Hořava gravity in the aftermath of GW170817: constraints and concerns

A. Emir Gümrükçüoğlu

ICG, University of Portsmouth

[*Based on collaborations with A. Coates, M. Colombo, M. Saravani and T. Sotiriou*]

[arXiv:1410.6360; 1503.07544; 1604.04215; 1711.08845]

Hot topics in Modern Cosmology, Cargèse 16 May 2018

Introduction

- Lorentz invariance is an empirical fact, but is it necessary?
- Constraints on LV in gravity far weaker than in matter.

Motivations for considering LV in gravity sector

- **Concrete framework for testing LI:** Æ [Gasperini'87; Jacobson, Mattingly'00]
- **Cosmological problems:** alternative to inflation [Magueijo '08], dark energy [Afshordi '08], dark matter [Mukohyama'09]
- **Quantum gravity:** NCFT [Douglas, Nekrasov '01], Hořava gravity [Hořava '09]

Hořava gravity: a self-consistent Lorentz violating gravity theory

- P.C. renormalisable [one version *is* renormalisable [Barvinsky et al.'16]]
- Low energy limit compatible with observations $\Longrightarrow$ [Part 1]
- LV in gravity sector (even only in the UV) can still impact the matter sector in the IR $\Longrightarrow$ [Part 2]

Hořava's idea
Anisotropic scaling

Higher order curvature corrections

- Modifying GR with high order curvature terms
$$\delta\mathcal{L} = \alpha\, R_{\mu\nu}R^{\mu\nu} + \beta R^2$$
Modified propagator has an improved UV behavior
$$\frac{1}{k^2 - \frac{k^4}{M^2}} = \frac{1}{k^2} - \frac{1}{k^2 - M^2}$$
[Stelle '77]
- ∂_i^4 improves UV $\Longleftrightarrow$ ∂_t^4 compromises unitarity

Anisotropic scaling

- Hořava's idea: *anisotropic scaling* in UV $\quad \vec{x} \to b^{-1}\vec{x}, \quad t \to b^{-z} t.$
- $\partial_t^2 \leftrightarrow \partial_i^{2z} \quad \Longrightarrow$ 2 time derivatives but higher spatial derivatives.
- $z \geq 3 \Rightarrow$Power-counting renormalisable (see [Visser'09] for detailed proof)
- Cost: violation of Lorentz invariance.

Generalised Hořava gravity

Building blocks

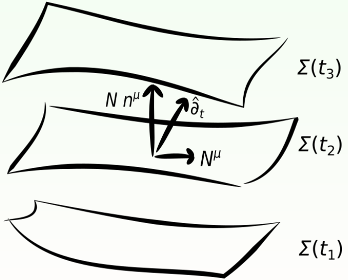
Symmetry

- Momentum dimensions from scaling: $[x] = -1$, $[t] = -z$.
- A compatible symmetry: foliation-preserving diffeos (FDiff)

$$t \to t'(t) \qquad \vec{x} \to \vec{x}'(t, \vec{x})$$

- ADM decomposition provides a natural parametrization

$$ds^2 = -N^2\, c^2 dt^2 + g_{ij}\, \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$



Building blocks

$$\begin{array}{lcccl} [\pounds_n] & = & z & \longleftrightarrow & \partial_t \\ [K_{ij}] & = & z & \longleftrightarrow & \dot{\phi} \\ [D_i] & = & 1 & \longleftrightarrow & \partial_i \\ [a_i] & = & 1 & \longleftrightarrow & \partial_i \phi \\ [R_{ij}] & = & 2 & \longleftrightarrow & \partial_i \partial_j \phi \end{array}$$

$$K_{ij} \equiv \pounds_n g_{ij}$$

$$a_i \equiv \partial_i \log N$$

Generalisation to Hořava gravity

Building the most general Lagrangian (i.e. without projectable functions) at $z = 3$

- $\underline{z = 3}$ Minimal model: All parity even FDiff scalar terms up to $2\,z = 6$ spatial derivatives.

$$\mathcal{L}_{HG} = (1-\beta)K_{ij}K^{ij} - (1+\gamma)\,K^2 + \alpha\,a_i a^i + R + \frac{1}{M_*^2}\mathcal{L}_4 + \frac{1}{M_*^4}\mathcal{L}_6$$

with

$$\begin{aligned} \mathcal{L}_4 &= \alpha_1\,R\,D_i a^i + \alpha_2 D_i a_j D^i a^j + \beta_1 R_{ij}R^{ij} + \beta_2 R^2 + \ldots\,, \\ \mathcal{L}_6 &= \alpha_3 D_i D^i R\,D_j a^j + \alpha_4 D^2 a_i D^2 a^i + \beta_3 D_i R_{jk} D^i R^{jk} + \beta_4 D_i R D^i R + \ldots\,. \end{aligned}$$

[Blas, Pujolàs, Sibiryakov '09-'10]

- Preferred foliation: gauge symmetry is less restrictive than full diffs. $t \to t'(t)$ not enough to remove 1 dof.
- 2 tensor gravitons + 1 scalar graviton.

