Inhomogeneity Factors for Regular Stellar Structures in Modified Gravity

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XII^{th} Spontaneous Workshop on Cosmology ≪Hot Topics in Modern Cosmology≫

Cosmology

It is the scientific study of large scale properties of the universe as a whole.

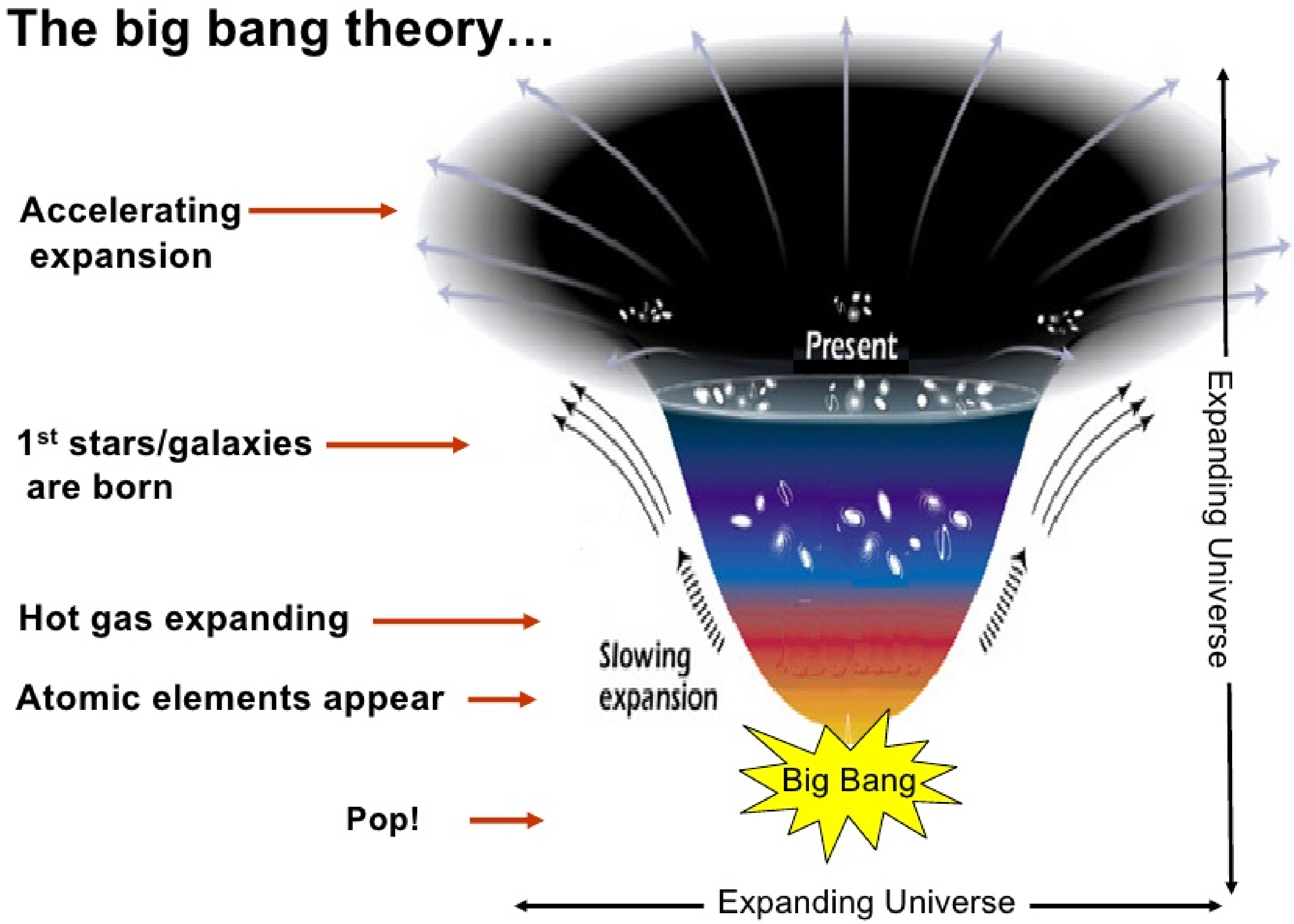
The Universe

- How did it begin?
- How is it organized and held together?
- What is it made of?
- How is it changing?

Big Bang Theory

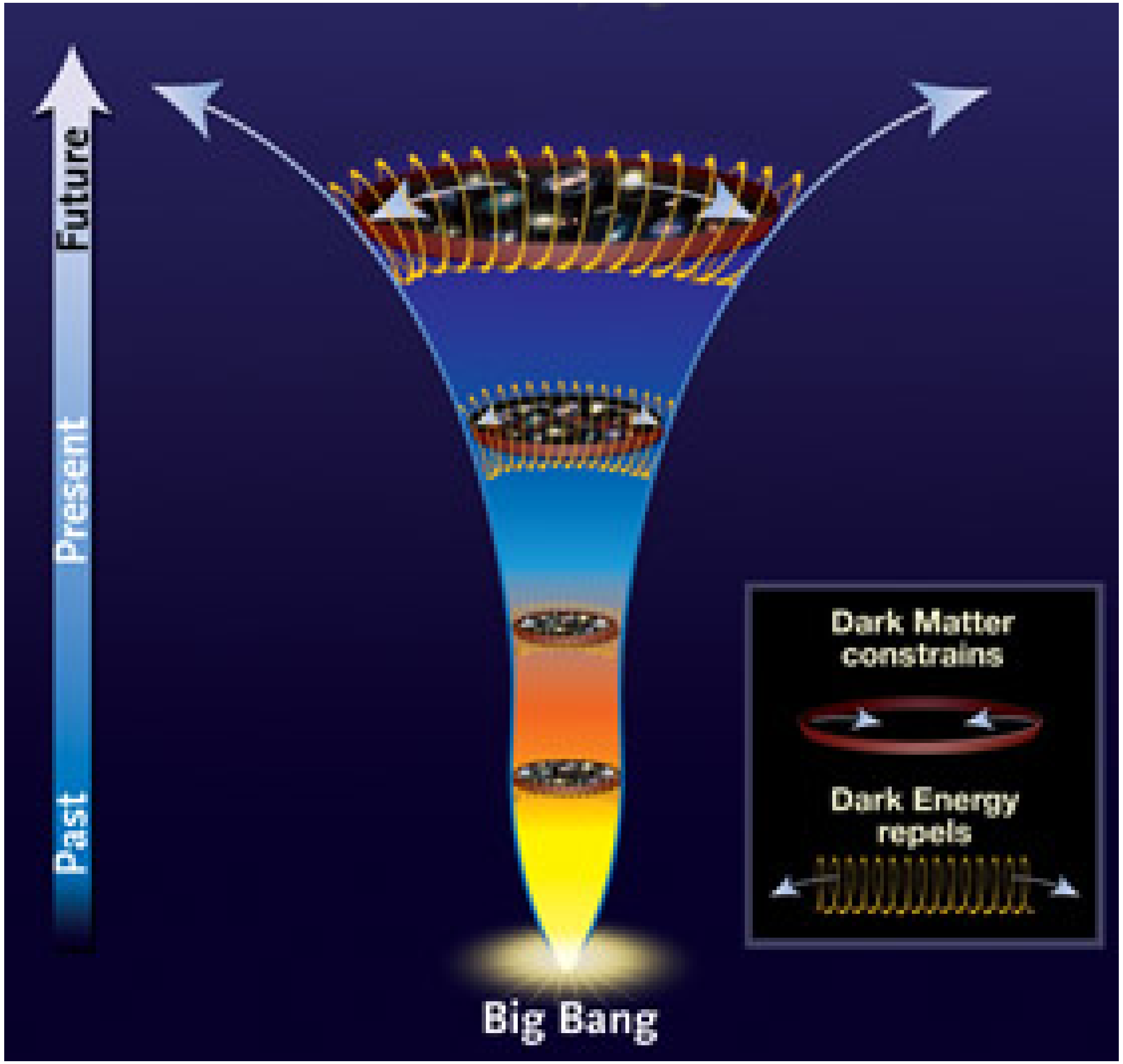
According to the Big Bang model, the universe expanded from an extremely dense and hot state and continues to expand today.

The Universe



Story of the Universe
At the beginning of time, space exploded out of nothingness to create the ever-expanding universe we inhabit now. It took billions of years for the story, depicted here, to unfold.
–Breanna Draxler
YOU ARE HERE
ACCELERATING EXPANSION
A little more than 5 billion years ago, dark energy caused the universe to expand increasingly fast.
INFLATION
In less than 10^{-30} of a second after the Big Bang, the universe burst open, expanding faster than the speed of light and flinging all the matter and energy in the universe apart in all directions.
BIG BANG
The universe expanded violently from an extremely hot and dense initial state some 13.7 billion years ago.

The Universe



The Evolution of the Universe

Composition

4.9% Baryonic Matter,

26.8% Dark Matter,

68.3% Dark Energy.

(ONLY 4.9% of the universe is visible to us!!!!)

[P. A. R. Ade *et al.*, Astron. Astrophys. **571**, 16 (2014).]

The Universe

The Motivation of GR

- The Special Theory of Relativity encompasses inertial frames of reference moving at uniform relative velocities.
- Einstein asked whether or not systems moving in non-uniform motion with respect to one another could be relative and came up with the idea of General Relativity.

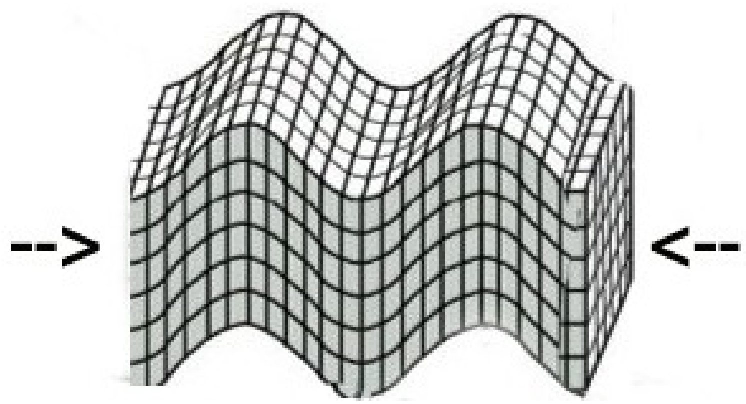
The History of GR

- 1907- published first paper applying SR to accelerating reference frames that also predicted gravitational time dilation.
- 1911- published paper predicting gravitational lensing.
- 1912- Einstein was focused on formulating a theory of spacetime that was purely geometrical.

- By 1915 Einstein had developed what are known as the Einstein Field Equations and published in *Annalen der Physik* in 1916.

A spacetime is a mathematical model that describes space and time as a single continuum having three spatial and one temporal dimension.

Warping of spacetime



▶ Due to different mass and gravity of distant planets and galaxies, spacetime is not just curved, but warped.

▶ Gravity and spacetime are linked to motion
⇒ increases in velocity and gravity can not only curve but shrink spacetime.

General Relativity in a Nutshell

Equivalence Principle:

The inertial mass of a body is equivalent to the gravitational mass.

General Covariance Principle:

Physical laws are invariant under general coordinate transformation.

