Distortion of Cosmological Evolution in R^2-gravity

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based on common work with A. Dolgov and R. Singh

arXiv:1803.01722

Hot Topics in Modern Cosmology

Spontaneous Workshop XII

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General Relativity (GR):

$$S_{EH} = -\frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\,R$$

describes basic properties of the universe in very good agreement with observations.

- $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass

Beyond the frameworks of GR:

$$S_F = -\frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\,[R + F(R)]$$

$F(R) = -R^2/(6m^2)$:

- was suggested by V.Ts. Gurovich and A.A. Starobinsky for elimination of cosmological singularity (JETP **50** (1979) 844).
- It was found that the the addition of the R^2-term leads to inflationary cosmology. (A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980))

As in any cosmological scenario the problem of graceful exit from inflation and the problem of the universe heating are of primary importance.

- Review: A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010)

We generalize and extend the analysis of our earlier paper

- ADR, "Cosmological evolution in R^2 gravity," JCAP **1202** (2012) 049.

starting from the inflationary stage till to large time: $mt \gg 1$.

I. Cosmological Equations in R^2-theory

- The term describing particle production is included as a source into equation for the energy density evolution.
- Modified EoM are rewritten in a convenient dimensionless form and solved numerically and analytically.

II. Solution *ab ovo* to $\Gamma t \lesssim 1$

1. Solution at inflationary epoch.
2. Numerical solutions at post-inflationary epoch
3. Asymptotic solution at $\tau \gg 1$ and $w = 1/3$ (RD)
4. Asymptotic solution at $\tau \gg 1$ and $w = 0$ (MD)
5. Energy influx to cosmological plasma from the scalaron decay

III. Solution at $\Gamma t \gtrsim 1$

II. Solution from inflation to $\Gamma t \lesssim 1$

One can find very simple analytical expressions for $H(t)$ and $R(t)$.

A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980); A. De Felice and S. Tsujikawa Living Rev. Rel. 13, 3 (2010); arXiv:1002.4928.

The system of the cosmological equations (in absence of the usual matter) was transformed into a single first order non-linear equation (Starobinsky)

ADS: The second order equation for R, the covariant law of conservation of the matter energy density, and the "kinematical" relation between R and the Hubble parameter in spatially flat universe are employed.

- We keep the matter effects from the very beginning.
- We found numerically that the onset of the simple asymptotic behavior of $R(t)$ and $H(t)$ started almost immediately after inflation.
- We have calculated the energy density of the usual matter, which drops down as $1/t$ with some weak superimposed oscillation.
- When $\Gamma t < 1$ the usual matter has very weak impact on the cosmological expansion which is determined by the oscillating R.
- During this time the universe evolution was quite different from the GR one.

I. Cosmological Equations in R^2-theory

II. Solution *ab ovo* to $\Gamma t \lesssim 1$, but $mt \gg 1$

III. Solution at $\Gamma t \gtrsim 1$

- We study the approach to the usual GR cosmology.
- GR is recovered when the energy density of matter becomes larger than that of the exponentially decaying scalaron.
- We argue that the approach is somewhat delayed. It takes place not at $\Gamma t \sim 1$, as it may be naively expected, but at $\Gamma t \sim \ln(m/\Gamma)$.

IV. Conclusions

A rather long regime during which the cosmological evolution differs from the standard FLRW cosmology could lead, in particular, to modification of high temperature baryogenesis scenarios, to a variation of the frozen abundances of heavy dark matter particles, and to necessity of reconsideration of the formation of primordial black holes.

I. Cosmological Equations in R^2-theory

Let us consider the theory described by the action:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{R^2}{6m^2} \right) + S_m$$

- m is a constant parameter with dimension of mass

The modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{3m^2} \left(R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} + g_{\mu\nu} D^2 - D_\mu D_\nu \right) R = \frac{8\pi}{m_{Pl}^2} T_{\mu\nu}$$

- $D^2 \equiv g^{\mu\nu} D_\mu D_\nu$ is the covariant D'Alembert operator.

The energy-momentum tensor of matter $T_{\mu\nu}$

$$T^\mu_\nu = diag(\varrho, -P, -P, -P)$$

where ϱ is the energy density, P is the pressure of matter.

The matter distribution is homogeneous and isotropic

$$P = w\varrho$$

- non-relativistic: $w = 0$, relativistic: $w = 1/3$, vacuum-like: $w = -1$

FRW: $ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2\right]$, $H = \dot{a}/a$

The curvature scalar:

$$R = -6\dot{H} - 12H^2$$

The covariant conservation condition $D_\mu T^\mu_\nu = 0$:

$$\dot{\varrho} = -3H(\varrho + P) = -3H(1+w)\varrho$$

Trace equation:

$$D^2 R + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}\, T^\mu_\mu$$

For homogeneous field, $R = R(t)$, and with $P = w\varrho$:

$$\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1-3w)\varrho$$

This is the Klein-Gordon (KG) type equation for massive scalar field R, which is sometimes called "scalaron". It differs from KG by the liquid friction term.

$$\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1 - 3w)\varrho$$

This equation:

- does not include the effects of particle production by the curvature scalar;
- is a good approximation at inflationary epoch, when particle production by $R(t)$ is practically absent, because R is large and friction is large, so R slowly evolves down to zero.

At some stage, when $\boldsymbol{H}$ becomes smaller than $\boldsymbol{m}$, $\boldsymbol{R}$ starts to oscillate efficiently producing particles.

