Dynamics and Cosmology of Self-Gravitating Media

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Outlines

- Hydrodynamics of Fluids
- Building a Lagrangian formalism for Hydrodynamical models
- Classification of the Media: Symmetries, Thermodynamical and Mechanical properties
- Perturbations on a FRW background
- The 6-th Dof of Massive gravity
- $\Lambda-$Media: Fluids with $p+\rho=0$
- Entropic Dark Perfect Fluids (DM+DE)

Relativistic Hydrodynamics equation of motion

The dynamical content of the hydrodynamic equation is simply conservation of energy and other conserved global charges

- $G_{\mu\nu} = R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R = T_{\mu\nu}$ Einstein equations
- $\nabla^{\nu} T_{\mu\nu} = 0$ EMT conservation
  $\nabla^{\mu} N_{\mu} = 0$ density conservation
  $\nabla^{\mu} S_{\mu} \geq 0$ second law of thermo.
- Thermodynamics:
  - Fundamental relation: $s = s(\rho, n)$ or $\rho = \rho(s, n)$
  - Euler Relation: $p + \rho = T\, s + \mu\, n$
  - First principle: $d\rho = T\, ds + \mu\, dn$
  - Gibbs-Duhem relation: $dp = s\, dT + n\, d\mu$

Perfect Fluids

variables: u^μ $(u^2 = -1)$, n, ρ, p

$$T_{\mu\nu} = \underbrace{(\rho + p)}_{\text{Euler: } T\, s + \mu\, n} u_\mu\, u_\nu + p\, g_{\mu\nu} = (T\, s_\mu + \mu\, n_\mu) u_\nu + p\, g_{\mu\nu}$$

$$n^\mu = n\, u^\mu, \qquad s^\mu = s\, u^\mu$$

$$EoM \quad : \quad \dot\rho + \theta(\rho + p) = 0 \to \quad \underbrace{T\, \nabla^\alpha s_\alpha + \mu\, \nabla^\alpha n_\alpha}_{0 \leftrightarrow 0} = 0\,,$$

$$(\rho + p)\dot u_\mu + (\delta^\nu_\mu + u_\mu u^\nu)\nabla_\nu p = 0$$

$$\nabla^\alpha n_\alpha = 0 \quad \Rightarrow \quad \frac{\dot s}{s} = \frac{\dot n}{n} = -\theta \quad \Rightarrow \quad u^\mu \nabla_\mu \left(\frac{s}{n}\right) \equiv \dot\sigma = 0\,;$$

$(\theta = \nabla_\mu u^\mu\,,\ \dot f = u^\mu \nabla_\mu f)$ entropy per particle $\boxed{\sigma = s/n}$ is conserved along the flow lines (adiabatic fluid).

Thermodynamics with broken shift symmetry?

Take a generic potential $\boxed{\boxed{U(X,\ \varphi)}}$, $X \equiv (\partial\varphi)^2$

$$\boxed{EMT} \quad T_{\mu\nu} = U\, g_{\mu\nu} - 2\, X\, U_X\, \partial_\mu\varphi\, \partial_\nu\varphi = (p+\rho)\, u_\mu\, u_\nu + p\, g_{\mu\nu}$$

$$\rho = -U + 2\, X\, U_X, \quad p = U, \quad u^\mu = -\frac{\partial^\mu\varphi}{\sqrt{-X}}$$

$$\boxed{Thermo}: \quad T^2 = -X, \; s = -2\, U_X \sqrt{-X}, \quad s_\mu = s\, u_\mu$$

$$\rho + p = T\, s, \quad d\rho = T\, ds \quad OK!$$

$$u_\nu\, \nabla_\mu\, T^{\mu\nu} = 0 \to T\, \nabla^\nu\, s_\nu = \sqrt{-X}\, \partial_\varphi U = 0!$$

a perfect fluid is adiabatic $\nabla^\nu\, s_\nu = 0$

$$\text{Fields} \quad \boxed{g_{\mu\nu} \oplus \varphi^A}, \qquad A = 0, 1, 2, 3$$

- Lagrangian in Newtonian gauge
(2 tensors + 4 phonons)

$$\int \sqrt{g}\ \left(R + \mathcal{L}(\partial \varphi^A)\right)$$

DoF: 2 (tensor gravitational modes) + 4 (scalar fields) =6

- Lagrangian in Unitary Gauge $\varphi^A = x^A$: Massive Gravity

$$\int \sqrt{g}\ \left(R + \mathcal{L}(g^{00},\ g^{0i},\ g^{ij})\right)$$

DoF:(2 tensors + 2 vectors + 2 scalars) gravitational modes =6

Lagrangian of 4 scalar fields $\mathcal{L}(\partial\varphi^{A=0,1,2,3})$

Stuckelberg fields for spont. broken space-time symm.
$\varphi^{a=1,2,3}$ comoving coordinates of the continuous medium

