# Fakeons, Lee-Wick Models And Quantum Gravity

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## PAPERS

### *** Reformulation of the Lee-Wick models as nonanalytically Wick rotated Euclidean theories ***

D. A. and M. Piva, **A new formulation of Lee-Wick quantum field theory**,
J. High Energy Phys. 06 (2017) 066 and arXiv:1703.04584 [hep-th]

D. A. and M. Piva, **Perturbative unitarity of Lee-Wick quantum field theory**,
Phys. Rev. D 96 (2017) 045009 and arXiv:1703.05563 [hep-th]

### *** Fakeons ***

D. A., **Fakeons and Lee-Wick models**,
J. High Energy Phys. 02 (2018) 141 and arXiv:1801.00915 [hep-th]

### *** Fakeons and quantum gravity ***

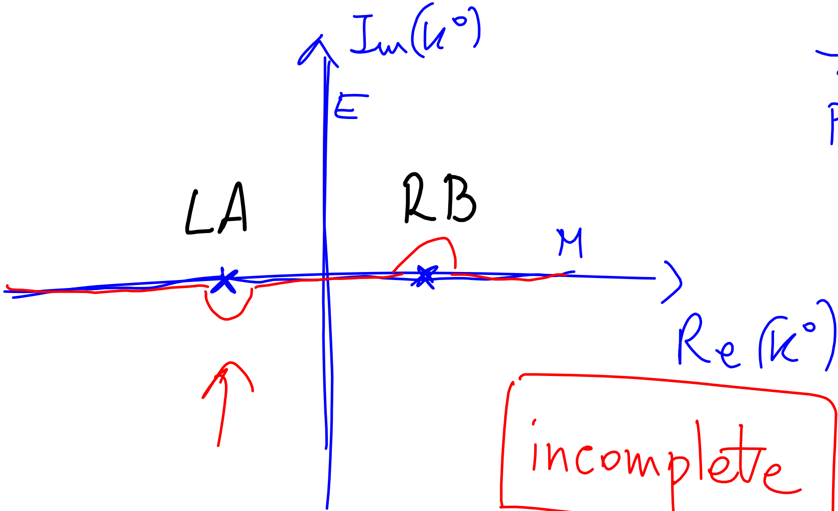
D. A., **On the quantum field theory of the gravitational interactions**,
J. High Energy Phys. 06 (2017) 086 and arXiv:1704.07728 [hep-th]

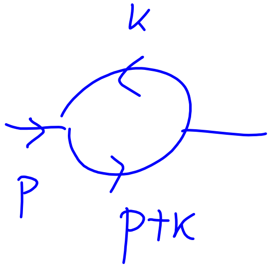
D. A. and M. Piva, **The ultraviolet behavior of quantum gravity**,
J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

### *** Inconsistency of Minkoswky HD theories ***

U.G. Aglietti and D. A., **Inconsistency of Minkowski higher-derivative theories**,
Eur. Phys. J. C 77 (2017) 84 and arXiv:1612.06510 [hepth]

$$S(k) = \frac{1}{k^2 - m^2} \qquad k^0 \in \mathbb{C} \qquad \vec{k} \in \mathbb{R}^3$$




$$\int dk^0 \int d^3\vec{k} \; S(k)\, S(p+k)$$

ok ?