- It commemorates the end of inflation, the heating of the universe, which was originally void of matter, and the transition from the accelerated expansion (inflation) to a de-accelerated one.
- The latter resembles the usual Friedmann matter dominated expansion regime but differs in many essential features.

For the harmonic potential the particle production can be approximately described by an additional friction term $\Gamma \dot{\boldsymbol{R}}$.

An account of Particle Production

- The effects of particle production by a scalar field for an arbitrary potential in KG equation in one loop approximation: A. Dolgov, S. Hansen, Nucl. Phys. **B548** (1999) 408-426.
- The case of particle production by the curvature scalar: EA, A. D. Dolgov and L. Reverberi, JCAP **1202** (2012) 049.
- Generally the one-loop effects on the particle production lead to non-local in time integro-differential equation.
- In the case of strictly harmonic oscillations the equation can be reduced to a simple differential equation with the liquid friction term $\Gamma \dot{R}$.

In the considered case the potential is harmonic and we can use the friction term approximation. The particle production rate:

$$\Gamma = \frac{m^3}{48 m_{Pl}^2}$$

- Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977); A. A. Starobinsky, Quantum Gravity, eds. M. A. Markov, P. C. West, Plenum Publ. Co., New York, 1982, pp. 103-128; A. Vilenkin, Phys. Rev. **D32**, 2511 (1985); EA, A. Dolgov, L. Reverberi, JCAP **1202** (2012) 049.

Equations with an account of Particle Production

Equation for R acquires an additional friction term:

$$\ddot{R} + (3H + \Gamma)\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1 - 3w)\varrho$$

Particle production leads to an emergence of the source term in Eq. for ϱ:

$$\dot{\varrho} = -3H(1 + w)\varrho + \frac{mR_{ampl}^2}{1152\pi}$$

where R_{ampl} is the amplitude of $R(t)$-oscillations.

- The state of the cosmological matter depends not only upon the spectrum of the decay products but also on the thermal history of the produced particles.
- Depending on that, the parameter w may be not exactly equal to 0 or 1/3. The equation of state can be not simple $P = w\varrho$ with constant w.
- Two limiting values $w = 0$ and $1/3$ are possible simple examples.
- Different values of w would not change the presented results significantly.

Dimensionless Equations

Dimensionless time variable and dimensionless functions

$$\tau = tm, \quad H = mh, \quad R = m^2 r, \quad \varrho = m^4 y, \quad \Gamma = m\gamma.$$

The system of dimensionless equations

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\, y = S[r]$$

- prime denotes derivative over τ, $\mu = m/m_{Pl}$, $\gamma = \mu^2/48$

The source term is taken as:

$$S[r] = \frac{\langle r^2 \rangle}{1152\pi}.$$

- $\langle r^2 \rangle$ means amplitude squared of harmonic oscillations, r^2_{ampl}, of the dimensionless curvature $r(\tau)$.
- For inharmonic oscillations we approximate $\langle r^2 \rangle$ as $2(r')^2$ or $(-2r''r)$.

Source term $S[r] = \langle r^2 \rangle / (1152\pi)$

The function $\langle r^2 \rangle$ slowly changes with time.

- Such a form for the description of the particle creation is true only during the epoch when $r(\tau)$ is a harmonically oscillating function with slowly varying amplitude.
- So it is surely inapplicable during inflation.
- In principle we can switch on this source only after inflation is over.

However, the ultimate result for y (or ϱ) does not depend on the history of the particle production. The reasons:

- During inflation the energy density of the normal matter very quickly red-shifted away and we arrive to the moment of the universe heating with essentially the same, vanishingly small, value of y (or ϱ).
- In other words, initial condition for the energy density of matter at the onset of the particle production is always $y = 0$ ($\varrho = 0$).

We show numerically that this is indeed true with very high precision.

II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Solution at inflationary epoch

Solutions at inflationary epoch: from the "very beginning" up to high τ ($\tau \gg 1$), but small $\gamma\tau \lesssim 1$

The initial conditions should be chosen in such a way that at least 70 e-foldings during inflation are ensured:

$$N_e = \int_0^{\tau_{inf}} h\, d\tau \geq 70$$

- τ_{inf} is the moment when inflation terminated.
- This can be achieved if the initial value of r is sufficiently large, practically independently on the initial values of h and y.

Solutions at inflationary epoch:

from the "very beginning" up to high τ ($\tau \gg 1$), but small $\gamma\tau \lesssim 1$

We can roughly estimate the duration of inflation neglecting higher derivatives in equations for h and r.

- A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980); A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010); arXiv:1002.4928;
- A. S. Koshelev et al., 2016, 2017,...

Simplified system to estimate the duration of inflation ($\boldsymbol{y = 0}$, $\gamma \ll \mathbf{1}$):

$$\boldsymbol{h^2 = -r/12, \quad 3hr' = -r}$$

Solutions:

$$\sqrt{-r(\tau)} = \sqrt{-r_0} - \tau/\sqrt{3}, \;\; h(\tau) = (\sqrt{-3r_0} - \tau)/6, \;\; r_0 = r(\tau = 0)$$

The duration of inflation is roughly determined by the condition $\boldsymbol{h = 0}$, i.e.