φ^0 internal time of the medium

- Internal Symmetries on scalars φ^A: shift, $SO(3)_\varphi$, Non Linear extensions
- Symmetries , Media (Supersolids, solids, superfluids, fluids) and Thermodynamics
- Media Lagrangian $\subset$ Massive Gravity

Building the Lagrangian

$$\textit{shift sym.} \quad : \quad \underline{\varphi^A \to \varphi^A + \partial c^A} : \; C^{AB} = \partial_\mu \varphi^A \, g^{\mu\nu} \, \partial_\nu \varphi^B ,$$

$$u^\mu \sim \epsilon^{\mu\alpha\beta\gamma} \, e_{abc} \partial_\alpha \varphi^a \partial_\beta \varphi^b \partial_\gamma \varphi^c$$

Global internal "spatial" $SO(3)_\varphi$ sym. $A,B = 0,1,2,3 \, , \quad a,b,c = 1,2,3$

$$\underline{\varphi^0 \to \varphi^0}, \quad \underline{\varphi^a \to R^a_b \, \varphi^b} \qquad R \, R^T = I$$

Operator	Definition
X	$\partial\varphi^0 \cdot \partial\varphi^0$
b	$\sqrt{\det \|\boldsymbol{B}\|} = \sqrt{\det \|\partial\varphi^a \cdot \partial\varphi^b\|}$
Y	$u^\mu \partial_\mu \varphi^0$
y_n	$\sum_{ab} \left(\varphi^0 \cdot \partial\varphi^a \, [\boldsymbol{B}^n]^{ab} \, \partial\varphi^b \cdot \partial\varphi^0 \right) , \qquad n = 0, 1, 2, 3$
τ_n	$\mathrm{Tr}\left(\boldsymbol{B}^n\right) , \qquad n = 1, 2, 3$

$$U_{SO(3)_\varphi \;\; invariant} = U \underbrace{\left(X , \; Y , \; y_n , \; \tau_n \right)}_{9 \;\; operators}$$

Evaluation of the Mass parameters (Perturbative)

Unitary gauge: $\varphi^A \equiv x^A$

$$U(C^{AB} = \partial_\alpha \varphi^A \, g^{\alpha\beta} \, \partial_\beta \varphi^B) \to U(g^{AB})$$

$g_{\mu\nu} = a^2 \, (\eta_{\mu\nu} + h_{\mu\nu})$ and expansion at second order $\mathcal{O}(h)^2$

$$\int \sqrt{g} \; U(g^{AB}) = \int M_0 \; h_{00}^2 + M_1 \; h_{0i}^2 + \underbrace{M_2 \, h_{ij}^2}_{Graviton-mass} + M_3 \; h_{ii}^2 + M_4 \; h_{00} \; h_{ii}$$

$$M_i \propto M_i(U, \; \partial_k U, \; \partial^2_{k,r} U)\Big|_{back} \, , \qquad k,r = b, Y, X, \tau_i, y_i$$

Mechanical properties (Non Perturbative)

$$T_{\mu\nu} = \underbrace{p\; g_{\mu\nu} + (p+\rho)\; u_\mu\; u_\nu}_{Perfect\ Fluid} + \underbrace{q_\mu\; u_\nu + q_\nu\; u_\mu + \overbrace{\Pi_{\mu\nu}}^{Solid}}_{Super\ Fluid/Solid}$$

EMT	Lagrangian	Medium	Masses
$q_\mu = 0,\ \Pi_{\mu\nu} = 0$	$U(b,\ Y)$	Perfect Fluid	$M_{1,\,2} = 0$
$q_\mu = 0,\ \Pi_{\mu\nu} \neq 0$	$U(b,\ \tau_n,\ Y)$	Solid	$M_1 = 0$
$q_\mu \neq 0,\ \Pi_{\mu\nu} \propto\ q_\mu\ q_\nu$	$U(b,\ Y,\ X)$	SuperFluid	$M_2 = 0$
$q_\mu \neq 0,\ \Pi_{\mu\nu} \neq 0$	$U(b,\ Y,\ X,\ \tau_n,\ y_n)$	SuperSolid	$M_{1,2} \neq 0$