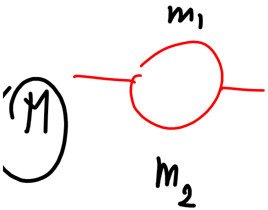
$$\int d^3\vec{k} \; f(k,p)$$

incomplete

Feynman : $\dfrac{1}{k^2 - m^2 + i\epsilon}$

$D=2$

$k = (k^0, k_x)$

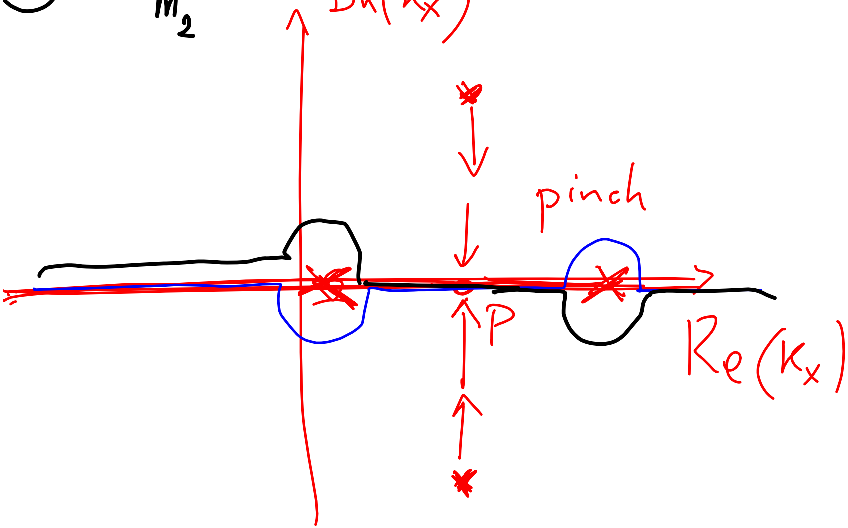



$\int dk^0$ ok

$$\int_{\mathbb{R}} dk_x \, f(k_x, p) = I(p)$$

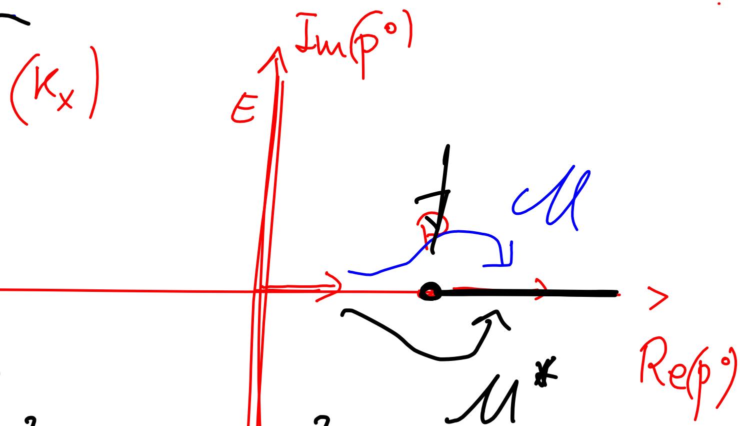
$k_x \in \mathbb{C}$

$p_x$ fixed




dis $\mathcal{M}$

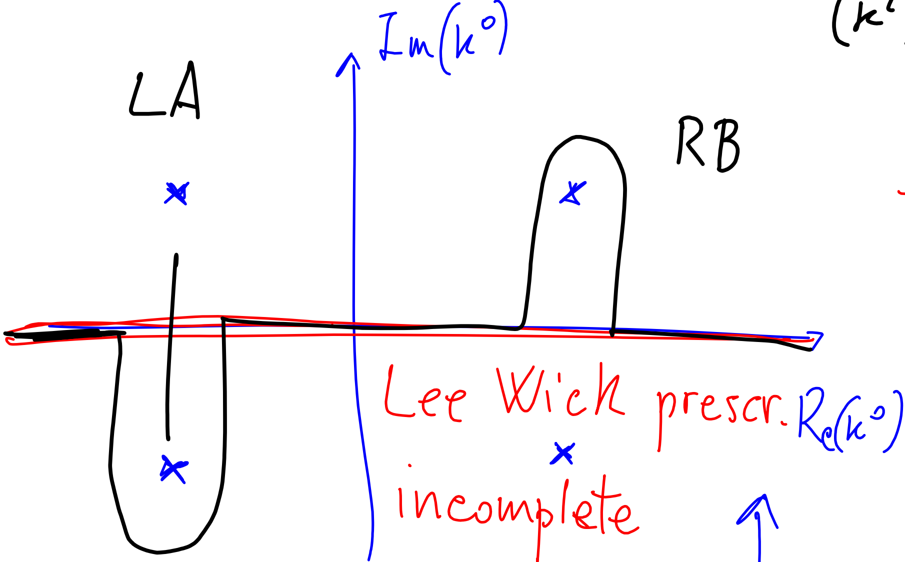
threshold $P$