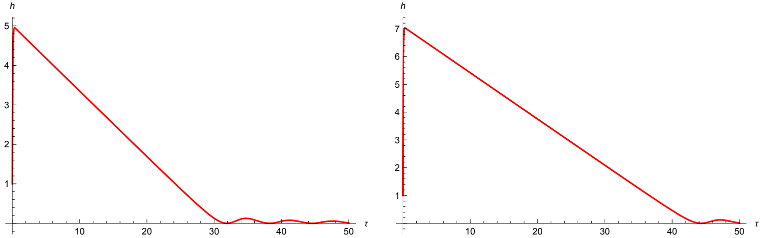
$$\tau_{inf} = \sqrt{-3r_0} \;\Rightarrow\; N_e \approx r_0/4$$

.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Numerical solutions: Evolution of $\boldsymbol{h(\tau)}$ at the inflationary stage



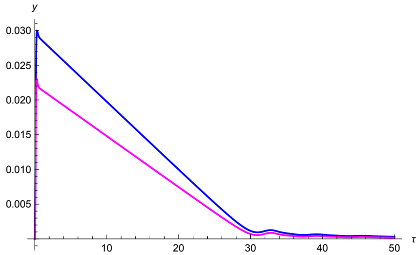
- Initial values of dimensionless curvature $r_0 = 300$ (left) and $r_0 = 600$ (right).
- Initially $h_{in} = 0$, but it quickly reaches the value $h(0) = \sqrt{-r_0/12}$.
- The numbers of e-foldings: $\boldsymbol{N_e \approx r_0/4 = 75}$ (left) and **150** (right).

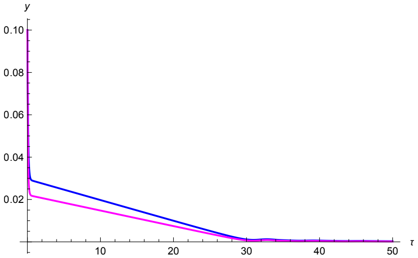
An excellent agreement with numerical solutions demonstrates high precision of the slow roll approximation and weak impact of particle production at (quasi)inflationary stage.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless energy density of matter $\boldsymbol{y(\tau)}$ during inflation for $w = 0$ (blue) and $w = 1/3$ (magenta).





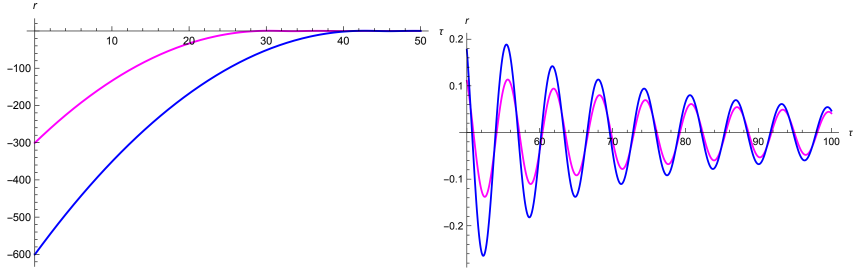
- *Left panel*: initially $y_{in} = 0$. *Right panel*: $y_{in} = 0.1$.

The initial fast rise of ϱ from zero during short time is generated by the $\boldsymbol{S[r]}$-term. The results are not sensitive to the form of $\boldsymbol{S[r]}$.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless curvature scalar $\boldsymbol{r(\tau)}$ for $\boldsymbol{r_{in} = -300}$ (magenta) and $\boldsymbol{r_{in} = -600}$ (blue)



- *Left panel*: evolution during inflation.
- *Right panel*: evolution after the end of inflation, the curvature scalar starts to oscillate.

II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Solution at inflationary epoch
- Solution at post-inflationary epoch

Numerical solutions at post-inflationary epoch

The behavior of $\boldsymbol{R}$, $\boldsymbol{H}$ and ϱ, or dimensionless quantities $\boldsymbol{r}$, $\boldsymbol{h}$, and $\boldsymbol{y}$ is drastically different at the vacuum-like dominated stage (inflation) and during scalaron dominated stage.

Now we will find the laws of evolution of $\boldsymbol{r}(\tau)$, $\boldsymbol{h}(\tau)$, and $\boldsymbol{y}(\tau)$ after inflation till $\gamma\tau \sim 1$.

The numerical solutions will be presented from the end of inflation to large $\tau \gg 1$, but not too large because the numerical procedure for huge $\tau \sim 1/\gamma$ becomes unstable.

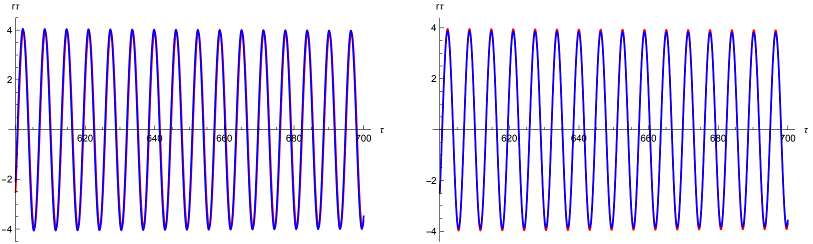
However, we can find pretty accurate analytical solution, asymptotically valid at any large τ up to $\tau \sim 1/\gamma$.

Very good agreement between numerical and analytical solutions at large but not huge τ allows to trust asymptotic analytical solution at huge τ.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1, \;\; \gamma = \mu^2/48$$

Evolution of the curvature scalar, $\boldsymbol{\tau r(\tau)}$, in post-infationary epoch.