Central Concept is Curvature:

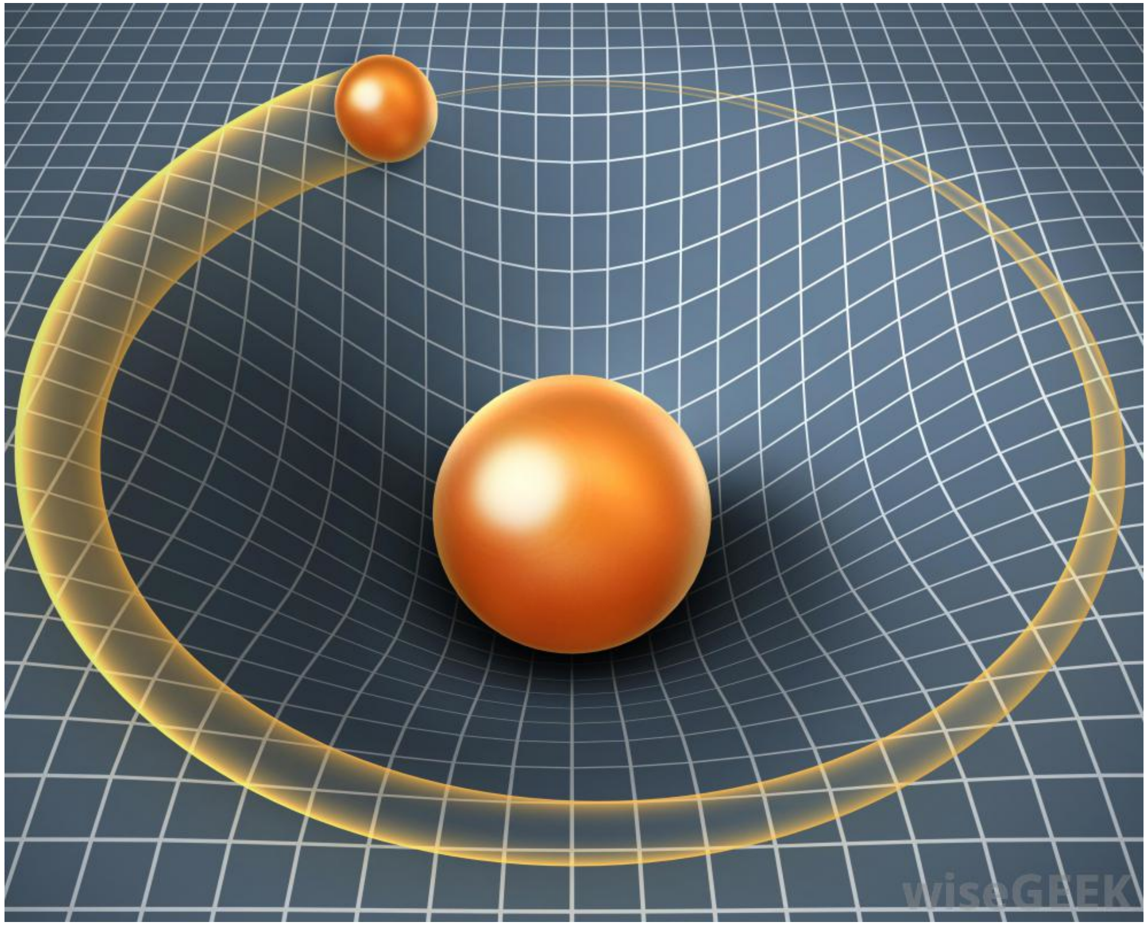
$$R^{\alpha}{}_{\beta\gamma\delta} = \Gamma^{\alpha}{}_{\beta\delta,\gamma} - \Gamma^{\alpha}{}_{\beta\gamma,\delta} + \Gamma^{\alpha}{}_{\upsilon\gamma}\Gamma^{\upsilon}_{\beta\delta} - \Gamma^{\alpha}{}_{\upsilon\delta}\Gamma^{\upsilon}_{\beta\gamma}.$$

According to this theory, gravity is the manifestation of the curvature in space and time.

General Relativity

Spacetime tells matter how to move and

Matter tells spacetime how to curve.



Energy-Momentum Tensor:

- Newton: mass.
- Einstein: mass, energy, pressure and momentum.

The quantity $T_{\alpha\beta}$ describes the matter distribution at any specific region of spacetime and is a natural candidate for the gravitation source in the field equations.

Energy-Momentum Tensor

The most general form of the energy-momentum tensor is given as

$$T_{\alpha\beta} = \rho(1+\epsilon)V_\alpha V_\beta + (P - \xi\,\Theta)h_{\alpha\beta} - 2\eta\,\sigma_{\alpha\beta} + q_\alpha V_\beta + q_\beta V_\alpha.$$

Einstein Field Equations:

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa \, T^{(m)}_{\alpha\beta} \, .$$

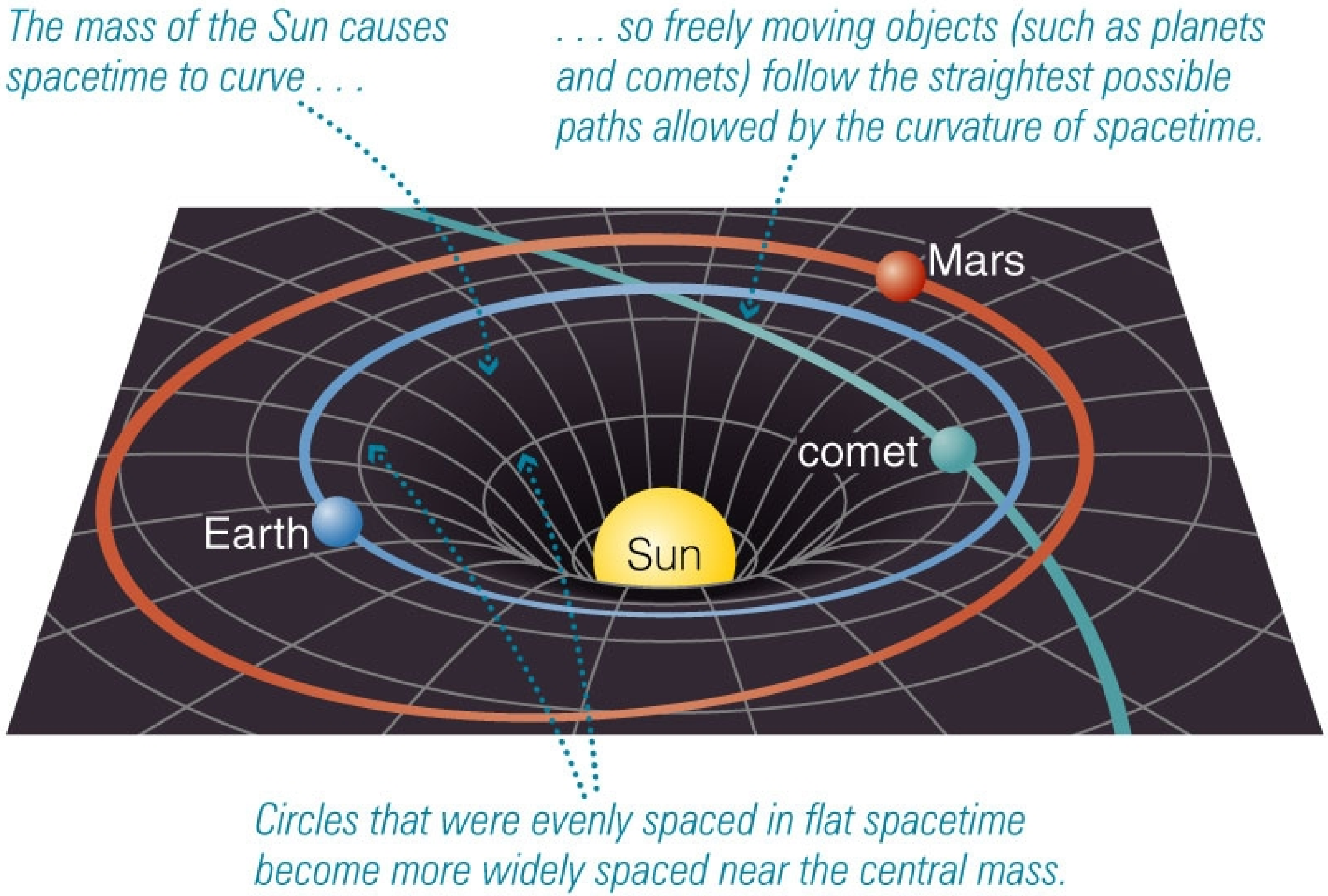
(spacetime curvature) (matter-energy)

Einstein Field Equation

Einstein-Hilbert Action:

$$S_{(\mathsf{GR})} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_M.$$

Curvature


The mass of the Sun causes spacetime to curve . . .
. . . so freely moving objects (such as planets and comets) follow the straightest possible paths allowed by the curvature of spacetime.
Mars
comet
Earth
Sun
Circles that were evenly spaced in flat spacetime become more widely spaced near the central mass.

Cosmological Constant:

It was believed that gravity contracts the universe. To overcome the attractive form of gravity, Einstein introduced cosmological constant in his field equations that would act as repulsive nature of gravity. The cosmological constant was rejected by Einstein himself when Hubble discovered that universe expands rather than contracts. Einstein felt that it was the biggest blunder that he had made in his life. However, the study of type Ia supernova have investigated that the expansion of the universe is accelerating. For the inclusion of this acceleration, it becomes necessary to add the cosmological constant in the Einstein field equation as

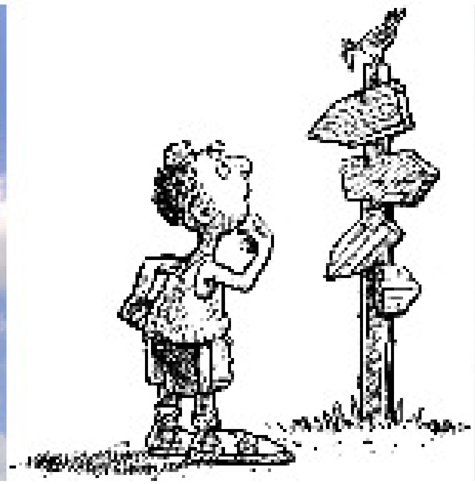
$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = \kappa\, T^{(m)}_{\alpha\beta}.$$

The problems related to the growth, structure and age of the universe can be solved by using the cosmological constant.

What is next?


what's
next


LOST
CONFUSED
UNSURE
UNCLEAR
PERPLEXED
DISORIENTED
BEWILDERED



Modified theories of gravity

General Relativity is to be modified to be compatible with the current observational data. It can be modified in two ways:

1. modify matter, *i.e.* change $T_{\alpha\beta}$,

2. modify gravity, *i.e.* change $G_{\alpha\beta}$.