Symmetries and mechanical properties

Four-dimensional media		
Symmetries of the action	**LO scalar operators**	**Type of medium**
$\varphi^A \to \varphi^A + f^A\,, \quad \partial_\mu f^A = 0$	$X,\ Y,\ \tau_n,\ y_n$	supersolids
$\varphi^a \to \varphi^a + f^a(\varphi^0)$	$X,\ w_n$	
$\varphi^0 \to \varphi^0 + f(\varphi^0)$	$\tau_n,\ w_n,\ \mathcal{O}_{\alpha\beta n}$	
$\varphi^a \to \varphi^a + f^a(\varphi^0)$ & $\varphi^0 \to \varphi^0 + f(\varphi^0)$	w_n	
$\varphi^0 \to \varphi^0 + f(\varphi^a)$	$Y,\ \tau_n$	solids
V_sDiff: $\varphi^a \to \Psi^a(\varphi^b)\,, \quad \det\|\partial\Psi^a/\partial\varphi^b\| = 1$	$b,\ Y,\ X$	superfluids
$\varphi^0 \to \varphi^0 + f(\varphi^0)$ & V_sDiff	$b,\ \mathcal{O}_\alpha$	
$\varphi^0 \to \varphi^0 + f(\varphi^a)$ & V_sDiff	$b,\ Y$	perfect fluid
$\varphi^A \to \Psi^A(\varphi^B)\,, \quad \det\|\partial\Psi^A/\partial\varphi^B\| = 1$	$b\,Y$	p. f. with $\rho + p = 0$

Thermodynamics + Mechanics (Non Perturbative)

$(s/n) \equiv \sigma$

- Adiabatic: $u^\alpha \nabla_\alpha \sigma \equiv \dot{\sigma} = 0$
- isentropic: $\nabla_\alpha \sigma = 0$
- barotropic: $p = p(\rho)$
- irrotational: $u^\alpha = \nabla^\alpha \phi$

$$u^\alpha \nabla_\alpha \sigma = -\frac{1}{b} \nabla^\alpha \left(\frac{q_\alpha}{Y} \right)$$

$$q_\mu = 2\, Y \left[\sum_{n=0}^{3} U_{y_n} \left[\partial\varphi^0 \cdot \partial\varphi^a \right] \left[\boldsymbol{B}^n \right]^{ab} \cdot \partial_\mu \varphi^b + U_X \, \xi_\mu \right]$$

Example: Superfluid $U(b, Y, X)$

$$T_{\mu\nu} = p\, g_{\mu\nu} + (p+\rho)\, u_\mu\, u_\nu + q_\mu\, u_\nu + q_\nu\, u_\mu + \Pi_{\mu\nu}$$

$$\rho = -U + Y\, U_Y - 2\, Y^2\, U_X, \quad p = U - b\, U_b,$$

$$\zeta_\mu = (\delta^\alpha_\mu + u_\mu\, u^\alpha)\, \partial_\alpha \varphi^0$$

$$q_\mu = 2\, Y\, U_X\, \zeta_\mu, \quad \Pi_{\mu\nu} \propto \frac{q_\mu\, q_\nu}{\dots}$$

$$T = Y, \quad s = U_Y - 2\, Y\, U_X, \quad n = b, \quad \mu = -U_b$$

$$\rho + p = T\, s + \mu\, n,$$

$$u^\alpha \nabla_\alpha \sigma = -\frac{1}{b} \nabla^\alpha \left(\frac{q_\alpha}{Y} \right) \to \dot{\sigma} \propto M_1 \propto U_X$$

Classification for Perfect Fluids

Operators: $(b,\ Y)$ $(M_{1,2}=0)$ and separately X

Lag	M_0	M_1	M_2	M_3	M_4	M_1^{eff}	DoF	Features
$U(b)$	0	0	0	$\neq 0$	0	$\neq 0$	1	Barotropic, Isentropic
$U(Y)$	$\neq 0$	0	0	0	$\neq 0$	$\neq 0$	2	Barotropic, Adiabatic
$U(b,\ Y)$	$\neq 0$	0	0	$\neq 0$	$\neq 0$	$\neq 0$	2	Adiabatic
$U(X)$	$\neq 0$	$\neq 0$	0	0	0	0	1	Barotropic, Isent., Irrot.

Table: *Masses and thermodynamical classification of Perfect Fluids*

$$M_1^{eff}=M_1+\rho+p$$

Classification for Superfluids

Operators: $(b,\ Y,\ X)$ $(M_2 = 0)$

Lagrangian	M_0	M_1	M_2	M_3	M_4	M_1^{eff}	DoF	Feature
$U(b,\ X)$	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	$\neq 0$	2	
$U(Y,\ X)$	$\neq 0$	$\neq 0$	0	0	$\neq 0$	$\neq 0$	2	
$U(b,\ Y,\ X)$	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	$\neq 0$	2	
$U(\mathcal{O}_\alpha)$	0	$\neq 0$	0	0	0	$\neq 0$	1	Isentropic
$U(b,\ \mathcal{O}_\alpha)$	0	$\neq 0$	0	$\neq 0$	0	$\neq 0$	1	Isentropic

Table: *Masses and features of superfluids*

Operators: $(Y,\ \tau_{n=1,2,3})$ $(M_1 = 0)$

Lagrangian	M_0	M_1	M_2	M_3	M_4	M_1^{eff}	DoF	Features
$U(\tau_n)$	0	0	$\neq 0$	$\neq 0$	0	$\neq 0$	1	Isentropic
$U(\tau_n,\ Y)$	$\neq 0$	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	2	Adiabatic

