Fine Tuning May Not Be Enough

arXiv:1506.07306
Hot Topics in Modern Cosmology
May 1,2017

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$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}$$

$$H(t) \equiv \dot{a}/a \quad \& \quad \epsilon(t) \equiv -\dot{H}/H^2 \text{(q = -1 + } \epsilon\text{)}$$

- Hard to avoid an early phase of accelerated expansion
  - but what caused it is a mystery
- Scalar potential models work

$$\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} - \tfrac{1}{2}\partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} \sqrt{-g} - V(\varphi)\sqrt{-g}$$

1. $$3H^2 = 8\pi G \left[\tfrac{1}{2}\dot{\varphi}_0^2 + V(\varphi_0) \right]$$
2. $$-2\dot{H} - 3H^2 = 8\pi G \left[\tfrac{1}{2}\dot{\varphi}_0^2 - V(\varphi_0) \right]$$

- (1)+(2) ➔ $\varphi_0(t) = \varphi_i - \int_{t_i}^{t} dt' \sqrt{\frac{-2\dot{H}(t')}{8\pi G}}$
- Rotate the graph ➔ $t(\varphi_0)$
- (1)-(2) ➔ $V(\varphi) = \frac{1}{8\pi G}\left[\dot{H} + 3H^2\right]_{t(\varphi)}$

Fine Tuning Problems

1. Initial Conditions
   - Potential energy domination over more than a Hubble volume
2. Duration
   - $N = -8\pi G \int_{\varphi_i}^{\varphi_f} d\varphi \frac{V(\varphi)}{V'(\varphi)} \geq 50$
3. Scalar Perturbations
   - $\frac{(GV)^3}{V'^2} \sim 10^{-11}$
4. Tensor Perturbations
   - $G^2V \leq 5 \times 10^{-12}$
5. Reheating
   - Reheating requires coupling to normal matter
6. Cosmological Constant
   - $G^2V_{min} \approx 10^{-123}$

Reheating $\rightarrow \Delta V(\varphi)$

& an obstacle to more fine tuning

- Flat Space: $\Delta V(\varphi) \sim \pm (cc \times \varphi)^4 ln\left[\frac{cc^2 \times \varphi^2}{\mu^2}\right]$
  - Not Planck suppressed & typically too steep
  - Just fine tune it away . . .
- On de Sitter: $\Delta V(\varphi) \sim \pm H^4 f\left[\frac{cc^2 \times \varphi^2}{H^2}\right]$
  - $f(x) \rightarrow x^2 \ln(x)$ gives Coleman-Weinberg
  - But small x is relevant: $f(x) = \alpha x + \beta x^2 + O(x^3)$
  - Set $\alpha = 0$ with $\delta\xi\varphi^2 R\sqrt{-g}$ & $\beta = 0$ with $\delta\lambda\varphi^4\sqrt{-g}$
- Factors of ``H^2" are not constant & not even local

  $\rightarrow$ stuck with $O(x^3)$ terms

Three Models

1. Another scalar ϕ (h is a coupling constant)

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{[1+\Delta\xi]}{12}\phi^2 R\sqrt{-g} - \tfrac{1}{4}h^2\phi^2\varphi^2\sqrt{-g}$$

2. A fermion ψ

$$\mathcal{L} = \bar{\psi}\gamma^b e_b^\mu\left(i\partial_\mu - \tfrac{1}{2}A_{\mu cd}J^{cd}\right)\psi\sqrt{-g} - f\varphi\bar{\psi}\psi\sqrt{-g}$$

3. A vector gauge boson A_μ (with complex inflaton)

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} - \left(\partial_\mu - ieA_\mu\right)\varphi(\partial_\nu + ieA_\nu)\varphi^* g^{\mu\nu}\sqrt{-g}$$

For each model:

- Give general (de Sitter) form for $\Delta V(\varphi)$
- Give large field expansion
- Give small field expansion

Relation between Re-heating & $V_{eff}(\varphi)$

- In Flat Space

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu} - \tfrac{1}{4}h^2\varphi^2\phi^2$$

- Re-heating needs NO (large) extra ϕ mass term
  - Shaking ϕ with $\varphi_0(t)$ generates ϕ quanta
  - If $\varphi_0(t)$ were large we'd just subtract its VEV ➔ small field limit
- $V_{eff}(\varphi)$ is the sum of $\frac{1}{2}\hbar\omega$ terms for ϕ 0-point energy
  - φ = constant ➔ $m_\phi^2 = \frac{1}{2}h^2\varphi^2$
  - $V_{eff}(\varphi) = \sum_{\vec{k}} \frac{1}{2}\sqrt{k^2 + m_\phi^2} =$
    $\sum_{\vec{k}} \frac{1}{2}k\left[1 + \frac{m_\phi^2}{2k^2} - \frac{m_\phi^4}{8k^4} + \cdots\right] \rightarrow \frac{m_\phi^4}{64\pi^2}\, ln\left[\frac{m_\phi^2}{\mu^2}\right]$
- It's MUCH more complicated in de Sitter!