Graviton propagation in vacuum

- Dispersion relation for tensor perturbations

$$\omega_T^2 = \overbrace{\frac{1}{1-\beta}}^{c_T^2} k^2 - \beta_1 \frac{k^4}{M_*^2} - \beta_3 \frac{k^6}{M_*^4}$$

- Scalar perturbations $\omega_S^2 \sim f(k)/g(k)$, but in the UV it goes $\omega_S^2 \propto \frac{k^6}{M_*^4}$, and in IR

$$\omega_S^2 = \underbrace{\frac{(2-\alpha)(\gamma+\beta)}{\alpha(1-\beta)(2+3\,\gamma+\beta)}}_{c_S^2} k^2 + \mathcal{O}(k^4)$$

- At low momenta ($k \ll M_*$), the IR effective theory contains three parameters α, β, γ. In the limit $\alpha, \beta, \gamma \to 0$, one recovers ∼GR.

Constraints on the IR theory

Theoretical consistency

1. *Unitarity*: Scalar kinetic term should be positive:

$$\frac{2+3\,\gamma+\beta}{\gamma+\beta} > 0$$

2. *Perturbative stability*: Real propagation speeds $c_T^2, c_S^2 > 0$

$$0 < \alpha < 2\,, \qquad \beta < 1$$

Constraints on the IR theory

Theoretical consistency

3. *Perturbative regime in IR*: Theory strongly coupled above scale M_{SC}

$$M_{SC} \simeq \sqrt{\alpha}\, M_p \begin{cases} c_S^{3/2} & , \; c_S^2 < 1 \\ c_S^{-1/2} & , \; c_S^2 > 1 \end{cases}$$

[Kimpton, Padilla '10; AEG, Saravani, Sotiriou '17]

We assume that IR theory stays perturbative. If the UV terms become relevant at a lower scale, strong coupling does not kick in:

$$M_* < M_{SC}$$

[Blas, Pujolàs, Sibiryakov '10]

An upper bound on UV physics! We will come back to this relation later.

Constraints on the IR theory
Observational constraints

4. *BBN*: Scalar graviton rescales gravitational constant differently in cosmology and Newtonian limit. Compared to GR, weak interactions freeze out later/earlier, modification in primordial helium abundance $\Delta Y_p = 0.08(G_C/G_N - 1)$ [Carroll, Lim'04]

$$\left|\frac{\alpha + 3\,\gamma + \beta}{2 + 3\,\gamma + \beta}\right| < \frac{1}{8}$$

5. *Gravi-Cherenkov*: Preventing UHECR from decaying into gravitons imposes

$$c_T^2 - 1 = \frac{\beta}{1-\beta} > -10^{-15}\,,$$

[Moore, Nelson '01]

For scalar modes, calculation in progress. Results for Æ suggest a subluminal margin of 10^{-15} is allowed [Elliott, Moore, Stoica '05]. For our purposes, $c_S^2 - 1 > 0$ is sufficiently accurate.

Constraints on the IR theory
Observational constraints

6. *ppN*: Preferred-frame effect parameters $|\alpha_1| < 10^{-4}, \ |\alpha_2| < 10^{-7}$ [Will'06]

$$\left|\frac{4(\alpha - 2\,\beta)}{1-\beta}\right| < 10^{-4}, \quad \left|\left(\frac{\alpha - 2\,\beta}{2\,\alpha}\right)\left(1 - \frac{(\alpha - 2\,\beta)(1+\beta+2\,\gamma)}{(1-\beta)(\beta+\gamma)}\right)\right| < 10^{-7}$$

- Most studies pre-LIGO focused on $\alpha = 2\,\beta$ plane.

7. *Binary pulsars*: Scalar graviton
   $\Rightarrow$ increased orbital decay
   due to dipolar radiation.