Table: *Mass spectrum and thermodynamical classification of Solids*

We have found that media can be classified according to the following scheme:

- Adiabatic media with $\delta\sigma(\vec{x})$ time independent, $M_1 = 0$:
  - Perfect Fluids at finite Temperature: $U(b,\ Y)$
  - Solids at finite Temperature: $U(\tau_n,\ Y)$
- Isentropic media with $\delta\sigma = 0$ are characterised by $M_0 = 0$:
  - Perfect Fluid: $U(b)$
  - Solids: $U(\tau_n)$
  - Superfluid: $U(\mathcal{O}_\alpha)$, $U(X + Y^2)$
  - Supersolids: $U(\mathcal{O}_\alpha,\ \tau_n,\ y_n)$, $U(\mathcal{O}_{\alpha\beta n})$, $U(X + Y^2,\ \tau_n,\ y_n)$, $\sqrt{-X}\,\mathcal{U}_1(\tau_n,\ y_n) + Y\,\mathcal{U}_2(\tau_n,\ y_n)$

$$\mathcal{O}_{\alpha\beta n} = \left(\frac{X}{Y^2}\right)^\alpha \left(\frac{y_n}{Y^2}\right)^\beta, \quad \mathcal{O}_\alpha = \left(\frac{X}{Y^2}\right)^\alpha$$

Thermodynamics + Mechanics

- The Lagrangian $U(X)$ describes an irrotational isentropic perfect fluid with $\delta\sigma = 0$ for a combination of factors : $M_1^{eff} = M_2 = 0$ and $c_b^2 = c_s^2$.
- Isotropic media have zero anisotropic perturbation tensor $\Pi_{\mu\nu} = 0$ in the EMT (the two Bardeen potentials are automatically equal: $\Phi = \Psi$) and are characterised by $M_2 = 0$ (massless spin two graviton).
    - All Perfect Fluids;
    - All Superfluids.
- Generically, superhorizon perturbations for all media becomes adiabatic ($\lim_{k/\mathcal{H}\to 0} \delta\sigma(k,t) = \delta\sigma(k)$) despite, in superfluids and supersolids, entropy perturbations have a non trivial dynamics.
- Media with non-dynamical entropy perturbations (the Bardeen potential determines his evolution: $\delta\sigma = f(\Phi)$) are characterised by $M_1^{eff} = 0$: $U(b\,Y, X,\ w_n)$.

DoF structure: Scalar Perturbations

Bardeen Potential: Φ,
Phonons: $\Phi^0 = x^0 + \pi_0$, $\Phi^i = x^i + \partial^i \pi_L$

M_0	M_1	M_1^{eff}	Propagating DoF	Eqs Φ – $\delta\sigma$
$\neq 0$	$\neq 0$	$\neq 0$	Φ, π_0 (or π_L)	$\Phi'' + \ldots = 0$, $\delta\sigma'' + \ldots = 0$
0	$\neq 0$	$\neq 0$	Φ	$\Phi'' + \ldots = 0$, $\delta\sigma = 0$
$\neq 0$	0	$\neq 0$	Φ, π_0	$\Phi'' + \ldots = 0$, $\delta\sigma' = 0$
$\neq 0$	$\neq 0$	0	Φ	$\Phi'' + \ldots = 0$, $\delta\sigma + \ldots = 0$

Table: *Structure of the scalar equations of motion and degrees of freedom (DoF) in terms of the masses*

Structure of Cosmological Perturbations

$$\delta p = c_s^2\,\delta\rho + \Gamma, \quad \Gamma \propto (c_b^2 - c_s^2)\,\delta\sigma$$

$$\boxed{\Pi \propto M_2}, \quad \boxed{\delta\sigma \propto M_{1,0}}$$

Master Equation

$$\Phi'' + 3\,\mathcal{H}\left(c_s^2+1\right)\Phi' + \Phi\left[k^2\,c_s^2 + 3\,\mathcal{H}^2\left(c_s^2 - w\right)\right] = -\frac{\Pi}{6}\left[k^2 - 9\,\mathcal{H}^2\left(c_s^2 - w\right)\right] + \frac{\mathcal{H}\,\Pi'}{2} - \frac{a^2\,\Gamma}{4}\,;$$

$$\delta\sigma' = \frac{M_1}{\phi'^2}\;k^2\,(\pi_0 - \phi'\,\pi_L')$$

The entropy per particle is not conserved ($M_1 \neq 0$) and, for example, for superfluids it satisfies the following equation ($M_1^{eff} = M_1 + \rho + p$)

$$\left[\frac{\phi'^2\,M_1^{eff}}{6\,a^2\,M_1(w+1)\,\mathcal{H}^2}\delta\sigma'\right]' = k^2\left[\frac{\phi'^2\,(c_s^2 - 2\,c_b^2)}{6\,a^2\,(w+1)\,\mathcal{H}^2} + \frac{\phi'^2}{2\,M_0}\right]\delta\sigma + \left[\frac{2\,k^4\,\phi'\,(c_b^2 - c_s^2)}{3\,(1+w)\,\mathcal{H}^2}\right]\Phi.$$