Left panel ($\boldsymbol{w = 1/3}$) : initially $\boldsymbol{r_{in} = -300}$ (red), $\boldsymbol{r_{in} = -600}$ (blue). There is absolutely no difference between the curves.

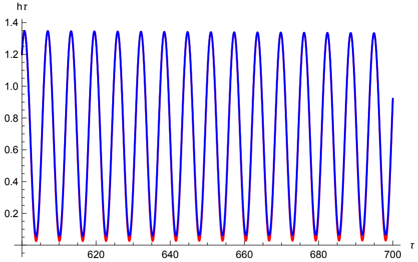
Right panel ($\boldsymbol{r_{in} = -300}$): $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue). The difference is minuscule.

The source term here is taken as $\boldsymbol{S[r] = (r')^2/1152\pi}$. The results are not sensitive to its form.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1$$

Evolution of the Hubble parameter, $\boldsymbol{h\tau}$, in post-inflationary epoch for $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)



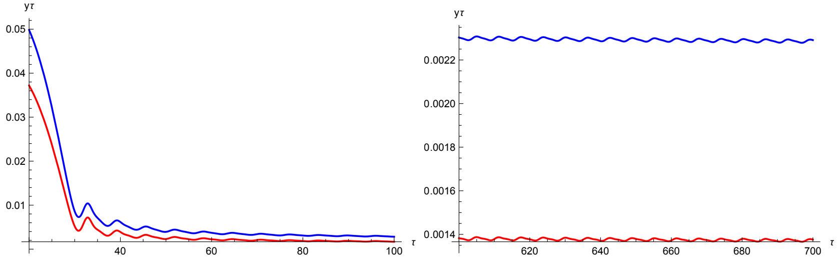
- The dependence on $\boldsymbol{w}$ is very weak, except for small values of $\boldsymbol{h}$.

If $\boldsymbol{h}$ is very close to zero, it may become negative because of numerical error due to insufficient precision.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Energy density of matter as a function of time for
$\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)



- Evolution of $\boldsymbol{y\tau}$ at small $\boldsymbol{\tau}$ (left) and at large $\boldsymbol{\tau}$ (right).

The magnitude of $\boldsymbol{\varrho}$ for these two values of $\boldsymbol{w}$ are noticeably different in contrast to other relevant quantities, $\boldsymbol{r}$ and $\boldsymbol{h}$, which very weakly depend upon $\boldsymbol{w}$.
The product $\boldsymbol{y\tau}$ tends to a constant value with rising $\boldsymbol{\tau}$ till $\boldsymbol{\gamma\tau}$ remains small.
This behavior much differs from the standard matter density evolution $\boldsymbol{\varrho \sim 1/t^2}$.

Asymptotic solution at $\tau \gg 1$, $\gamma\tau \lesssim 1$ and $w = 1/3$

Simple form of numerical solutions at large τ:

- r oscillates with the amplitude decreasing as $1/\tau$ around zero
- h also oscillates almost touching zero with the amplitude also decreasing as $1/\tau$ around some constant value close to 2/3.

In the case $w = 1/3$ we have the system of equations

$$h' + 2h^2 = -r/6, \qquad (1)$$

$$r'' + 3hr' + r = 0, \qquad (2)$$

$$y' + 4h\, y = \frac{< r^2 >}{1152\pi},$$

We search for the asymptotic expansion of h and r at $\tau \gg 1$ in the form:

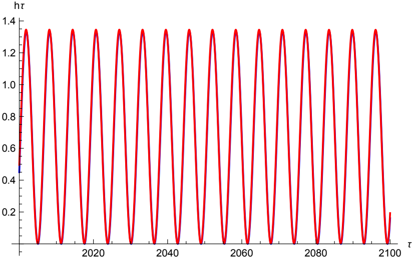
$$r = \frac{r_1 \cos(\tau + \theta_r)}{\tau} + \frac{r_2}{\tau^2}, \quad h = \frac{h_0 + h_1 \sin(\tau + \theta_h)}{\tau}$$

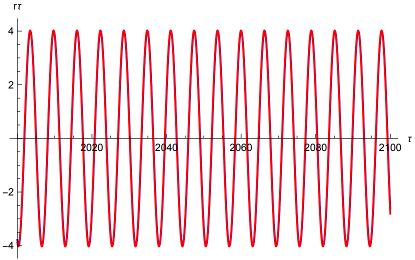
- r_j and h_j are some constant coefficients to be calculated from Eqs.(1)-(2)
- the constant phases θ_j are determined through the initial conditions and will be adjusted by the best fit of the asymptotic solution to the numerical one

Finally we find:

$$h = \frac{2}{3\tau}\left[1 + \sin(\tau + \theta)\right], \quad r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

Comparison of numerical calculations with analytical estimates for the adjusted "by hand" phase $\theta = -2.9\pi/4$





- *Left panel*: comparison of numerical solution for $h\tau$ (red) with analytic estimate (blue).
- *Right panel*: the same for numerically calculated $r\tau$.

The difference between the red and blue curves is not observable.

Equation for energy density: $y' + 4h\, y = < r^2 > /(1152\pi)$

$\langle r^2 \rangle = 16/\tau^2$ is the square of the amplitude of the harmonic oscillations.

Analytical integration gives:

$$y(\tau) = \frac{1}{72\pi} \int_{\tau_0}^{\tau} \frac{d\tau_2}{\tau_2^2} \exp\left[-4\int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

- $\tau_0 \ll \tau$ is some initial value of the dimensionless time.