Modified theories of gravity

- Modified Matter: (candidates for dark energy)
  -Cosmological Constant
  -Scalar-Field Models (quintessence, k-essence, tachyon,etc.)
  -Chaplygin Gas

Modified Theories of Gravity

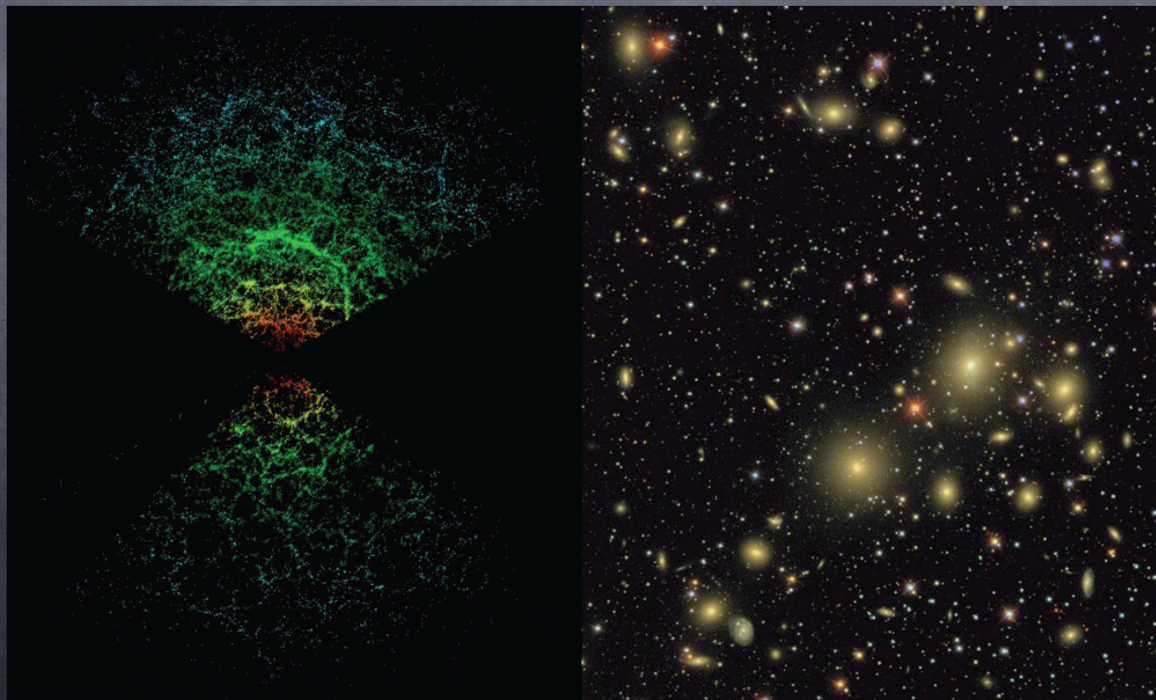
There exist a family of modified theories of gravity which extend GR, for example,

- Scalar-tensor theory
- Brans-Dick theory
- Gauss-Bonnet theory
- $f(R)$ theory

Homogeneity and Isotropy

The universe is homogeneous and isotropic

- Homogeneous: appears the same everywhere in space
- Isotropic: appears the same in every direction



Kinematical Variables

The description of the motion of the bodies can be found by viewing kinematical variables. There are four kinematical variables for the discussion of matter, i.e. expansion scalar, four acceleration, shear tensor and vorticity tensor.

♦ The expansion scalar measures the fractional change of volume per unit time.

$$\Theta = u^{\alpha}{}_{;\beta}h^{\beta}_{\alpha} = u^{\alpha}{}_{;\alpha}$$

- $\Theta > 0 \Rightarrow$ expanding universe.
- $\Theta < 0 \Rightarrow$ decelerating universe.
- $\Theta = 0 \Rightarrow$ formation of the vacuum cavity.

♦ The four acceleration is related with the elements of four velocity through covariant derivative.

$$\dot{u}_\alpha = a_\alpha = u_{\alpha;\beta}u^\beta.$$

♦ The shear tensor characterizes the tendency of the initial sphere to deform into an ellipsoidal shape.

$$\sigma_{\alpha\beta} = u_{(\alpha;\beta)} + \dot{u}_{(\alpha}u_{\beta)} - \frac{1}{3}\Theta h_{\alpha\beta}.$$

The shearing motion can be characterized by the shear scalar σ as

$$2\sigma^2 = \sigma_{\alpha\beta}\sigma^{\alpha\beta} \geq 0.$$

Kinematical Variables

♦ The vorticity tensor measures the rotational and irrotational motion of the body. It can be given as

$$\omega_{\alpha\beta} = u_{[\alpha;\beta]} + \dot{u}_{[\alpha}u_{\beta]}.$$

It is antisymmetric in its indices. The rotational motion can be characterized by the scalar ω given by

$$\omega^2 \equiv -\omega_a\omega^a = \frac{1}{2}\omega_{ab}\omega^{ab} \geq 0.$$

Gravitational Collapse:

The implosion of a stellar object under the influence of its own gravitational pull.

★ What is the reason behind the gravitational collapse?

★ Outcomes of gravitational collapse phenomena?

Gravitational Collapse

▷Pressure⇔gravity (Equilibrium)

Pressure<gravity

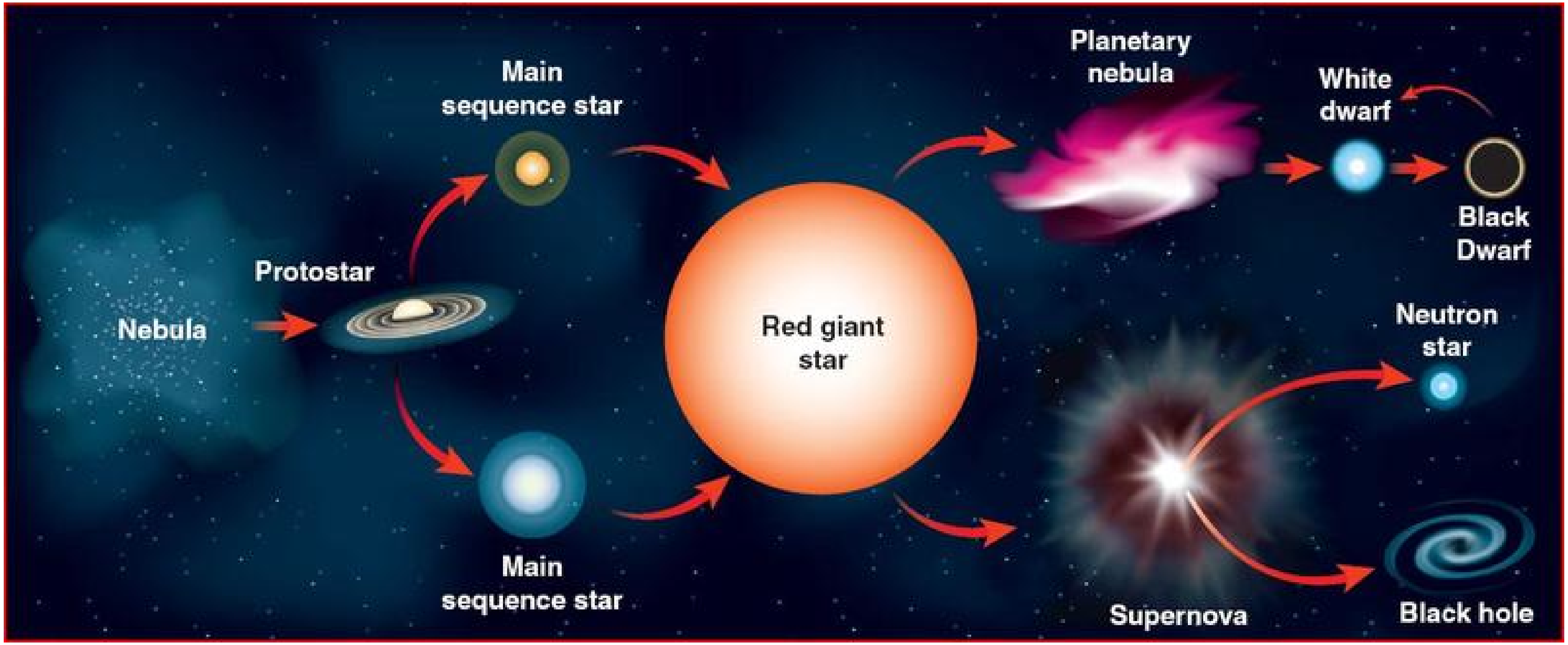
⇒ Continuous catastrophic contraction...

Gravitational Collapse ⇒ Out product...?

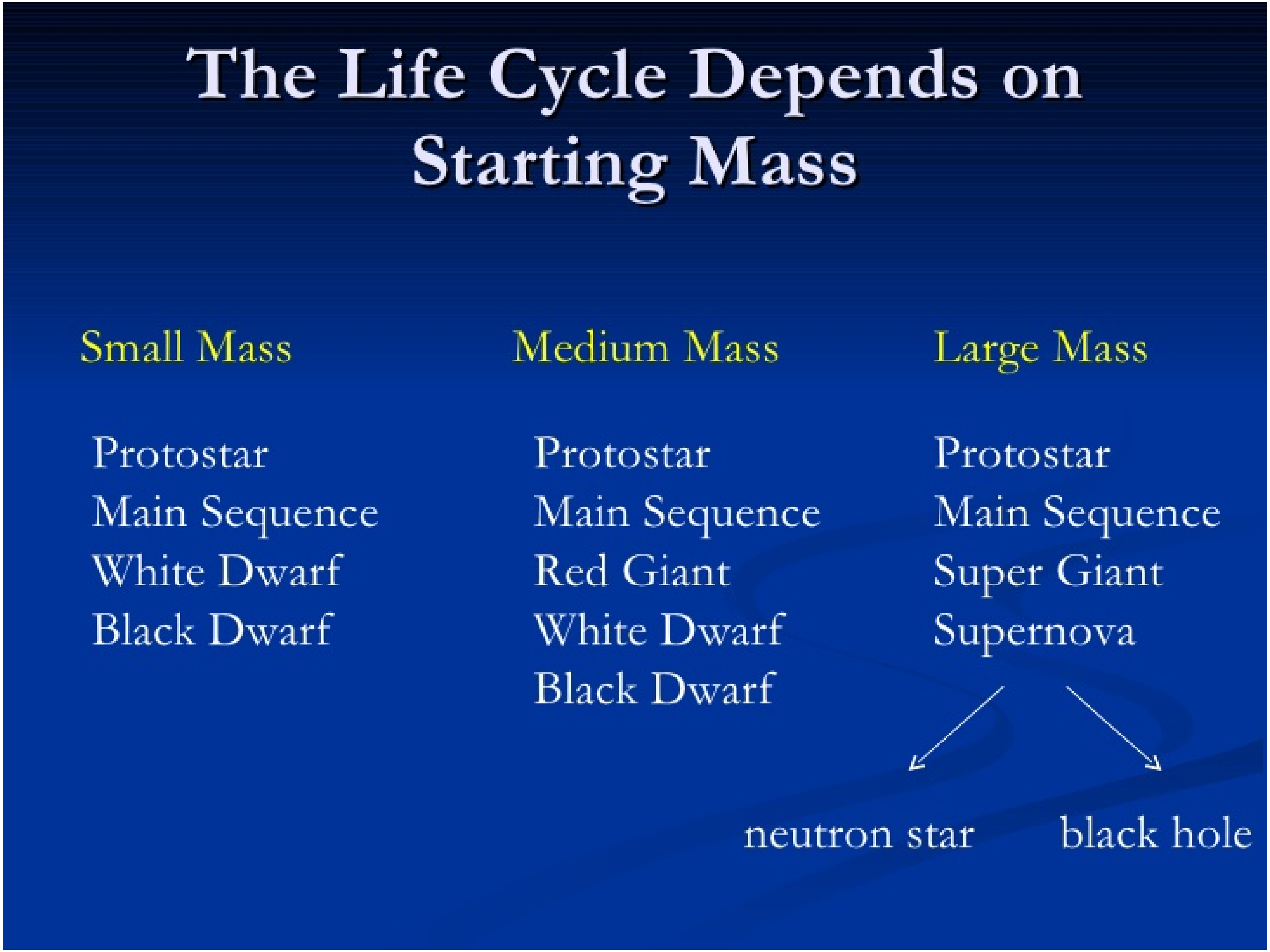
Identification of collapse out product

- $1.0M_\odot - 1.4M_\odot \Rightarrow$ White dwarf
- $1.5M_\odot - 3.2M_\odot \Rightarrow$ Neutron star
- Mass$> 3.2M_\odot \Rightarrow$ Black hole

Gravitational Collapse



Gravitational Collapse



Review

- Our current understanding of gravitation is well established on the fundamental building blocks of General Relativity published by Albert Einstein in 1915. It generalizes Special Relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time.