Limit on backgrounds is PROPAGATORS

1. Mass M scalar with conformal coupling ξ

$$\sqrt{-g}[\square - \xi R - M^2] i\Delta[\xi, M^2](x;x') = i\delta^D(x-x')$$

2. Mass m fermion on $ds^2 = a^2[-d\eta^2 + d\vec{x}\cdot d\vec{x}]$

$$i[{}_iS_j](x;x') = \frac{1}{a^{(D-1)/2}}[i\gamma^\mu \partial_\mu + am]\frac{a^{(D-1)/2}}{\sqrt{aa\prime}}\mathcal{S}(x;x')$$

$$\mathcal{S} = \tfrac{1}{2}(1+\gamma^0) i\Delta[\xi_c, M_+^2](x;x') + \tfrac{1}{2}(1-\gamma^0) i\Delta[\xi_c, M_-^2](x;x')$$

- With $\xi_c \equiv \frac{1}{2(D-1)}$ & $M_\pm^2 \equiv m(m \mp iH)$

3. Mass M_V vector with $\xi_v \equiv \frac{(D-2)}{D(D-1)}$

$$i[{}_\mu\Delta_\rho](x;x') = \left[\square_\mu^\nu - D^\nu D_\mu\right]\left[\square'^\sigma_\rho - D'^\sigma D'_\rho\right]\left[\frac{\partial^2 \ell^2(x;x')}{\partial x^\nu \partial x'^\sigma} S(x;x')\right]$$

$$S(x;x') = \frac{i\Delta[\xi_v, M_V^2] - i\Delta[\xi_v, 0]}{M_V^4} - \frac{1}{M_V^2}\frac{\partial i\Delta[\xi_v, N^2]}{\partial N^2}\Big|_{N^2=0}$$

$$\Delta V'_{scalar} = \delta\xi R\varphi + \frac{1}{6}\delta\lambda\varphi^3 + \frac{1}{2}h^2\varphi i\Delta\left[\frac{1+\Delta\xi}{12}, \frac{1}{2}h^2\varphi^2\right](x;x)$$

- $\nu_{\pm} \equiv \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 8\Delta\xi}$, $z^2 \equiv \frac{h^2\varphi^2}{H^2}$
- $\psi(x) \equiv \frac{d}{dx} ln[\Gamma(x)]$
- General form →

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{-\Big[\psi(\nu_+)+\psi(\nu_-)\Big]\Big[2\Delta\xi z^2+\frac{z^4}{4}\Big]+\Big[\psi'(\nu_+)-\psi'(\nu_-)\Big]\frac{\frac{1}{2}\Delta\xi z^4}{\sqrt{1-8\Delta\xi}} + \int_0^{z^2} dx\,\Big(2\Delta\xi+\frac{x}{2}\Big)\Bigg[\psi\Bigg(\frac{1}{2}+\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Bigg)+\psi\Bigg(\frac{1}{2}-\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Bigg)\Bigg]\Bigg\}$$

Large field and small field expansions

- Large field →

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{\frac{1}{4}z^4\ln\Big(\frac{1}{2}z^2+2\Delta\xi\Big) - \Bigg[\frac{1}{8}+\frac{[\psi(\nu_+)+\psi(\nu_-)]}{4} - \frac{\Delta\xi[\psi'(\nu_+)-\psi'(\nu_-)]}{2\sqrt{1-8\Delta\xi}}\Bigg]z^4 + 2\Delta\xi z^2\ln\Big(\frac{1}{2}z^2+2\Delta\xi\Big) - \Bigg[\frac{1}{3}+\Delta\xi + 2\Delta\xi\Big[\psi(\nu_+)+\psi(\nu_-)\Big]\Bigg]z^2 + \Big[4\Delta\xi^2-\frac{2}{15}\Big]\ln\Big(\frac{1}{2}z^2+2\Delta\xi\Big) + O(z^0)\Bigg\}$$

