\mu\nu}D^2 - D_\mu D_\nu\right]D_\alpha J^\alpha + \frac{f}{2m_0^2}(J_\mu\partial_\nu R + J_\nu\partial_\mu R)$$

where $\mathbf{D}_\mu$ is the usual tensor covariant derivative in background metric.

Gravitational EoM for Trace

Taking trace with respect to μ and ν we obtain:

$$
\begin{aligned}
-\frac{m_{Pl}^2}{8\pi} R &= m_Q \bar{Q} Q + m_L \bar{L} L + \frac{2g}{m_X^2}\left[(\bar{Q}\, Q^c)(\bar{Q} L) + (\bar{Q}^c Q)(\bar{L} Q)\right] \\
&- \frac{2f}{m_0^2}(R + 3D^2) D_\alpha J^\alpha + T_{other}
\end{aligned}
$$

- T_{other} is the trace of the energy momentum tensor of all other fields.
- At relativistic stage we can take $T_{other} = 0$.
- The average expectation value of the interaction term, $\sim g$, is also small, especially at $T < m_X$, so contribution of all matter fields may be neglected.

Kinetic equation leads to an explicit dependence on R of the current divergence, $D_\alpha J^\alpha$, if the current is not conserved. As a result we obtain high (fourth) order equation for R.

We study solutions in cosmology in homogeneous and isotropic FRW background:

$$
ds^2 = dt^2 - a^2(t) dr^2 \,, \quad D_\alpha J^\alpha = (\partial_t + 3H) J^t .
$$

Kinetic Equation: the reaction $q_1 + q_2 \leftrightarrow \bar{q}_3 + l_4$

The kinetic equation for the variation of the baryonic number density $n_B \equiv J^t$ through this reaction in the FRW background has the form:

$$(\partial_t + 3H) n_B = I_B^{coll}$$

where the collision integral for space and time independent interaction is equal to:

$$I_B^{coll} = -3B_q (2\pi)^4 \int d\nu_{q_1,q_2}\, d\nu_{\bar{q}_3,l_4} \delta^4(q_1 + q_2 - q_3 - l_4)$$
$$\left[|A(q_1 + q_2 \to \bar{q}_3 + l_4)|^2 f_{q_1} f_{q_2} - |A(\bar{q}_3 + l_4 \to q_1 + q_2)|^2 f_{\bar{q}_3} f_{l_4} \right]$$

- **A(a → b)** is the amplitude of the transition from state **a** to state **b**
- $\mathbf{B_q}$ is the baryonic number of quark, $\mathbf{f_a}$ is the phase space distribution (the occupation number), and the element of phase space is:

$$d\nu_{q_1,q_2} = \frac{d^3 q_1}{2E_{q_1}(2\pi)^3} \frac{d^3 q_2}{2E_{q_2}(2\pi)^3}$$

where $E_q = \sqrt{q^2 + m^2}$ is the energy of particle with three-momentum **q** and mass **m**.

Equilibrium w.r.t. Elastic Scattering and Annihilation

Equilibrium distribution functions of quarks and leptons:

$$\mathbf{f = \frac{1}{e^{(E/T-\xi)}+1} \approx e^{-E/T+\xi}}$$

- $\xi = \mu/T$ is dimensionless chemical potential, different for quarks and leptons

The assumption of kinetic equilibrium is well justified since it is usually enforced by very efficient elastic scattering.

However, if we use the original representation for quark fields, which satisfy EoM:

$$\mathbf{i\gamma^\mu \nabla_\mu Q = m_Q Q - \frac{f}{m_0^2}(\partial_\mu R)\gamma^\mu Q - \frac{g}{m_X^2}\left[2Q^c(\bar{Q}L) + (\bar{Q}Q^c)L\right]}$$

the conclusion of kinetic equilibrium is not evident because the quark evolution depends upon $\mathbf{R(t)}$, which may be quickly varying.
This problem is absent in the representation:

$$\mathbf{Q_2 = \exp(ifR/m_0^2)\, Q}$$

analogously to what is done in:

- EA, A.D. Dolgov, V.A. Novikov, Phys. Rev. D**94** (2016) 123501
  "General properties and kinetics of spontaneous baryogenesis"

For Q_2-representation: $Q_2 = \exp(ifR/m_0^2)\, Q$

- Elimination of the term $\mathbf{f(\partial_\mu R) J^\mu / m_0^2}$ in the Lagrangian (but the dependence on the curvature reappears in the interaction term)
- $\mathbf{R(t)}$ neither enters EoM for quarks, nor the amplitudes of elastic scattering and annihilation.
- The assumption of kinetic equilibrium is well justified.