Situation pre-GW170817$\longrightarrow$

[Yagi, Blas, Barausse, Yunes '14]

$[\alpha, \beta \lesssim 10^{-2}, \ \gamma \lesssim 10^{-1}]$

On the $\alpha = 2\,\beta$ plane, binary pulsars provide the strongest constraints

Constraints on the IR theory
Observational constraints

8. *ppN*: Preferred-frame effect parameters $|\alpha_1| < 10^{-4}$, $|\alpha_2| < 10^{-7}$ [Will'06]

$$\left|\frac{4(\alpha - 2\,\beta)}{1-\beta}\right| < 10^{-4}, \quad \left|\left(\frac{\alpha - 2\,\beta}{2\,\alpha}\right)\left(1 - \frac{(\alpha - 2\,\beta)(1+\beta+2\,\gamma)}{(1-\beta)(\beta+\gamma)}\right)\right| < 10^{-7}$$

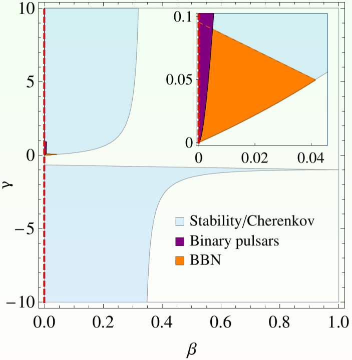
- Most studies pre-LIGO focused on $\alpha = 2\,\beta$ plane.

9. *Binary pulsars*: Scalar graviton
$\Rightarrow$ increased orbital decay
due to dipolar radiation.

Situation pre-GW170817$\longrightarrow$

[Yagi, Blas, Barausse, Yunes '14]

$$[\alpha, \beta \lesssim 10^{-2},\ \gamma \lesssim 10^{-1}]$$



On the $\alpha = 2\,\beta$ plane, binary pulsars provide the strongest constraints

Constraints on the IR theory

Aftermath of GW170817/GRB170817A

10 *GW*: GW with EM counterpart imposes a strong bound on c_T

$$-3 \times 10^{-15} \leq c_T - 1 \leq 7 \times 10^{-16}$$

[Abbott et al. '17]

- For Hořava gravity this implies $|\beta| \lesssim 10^{-15}$
  [*c.f.* bounds from 2014, $\beta \lesssim 10^{-2}$]
- Although no direct impact on other bounds, the "conventional" $\alpha = 2\beta$ plane no longer relevant. Theory now confined to the $\beta = 0$ plane with a thickness of 10^{-15}.
- Bounds on modified dispersion:

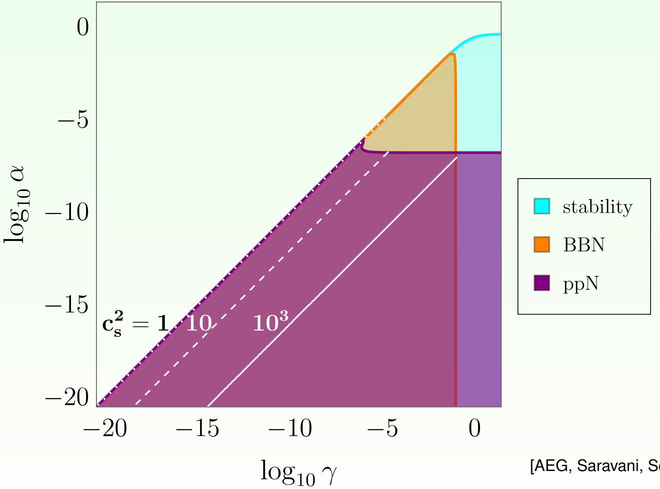
$$\omega_T^2 = k^2 + \frac{1}{M_*^2} k^4 + \mathcal{O}(k^6)\,,$$

Mild lower bound from mergers: $M_* \gtrsim \mathrm{meV}$ [Yunes, Yagi, Pretorius'16]
Not competitive with sub-mm searches:

$M_* \gtrsim 10\,\mathrm{meV}$, see e.g.[Adelberger et al.'09]