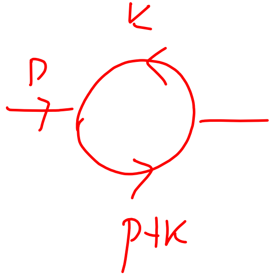



$$\underline{p^2 = (m_1+m_2)^2}$$

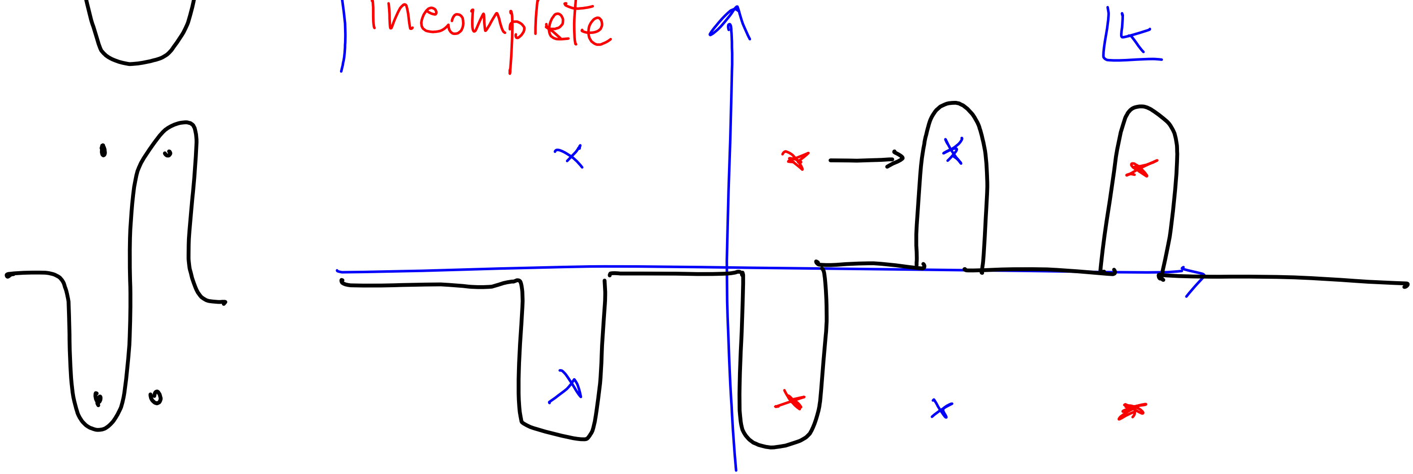
LW models

$$S(k) = \frac{1}{(k^2)^2 + M^4} = \frac{1}{(k^2 + iM^2)(k^2 - iM^2)}$$



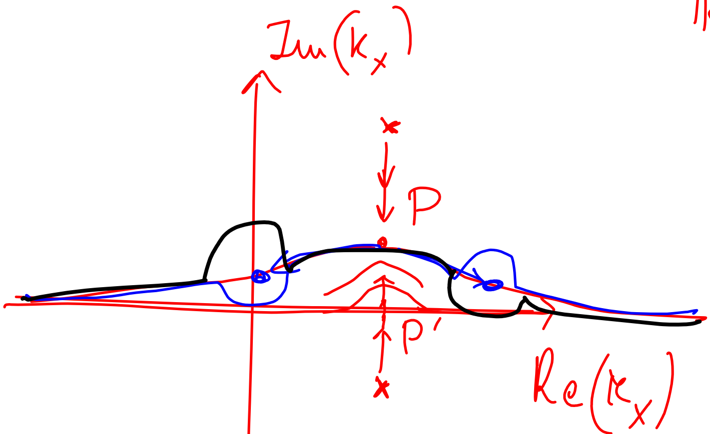

$$= \int_{\mathbb{R}} dk^0 \int_{\mathbb{R}^3} d^3k \cdot S(k)\, S(p+k)$$

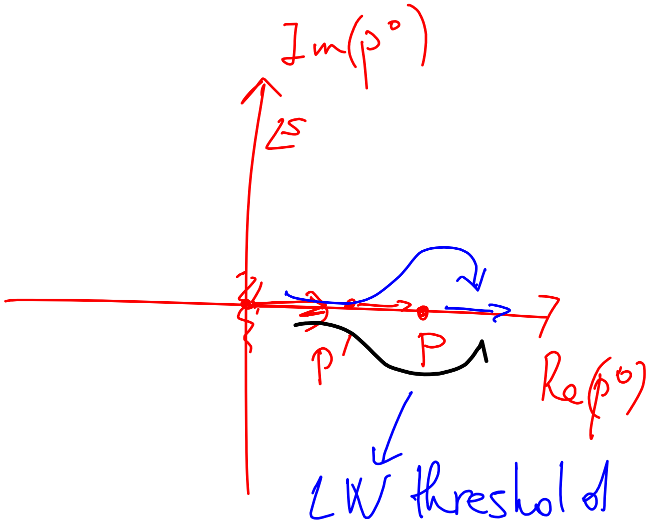


$D=2$

$$\underset{\text{LW ok}}{\int dk_0} \; \underset{\mathbb{R}\,?}{\int dk_x} \; S(k)\, S(p+k) =$$