The 6-th Mode

General structure of the quadratic Lagrangian ($\int \sqrt{g}\,(R+U)$)
Phonon pert: $\Phi^0 \equiv x^0 + \pi^0$, $\Phi^i \equiv x^i + \pi_i$,
$\pi_i \equiv \partial_i \pi_L + v_i \;\; (\partial^i v_i = 0)$

Metric pert: $g_{\mu\nu} = \eta_{\mu\nu} + \Phi\; \delta^0_\mu\, \delta^0_\nu + \Psi\; \delta^i_\mu\, \delta^i_\nu$

$$\mathcal{L}_2 = \mathcal{L}_U(\pi^A) + \mathcal{L}_U(\pi^A,\; \Phi,\; \Psi) + \mathcal{L}_R(\Phi,\; \Psi)$$

General Behaviours

(1) Phonons on Minkowski: stable for $\rho + p \neq 0$

(2) Phonons + GR perturbaz on Minkowski: Ghost

(3) Phonons + GR perturbaz on FRW and
$k^2 \gg \mathcal{H}^2$: stable (same stability conditions of (1))

The 6-th Mode: Phonons on Minkowski

Transverse vectors $\vec{V} = (V^1, V^2, V^3)$ one gets

$$L_\pi^{(V)} = \frac{1}{2} (M_1 + p + \rho) \; \vec{V}'^2 - k^2 \; M_2 \; \vec{V}^2 \,.$$

Scalars

$$E_s = M_0 \; \pi_0'^{\,2} + \frac{k^2}{2} \; (M_1 + p + \rho) \, \pi_L'^{\,2} + k^4 \; (M_2 - M_3) \, \pi_L^2 - \frac{k^2}{2} M_1 \, \pi_0^2 \,.$$

Imposing that the energy is bounded from below in both the scalar and vector sectors leads to

$$M_0 > 0 \,, \quad \boxed{-(p + \rho) < M_1 < 0} \,, \qquad M_2 > 0 \,, \qquad M_2 > M_3 \,.$$

Clearly, when $\rho + p > 0$, stability is possible, as it should be.

The 6-th Mode in Minkowski

The linear expansion of the medium action gives

$$S_\pi^{(1)} = \int d^4x \, \left[\sigma \, \dot{\pi}^0 + (\sigma - p - \rho) \, \partial_i \pi^i\right] = 0 \, ; \quad {\scriptstyle p,\ \rho,\ \sigma = const}$$

$$S_h^{(1)} = \int d^4x \, \left[T^{\mu\nu}_{Mink} \, h_{\mu\nu}\right] = 0, \quad {\scriptstyle T^{\mu\nu}_{Mink} = p \, \eta_{\mu\nu} + (\rho + p) \, u_\mu \, u_\nu = 0}$$

$p, \ \rho = 0 \to$ Ghost!!

1. $R \sim \frac{\rho,\, p}{M_{pl}^2} \ll k^2$ the flat space picture is adequate and the fluctuations of the spacetime metric can be neglected.
2. $R \sim \frac{\rho,\, p}{M_{pl}^2} \simeq k^2$ the background solution has to amended and the metric fluctuations are important.

The 6-th Mode

- Phonons on Minkowski space show no instability.
- If we Add dynamical gravity we get a massive gravity model with six degrees of freedom.
- massive gravity theories with six DoF on Minkowski are plagued by ghost instabilities.

WAY OUT:

- Mink. space is not a consistent background for a self-gravitating medium - massive gravity
- Taking a FLRW Universe; the stability analysis shows that generically no ghost instability is present and actually for modes with $k \gg \mathcal{H}$ the flat space results with gravity switched off are recovered.
- The fluid picture is incompatible with the requirement of Lorentz invariance of the medium energy-momentum tensor and leads again to $p = \rho = 0$ and inevitably the Lagrangian has to be tuned to get less than six DoF.

Λ-Media

Suppose that we mesure $w \equiv p/\rho \equiv -1$, can we conclude then that dark energy behaves as a CC?