Review

- In particular, the curvature of spacetime is directly related to the energy and momentum of whatever matter and radiation are present which is specified by the Einstein field equations. GR paved the way to understand dynamical nature of the universe through its well-known field equations. Einstein discovered that there is a relationship between mass, gravity and spacetime. Mass distorts spacetime, causing it to curve.

Review

- GR is in fact one of the most significant gravitational theory of recent era but still it does not provide a satisfactory explanation for latest cosmological and astrophysical observations in the presence of usual matter. The theoretical cosmology has been put into crisis due to the concept of DE and observational evidences of expanding universe.

- Modified gravity theories provide a useful way to demonstrate the problem of DE and even the inflationary phases with late time acceleration. To achieve the cosmic acceleration with different ways, the most popular investigation is focused on the fourth order gravity in which the corrections are in the gravitational Lagrangian.

- Recent observations propose that matter distribution is homogeneous and isotropic at present state of the universe. But on theoretical grounds, the universe does not have such a smooth picture. It was not homogeneous initially and extremely dense in certain areas. Thus, the formation of our universe at very early stages and its exact physical modeling has provoked interest of many researchers. The inhomogeneous matter density is illustrated in many ways like formation of galaxies with different sizes and fluctuation of the local density etc.

- Higher order curvature invariants due to FOG shed some light on the significant issue of irregular matter distribution and are helpful to understand the irregular distribution of matter over the universe.

- Many researchers considered spherical geometries which gave a way to study inhomogeneous earlier universe.
- Self-gravitating objects pass through different intense phases during the evolution of the fluid model.

The problem of inhomogeneity is closely associated with formation and evolution of self-gravitating objects which has importance in the light of Newtonian and general relativistic models. It is well-known that different physical variables like pressure anisotropy, heat radiations, viscosity etc effect the homogeneity of the system.

- The anisotropy plays a key role in understanding the gravitation of those objects which have higher densities than neutron stars. Lemaître [Lemaître, G.: Ann. Soc. Sci. Bruxells A **53**(1933)51] was the first who gave the idea about the anisotropic pressures.

- There are many attempts to understand interaction between electromagnetic and gravitational fields. The magnetic field is witnessed in compact objects like neutron stars, white dwarfs or magnetized strange quark stars.

Motivation

- Highly energetic explosions in self-gravitating fluid distributions are common events in relativistic astrophysics. The utmost importance of the gravitational collapse lies in the structure formation of the universe. During this evolution, objects may move through immense phases of dynamical activities under which system may be homogeneous or inhomogeneous.

Motivation

- **Penrose and Hawking (CUP, 1979)** described the importance of energy density inhomogeneity for perfect fluids by relating inhomogeneous density with Weyl tensor.

- **Herrera *et al.*(PRD, 2004)** investigated density inhomogeneity effects on the evolutionary phases of dissipative anisotropic spherical systems by evaluating a link between the Weyl tensor and local anisotropic pressure.

Motivation

- **Herrera** [Herrera, L.: Int. J. Mod. Phys. D. 20(2011)1689] explored particular conditions and described different factors on self-gravitating objects including the inhomogeneity in the fluid distribution.

Metric:

$$ds^2 = + A^2dt^2 - B^2dr^2 - C^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (1)$$

Matter:

$$T_{\alpha\beta} = (\mu + P_\perp)V_\alpha V_\beta + (P_r - P_\perp)\chi_\alpha\chi_\beta - P_\perp g_{\alpha\beta} + q_\alpha V_\beta + \varepsilon l_\alpha l_\beta + V_\alpha q_\beta, \qquad (2)$$

satisfying

$$V^\alpha V_\alpha = 1, \quad q_\alpha V^\alpha = 0.$$

The Misner-Sharp mass function is given by

$$m(t,r) = \left\{1 + \left(\frac{\dot{C}}{A}\right)^2 - \left(\frac{C'}{B}\right)^2\right\}\frac{C}{2}. \qquad (3)$$

The metric $f(R)$ field equations for Eq.(1)

Research Work

become

$$\frac{\kappa}{f_R}\left[A^2(\mu+\varepsilon)+\frac{A^2}{\kappa}\left\{\frac{f'_R}{B^2}\left(\frac{B'}{B}+\frac{2C'}{C}\right)-\frac{f_R}{2}\left(R-\frac{f}{f_R}\right)+\frac{f''_R}{B^2}-\frac{\dot{f}_R}{A^2}\right.\right.$$
$$\left.\left.\times\left(\frac{\dot{B}}{B}+\frac{2\dot{C}}{C}\right)\right\}\right]=\left(\frac{\dot{C}}{C}\right)^2+\frac{2\dot{C}\dot{B}}{CB}+\left\{\frac{C'}{C}\left(\frac{2B'}{B}-\frac{C'}{C}\right)+\left(\frac{B}{C}\right)^2\right.$$
$$\left.-\frac{2C''}{C}\right\}\left(\frac{A}{B}\right)^2, \tag{4}$$

$$\frac{\kappa}{f_R}\left[BA(q+\varepsilon)-\frac{1}{\kappa}\left(\dot{f}'_R-\frac{\dot{B}f'_R}{B}-\frac{A'\dot{f}_R}{A}\right)\right]=2\left(\frac{\dot{C}'}{C}-\frac{A'\dot{C}}{CA}-\frac{C'\dot{B}}{BC}\right), \tag{5}$$

$$\frac{\kappa}{f_R}\left[B^2(P_r+\varepsilon)-\frac{B^2}{\kappa}\left\{\frac{f'_R}{B^2}\left(\frac{A'}{A}+\frac{2C'}{C}\right)-\frac{f_R}{2}\left(R-\frac{f}{f_R}\right)+\left(\frac{\dot{A}}{A}-\frac{2\dot{C}}{C}\right)\right.\right.$$
$$\left.\left.\times\frac{\dot{f}_R}{A^2}-\frac{\ddot{f}_R}{A^2}\right\}\right]=\left\{\left(\frac{2\dot{A}}{A}-\frac{\dot{C}}{C}\right)\frac{\dot{C}}{C}-\frac{2\ddot{C}}{C}\right\}\frac{B^2}{A^2}-\frac{B^2}{C^2}+\frac{C'}{C}\left(\frac{C'}{C}+\frac{2A'}{A}\right), \tag{6}$$

Research Work

$$\frac{\kappa}{f_R}\left[P_\perp C^2-\frac{C^2}{\kappa}\left\{\frac{f_R''}{B^2}-\frac{\ddot{f}_R}{A^2}+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\frac{\dot{f}_R}{A^2}-\frac{f_R}{2}\left(R-\frac{f}{f_R}\right)\right.\right.$$
$$\left.\left.+\left(\frac{C'}{C}-\frac{B'}{B}+\frac{A'}{A}\right)\frac{f_R'}{B^2}\right\}\right]=\left\{\frac{\dot{C}}{C}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)-\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{A}}{BA}-\frac{\ddot{B}}{B}\right\}\frac{C^2}{A^2}$$
$$+\left\{\frac{A'}{A}\left(\frac{C'}{C}-\frac{B'}{B}\right)\frac{C''}{C}-\frac{B'C'}{BC}+\frac{A''}{A}\right\}\frac{C^2}{B^2}. \tag{7}$$

The time and radial mass variations can be followed from Eqs.(3)-(6) as

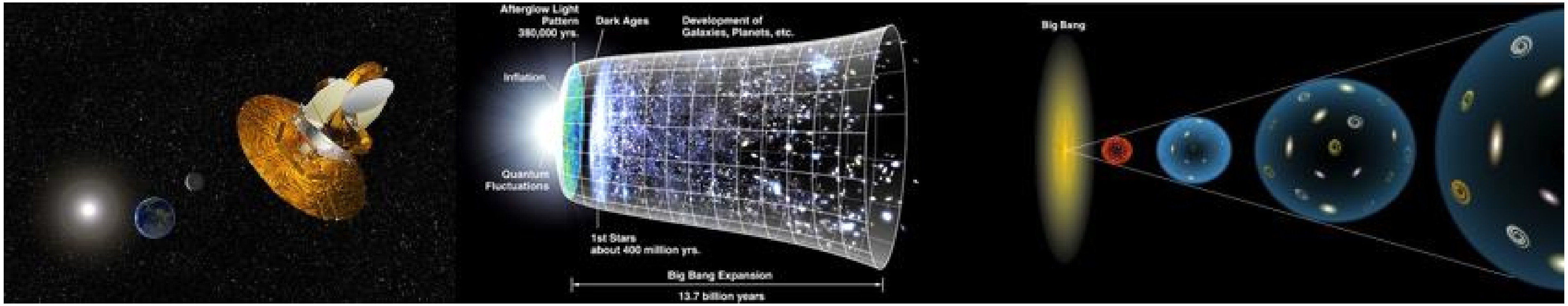
$$D_T m = -\frac{\kappa}{2f_R}\left\{U\left(\hat{P}_r + \frac{T_{11}^{(D)}}{B^2}\right) + E\left(\hat{q} - \frac{T_{01}^{(D)}}{AB}\right)\right\}C^2, \tag{8}$$

$$D_C m = \frac{\kappa}{2f_R}\left\{\hat{\mu} + \frac{T_{00}^{(D)}}{A^2} + \frac{U}{E}\left(\hat{q} - \frac{T_{01}^{(D)}}{AB}\right)\right\}C^2. \tag{9}$$

The shear scalar for Eq.(1) is given by

$$\sigma = -\frac{1}{A}\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right). \tag{10}$$

Polynomial Inflationary Model



$f(R)$ model:

$$f(R) = R + \frac{R^2}{6M^2} + \frac{\lambda_n R^n}{2n(3M^2)^{n-1}}. \qquad (11)$$

Structure Scalars and $f(R)$ Ellis Equations

We propose tensors $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ by splitting Riemann tensor as (Herrera et al. 2011)

$$X_{\alpha\beta}=\frac{1}{3}X_T h_{\alpha\beta}+X_{TF}\left(\chi_\alpha\chi_\beta-\frac{1}{3}h_{\alpha\beta}\right), \qquad (12)$$

$$Y_{\alpha\beta}=\frac{1}{3}Y_T h_{\alpha\beta}+Y_{TF}\left(\chi_\alpha\chi_\beta-\frac{1}{3}h_{\alpha\beta}\right). \qquad (13)$$