- Small field →

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{\Bigg[\frac{(1-6\Delta\xi)[-\psi'(\nu_+)+\psi'(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}} + \frac{\Delta\xi[\psi''(\nu_+)+\psi''(\nu_-)]}{1-8\Delta\xi}\Bigg]\frac{z^6}{12} + \Bigg[\frac{3(1-4\Delta\xi)[-\psi'(\nu_+)+\psi'(\nu_-)]}{(1-8\Delta\xi)^{\frac{5}{2}}} + \frac{3(1-4\Delta\xi)[\psi''(\nu_+)+\psi''(\nu_-)]}{2(1-8\Delta\xi)^2} + \frac{\Delta\xi[-\psi'''(\nu_+)+\psi'''(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}}\Bigg]\frac{z^8}{96} + O(z^{10})\Bigg\}.$$

$$\Delta V'_{fermion} = \delta\xi R\varphi + \tfrac{1}{6}\delta\lambda\varphi^3 - fi[{}_iS_i](x;x)$$

- $z^2 \equiv \frac{f^2\varphi^2}{H^2}$

$\Delta V_{fermion}$(general form)

$$= -\frac{H^4}{8\pi^2}\Big\{2\gamma z^2 - [\zeta(3)-\gamma]z^4$$

$$+ 2\int_0^z dx(x+x^3)[\psi(1+ix)+\psi(1-ix)]\Big\}$$

$\Delta V_{fermion}$(large field)

$$= -\frac{H^4}{8\pi^2}\Big\{\tfrac{1}{2}z^4\ln(z^2+1) - \big[\zeta(3)+\tfrac{1}{4}-\gamma\big]z^4 + z^2\ln(z^2+1)$$

$$-\big[\tfrac{4}{3}-2\gamma\big]z^2 + \frac{11}{60}\ln(z^2+1) + O(z^0)\Big\}$$

$$\Delta V_{fermion}(\text{small field}) = -\frac{H^4}{8\pi^2}\Big\{\tfrac{2}{3}[\zeta(3)-\zeta(5)]z^6 - \tfrac{1}{2}[\zeta(5)-\zeta(7)]z^8 + O(z^{10})\Big\}$$

$$\Delta V'_{vector}(\varphi^*\varphi) = \delta\xi R + \tfrac{1}{2}\delta\lambda\varphi^*\varphi + e^2 g^{\mu\nu} i[{}_\mu\Delta_\nu](x;x)$$

- $z^2 \equiv \frac{e^2\varphi^*\varphi}{H^2}$
- General form →

$$\Delta V_{vector} = \frac{3H^4}{8\pi^2}\Big\{[2\gamma - 1]z^2 + \big[\gamma - \tfrac{3}{2}\big]z^4 + \int_0^{z^2} dx(1+x)\left[\psi\left(\frac{3}{2} + \frac{1}{2}\sqrt{1-8x}\right) + \psi\left(\frac{3}{2} - \frac{1}{2}\sqrt{1-8x}\right)\right]\Big\}$$

- Large field →

$$\Delta V_{vector} = \frac{3H^4}{8\pi^2}\left\{\frac{1}{2}z^4\ln(z^2+1) + \left[-\frac{7}{4} + \frac{1}{2}\ln(2) + \gamma\right]z^4 + z^2\ln(z^2+1) + \left[-\frac{13}{6} + \ln(2) + 2\gamma\right]z^2 + \frac{19}{60}\ln(z^2+1) + O(z^0)\right\}$$

- Small field →

$$\Delta V_{vector} = \frac{3H^4}{8\pi^2}\left\{\left[\frac{10}{3} - \frac{8}{3}\zeta(3)\right]z^6 + [12 - 10\zeta(3)]z^8 + O(z^{10})\right\}$$

Factors of ``H^2" are not constant

- Evaluate $\langle \Delta T_{\mu\nu} \rangle = -\frac{2}{\sqrt{-g}} \frac{\delta \Delta\Gamma[g]}{\delta g^{\mu\nu}}$ on de Sitter
  - $\Delta\Gamma[g] = -\int dx' \Delta V(g(x'))\sqrt{-g(x')}$
- If $H^2 = \frac{\Lambda}{3}$ ➔ $\langle \Delta T_{\mu\nu} \rangle_{dS} = -g_{\mu\nu} H^4 \times f(z^2)$
  - Recall $\Delta V = H^4 f(z^2)$ & $z^2 = \frac{cc^2|\varphi|^2}{H^2}$
- If $H^2 = \frac{R}{12}$ ➔ $\langle \Delta T_{\mu\nu} \rangle_{dS} = -g_{\mu\nu} H^4 \times \frac{1}{2} z^2 f'(z^2)$
- Actually consistent with ``H^2'' $= \frac{R}{12}$
  - Except for $\mu^{D-4} \rightarrow H^{D-4}$ in $\delta\xi$ and $\delta\lambda$
  - ΔV could be removed because $F(R)$ are ok