Taking asymptotical $h(\tau)$ we can partly perform integration over $d\tau_1$ as

$$\int_{\tau_2}^{\tau} d\tau_1 h(\tau_1) = \frac{2}{3}\ln\frac{\tau}{\tau_2} + \int_{\tau_2}^{\tau} \frac{d\tau_1}{\tau_1} \sin(\tau_1 + \theta)$$

It is convenient to introduce new integration variables:

$$\eta_1 = \tau_1/\tau, \quad \eta_2 = \tau_2/\tau$$

In terms of these variables we lastly obtain:

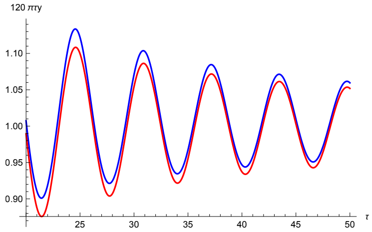
$$y(\tau) = \frac{1}{72\pi\tau} \int_{\eta_0}^{1} d\eta_2\, \eta_2^{2/3} \exp\left[-\frac{8}{3}\int_{\eta_2}^{1} \frac{d\eta_1}{\eta_1} \sin(\tau\eta_1 + \theta)\right]$$

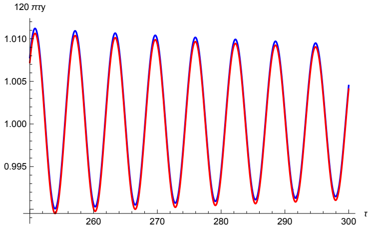
- The integral in the exponent is small, so the exponential factor is close to unity

Asymptotic behavior of the solution for $w = 1/3$

$$y_{1/3} = \frac{1}{120\pi\tau} + \frac{1}{45\pi}\frac{\cos(\tau+\theta)}{\tau^2} - \frac{1}{27\pi\tau^2}\int_\epsilon^1 \frac{d\eta_2}{\eta_2^{1/3}}\cos(\tau\eta_2+\theta)$$

- the subindex (1/3) indicates that $w = 1/3$
- $\epsilon = \tau_0/\tau \ll 1$. The last integral is proportional to $1/\tau^{2/3}$ and is subdominant.





Comparison of the integral solution

$$y(\tau) = \frac{1}{72\pi}\int_{\tau_0}^{\tau}\frac{d\tau_2}{\tau_2^2}\exp\left[-4\int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

for the dimensionless energy density $120\pi\tau\, y(\tau)$ (blue) with the asymptotic expression $120\pi\tau\, y_{1/3}(\tau)$ (red) for moderately large τ (*left panel*) and very large τ (*right panel*).

Asymptotic solution at $\tau \gg 1$, $\gamma\tau \lesssim 1$ and $w = 0$

For $w = 0$ equations take the form

$$h' + 2h^2 = -r/6$$
$$r'' + 3hr' + r = -8\pi\mu^2 y$$
$$y' + 3h\,y = S[r]$$

$\mu \ll 1 \Rightarrow$ the impact of the r.h.s. in Eq. for r is not essential $\Rightarrow$ we can use:

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right], \quad r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

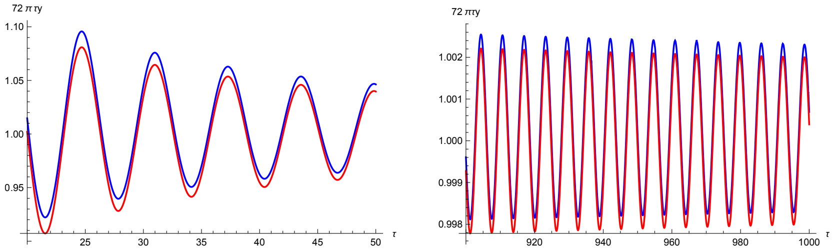
The only essential difference with the $w = 1/3$ case arises in the equation governing the evolution of the energy density, $y(\tau)$.

There appears coefficient (-3) in the exponent, instead of (-4):

$$y(\tau) = \frac{1}{72\pi}\int_{\tau_0}^{\tau}\frac{d\tau_2}{\tau_2^2}\exp\left[-3\int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

Asymptotic behavior of the solution for $w = 0$

$$y_0 = \frac{1}{72\pi\tau} + \frac{\cos(\tau+\theta)}{36\pi\tau^2}$$



Comparison of the integral solution

$$y(\tau) = \frac{1}{72\pi}\int_{\tau_0}^{\tau}\frac{d\tau_2}{\tau_2^2}\exp\left[-3\int_{\tau_2}^{\tau}d\tau_1 h(\tau_1)\right]$$

for the dimensionless energy density $72\pi\tau\, y(\tau)$ (blue) with the asymptotic expression $72\pi\tau\, y_0(\tau)$ (red) for moderately large τ (*left panel*) and very large τ (*right panel*).

II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Energy influx to cosmological plasma from the scalaron decay

Energy influx to cosmological plasma from the scalaron decay

Energy conservation demands equality of the energy influx induced by

$$S[r] = \frac{\langle r^2 \rangle}{1152\pi}$$

to the loss of scalaron energy density due to its decay with the width $\Gamma = m^3/(48 m_{Pl}^2)$.