We obtain the same fourth order equation for the evolution of curvature, as for non-rotated field $\mathbf{Q}$.

The baryonic number density is given by the expression:

$$\begin{aligned} n_B &= \int \frac{d^3q}{2E_q\,(2\pi)^3}(f_q - f_{\bar{q}}) \\ &= \frac{g_S B_q}{6}\left(\mu T^2 + \frac{\mu^3}{\pi^2}\right) = \frac{g_S B_q T^3}{6}\,\left(\xi + \frac{\xi^3}{\pi^2}\right) \end{aligned}$$

- $\mathbf{T}$ is the cosmological plasma temperature
- $\mathbf{g_S}$ and $\mathbf{B_q}$ are respectively the number of the spin states and the baryonic number of quarks.

Energy Conservation

Dependence on terms $\sim f\,R(t)/m_0^2$ would reappear in the interaction term:

$$\mathcal{L}_{int} = \frac{2g}{m_X^2}\left[e^{-3ifR/m_0^2}\,(\bar{Q}_2\,Q_2^c)(\bar{Q}_2 L) + e^{3ifR/m_0^2}\,(\bar{Q}_2^c Q_2)(\bar{L} Q_2)\right]$$

- The transition amplitudes, which enter $\mathbf{I^{coll}}$, are obtained by integration over time of $\mathcal{L}_{int}$ operator, taken between $|\mathbf{in}\rangle$ and $|\mathbf{fin}\rangle$ states $\Longrightarrow$
- The energy conservation δ-function in $\mathbf{I^{coll}}$ would be modified due to time dependent factors $\mathbf{exp\,[\pm 3ifR(t)/m_0^2]}$.

The simplest case: slowly changing $\dot{R}$:

$$\mathbf{R(t) \approx \dot{R}(t)\,t.}$$

The energy is not conserved but the energy conservation condition is trivially modified:

$$\delta[E(q_1) + E(q_2) - E(q_3) - E(l_4)] \Rightarrow$$
$$\Rightarrow \delta[E(q_1) + E(q_2) - E(q_3) - E(l_4) - 3f\,\dot{R}(t)/m_0^2\,].$$

Thus the energy is non-conserved due to the action of the external field $\mathbf{R}(t)$.

Collision Integral

If the dimensionless chemical potentials for quarks, ξ_q, and leptons, ξ_l, as well as $f\dot{R}(t)/m_0^2/T$, are small, and the energy balance is ensured by the delta-function

$$\delta[\mathbf{E(q_1) + E(q_2) - E(q_3) - E(l_4) - 3f\dot{R}(t)/m_0^2}]$$

then the collision integral can be approximated as:

$$\mathbf{I_B^{coll} \approx \frac{C_I g^2 T^8}{m_X^4} \left[\frac{3f\dot{R}(t)}{m_0^2\,T} - 3\xi_q + \xi_l\right]}$$

- $\mathbf{C_I}$ is a positive dimensionless constant
- The factor $\mathbf{T^8}$ appears for reactions with massless particles from dimensional consideration.

The conservation of the sum of baryonic and leptonic numbers gives

$$\mathbf{\xi_l = -\xi_q/3.}$$

The case of an essential variation of $\dot{R}(t)$ is much more complicated technically. Here we consider only the simple situation with quasi-stationary background.

Stationary Point Approximation

For small chemical potential the baryonic number density is equal to

$$n_B \approx \frac{g_s B_q}{6} \xi_q T^3$$

If the temperature adiabatically decreases in the course of the cosmological expansion, according to $\dot{T} = -HT$, the kinetic equation

$$(\partial_t + 3H) n_B = I_B^{coll}$$

turns into

$$\dot{\xi}_q = \Gamma \left[\frac{9 f \dot{R}(t)}{10 m_0^2 T} - \xi_q \right], \quad \Gamma \sim g^2 T^5 / m_X^4$$

- where Γ is the rate of B-nonconserving reactions.