Cosmological Constant : $\quad T_{\mu\nu} = \Lambda\, g_{\mu\nu}, \quad \delta\rho = -\delta p = 0$

Improved CC : $\quad T_{\mu\nu} = \Lambda\, g_{\mu\nu} + (q_\mu\, u_\nu + q_\nu\, u_\mu + \pi_{\mu\nu}), \quad \delta\rho = -\delta p \neq 0$

- Adiabatic: Perfect Fluid and Solid

$$q_\mu = \pi_{\mu\nu} = 0 \qquad U^\Lambda_{PF}(b\, Y), \qquad \varphi^A \to \psi^A(\varphi^B)\,, \qquad \det\left|\partial\psi^A/\partial\varphi^B\right| = 1$$

$$q_\mu = 0,\ \pi_{\mu\nu} \neq 0 \qquad U^\Lambda_S = U\left(Y\tau_1^{3/2}, \frac{\tau_2}{\tau_1^2}, \frac{\tau_3}{\tau_1^3}\right), \quad \varphi^0 \to \varphi^0 + f(\varphi^j)$$

$$q_\mu \neq 0,\ \pi_{\mu\nu} \neq 0 \qquad U^\Lambda_{SS} = U\left(Xw_1^3, \frac{w_2}{w_1^2}, \frac{w_3}{w_1^3}\right) \quad \varphi^j \to \varphi^j + f^j(\varphi^0)\,.$$

- Isentropic: Special Solids

$$q_\mu = 0,\ \pi_{\mu\nu} \neq 0 \qquad U^\Lambda_{IS} = U\left(\frac{\tau_2}{\tau_1^2}, \frac{\tau_3}{\tau_1^3}\right), \quad \varphi^j \to \varphi^j + f^j(\varphi^0)$$

$$q_\mu \neq 0,\ \pi_{\mu\nu} \neq 0 \qquad U^\Lambda_{ISS} = U\left(\frac{w_2}{w_1^2}, \frac{w_3}{w_1^3}\right), \quad \varphi^0 \to \varphi^0 + f(\varphi^0)$$

Λ-Media Cosmological Perturbations

Λ-Medium	$\delta\rho = -\delta p,\ \delta\sigma$	Ψ	Φ	δ_m	m_g
CC	0	0	0	const.	0
U^{Λ}_{PF}	0	0	0	const.	0
U^{Λ}_{S} & U^{Λ}_{SS}	$\delta\rho_0 = \delta\sigma_0$	$-2\ \Phi$	$\Phi_0\ a^2$	$\propto a^2$	$\neq 0$
U^{Λ}_{IS} & U^{Λ}_{ISS}	0	0	0	const.	$\neq 0$

Table: Features of the different Λ-Media. The quantities Φ_0 and $\delta\sigma_0$ are time independent.

$$\delta\rho = -\delta p = \delta\sigma = \frac{2\ k^2\ \Pi}{3\ a^2} = const$$

$$\Pi = -\frac{2\ M_2}{a^2}\ \pi_L \quad \text{anisotropic stress}$$

Entropic Dark Fluids: The Potential

Simplest Cosmological potentials

$$\boxed{p = w\ \rho}, \text{ with } \ w' = 0 \to c_s^2 = w$$

$$U = b^{1+w}\ f\left(\frac{Y}{b^w}\right), \quad U = b^{1+w}\ f\left(\frac{Y}{b^w}\right)$$

- Radiation Era
  $w = 1/3:\ \ U = \lambda_y\ Y^4 + \lambda_b\ b^{4/3}$ or $U = b^{4/3}\ f(Y\ b^{-1/3})$
- Matter Era $w = 0$: $U = \lambda_b\ b$ or $U = b\ f(Y)$
- Vacuum energy Era $w = -1$: $U = f(b\ Y)$.

Entropic Dark Fluids: Features

$$-U(b,\ Y) = Free\ Energy = \rho - T\ s$$

$$p = U - b\ U_b, \quad \rho = -U + Y\ U_Y,$$

$$\boxed{T = Y,\quad s = U_Y},\quad \boxed{b = n\,\mu = -U_b},\ \sigma \equiv \frac{s}{n} = \frac{U_Y}{b}$$

$$\nabla^\mu\,(n\,u_\mu) = \nabla^\mu\,(s\,u_\mu) = 0 \ \rightarrow\ \dot{Y} + c_b^2\,\theta\ Y = 0,\quad \dot{\sigma} = 0$$

$$\delta p = c_s^2\ \delta\rho + \Gamma \qquad (\Gamma\text{:intrinsic entropy})$$

$$\Gamma = \underbrace{b\ Y\ (c_b^2 - c_s^2)}_{t-dependent}\quad \underbrace{\delta\sigma}_{k-dependent}$$

$$c_s^2 = \frac{(U_Y - b\ U_{bY})^2 - b^2\ U_{b^2}\ U_{Y^2}}{U_{Y^2}\ (\rho + p)},\quad c_b^2 = \frac{U_Y - b\ U_{bY}}{Y\ U_{Y^2}}$$