To check that let us consider a simplified model:

$$A_R = \frac{m_{Pl}^2}{48\pi m^4} \int d^4x \sqrt{-g} \left[\frac{(DR)^2}{2} - \frac{m^2 R^2}{2} - \frac{8\pi m^2}{m_{Pl}^2} T_\mu^\mu R \right]$$

which leads to the proper equation of motion

$$D^2 R + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2} T_\mu^\mu$$

To determine the energy density of the scalaron field we have to redefine this field in such a way that the new field is canonically normalized, that is its kinetic term enters the action with the coefficient $1/2$.

Energy influx to cosmological plasma from the scalaron decay

Canonically normalized scalar field:

$$\Phi = \frac{m_{Pl}}{\sqrt{48\pi}\, m^2} R$$

Correspondingly, the energy density of the scalaron field:

$$\varrho_R = \frac{\dot{\Phi}^2 + m^2 \Phi^2}{2} = \frac{m_{Pl}^2 (\dot{R}^2 + m^2 R^2)}{96\pi m^4}$$

The energy production rate is given by:

$$\dot{\varrho}_R = 2\Gamma \varrho_R = \frac{\dot{R}^2 + m^2 R^2}{2304\pi m} = \frac{m^3}{72\pi t^2}$$

- The coefficient 2 in front of Γ appears because a pair of particles is produced in the scalaron decay.
- We take $\Gamma = m^3/(48 m_{Pl}^2)$, $r = -4\cos(\tau + \theta)/\tau - 4/\tau^2$ and differentiate only the quickly oscillating factor.

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Let us compare result

$$\dot{\varrho}_R = 2\Gamma\varrho_R = \frac{\dot{R}^2 + m^2 R^2}{2304\pi m} = \frac{m^3}{72\pi t^2}$$

with

$$\dot{\varrho} = -3H(1+w)\varrho + \frac{mR^2_{ampl}}{1152\pi} \quad or \quad S[r] = \frac{\langle r^2 \rangle}{1152\pi}$$

- we take the amplitude of harmonic oscillations of R equal to $R_{ampl} = 4m/t$.

The contribution of the particle production is exactly the same as above:

$$\dot{\varrho}_{source} = \frac{mR^2_{ampl}}{1152\pi} = \frac{m^3}{72\pi t^2}.$$

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- Firstly, the energy density of matter in R^2 modified gravity at RD stage drops down as

$$\varrho_{R^2} = \frac{m^3}{120\pi t} \quad \textit{instead of} \quad \varrho_{GR} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{3m_{Pl}^2}{32\pi t^2}$$

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- Secondly, the Hubble parameter quickly oscillates with time

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right]$$

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- The curvature scalar drops down as m/t and oscillates changing sign

$$r = -\frac{4\cos(\tau+\theta)}{\tau} - \frac{4}{\tau^2}$$

instead of being proportional to the trace of the energy-momentum tensor of matter, which is identically zero at RD stage and monotonically decreases with time, as $1/t^2$ at MD stage.

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  instead of being proportional to the trace of the energy-momentum tensor of matter, which is identically zero at RD stage and monotonically decreases with time, as $1/t^2$ at MD stage.
- It is noteworthy that R is not related to the energy density of matter as is true in GR.

Comments on the cosmological evolution at $\tau \lesssim 1/\gamma$

Because of this difference between the cosmological evolution in the R^2-theory and GR, the conditions for thermal equilibrium in the primeval plasma also very much differ.

Assuming that the equilibrium with temperature T is established, we estimate the particle reaction rate as

$$\Gamma_{part} \sim \alpha^2 T,$$

- α is the coupling constant of the particle interactions, $\alpha \sim 10^{-2}$

Equilibrium is enforced if $\Gamma_{part} > H$ or $\alpha^2 T t > 1$.

The energy density of relativistic matter in thermal equilibrium:

$$\varrho_{therm} = \frac{\pi^2 g_*}{30} T^4$$

- g_* is the number of relativistic species in the plasma, $g_* \sim 100$

The equilibrium condition for R^2 cosmology:

$$\left(\alpha^2 tT\right)_{R^2} = \frac{30\alpha^2}{120\pi^3 g_*}\left(\frac{m}{T}\right)^3 = 8\cdot 10^{-9}\left(\frac{m}{T}\right)^3 > 1$$

Analogously for GR-cosmology:

$$\left(\alpha^2 tT\right)_{GR} = \alpha^2\left(\frac{90}{32\pi^3 g_*}\right)^{1/2}\frac{m_{Pl}}{T} = 3\cdot 10^{-6}\frac{m_{Pl}}{T} > 1$$

Equilibrium between light particles in R^2-cosmology is established, when $T_{R^2} < 2\cdot 10^{-3} m$, while $T_{GR} < 3\cdot 10^{-6} m_{Pl}$.

- Expressions above determine the temperature below which thermal equilibrium is established in the primeval plasma.
- This temperature is not the same as the so called heating temperature T_h, which is defined by the condition that all energy of the scalaron field is transferred into the energy of the plasma. This takes place at $t\Gamma \approx 1$.

$$T_h \approx \frac{m}{(192\pi^2)^{1/4}}\sqrt{\frac{m}{m_{Pl}}}$$

For $m = 3\cdot 10^{13}$ Gev $T_h \approx 6\cdot 10^8$ GeV.

III. Solution at $\Gamma t \gtrsim 1$

Solution at $\gamma\tau \gtrsim 1$, $\gamma = \mu^2/48$ and $\mu = m/m_{Pl}$

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\,y = S[r]$$

A straightforward numerical solution of this system quickly becomes unreliable due to very small exponential suppression factor $\exp(-\gamma\tau/2)$, when $\gamma\tau \gg 1$.