If Γ is large, this equation can be solved in stationary point approximation as

$$\xi_q = \xi_q^{eq} - \dot{\xi}_q^{eq} / \Gamma, \quad where \quad \xi_q^{eq} = \frac{9}{10} \frac{f \dot{R}}{m_0^2 T}.$$

If we substitute ξ_q^{eq} into EoM for trace we arrive to the 4th order equation for **R**.

Curvature Instability

The contribution of thermal matter into EoM for trace can be neglected

$$\frac{m_{Pl}^2}{8\pi} R = \frac{2f}{m_0^2} (R + 3D^2)(\partial_t + 3H) n_B$$

From the kinetic equation

$$n_B \sim \xi_q^{eq} = \frac{9}{10} \frac{f\dot{R}}{m_0^2 T}$$

Neglecting the H-factor in comparison with time derivatives of R, we arrive to:

$$\frac{d^4R}{dt^4} = \lambda^4 R \qquad (*)$$

where

$$\lambda^4 = C_\lambda m_{Pl}^2 m_0^4 / T^2 \;\; \text{with} \;\; C_\lambda = 5/(36\pi f^2 g_s B_q).$$

Equation $(*)$ has extremely unstable solution with instability time by far shorter than the cosmological time.

This instability would lead to an explosive rise of **R**, which may possibly be terminated by the nonlinear terms $\sim$to the product of H to lower derivatives of R.

Possible Stabilization

Correspondingly one may expect stabilization when

$$\mathbf{HR \sim \dot{R}\,, \quad i.e. \quad H \sim \lambda}$$

Since

$$\mathbf{\dot{H} + 2H^2 = -R/6}$$

H would also exponentially rise together with **R**,

$$\mathbf{H \sim \exp(\lambda t) \quad and \quad \lambda H \sim R}$$

Thus stabilization may take place at

$$\mathbf{R \sim \lambda^2 \sim m_{Pl} m_0^2 / T}$$

which is much larger than the normal General Relativity value

$$\mathbf{R_{GR} \sim T_{matter} / m_{Pl}^2}$$

where $\mathbf{T_{matter}}$ is the trace of the energy-momentum tensor of matter.

Some Comments

The considered here effect of strong instability in high order differential equations with small coefficient, ϵ, in front of the highest derivative term is well known in mathematics but might be unexpected for a physicist.

Even more surprising is a discontinuity of the limit $\epsilon \to \mathbf{0}$

- If we take $\epsilon = \mathbf{0}$ at the very beginning, then the instability does not appear and the theory is reduced to the normal lower order one
- With any small but non-zero ϵ the equation of motion has solutions which are absent in the limit $\epsilon = \mathbf{0}$. Moreover, the smaller is ϵ, the faster is the rise of the unstable solution.

A counterexample: the decoupling of heavy modes in field theory

- As well known, a low energy limit of a normal field theory is not sensitive to existence of very high mass particles.
- This is true for stable 2nd order eqs., which is not the case considered here.

Conclusions

- The derivative coupling of baryonic current to the curvature scalar in GBG scenarios leads to higher (4th) order equations for gravitational field.
- These equations are unstable with respect to small perturbations of the FRW background and such instability leads to an exponential rise of the curvature.
- For a large range of cosmological temperatures the development of the instability is much faster than the universe expansion rate.
- The rise of $\mathbf{R}$ could be terminated by the effects of non-linear terms in the equations of motion. Evidently $\mathbf{R}$ would stop rising if the non-linear terms become comparable by magnitude with the linear ones.
- It means that the rise terminates when $\mathbf{R}$ by far exceeds the value found in the normal General Relativity.
- Simple versions of GBG based on the coupling, $\sim (\partial_\mu \mathbf{R}) \mathbf{J}_\mathbf{B}^\mu$, would be not compatible with observations and some stabilization mechanism is desirable.
- Probably stabilization may be achieved in a version of $\mathbf{F(R)}$-gravity or by an introduction of a form-factor $\mathbf{g(R)}$ into the coupling $\mathcal{L}_{\mathbf{GBG}}$, such as $\mathbf{g(R)}$ drops down with rising $\mathbf{R}$.

THE END

THANK YOU FOR YOUR ATTENTION!