Factors of ``H^2'' are not $\frac{R}{12}$

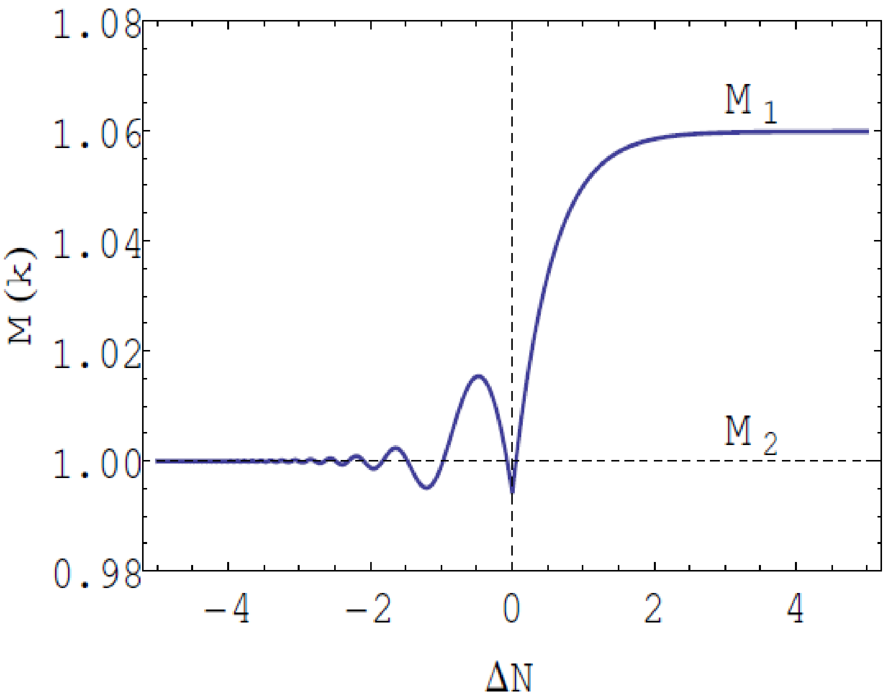
- Small $M^2 = cc^2|\varphi|^2$ relevant ➔ expand in M^2
  $i\Delta[\xi, M^2](x;x)$
  $$= i\Delta[\xi,0](x;x) - iM^2 \int d^D w \sqrt{-g(w)} [i\Delta[\xi,0](x;w)]^2 + \cdots$$
- $i\Delta[\xi, 0](x;x')$ is known for constant $\epsilon(t) \equiv -\frac{\dot{H}}{H^2}$
  - Indices ν depend on ϵ, e.g., $\nu = \frac{1}{2}\left(\frac{D-1-\epsilon}{1-\epsilon}\right)$
- Interpolating these factors of ``ϵ'' requires higher curvatures
  - E.g., $\frac{2}{2-\epsilon} = 1 + \sqrt{1 - \frac{6G}{R^2}}$
  - $G \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$
  - Functions of these are not allowed in classical actions

Factors of ``H^2“ are not even local!

- E.g., ``ringing’’ in $\Delta^2_{\mathcal{R}}(k)$
  - $\epsilon_1 = \frac{1}{200} \rightarrow \epsilon_2 = \frac{1}{10}$
  - Starobinsky 1992
- Good analytic form in arXiv:1507.07452

$$h(t,k) = \int_0^N dn \, \sin\Big[\int_n^N dn' \, \omega(n', N_k)\Big] \frac{\mathcal{S}(n, N_k)}{\omega(n, N_k)}$$

- This stuff also can’t be part of a classical action



How bad is it in practice?

- Could tune $V(\varphi)$ to cancel ΔV at one instant
  - But $H(t)$ typically changes a lot
- For $V(\varphi) = A\varphi^{\alpha}$ we have $\frac{H(t)}{H_i} = \left[\frac{\epsilon_i}{\epsilon(t)}\right]^{\frac{\alpha}{4}}$
- Could cancel with ``H^2'' → $\frac{R}{12}$
  - But then a modified gravity theory
  - E.g., Friedman equation changes
- Need a careful numerical study

Conclusions

- $V(\varphi)$ models need heavy fine tuning
- Reheating requires couplings to light matter
- These induce corrections ΔV to $V(\varphi)$
  - Not Planck-suppressed
  - Can't compute for general $\epsilon(t) = -\frac{\dot{H}}{H^2}$
  - $\epsilon = 0$ ➔ $\Delta V = H^4 f\left(\frac{cc^2|\varphi|^2}{H^2}\right)$
- Factors of ``H^2'' are not constant, or $\frac{R}{12}$, or even local

  ➔ cannot remove ΔV by more fine tuning of $V(\varphi)$
- This looks bad
  - At the least, a new constraint on model-building