$$\longrightarrow \int_{\mathbb{R}\,?} dk_x \; \underline{f(k_x, p)} = I(p)$$

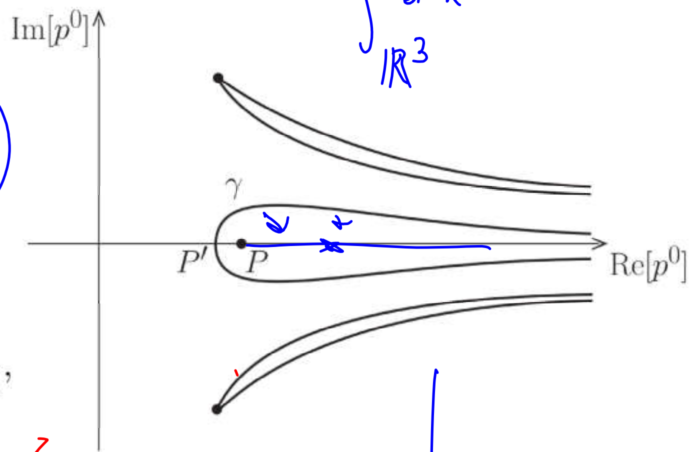


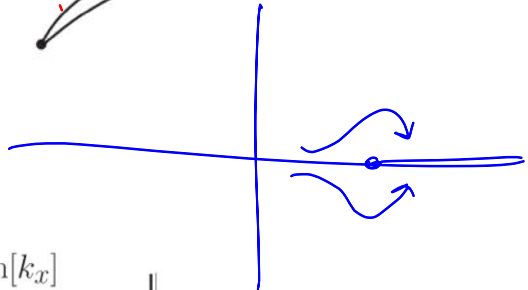



$$\mathcal{B}(p) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} S(k, m_1) S(k-p, m_2),$$

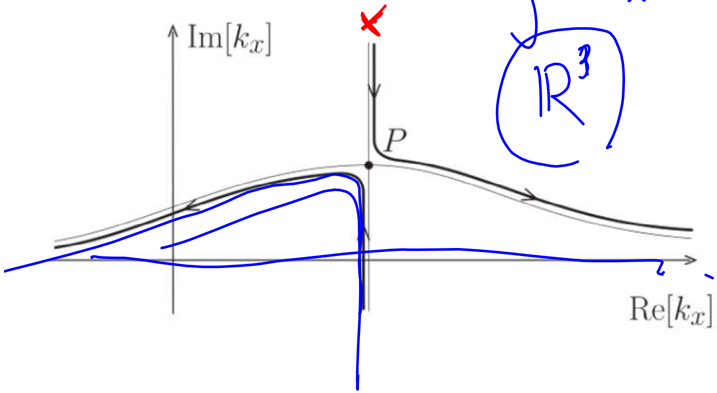
$$S(p, m) = \cancel{\frac{1}{p^2 - m^2 + i\epsilon}} \frac{M^4}{(p^2 - \cancel{\mu^2})^2 + M^4},$$