The case of relativistic matter: $w = 1/3$, homogeneous Eq. for r.

Eliminating the first derivative r' by introducing the new function v according to:

$$r = \exp\left[-\gamma(\tau - \tau_0)/2 - (3/2)\int_{\tau_0}^{\tau} d\tau_1 h(\tau_1)\right] v$$

we come to the equation

$$v'' + \left[1 - \frac{(\gamma + 3h)^2}{4}\right] v = 0$$

Since in realistic case $\gamma \ll 1$ and $h \lesssim \gamma$, as $h \sim 1/\tau$, and by assumption $\gamma\tau \gtrsim 1$, the second term in square brackets can be neglected and we find: $v = -4\gamma\cos(\tau + \theta)$

Large $\gamma\tau/2$ limit

Curvature r exponentially vanishes at large $\gamma\tau/2$, so the r.h.s. of

$$h' + 2h^2 = -r/6 \to 0$$

$\Rightarrow$ Hubble parameter $h \to 1/(2\tau)$ as in the standard cosmology at RD stage.

The energy density in this limit satisfies

$$y' + 3(1+w)h\,y = S[r]$$

with vanishing r.h.s. $\Rightarrow$ y drops down as $1/a^4$ as expected.

NB: It is not clear if the standard relation between H and ϱ is fulfilled?

$$H^2 = \frac{8\pi}{3}\,\frac{\varrho}{m_{Pl}^2}$$

00-component of the R^2-modified gravity equation:

$$H^2 + \frac{1}{m^2}\left[2\ddot{H}H - \dot{H}^2 + 6\dot{H}H^2\right] = \frac{8\pi\varrho}{3m_{Pl}^2}$$

Curvature exponentially disappeared at $\gamma\tau > 1 \Rightarrow \dot{H} + 2H^2 = 0 \Longrightarrow$
The term in square brackets vanishes and the normal cosmology is restored.

The case of nonrelativistic dominance: $w = 0$ or some deviations from the strict $w = 1/3$

We study

$$r'' + (3h + \gamma) r' + r = -8\pi\mu^2 (1 - 3w) y$$

- non-zero r.h.s. might change the asymptotical exponential decrease of r.

Making the transformation

$$r = \exp\left[-\gamma(\tau - \tau_0)/2 - (3/2)\int_{\tau_0}^{\tau} d\tau_1 h(\tau_1)\right] v(\tau)$$

we arrive to

$$v'' + v = -8\pi\mu^2 (1 - 3w) y(\tau) \exp\left[\gamma(\tau - \tau_0)/2 + (3/2)\int_{\tau_0}^{\tau} d\tau_1 h(\tau_1)\right]$$

- The value of $(1 - 3w)$ is not yet specified here, we only assume that it is nonzero.

The unhomogeneous part of the solution:

$$v(\tau) = -8\pi\mu^2 (1 - 3w) \int_{\tau_0}^{\tau} d\tau_1 \sin(\tau - \tau_1) y(\tau_1) \exp\left[\frac{\gamma(\tau_1 - \tau_0)}{2} + \frac{3}{2}\int_{\tau_0}^{\tau_1} d\tau_1 h(\tau_1)\right]$$

We find for the curvature scalar:

$$r = -8\pi\mu^2(1-3w)\int_{\tau_0}^{\tau} d\tau_1 y(\tau_1)\sin(\tau-\tau_1)\exp\left[-\frac{\gamma}{2}(\tau-\tau_1)-\frac{3}{2}\int_{\tau_1}^{\tau} d\tau_2 h(\tau_2)\right]+r_h$$

where r_h is a solution of the homogeneous equation:

$$r_h = r_0\cos(\tau+\theta_r)\,\mathbf{exp}\left[-\frac{\gamma}{2}(\tau-\tau_0)-\frac{3}{2}\int_{\tau_0}^{\tau} d\tau_2 h(\tau_2)\right].$$

- the solution of the homogeneous equation, r_h, drops down exponentially as $e^{-\gamma\tau/2}$
- the inhomogeneous part does not; integral for r is dominated by τ_1 close to τ.

Let us assume that the standard GR became valid after sufficiently long cosmological time and check if this assertion is compatible with equations above.

So we take, according to the standard cosmological laws with $w > -1$:

$$a \sim t^{\frac{2}{3(1+w)}}, \quad H = \frac{2}{3(1+w)t}, \quad \varrho = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{m_{Pl}^2}{6\pi(1+w)^2 t^2}$$

Introducing new integration variables $x = \tau_1/\tau$, $x_2 = \tau_2/\tau$ and taking integral over dx_2 we obtain:

$$r_{inh} = -\frac{4(1-3w)}{3(1+w)^2\tau}\int_\epsilon^1 dx \sin\left[\tau(1-x)\right]\left(\frac{1}{x}\right)^{\frac{1+2w}{1+w}} \exp\left[-\frac{\gamma\tau}{2}(1-x)\right],$$

- r_{inh} is the contribution to r from the inhomogeneous term and $\epsilon = \tau_0/\tau \ll 1$.

