Duality and the generation of scale invariant primordial magnetic fields in bouncing scenarios

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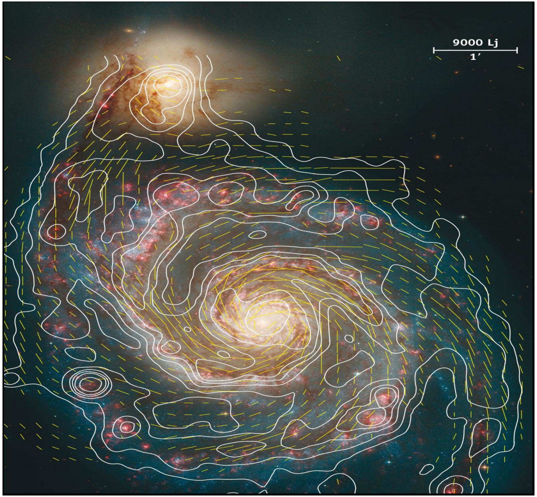
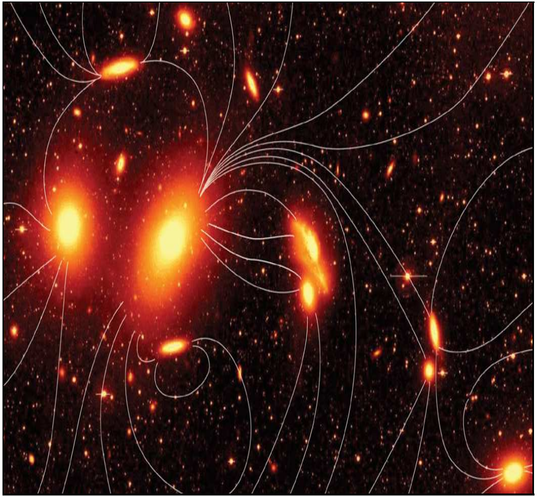
Magnetic fields in the universe



Whirlpool galaxy[1]



Virgo cluster[2]

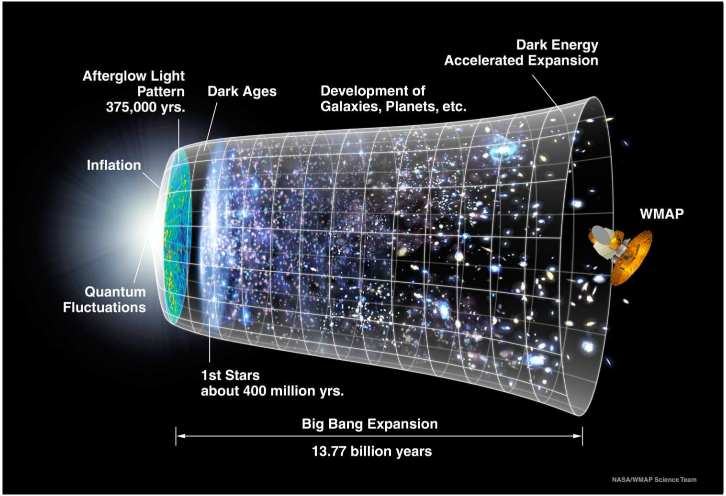
[1] Image from http://www.mpifr-bonn.mpg.de/research/fundamental/cosmag.

[2] Image from http://science.sciencemag.org/content/311/5762/787.full.pdf+html.

Outline of the talk

1. The early universe
2. Modeling a bounce
3. Generating magnetic fields in bouncing universes
4. Summary

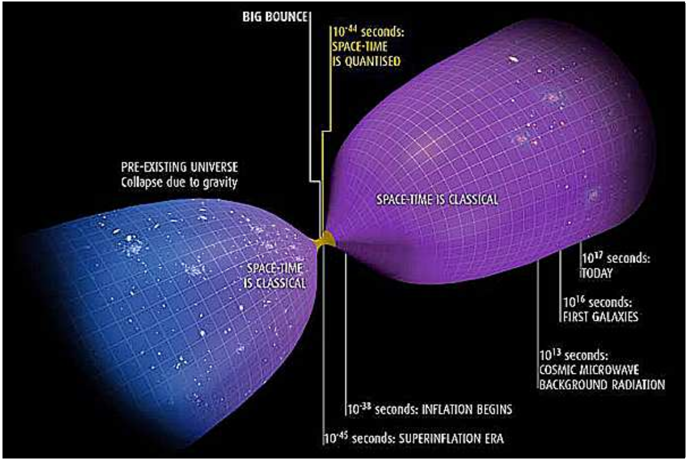
Inflation



Inflation refers to a period of accelerated expansion at very early times[3].