Research Work

We use Eqs.(4), (6), (7) and (11)-(13) with some manipulations to obtain the following scalar structures

$$X_T = \frac{4\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\mu}+\frac{\varphi_\mu}{A^2}\right), \tag{14}$$

$$X_{TF} = -\mathcal{E}-\frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\Pi}-2\sigma\eta+\frac{\varphi_{P_r}}{B^2}-\frac{\varphi_{P_\perp}}{C^2}\right), \tag{15}$$

$$Y_T = \frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\mu}+\frac{\varphi_\mu}{A^2}+\frac{\varphi_{P_r}}{B^2}+\frac{2\varphi_{P_\perp}}{C^2}+3\hat{P}_r-2\hat{\Pi}\right), \tag{16}$$

$$Y_{TF} = \mathcal{E}-\frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\Pi}-2\eta\sigma+\frac{\varphi_{P_r}}{B^2}-\frac{\varphi_{P_\perp}}{C^2}\right), \tag{17}$$

where φ_μ, φ_{P_r} and $\varphi_{P_\perp}$ are

$$\varphi_\mu = \frac{A^2}{\kappa}\left[\frac{2\epsilon R''}{B^2} + \frac{\lambda_n(n-1)(2\epsilon R)^n}{\epsilon(2BR)^2}\left\{\frac{(n-2)R'^2}{R} + R''\right\} - \left(\frac{\dot{B}}{B} + 2\frac{\dot{C}}{C}\right)\right.$$
$$\times\left\{\frac{2\epsilon\dot{R}}{A^2} + \frac{\lambda_n(n-1)(2\epsilon R)^n\dot{R}}{2\epsilon R^2A^2}\right\} - \frac{\epsilon R^2}{2} + \frac{\lambda_n(n-1)(2\epsilon R)^n}{8n\epsilon} - \left\{\frac{2\epsilon R'}{B^2}\right.$$
$$\left.\left.+\frac{\lambda_n(n-1)(2\epsilon R)^n R'}{2\epsilon R^2B^2}\right\}\left(\frac{B'}{B} - \frac{2C'}{C}\right)\right], \tag{18}$$

$$\varphi_{P_r} = -\frac{B^2}{\kappa}\left[\left\{\frac{2\epsilon\dot{R}}{A^2} + \frac{\lambda_n(n-1)(2\epsilon R)^n\dot{R}}{2\epsilon R^2A^2}\right\}\left(\frac{\dot{A}}{A} - \frac{2\dot{C}}{C}\right) + \frac{\lambda_n(1-n)(2\epsilon R)^n}{8\epsilon n}\right.$$
$$-\frac{\epsilon R^2}{2} - \frac{2\epsilon\ddot{R}}{A^2} + \frac{\lambda_n(1-n)(2\epsilon R)^n}{\epsilon(2AR)^2}\left\{\ddot{R} + \frac{(n-2)\dot{R}^2}{R}\right\} + \left(\frac{A'}{A} + \frac{2C'}{C}\right)$$
$$\left.\times\left\{\frac{2\epsilon R'}{B^2} + \frac{\lambda_n(n-1)(2\epsilon R)^n R'}{2\epsilon R^2B^2}\right\}\right], \tag{19}$$

$$\varphi_{P_\perp} = -\frac{C^2}{\kappa}\left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\left\{\frac{2\epsilon\dot{R}}{A^2} + \frac{\lambda_n(n-1)(2\epsilon R)^n\dot{R}}{2\epsilon R^2 A^2}\right\} + \frac{\lambda_n(1-n)}{8\epsilon n}\right.$$
$$\times(2\epsilon R)^n + \left\{\frac{2\epsilon R'}{B^2} + \frac{\lambda_n(n-1)(2\epsilon R)^n R'}{2\epsilon R^2 B^2}\right\}\left(\frac{C'}{C} - \frac{B'}{B} + \frac{A'}{A}\right) + 2\epsilon\left(\frac{R''}{B^2}\right.$$
$$\left.\left. - \frac{\ddot{R}}{A^2}\right) + \frac{\lambda_n(n-1)(2\epsilon R)^n}{4\epsilon R^2}\left\{\frac{R''}{B^2} - \frac{\ddot{R}}{A^2} + \frac{(n-2)R'^2}{RB^2} - \frac{(n-2)\dot{R}^2}{RA^2}\right\} - \frac{\epsilon R^2}{2}\right], \tag{20}$$

The two independent components of the contracted Bianchi identities are

$$\dot{\hat{\mu}} + \frac{A\hat{q}'}{B} + (\hat{P}_r + \hat{\mu})\frac{\dot{B}}{B} + \frac{2A\hat{q}C'}{BC} + 2(P_\perp + \hat{\mu})\frac{\dot{C}}{C} + D_0(t,r) = 0, \quad (21)$$

$$\frac{A\hat{P}_r'}{B} + \dot{\hat{q}} + (\hat{P}_r + \hat{\mu})\frac{A'}{B} + 2\left(\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\right)\hat{q} + 2\hat{\Pi}\frac{(AC)'}{BC} + D_1(t,r) = 0, \quad (22)$$

where

$$D_0=\frac{1}{\kappa}\Bigg[\left\{\left(\frac{A'}{A}\dot{f}_R+\frac{\dot{B}}{B}f'_R-\dot{f}'_R\right)\frac{1}{A^2B^2}\right\}_{,1}+\frac{1}{A^2}\left\{\frac{f_R}{2}\left(\frac{f}{f_R}-R\right)\right.$$
$$+\left(\frac{2C'}{C}+\frac{B'}{B}\right)\frac{f'_R}{B^2}+\frac{f''_R}{B^2}-\frac{\dot{f}_R}{A^2}\left(\frac{2\dot{C}}{C}+\frac{\dot{B}}{B}\right)\Bigg\}_{,0}+\frac{1}{A^2}\left\{\frac{\ddot{f}_R}{A^2}\right.$$
$$-\left(\frac{B'}{B}+\frac{A'}{A}\right)\frac{f'_R}{B^2}+\frac{f''_R}{B^2}-\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)\frac{\dot{f}_R}{A^2}\Bigg\}\frac{\dot{B}}{B}+\frac{2}{A^2}\left\{-\left(\frac{A'}{A}\right.\right.$$
$$\left.-\frac{C'}{C}\right)\frac{f'_R}{B^2}+\frac{\ddot{f}_R}{A^2}-\frac{\dot{f}_R}{A^2}\left(\frac{3\dot{C}}{C}+\frac{\dot{A}}{A}\right)\Bigg\}\frac{\dot{C}}{C}+\left(\frac{\dot{B}}{B}f'_R-\dot{f}'_R+\frac{A'}{A}\dot{f}_R\right)$$
$$\times\frac{1}{A^2B^2}\left(\frac{3A'}{A}+\frac{B'}{B}+\frac{2C'}{C}\right)\Bigg], \tag{23}$$

$$
\begin{aligned}
D_1 = \frac{1}{\kappa}\Bigg[&\left\{\frac{1}{B^2A^2}\left(\frac{A'}{A}\dot{f}_R-\dot{f}'_R+\frac{\dot{B}}{B}f'_R\right)\right\}_{,0}+\frac{1}{B^2}\left\{\frac{\ddot{f}_R}{A^2}-\frac{f_R}{2}\left(\frac{f}{f_R}\right.\right.\\
&\left.-R\right)-\frac{f'_R}{B^2}\left(\frac{A'}{A}+2\frac{C'}{C}\right)-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}-2\frac{\dot{C}}{C}\right)\Bigg\}_{,1}+\frac{1}{B^2}\left\{\frac{f''_R}{B^2}+\frac{\ddot{f}_R}{A^2}\right.\\
&\left.-\left(\frac{A'}{A}+\frac{B'}{B}\right)\frac{f'_R}{B^2}-\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\frac{\dot{f}_R}{A^2}\right\}\frac{A'}{A}+\frac{2}{B^2}\left\{-\frac{f'_R}{B^2}\left(\frac{B'}{B}+\frac{C'}{C}\right)\right.\\
&\left.+\frac{f''_R}{B^2}-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{B}}{B}+\frac{3\dot{C}}{C}\right)\right\}\frac{C'}{C}-\frac{1}{(AB)^2}\left(\dot{f}'_R-\frac{\dot{B}}{B}f'_R-\frac{A'}{A}\dot{f}_R\right)\\
&\times\left(\frac{3\dot{B}}{B}+\frac{\dot{A}}{A}+\frac{2\dot{C}}{C}\right)\Bigg]. \qquad (24)
\end{aligned}
$$

Research Work

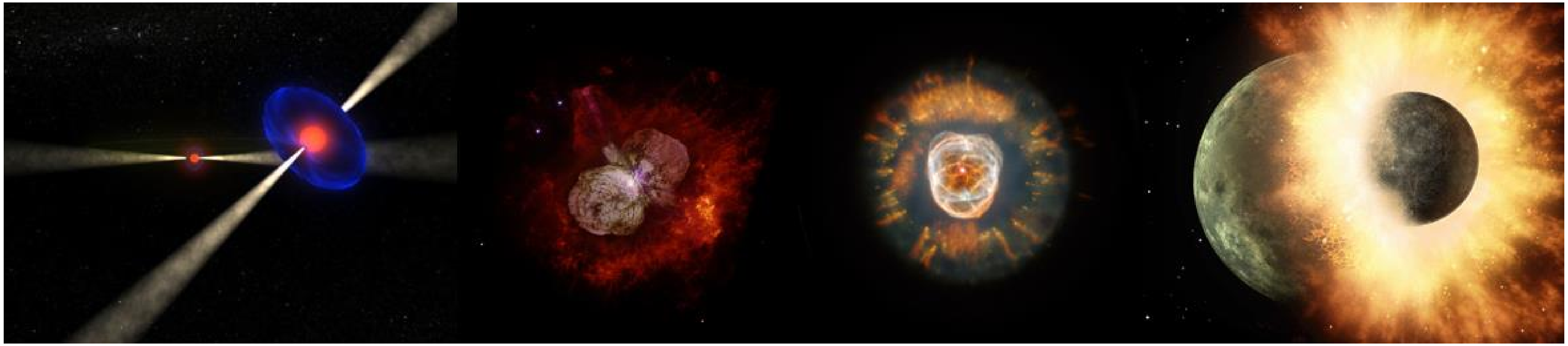
The very important equations, i.e., modified Ellis equation can be obtained by using Eqs.(4)-(9) and (11) as

$$\left[\mathcal{E}-\frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\mu}-\hat{\Pi}+\frac{\varphi_\mu}{A^2}-\frac{\varphi_{P_r}}{B^2}+\frac{\varphi_{P_\perp}}{C^2}\right)\right]_{,0}$$
$$=\frac{3\dot{C}}{C}\left[\frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\mu}+\hat{P}_\perp+\frac{\varphi_\mu}{A^2}+\frac{\varphi_{P_\perp}}{C^2}\right)-\mathcal{E}\right]$$
$$+\frac{6\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\frac{AC'}{BC}\right)\left(\hat{q}-\frac{\varphi_q}{AB}\right), \qquad (25)$$

$$\left[\mathcal{E}-\frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\mu}-\hat{\Pi}+\frac{\varphi_\mu}{A^2}-\frac{\varphi_{P_r}}{B^2}+\frac{\varphi_{P_\perp}}{C^2}\right)\right]_{,1}$$
$$=-\frac{3C'}{C}\left[\frac{2\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\hat{\mu}+\frac{\varphi_\mu}{A^2}\right)-\frac{3m}{C^3}\right]$$
$$-\frac{6\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\frac{B\dot{C}}{AC}\right)\left(\hat{q}-\frac{\varphi_q}{AB}\right), \qquad (26)$$

where

$$\varphi_q = \frac{1}{\kappa}\left[\frac{\lambda_n(n-1)(2\epsilon R)^n}{4\epsilon R^2}\left\{\frac{(n-2)\dot{R}R'}{R}+\dot{R}'\right\}-2\epsilon\left(\frac{R'\dot{B}}{B}+\frac{\dot{R}A'}{A}\right)\right.$$
$$\left.-\frac{\lambda_n(n-1)(2\epsilon R)^n}{2\epsilon R^2}\left(\frac{R'\dot{B}}{B}+\frac{\dot{R}A'}{A}\right)+2\epsilon\dot{R}'\right].$$