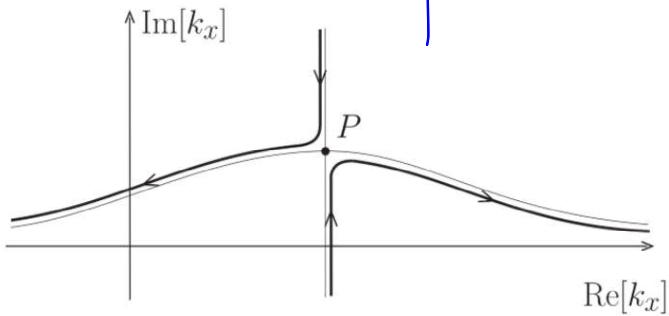
$\int_{\mathbb{R}^3} d^3\vec{k}$



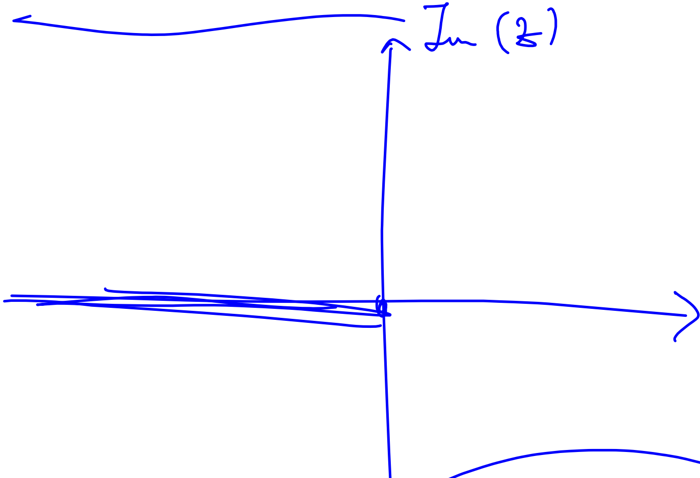
$\int_{\mathbb{R}^3} d^3 k_x$

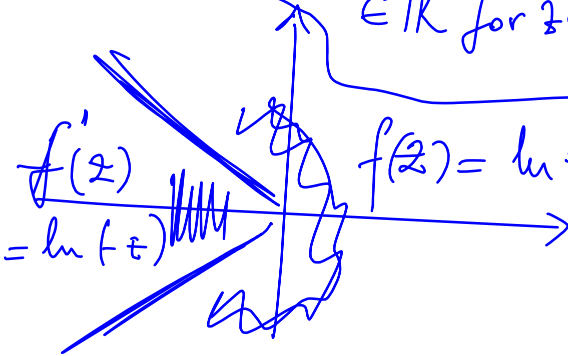


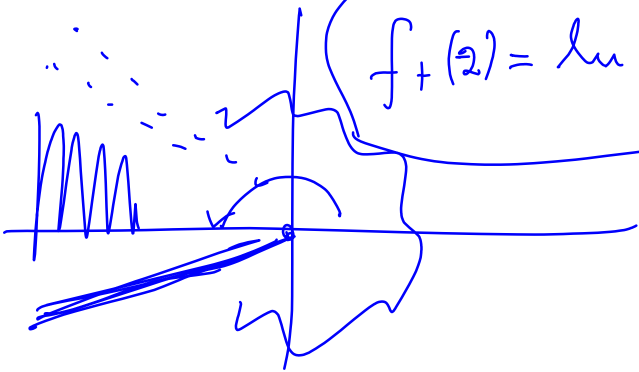
# AV continuation $\quad \ln(z)$

$$\boxed{f_{AV}(z) = \frac{1}{2}\ln z^2 \in \mathbb{R} \text{ for } z \in \mathbb{R}}$$