[3] Image from http://wmap.gsfc.nasa.gov/media/060915/060915_CMB_Timeline150.jpg.

Bouncing models

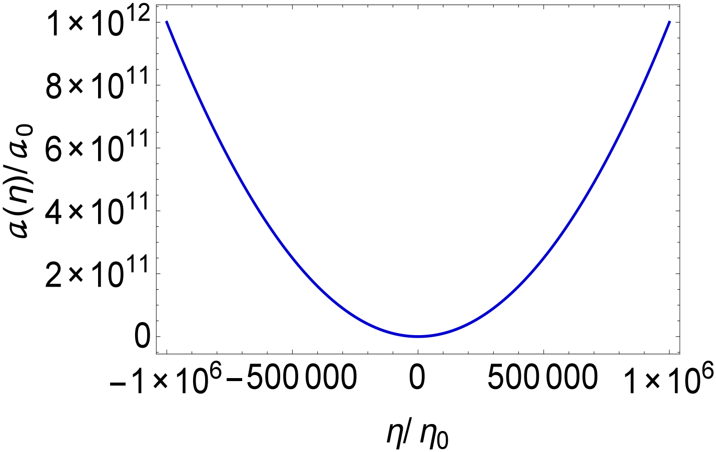


The currently expanding universe is connected to a previously contracting phase through a bounce[4].

[4] Image from http://www.bigbangcentral.com/before_bb_page.html.

The scale factor in a bounce

We assume the scale factor behaves as: $a(\eta) = a_0\, \left(1 + \eta^2/\eta_0^2\right)^q = a_0\, \left(1 + k_0^2\, \eta^2\right)^q$.



The behavior of $a(\eta)/a_0$ has been plotted as a function of η/η_0 for $q = 1$, which corresponds to a matter bounce.

The non-minimal action

We consider the action

$$S[\phi, A^\mu] = -\frac{1}{16\,\pi} \int \mathrm{d}^4x\, \sqrt{-g}\, J^2(\phi)\, F_{\mu\nu} F^{\mu\nu},$$

where $F_{\mu\nu} = \partial_\mu\, A_\nu - \partial_\nu\, A_\mu$.

The coupling function $J(\phi)$ breaks the conformal invariance of the standard electromagnetic action.

On quantization, the vector potential $\hat{A}_i$ can be Fourier decomposed as follows:[5]

$$\hat{A}_i\,(\eta, \boldsymbol{x}) = \sqrt{4\,\pi} \int \frac{\mathrm{d}^3\,\boldsymbol{k}}{(2\,\pi)^{3/2}} \sum_{\lambda=1}^{2} \tilde{\epsilon}_{\lambda i}(\boldsymbol{k}) \left[\hat{b}_{\boldsymbol{k}}^{\lambda}\, \bar{A}_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}_{\boldsymbol{k}}^{\lambda\dagger}\, \bar{A}_k^{*}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right].$$

We consider the following form of the coupling function:

$$J(\eta) = J_0\, a^n(\eta).$$

[5] J. Martin and J. Yokoyama, JCAP **0801**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110-120 (2010).

Evolution of the modes

The evolution of the Fourier modes of the electromagnetic vector potential is given by

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right) \mathcal{A}_k = 0,$$

where $\mathcal{A}_k = J\, \bar{A}_k$.

We divide the time period before the bounce into two domains – one far away from the bounce ($-\infty < \eta < -\alpha\, \eta_0$) and another one closer to the bounce ($-\alpha\, \eta_0 < \eta < 0$). We evaluate the modes in both these domains.

In the first domain, the coupling function behaves as:

$$J(\eta) \propto \eta^{\gamma},$$

where $\gamma = 2\, n\, q$.