Constraints on the IR theory
Summary

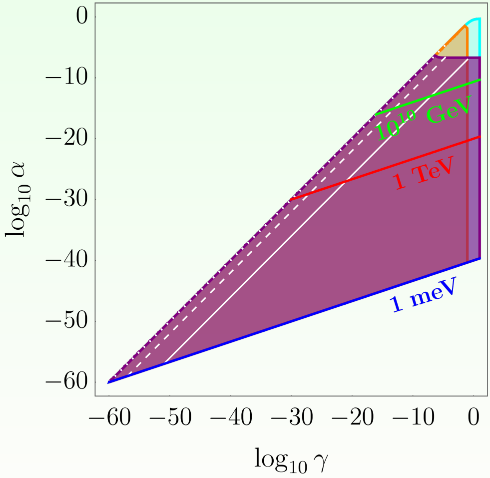
$\beta = 0$ surface



[AEG, Saravani, Sotiriou '17]

Constraints on the IR theory

Including the strong coupling scale



- ***M_* linked to IR parameters***
  $\text{meV} < M_* < M_{SC}$
  $\Rightarrow$ Improving bounds on M_* would reduce the parameter space (or rule out the theory).
- Allowed parameter space is a finite region
- Post-GW bounds stronger, but c_S remains unconstrained. Even a mild constraint on c_S would rule out a vast portion of parameter space [scalar GW counterpart?].

[AEG, Saravani, Sotiriou '17]

End of Part One

- Bounds presented here are independent of the model. We are testing the vacuum theory.
- The cancellation of ppN parameters for $\alpha = 2\,\beta$ is irrelevant in the aftermath of GW170817. Current bounds:

$$\alpha \lesssim 10^{-7}\ (\mathrm{ppN})\,, \qquad |\beta| \lesssim 10^{-15}\ (\mathrm{GW})\,, \qquad \gamma < 0.1\ (\mathrm{BBN})$$

- Relevant parameter range is $\beta = 0$ surface with a thickness of 10^{-15}
- Parameters further confined to a finite region, but c_S virtually unconstrained.
- New bounds on modified dispersions can impose further restrictions.
- Advantage: Information on UV scale from bounds on IR parameters!

Constraints on the IR theory

The weak bounds on M_*

- The IR theory is compatible with observations. But to adopt it as a fundamental description of gravity beyond the effective level, it should fit into the picture of rest of physics. This is not a trivial task.
- Current bound on the UV scale leaves an enormous window

$$\mathrm{meV} < M_* < 10^{15}\mathrm{GeV}$$

- Conversely, room for LV in matter sector *quite* narrow!

LV in the matter sector

Bounds on dim 4 operators

Constraints on maximum attainable velocity for different species
e.g. [Coleman, Glashow '98]

- Cherenkov radiation bound: $c_p - c_\gamma < 10^{-23}$
- Frame of CMB: $|c_m - c_\gamma| < 6 \times 10^{-22}$
- Neutrino oscillations: $|c' - c|_{\nu_e \nu_\mu} < 6 \times 10^{-22}$
- Radiative muon decay: $|c' - c|_{e\mu} < 4 \times 10^{-21}$
- Neutral kaons: $|c_{K_L} - c_{K_R}| < 3 \times 10^{-21}$.

$$\delta c \lesssim 10^{-23} \div 10^{-21}$$

Different SM species effectively see the same light cone.

LV in the matter sector

Effect of LV operators in matter

- Naïve estimate for LV matter gives $\Rightarrow \delta c^2 \sim 1\%$
  [Collins, Perez, Sudarsky, Urrutia, Vucetich '04]
- Toy model with 2 Lifshitz fields: $\delta c^2 = 0$ attractive IR fixed point (good), but RG flow too slow (not good). Unnaturally strong fine-tuning unavoidable.
  [Iengo, Russo, Serone '09]
- Assume dim 4 LV operators fine tuned away. Matter dispersion relation will get high order modification above some scale $M_{*,m}$, e.g.

$$E^2 = m^2 + p^2 + \frac{p^4}{M_{*,m}^2}$$

- Constraints from UHECR:
  $M_{*,m} \gg M_p$ [Liberati, Maccione'09]; [Saveliev, Maccione, Sigl'11]
  Synchrotron radiation constraints from Crab nebula:
  $M_{*,m} > 10^{-3} M_p$ [Liberati, Maccione, Sotiriou'12]
- For a universal LV scale $M_{*,m} \sim M_*$, the bound is in conflict with the allowed region for M_* in Hořava gravity ($< 10^{-4} M_p$).

Hiding LV from matter sector

- ***What if matter sector is Lorentz invariant at tree level?***
  Graviton loops will still communicate the violation in UV to dim 4 matter operators.

How to circumvent this issue?