Stability of Homogeneous Energy Density in Spherical Stars

Dust Cloud

By making use of Eqs.(10) and (18)-(21) in Eqs.(25) and (26), we obtain

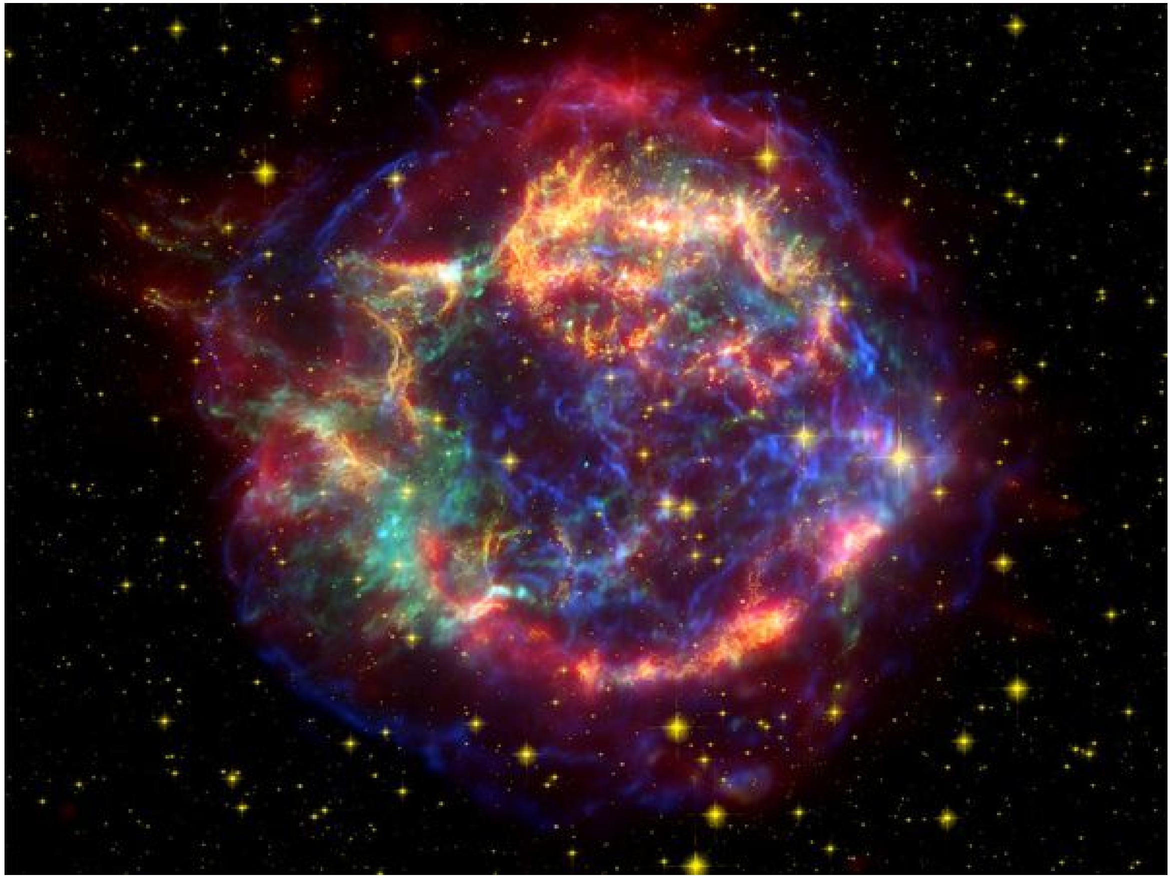
$$\dot{\mathcal{E}}+\frac{3\dot{C}}{C}\mathcal{E}=\frac{-2\kappa\epsilon A\sigma\mu\tilde{R}}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n}, \qquad (26)$$

$$\mathcal{E}'+\frac{3C'}{C}\mathcal{E}=\frac{2\kappa\epsilon\tilde{R}\mu'}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n}, \qquad (27)$$

which yield

$$\mathcal{E}=\frac{-2\kappa\epsilon\tilde{R}\int_0^t A\sigma\mu C^3dt}{[4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n]C^3}. \qquad (28)$$

Dust Cloud



Isotropic Matter

Equation (25) after using Eqs.(10) and (21) provides

$$\dot{\mathcal{E}} + \frac{3\dot{C}}{C}\mathcal{E} = \frac{-2\kappa\epsilon A\sigma(\mu + P)\tilde{R}}{4\epsilon\tilde{R}(1 + 2\epsilon\tilde{R}) + \lambda_n(2\epsilon\tilde{R})^n}, \qquad (29)$$

whose solution is

$$\mathcal{E} = \frac{-2\kappa\epsilon\tilde{R}\int_0^t A\sigma(\mu + P)C^3 dt}{[4\epsilon\tilde{R}(1 + 2\epsilon\tilde{R}) + \lambda_n(2\epsilon\tilde{R})^n]C^3}. \qquad (30)$$

Using Eqs.(10) and (21) in Eqs.(25) and (26), it follows that

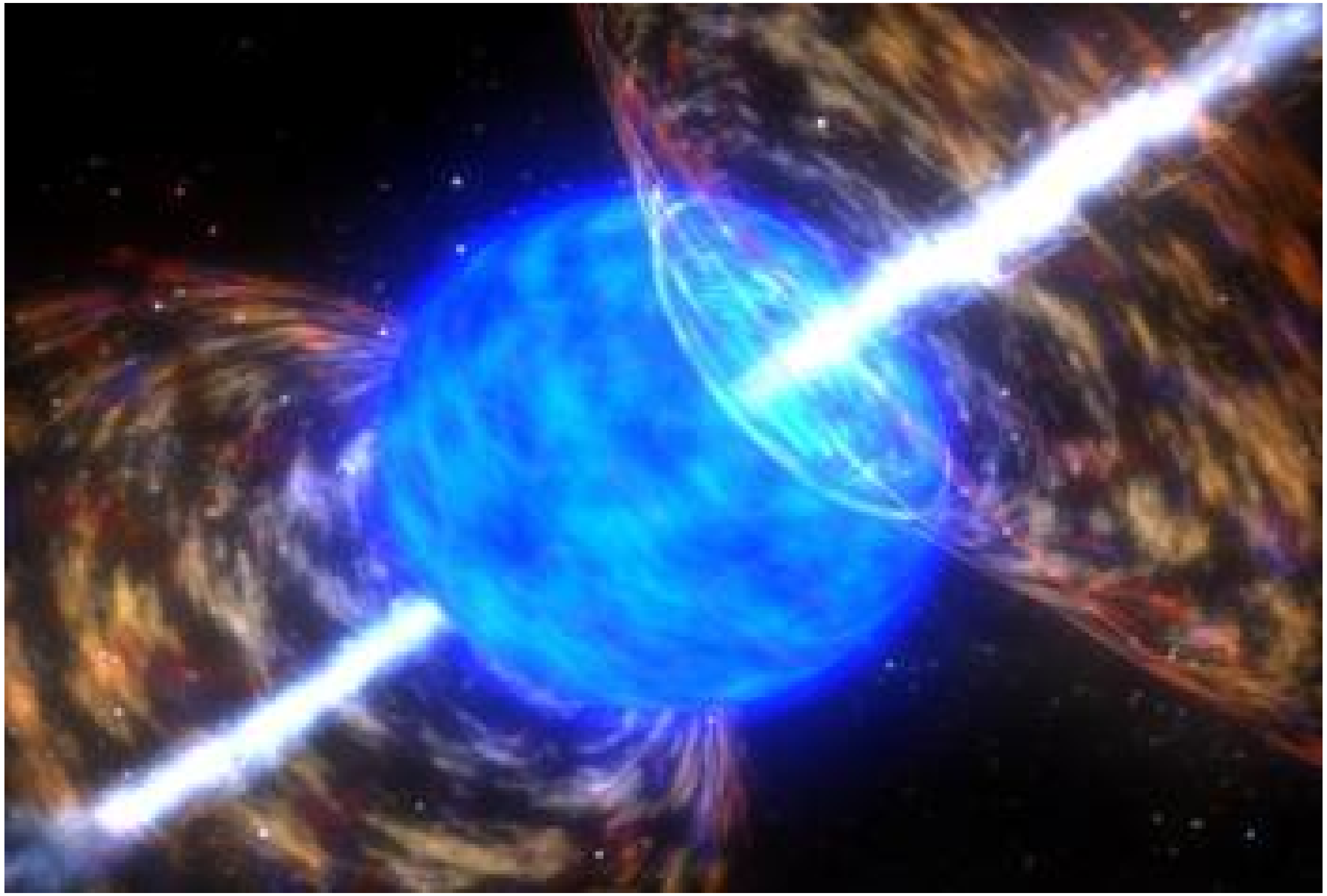
$$\dot{X}_{TF}+\frac{3X_{TF}\dot{C}}{C}=\frac{2\kappa\epsilon\tilde{R}}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n}\left\{A\sigma(\mu+P_r)-2\Pi\frac{\dot{C}}{C}\right\},$$

$$X'_{TF}+\frac{3X_{TF}C'}{C}=-\frac{2\kappa\epsilon\tilde{R}\mu'}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n},$$

Anisotropic Collapsing Star

whose general solution is

$$X_{TF} = -\frac{2\kappa\epsilon\tilde{R}\int_0^t[2\Pi\dot{C} - AC\sigma(\mu + P_r)]C^2 dt}{C^3[4\epsilon\tilde{R}(1 + 2\epsilon\tilde{R}) + \lambda_n(2\epsilon\tilde{R})^n]}. \tag{31}$$



Dissipative Collapsing Dust Cloud

We use Eqs.(10) and (21) in Eq.(25) to obtain Φ evolution equation as follows

$$\dot{\Phi}-\frac{\dot{\Psi}}{C^3}=\frac{2\kappa\epsilon\tilde{R}}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n}\times\left(\frac{\tilde{q}C'}{BC}-\tilde{\mu}\sigma-\frac{\tilde{q}'}{B}\right)-\frac{3\dot{C}}{C}\Phi, \quad (32)$$

where $\Psi=\frac{6\kappa\epsilon\tilde{R}}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n}\int_0^r BC^2\tilde{q}\dot{C}dr.$

Dissipative Collapsing Dust Cloud

The solution of above differential equation is

$$\Phi = \frac{\int_0^t \left[\dot{\Psi} + \frac{2\kappa\epsilon C^2 \tilde{R}}{4\epsilon\tilde{R}(1+2\epsilon\tilde{R})+\lambda_n(2\epsilon\tilde{R})^n} \left(\frac{\tilde{q}C'}{B} - \tilde{\mu}C\sigma - \frac{\tilde{q}'C}{B}\right)\right] dt}{C^3}. \tag{33}$$

Dissipative Collapsing Dust Cloud

Concluding Remarks

- For non-dissipative dust and isotropic object, it is seen that the system will encapsulate homogeneous energy density if and only if the system is conformally flat.