The magnetic and electric power spectra

The spectral energy density $\mathcal{P}_{_{\rm B}}(k)$ for the magnetic field is given by

$$\mathcal{P}_{_{\rm B}}(k) = \frac{{\rm d}\langle 0|\hat{\rho}_{_{\rm B}}|0\rangle}{{\rm d}\ln k} = \frac{1}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\mathcal{A}_k(\eta)|^2$$

and spectral energy density $\mathcal{P}_{_{\rm E}}(k)$ for the electric field is given by

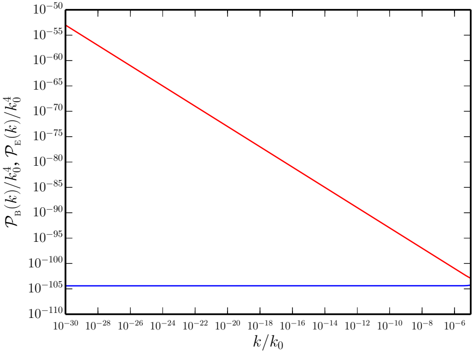
$$\mathcal{P}_{_{\rm E}}(k) = \frac{{\rm d}\langle 0|\hat{\rho}_{_{\rm E}}|0\rangle}{{\rm d}\ln k} = \frac{1}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,\left|\mathcal{A}_k'(\eta) - \frac{J'(\eta)}{J(\eta)}\,\mathcal{A}_k(\eta)\right|^2,$$

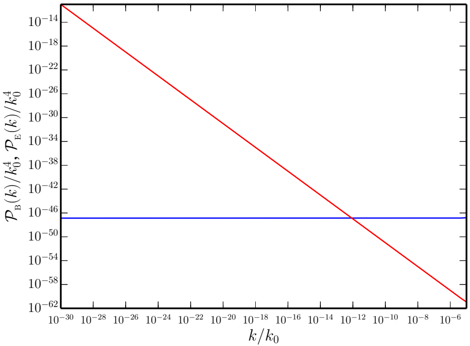
where $\hat{\rho}_{_{\rm E}}$ and $\hat{\rho}_{_{\rm B}}$ denote the operators corresponding to the energy densities associated with the electric and magnetic fields.

We evaluate the magnetic and electric power spectra both before and after the bounce.

For $\gamma = 3$ or $\gamma = -2$, the magnetic power spectrum is scale invariant.

The power spectra before and after the bounce





The power spectra of the magnetic field (in blue) and the electric field (in red), evaluated before the bounce, have been plotted on the left as a function of k/k_0 for $q = 1$ and $\gamma = 3$.

The power spectra of the electric field (in red) and the magnetic field (in blue), evaluated after the bounce, have been plotted on the right for the same set of values of the parameters as in the adjoining figure[6].

[6] D. Chowdhury, L. Sriramkumar and R. K. Jain, PRD **94**, 083512 (2016).

Duality invariance of the power spectra

The quantity J''/J determines the evolution of the perturbations.

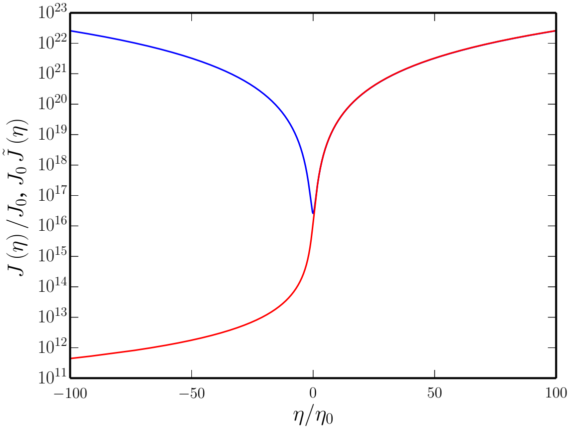
Hence, two different forms of the coupling function J that lead to the same J''/J also lead to the same form for the magnetic field power spectra.

Given a coupling function J, its dual function, say, $\tilde{J}$, which leads to the same $\tilde{J}''/\tilde{J}$ is found to be

$$J(\eta) \to \tilde{J}(\eta) = C\, J(\eta) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{J^2(\bar{\eta})},$$

where C and η_* are constants.

The coupling function and its dual form



The coupling function J (in blue) and its dual $\tilde{J}$ (in red) have been plotted as a function of η/η_0 for $\gamma = 3$. It is evident that even though the original coupling function was symmetric about the bounce, its dual form is not[7].

[7] D. Chowdhury, L. Sriramkumar and R. K. Jain, PRD **94**, 083512 (2016).

Summary

- Magnetic fields have been observed over a wide range of scales in the universe.
- We have analytically evaluated the spectra of magnetic and electric fields in a certain class of bouncing models.
- We have illustrated that the shapes of the spectra are preserved across the bounce.
- We have also shown that the power spectra of the magnetic fields remain invariant under a two parameter family of solutions of the coupling function.

Thank you!