For large τ and $\gamma\tau$ this integral can be estimated as:

$$r_{inh} = \frac{2(1-3w)}{3(1+w)^2\tau}\int_0^\infty d\zeta\, e^{-\tau\zeta} \left[\left(\frac{1}{\epsilon+i\zeta}\right)^{\frac{1+2w}{1+w}} \exp\left(-i\tau(1-\epsilon) - \frac{\gamma\tau(1-\epsilon-i\zeta)}{2}\right) - \left(\frac{1}{1+i\zeta}\right)^{\frac{1+2w}{1+w}} \exp\left(i\frac{\gamma\tau\zeta}{2}\right) + h.c.\right]$$

- Since $\tau \gg 1$, the integrals effectively "sit" at $\zeta \sim 1/\tau$ because of $\exp(-\zeta\tau)$
- In the leading order the second term in the square brackets is equal to (-1) and together with the hermitian conjugate after integration they give $-2/\tau$.
- The first term is exponentially suppressed at large $\gamma\tau/2$, as $\sim \exp(-\gamma\tau/2)$.
- A large pre-exponential factor $\sim \tau^{(1+2w)/(1+w)}$ slows down the approach to GR

$$R_{GR} = -\frac{8\pi}{m_{Pl}^2}(1-3w)\varrho = \frac{4(1-3w)}{3(1+w)^2 t^2}$$

More general anzatz: $h_{test}(\tau) = (h_1 + h_2 \sin(\tau + \theta_h))/\tau, \; y(\tau) = y_1/\tau^\beta$

Is it possible to adjust the constants $\boldsymbol{h_1}$, $\boldsymbol{h_2}$, $\boldsymbol{y_1}$, and β to restore GR?

$$r_{inh} \approx \frac{8\pi\mu^2 y_1(1-3w)}{\tau^{\beta-1}} \int_0^\infty d\zeta\, e^{-\tau\zeta}$$

$$\left[\left(\frac{1}{\epsilon + i\zeta}\right)^{\beta - \frac{3h_1}{2}} \exp\left(-i\tau(1-\epsilon) - \frac{\gamma\tau(1-\epsilon-i\zeta)}{2}\right) - \left(\frac{1}{1+i\zeta}\right)^{\beta-\frac{3h_1}{2}} \exp\left(i\frac{\gamma\tau\zeta}{2}\right) + h.$$

Keeping in mind that $\zeta \sim 1/\tau$ and that $\epsilon = \tau_0/\tau \gg 1/\tau$ we simplify the result as:

$$r_{inh} \approx \frac{16\pi\mu^2 y_1(1-3w)}{\tau^\beta}\left[\left(\frac{\tau}{\tau_0}\right)^{\beta-3h_1/2} e^{-\gamma(\tau-\tau_0)/2}\cos(\tau-\tau_0) - 1\right]$$

- This expression is a small correction to homogeneous solution

$$r = -\frac{4\cos(\tau+\theta)}{\tau} - \frac{4}{\tau^2},$$

  for which $\beta = 1$, $y_1 = 1/(72\pi)$, and $h_1 = 2/3$.
- At $\gamma\tau > 1$, the first term dies down and only the last non-oscillating term survives. In this limit the particle production by R vanishes, or strongly drops down.

The transition from the modified R^2-regime to GR

According to

$$y' + 3(1+w)h\,y = S[r] \Longrightarrow 0$$

in the absence of particle production the dimensionless energy density drops down as

$$y \sim \frac{1}{a^{3(1+w)}}.$$

Since the oscillations exponentially disappear, the derivatives of H in

$$H^2 + \frac{1}{m^2}\left[2\ddot{H}H - \dot{H}^2 + 6\dot{H}H^2\right] = \frac{8\pi\varrho}{3m_{Pl}^2}$$

can be neglected and H satisfies the GR relation

$$H^2 = \frac{8\pi}{3}\,\frac{\varrho}{m_{Pl}^2}$$

with $\varrho = m^4 y$ decreasing as $1/\tau^2$ independently of the value of w.

The transition from the modified R^2-regime to GR

$$r_{inh} \approx \frac{16\pi\mu^2 y_1(1-3w)}{\tau^\beta}\left[\left(\frac{\tau}{\tau_0}\right)^{\beta-3h_1/2} e^{-\gamma(\tau-\tau_0)/2}\cos(\tau-\tau_0) - 1\right]$$

- due to the inhomogeneous part of the solution for r which does not drop down exponentially, i.e. due to the last term in the square brackets.

It is natural to expect that the GR regime starts roughly at $\tau \gtrsim 1/\gamma$.

Simple estimate for $w \neq 1/3$:

- We have to compare the value of the curvature scalar $r = 2\mu^2/(9\tau)$ with homogeneous solution for the curvature: $r \sim 4\exp(-\gamma\tau/2)/\tau$.
- These expressions become comparable at $\gamma\tau \approx 2\ln(1/\mu^2)$, where $\ln(1/\mu^2)$ may be much larger than unity.

Similar arguments cannot be applied to $w = 1/3$, because in this case $R_{GR} \equiv 0$.
In realistic case w differs from zero either due to presence of massive particles in the primeval plasma or because of the conformal anomaly.

Cosmological history in R^2-gravity: **4** distinct epoch

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$$R \sim m \cos(mt)/t$$

The curvature oscillations resulted in the onset of creation of usual matter, which remains subdominant.

The universe expansion is described by unusual law with the Hubble parameter

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Such a regime was realised asymptotically for large time, $mt \gg 1$, but $\Gamma t \lesssim 1$.

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4. **After this time we arrive to the usual GR cosmology.**

The END

Thank You for Your Attention