ΛCDM	$U(b,\ Y)$	Radiation $+\ U(b,\ Y)$
Γ_{rel}	Γ_{int}	$\Gamma_{int} + \Gamma_{rel}$

$$\Gamma_i = \delta p_i - c_s^2\ \delta\rho_i\,, \quad \delta p - c_s^2\ \delta\rho = \Gamma_{tot} = \sum_i^{all\ fluids} \left[\delta p_i - c_s^2\ \delta\rho_i\right] = \underbrace{\sum_i \Gamma_i}_{\Gamma_{int}} + \underbrace{\sum_i (c_{s\ i}^2 - c_s^2)\ \delta\rho_i}_{\Gamma_{rel}}$$

$$\boxed{\Gamma_i = \left.\frac{\partial p_i}{\partial\sigma_i}\right|_{\rho_i} \delta\sigma_i} = 0\ for\ \begin{cases} Barotropic\ fluids & \frac{\partial p_i}{\partial\sigma_i}|_{\rho_i} = 0 \to p = p(\rho) \\ Isentropic\ fluids & \delta\sigma_i = 0 \end{cases}$$

Superhorizon evolution comoving curvature perturbations $\mathcal{R}$

$$\mathcal{R} \underbrace{=}_{k\to 0} \underbrace{\int_0^a \frac{\Gamma_{tot}}{(\rho+p)\ a'}}_{entropic}\ da' + \underbrace{\mathcal{R}_0}_{adiabatic}$$

Example: Single Fluid Universe

Potentials that fit exactly the ΛCDM background parameters: $w = \frac{p}{\rho}$ and $c_s^2 = \frac{p'}{\rho'}$

- $$U(b) = -6\,\mathcal{H}_0^2\,(\underbrace{\Omega_r\ b^{4/3}}_{Radiation} + \underbrace{\Omega_m\ b}_{Matter} + \underbrace{\Omega_\Lambda}_{Vacuum:\ CC}):$$

  $s = 0 \rightarrow \mathcal{R} = 0$

- $$U(b, Y) = 6\,\mathcal{H}_0^2\,(\underbrace{\frac{\Omega_r}{3}\ Y^4}_{Radiation} - \underbrace{\Omega_m\ b}_{Matter} - \underbrace{\Omega_\Lambda}_{Vacuum:\ CC}),$$

  $T = \frac{T_0}{a}$, $\mathcal{R} = \propto \delta\sigma_0\ \frac{a}{3\ a + 4\ a_{eq}}$

- $$U(b, Y) = 6\,\mathcal{H}_0^2\,(-\underbrace{\Omega_r\ b^{4/3}}_{Radiation} - \underbrace{\Omega_m\ b}_{Matter} + \underbrace{\Omega_\Lambda\ (b\ Y)^2}_{Vacuum}),$$

  $T = T_0\ a^3$, $\mathcal{R} \propto \delta\sigma_0\ \frac{a^4}{3\ a + 4\ a_{eq}}$

Example: Two Fluid Universe: Rad+ Dark Fluid

Background $\equiv$ ΛCDM
Perturbations: ΛCDM + Entropic DF=DM+DE

$$\int \sqrt{g}\left(R + U_{rad} + U\underbrace{DF}_{DM+DE}\right)$$

$$U_{rad} = 6\,\mathcal{H}_0^2\left(\frac{\Omega_r}{3}\,\hat{\mathbf{Y}}^4\right)$$

$$U_{DM} = 6\,\mathcal{H}_0^2\,(-\underbrace{\Omega_m\,b}_{Matter} + \underbrace{\Omega_\Lambda\,(b\,Y)^2}_{Vacuum}),$$

$$T_{DF} = T_0\,a^3$$

$$\mathcal{R} \propto \underbrace{\delta\sigma_0(k)\,\frac{a^4}{3\,a + 4\,a_{eq}}}_{Intrinsic\ entropy} + \underbrace{s_0(k)\,\frac{a}{3\,a + 4\,a_{eq}}}_{relative\ entropy\ \equiv \Lambda CDM}$$

Conclusions

DoF: four Goldstone Modes $\Phi^A \Rightarrow U(\partial\Phi^A)$

Hydrodynamical + Thermodynamical eqs $= EoM_{\Phi^A}$

Internal Symmetries $\leftrightarrow$ Mechanics $\leftrightarrow$ Thermodynamics

Applications:
- *FRW Perturbations*
  - *Bardeen pert.* Φ
  - *entropy pert.* $\delta\sigma$
- $6-$ *th DoF of Massive GR*
- Λ *Media*
- *Entropic DM*