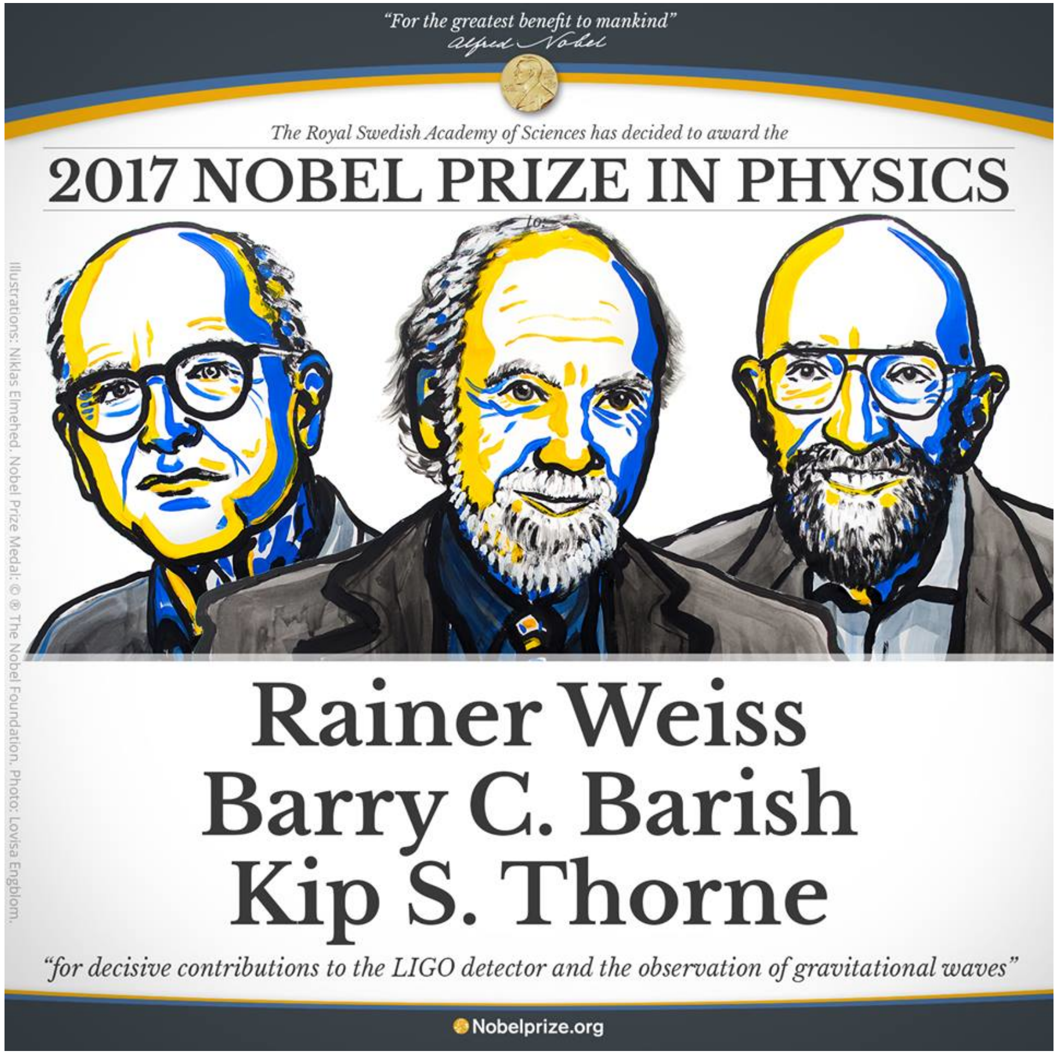
- In an adiabatic anisotropic spherical system, the density inhomogeneity is described in terms of $f(R)$ structure scalar, X_{TF}.

Concluding Remarks

- The quantity Φ is explored and identified to be responsible for the emergence of inhomogeneity in collapsing radiating dust cloud.


THANK YOU FOR
YOUR ATTENTION!!!

Nobel Prize in Physics, 2017



BREAKING NEWS

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics, 2017 with one half to Rainer Weiss and the other half jointly to Barry C. Barish and Kip S. Thorne at LIGO/VIRGO COLLABORATION LIGO Scientific Collaboration for decisive contributions to the LIGO detector and the observation of gravitational waves.

On 14 September 2015, the universes gravitational waves were observed for the very first time. The waves, which were predicted by Albert Einstein a hundred years ago, came from a collision between two black holes. It took 1.3 billion years for the waves to arrive at the LIGO detector in the USA.

Nobel Prize in Physics, 2017

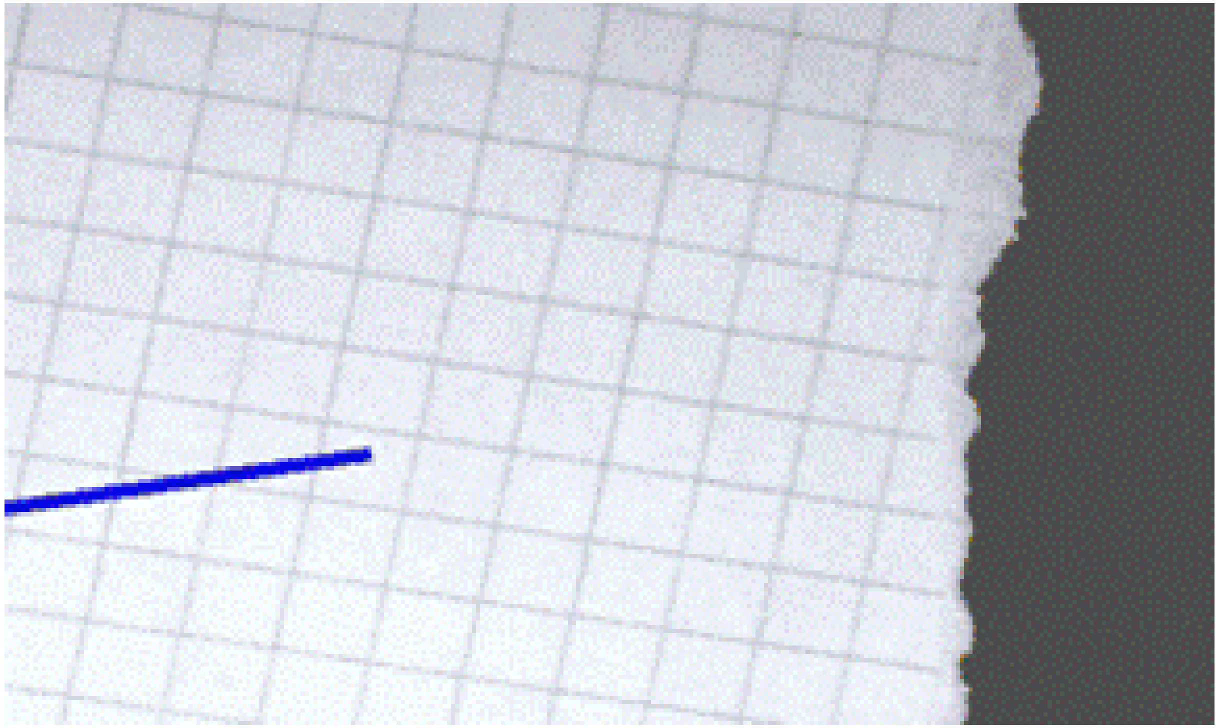
The signal was extremely weak when it reached Earth, but is already a promising revolution in astrophysics. Gravitational waves are an entirely new way of observing the most violent events in space and testing the limits of our knowledge.

LIGO, the Laser Interferometer Gravitational-Wave Observatory, is a collaborative project with over one thousand researchers from more than twenty countries. Together, they have realized a vision that is almost fifty years old. Pioneers Rainer Weiss and Kip S. Thorne, together with Barry C. Barish, the scientist and leader who brought the project to completion, ensured that four decades of effort led to gravitational waves finally being observed.

Nobel Prize in Physics, 2017

Singularity:

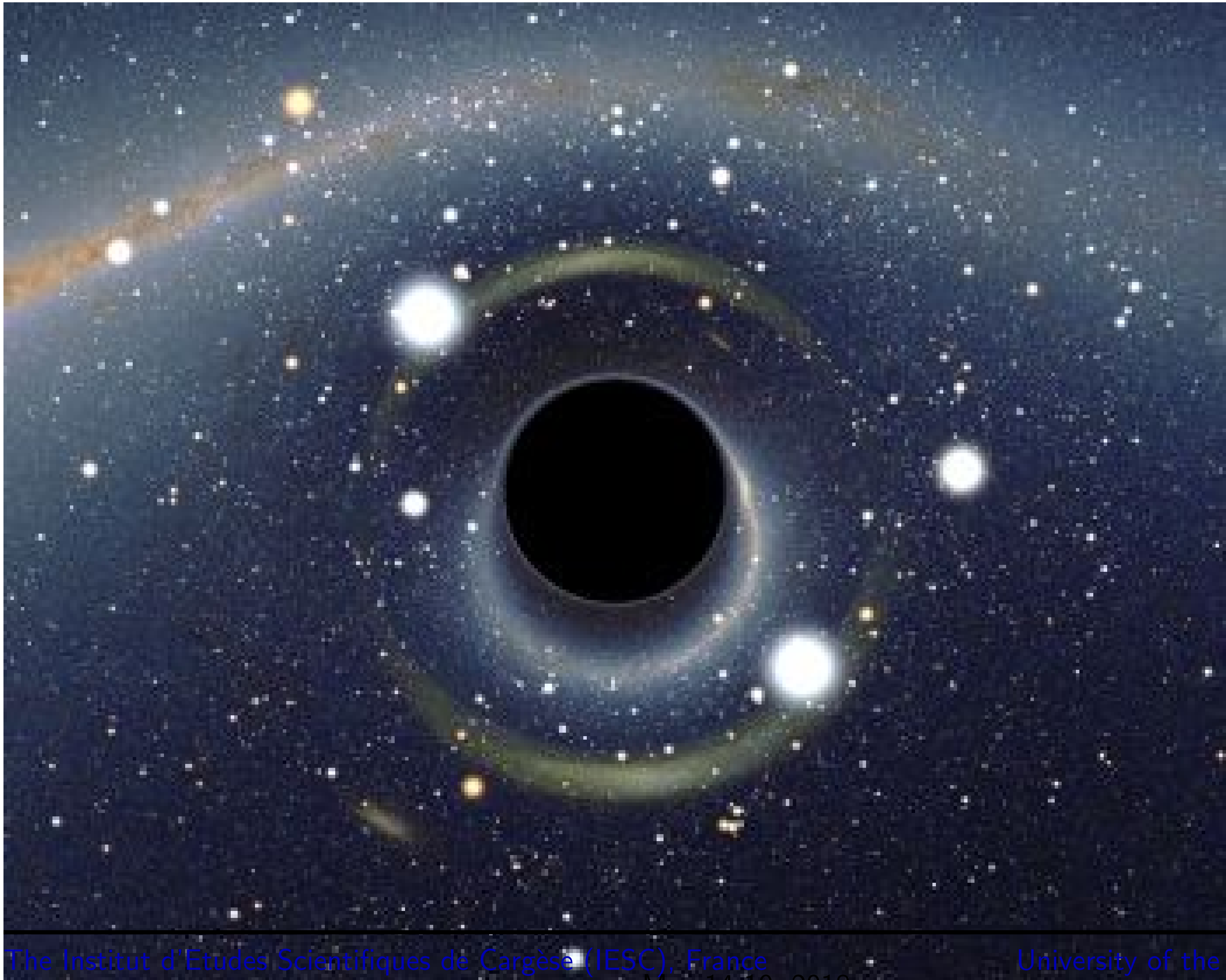
Point at the center of a black hole where spactime becomes infinite.



Black Hole:

Our solar system is filled with a wide range of celestial objects. Black hole is one of such objects, having so strong gravitational pull that no nearby matter, not even light can escape from its gravitational field. It is believed that collapse of a massive star under its own gravity leads to the formation of black hole.