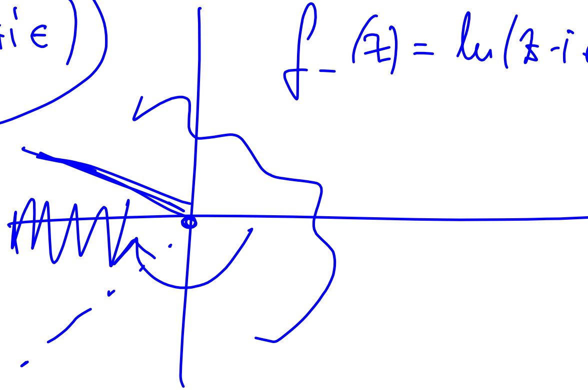


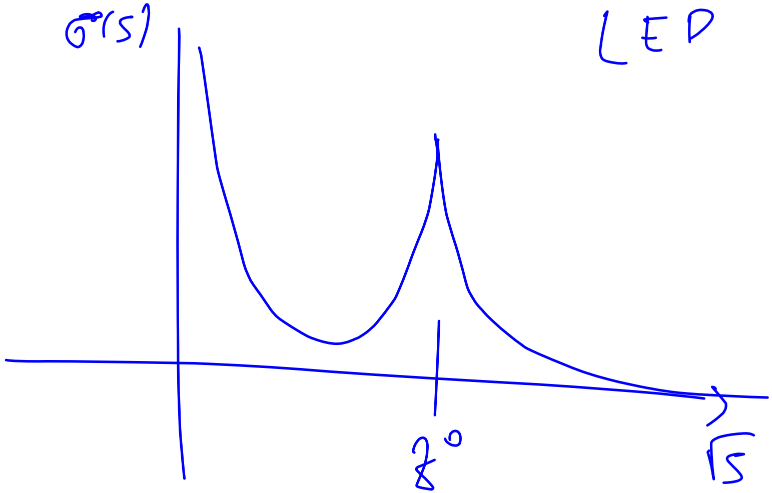




$f_+(z) = \ln(z + i\epsilon)$

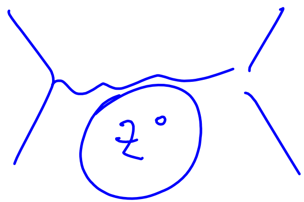


$f_-(z) = \ln(z - i\epsilon)$

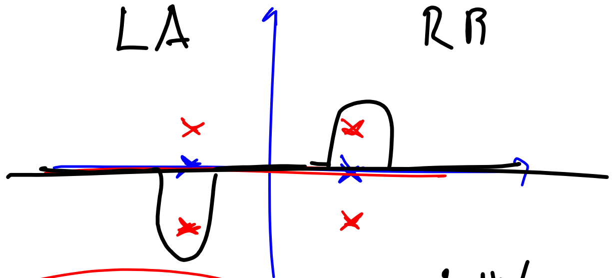





LEP




+ domain deform




Export the idea

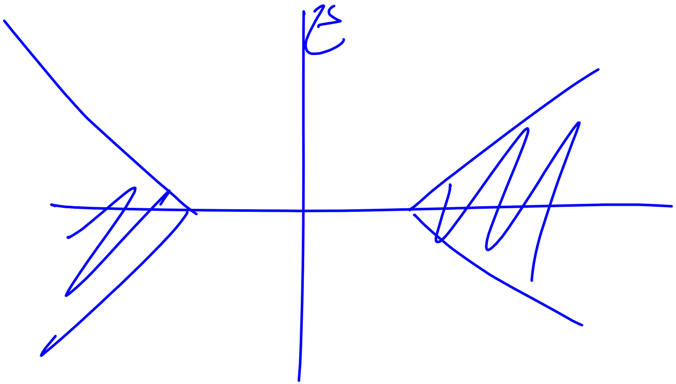
$$\frac{1}{p^2} = \frac{p^2}{(p^2)^2} = \lim_{\varepsilon \to 0} \frac{p^2}{(p^2)^2 + \varepsilon^4}$$

LW

double pole

$$= \lim_{\epsilon \to 0} \frac{1}{2} \left( \frac{1}{p^2 + i\varepsilon^2} + \frac{1}{p^2 - i\varepsilon^2} \right)$$

$$S_{\mathrm{HD}} = -\frac{\mu^{-\varepsilon}}{2\kappa^2}\int\sqrt{-g}\left[\cancel{2\Lambda_C} + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right]$$