Building the Lagrangian

$$\varphi^{A=0,1,2,3} \underbrace{\Rightarrow}_{\text{shift sym.}:\underline{\varphi^A \to \varphi^A + \partial c^A}} \partial\varphi^A$$

$$\underbrace{\Rightarrow}_{\text{Lorentz Scalar}} C^{AB} = \partial_\mu\varphi^A g^{\mu\nu} \partial_\nu\varphi^B \underbrace{\rightarrow}_{SO(3)_\Phi} \underbrace{C^{00} \equiv \mathbf{S}}_{Scalar}, \underbrace{C^{0a} \equiv \mathbf{V}}_{Vector}, \underbrace{C^{ab} \equiv \mathbf{T}}_{Tensor}:$$

$$\mathbf{S}, \ Tr[\mathbf{T}^{n=1,2,3}], \ \mathbf{V} \cdot \mathbf{T}^{n=0,1,2,3} \cdot \mathbf{V}$$

$$\underbrace{\Rightarrow}_{\text{Lorentz Vector},\ SO(3)_\Phi \text{ Scalar}} u^\mu \sim \epsilon^{\mu\alpha\beta\gamma} e_{abc}\partial_\alpha\varphi^a\partial_\beta\varphi^b\partial_\gamma\varphi^c \underbrace{\rightarrow}_{\text{Lorentz Scalar}} u^\mu\partial_\mu\varphi^0$$

9 operators

$A, B = 0, 1, 2, 3\,, \quad a, b, c = 1, 2, 3$

- At lowest order in derivatives: $C^{AB} = \partial_\mu \varphi^A \, g^{\mu\nu} \, \partial_\nu \varphi^A$

$$S = \int d^4x \sqrt{-g} \, \left(M_{pl}^2 \, R \, + \, U(C^{AB})\right)$$

global internal spatial $SO(3)_\Phi$ symmetry

$$\varphi^0 \to \varphi^0, \quad \varphi^a \to R^a_b \, \varphi^b \qquad R \, R^T = I, \quad a = 1, 2, 3$$

$\underline{SO(3)_\Phi \text{ Tensors}}$: C^{ab}, C^{a0}, C^{00}

Operator	**Definition**
C^{AB}	$g^{\mu\nu} \, \partial_\mu \varphi^A \, \partial_\nu \varphi^B$, $\quad A, B = 0, 1, 2, 3$
B^{ab}	$C^{ab} = g^{\mu\nu} \partial_\mu \varphi^a \, \partial_\mu \varphi^b$, $\quad a, b = 1, 2, 3$
Z^{ab}	$C^{a0} \, C^{b0}$
X	C^{00}
W^{ab}	$B^{ab} - Z^{ab}/X$

Lagrangian of 4 scalar + shift $+SO(3)_\Phi$ symm.

$SO(3)_\Phi$ Scalars

$$L.V.: \quad u^\mu = -\frac{\epsilon^{\mu\alpha\beta\gamma}}{6\, b\, \sqrt{-g}} \epsilon_{abc}\, \partial_\alpha \Phi^a\, \partial_\beta \Phi^b\, \partial_\gamma \Phi^c, \qquad \mathcal{V}^\mu = -\frac{\nabla^\mu \Phi^0}{(-X)^{1/2}}$$

L.S.:

Operator	Definition
X	C^{00}
b	$\sqrt{\det \boldsymbol{B}} = \sqrt{\det \partial\varphi^a\, \partial\varphi^b}$
Y	$u^\mu \partial_\mu \Phi^0$
y_n	$\mathrm{Tr}\left(\boldsymbol{B}^n \cdot \boldsymbol{Z}\right), \quad n = 0, 1, 2, 3$
τ_n	$\mathrm{Tr}\left(\boldsymbol{B}^n\right), \quad n = 1, 2, 3$
w_n	$\mathrm{Tr}\left(\boldsymbol{W}^n\right), \quad n = 0, 1, 2, 3$
$\mathcal{O}_{\alpha\beta n}$	$\left(X/Y^2\right)^\alpha \left(y_n/Y^2\right)^\beta$

$$U_{SO(3)_\Phi\ invariant} = U \underbrace{\left(X,\ Y,\ y_n,\ \tau_n\right)}_{9\ operators}$$

The 6-th Mode

- Low energy phonon-like excitations of generic media on Minkowski space showing that typically no instability is present.
- Adding dynamical gravity, in the unitary gauge, self-gravitating media are equivalent to massive gravity and six degrees of freedom are present.
- massive gravity theories with six DoF on Minkowski are plagued by ghost instabilities and a great effort has been devoted trying to find a non-perturbative way to project out the unwanted (ghost) sixth mode.
- The point is that flat space is not a consistent background for a self-gravitating medium and for massive gravity, unless the background pressure and energy density is set to zero.
- Taking a FLRW Universe; the stability analysis shows that generically no ghost instability is present and actually for modes with $k \gg \mathcal{H}$ the flat space results with gravity switched off are recovered.
- The fluid picture is incompatible with the requirement of Lorentz invariance of the medium energy-momentum tensor and leads again to $p = \rho = 0$ and inevitably the Lagrangian has to be tuned to get less than six DoF.