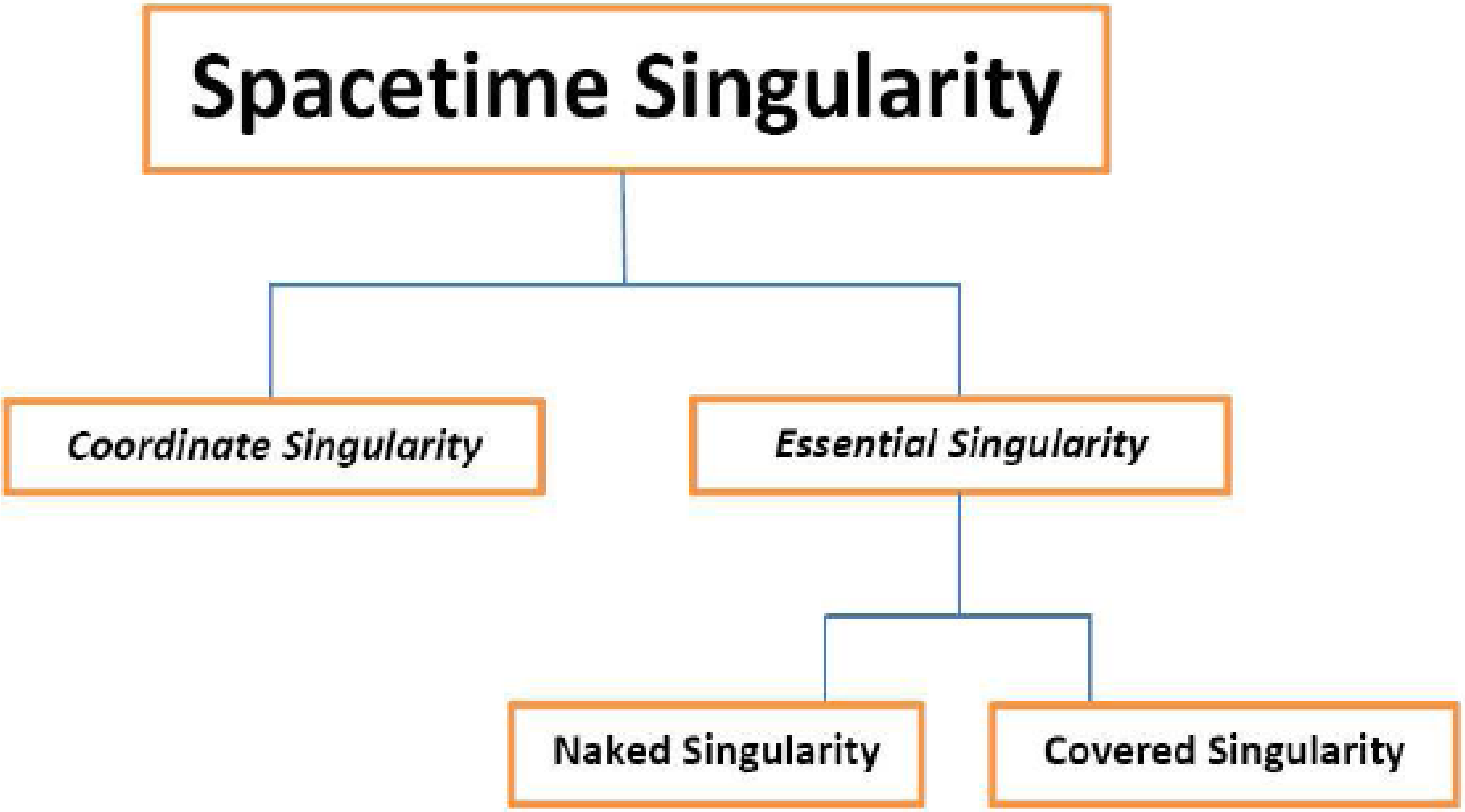
Black Hole



Spacetime Singularity:

The spacetime singularity is a region in spacetime where the physical quantities such as mass, density and spacetime curvature become infinite and usual laws of physics breaks down at such singularity. In other words trajectories of material particles will come to a sudden end (disappear from the space-time).

Classification of Spacetime Singularity:



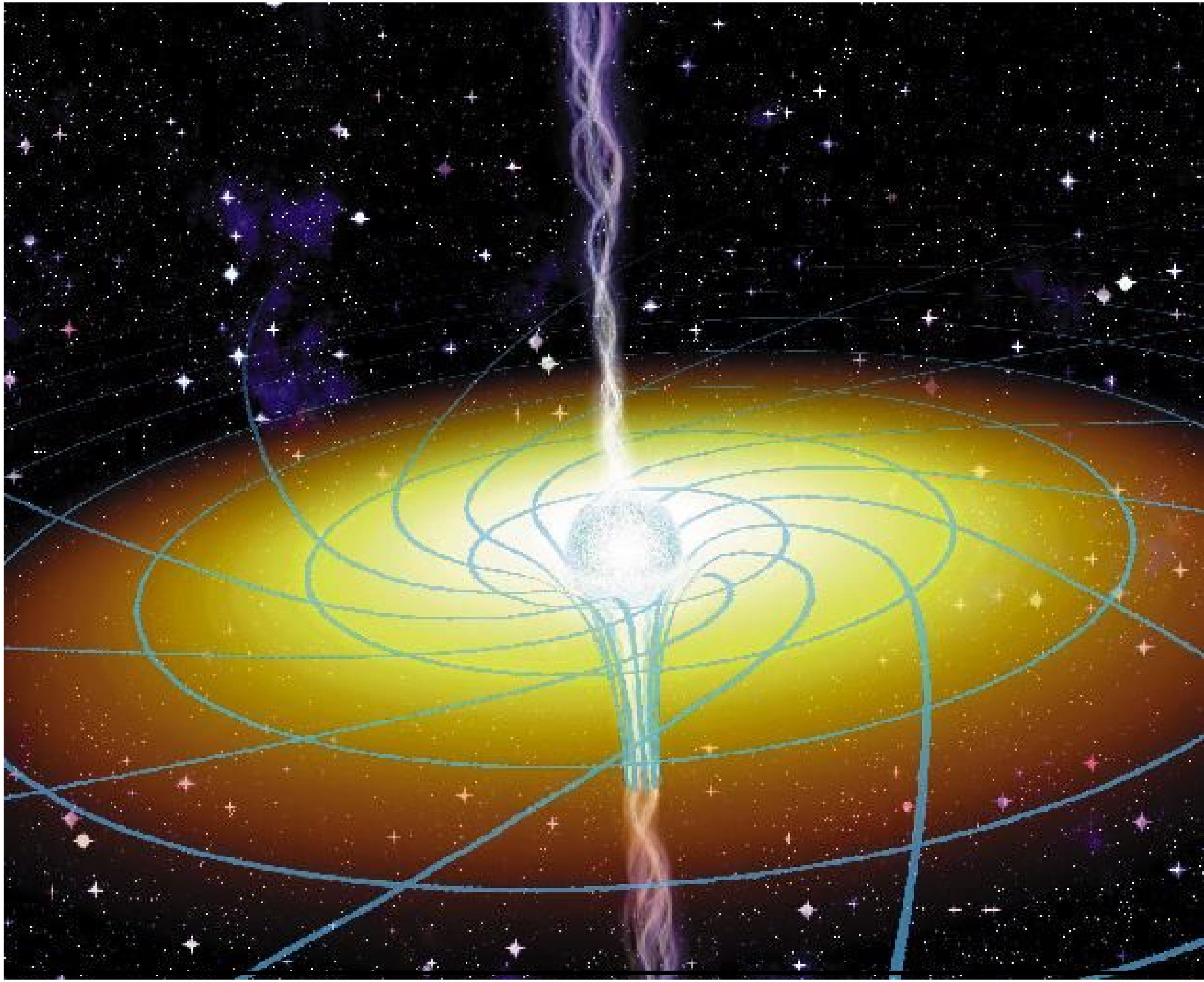
Naked Singularity:

A singularity that is not enclosed by a event horizon is called a naked singularity. These singularities can be visualized by distant observers.

- It represents the formation of high curvature and strong gravity regions.
- It also provides a source of gravitational waves.

Singularity

Singularity



Cosmic Censorship Hypothesis:

It states that end state of gravitational collapse of a massive star must be a black hole covered by an apparent horizon i.e. not observable singularity. It has two versions:

- Weak Conjecture
  - It states that singularity must be hidden within a black hole at the end state of collapse. More precisely, the singularity may not be

globally

naked.

- Strong Conjecture
  - It states that singularity can never be a locally naked at the end state of the collapse.

There is no mathematical formulations for either version of CCC.

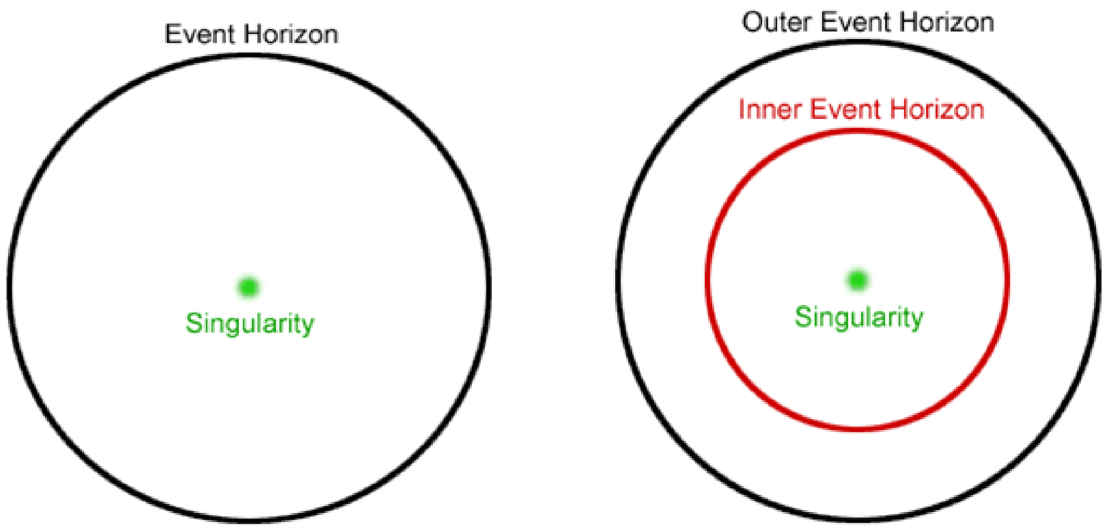


Figure 1: Schwarzschild Black Hole Reissner-Nordström Black Hole

Event Horizon:

Spherical boundary around the black hole, surrounding the singularity is said to be event horizon.

Event Horizon

Apparent Horizon:

The outer most boundary of the region containing trapped surface is called apparent horizon. In de-Sitter spacetime, the event horizon coincides with apparent horizon. For general cosmological models, apparent horizon exists necessarily while event horizon may not exist. The gravitational collapse of a massive star that leads to the formation of black hole, predicts that the event horizons are formed earlier than the apparent horizons. The existence of apparent horizons can be explained by the slicing of spacetime. For example, the slicing of Schwarzschild geometry would describe the existence of event horizon but no apparent horizon.

Trapped Surface:

In 1965, Penrose introduced the concept of trapped surface in GR for the development of singularity theorem to study the gravitational collapse, black hole and many types of horizons. In four dimensional spacetime trapped surface is defined as a two dimensional imbedded surface such that any portion of it has at least initially, a decreasing area along any future evolution direction.

Curvature:

The rate of rotation of curve at any particular point is called curvature of that curve .The curvature is the measure of change of direction of curve. There are two types of curvature:

(i) Extrinsic curvature, (ii) Intrinsic curvature.

Extrinsic Curvature:

The curvature of a surface, which is embedded in a surface of one more dimension is called extrinsic curvature. The measure of the extrinsic curvature is obtained by specifying the direction of unit normal to the hypersurface.

Intrinsic Curvature:

The deviation of a surface from the flat geometry within the whole surface is called intrinsic curvature of a surface. This type of curvature can be detectable by the inhabitants of the surface as well as by the outside observer. It is defined over each point of spacetime and depends on the metric tensor of the spacetime.

Comet:

A comet is an icy small Solar System body that, when passing close to the Sun, heats up and begins to outgas, displaying a visible atmosphere or coma, and sometimes also a tail. These phenomena are due to the effects of solar radiation and the solar wind upon the nucleus of the comet.