Reno ok in ε

$$\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_0 = \frac{i\mathcal{I}_{\mu\nu\rho\sigma}}{2p^2(\zeta - \alpha p^2)} + \frac{i(\alpha-\xi)\varpi_{\mu\nu}\varpi_{\rho\sigma}}{6(p^2)^2(\zeta-\alpha p^2)(\zeta-\xi p^2)}$$

$p^2 = 0$

$$\mathcal{I}_{\mu\nu\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}, \qquad \varpi_{\mu\nu} = p^2\eta_{\mu\nu} + 2p_\mu p_\nu.$$

GRAVIT / FAKEON PRESCR.

(a) replace $p^2$ with $p^2 + i\epsilon$ everywhere in the denominators of the propagators;

(b) turn the massive poles into fakeons by means of the replacement

$$\frac{1}{\zeta - up^2} \to \frac{\zeta - up^2}{(\zeta - u(p^2+i\epsilon))^2 + \mathcal{E}^4},$$

$\mathcal{E} \to 0$

LW

where $u$ is equal to $\alpha$ or $\xi$.

~ AV continuation

$$\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_0 = \langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_{0\text{grav}} + \langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_{0\text{fake}}$$

$$\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_{0\text{grav}} = \frac{i}{2\zeta(p^2+i\epsilon)}\left[\mathcal{I}_{\mu\nu\rho\sigma} + \frac{(\alpha-\xi)\varpi_{\mu\nu}\varpi_{\rho\sigma}}{3\zeta^2}\left(\frac{\zeta}{p^2+i\epsilon}+\alpha+\xi\right)\right],$$

$$\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_{0\text{fake}} = \frac{i\alpha\mathcal{I}_{\mu\nu\rho\sigma}(\zeta-\alpha p^2)}{2\zeta[(\zeta-\alpha(p^2+i\epsilon))^2+\mathcal{E}^4]} + \frac{i\varpi_{\mu\nu}\varpi_{\rho\sigma}}{6\zeta^3}\left(\frac{\alpha^3(\zeta-\alpha p^2)}{(\zeta-\alpha(p^2+i\epsilon))^2+\mathcal{E}^4} - \frac{\xi^3(\zeta-\xi p^2)}{(\zeta-\xi(p^2+i\epsilon))^2+\mathcal{E}^4}\right)$$

At $E \gg m_{\text{fakeon}}$

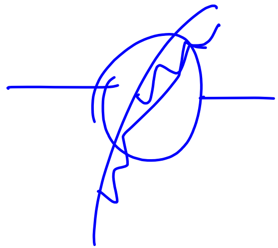
$$\Gamma_{\rm abs} = \frac{i\mu^{-\varepsilon}}{16\pi} \int \sqrt{-g} \left[ c \left( R_{\mu\nu} \theta(-\Box_c) R^{\mu\nu} - \frac{1}{3} R \theta(-\Box_c) R \right) + \frac{N_s \eta^2}{36} R \theta(-\Box_c) R \right]$$
$$- \int \frac{\delta S_{\rm HD}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu},$$

$$c = \frac{1}{120}(N_s + 6N_f + 12N_v)$$

$$S_s = \frac{1}{2} \sum_{i=1}^{N_s} \int \sqrt{-g} \left[ g^{\mu\nu} (\partial_\mu \varphi^i)(\partial_\nu \varphi^i) + \frac{1}{6}(1 + 2\eta) R \varphi^{i2} \right]$$

# PRICE TO PAY

- Renorm
- $\boxed{\text{Unitarity}}$



Violations of microcausality

$\nexists \quad H =$ posit. $-$ fakeons

$Q \quad Q^{\dagger}$

$$\Box(\alpha+\beta\Box)\varphi = J$$

cf. fakeon

$$\Box\varphi = \frac{1}{\alpha+\beta\Box} J = \langle J \rangle$$

Green

Projection:

analiticity in $\beta$

$\alpha \neq 0$

$\sim$ LOW En. exp.