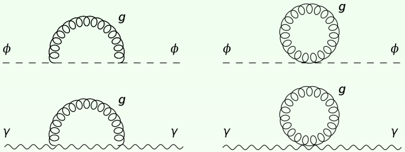
1. Accidental Lorentz invariance from common symmetry? e.g. SUSY. An extension of MSSM has LV operators at dim ≥ 5.
   [Groot-Nibbelink, Pospelov '05]
   For Hořava gravity, no known SUSY extension.
   [Xue '10; Redigolo '12; Pujolàs, Sibiryakov '12]
2. Strong dynamics in IR? Non-perturbative behavior may accelerate $\delta c^2 \to 0$, screening LV in IR. [Bednik, Pujolàs, Sibiryakov '13]
3. LV gravity theory with UV stabilization at M_* & LI matter sector
   [Pospelov, Shang '10]

$$M_p^{4-k-n} \mathcal{O}_{\rm LV}^{(n)} \mathcal{O}_{\rm SM}^{(k)} \xrightarrow[\text{gravitational dof}]{\text{integrate out}} M_p^{4-k} \left(\frac{M_*}{M_p}\right)^{\alpha} \mathcal{O}_{\rm SM,LV}^{(k)}$$

For $M_* \ll M_p$, the LV contributions can be under control.

Scale separation mechanism in Hořava gravity

- Hořava gravity with $z = 3$. A canonical scalar and a vector field, coupled minimally to gravity. 1-loop graviton corrections to scalar&vector propagators:



$$\mathcal{L}_\phi = -\frac{1}{2}\sqrt{-g}\partial^\mu\phi\partial_\mu\phi$$

$$\mathcal{L}_\gamma = -\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

$$c_v^2 - c_s^2 = (\dots)\frac{M_*^2}{M_p^2}\log\frac{\Lambda_{\rm UV}^2}{M_*^2} + (\dots)\frac{\Lambda_{UV}^2}{M_p^2}$$

[Pospelov, Shang '10]

- 2nd term divergent⇒Naturalness problem from vector graviton loops.
- Vector part of HG = Vector part of GR. Propagator $\sim \frac{1}{\vec{k}^2}$, not enough to stabilize the UV.

A quick fix

- Technically: $R_{ij} \propto \mathcal{O}(\text{vector}^2)$. Vector propagator contribution only from R and $K_{ij}K^{ij}$. Extensions with K_{ij}.
- Ad hoc resolution by Pospelov & Shang: $\frac{1}{M_*^2} D^i K_{ik} D_j K^{jk} \Longrightarrow \partial_t^2 \partial_i^2$. Keeps Scalar & Tensor $\omega_{UV}^2 \sim k^6$, but vector propagator becomes $\frac{1}{\vec{k}^4}$.
- The degree of non-universality of speeds:

$$c_v^2 - c_s^2 = (\dots)\frac{M_*^2}{M_p^2} \log \frac{\Lambda_{\rm UV}^2}{M_*^2}$$

- Good enough for the mechanism, but term of is beyond Hořava's counting. These are dominant kinetic terms in the UV

$$k^2 \omega_{UV}^2 \sim k^6$$

[Colombo, AEG, Sotiriou '14]

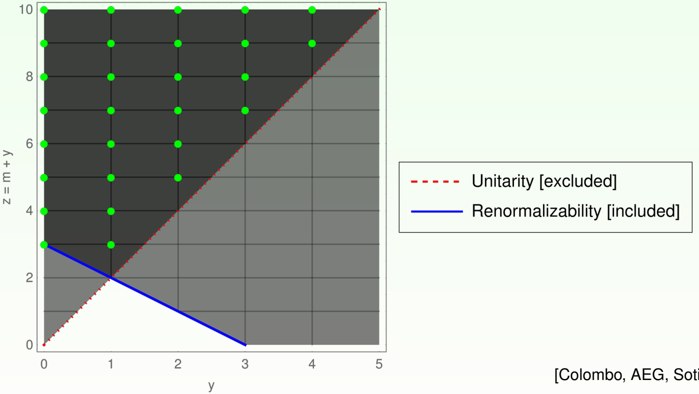
- Counting modified, i.e. $\partial_t \leftrightarrow \partial_i^2$. Can we use the new counting to construct mixed-derivative theories?

A class of Lifshitz–like extended theories

A Lifshitz scalar with mixed derivatives

$$S = \int dt\, d^3x \left[\dot\phi(-\triangle)^y\,\dot\phi - \phi(-\triangle)^z\phi + \lambda(\partial_t^{p_t}, \partial_i^{p_x}, \phi^n)\right]$$

Power-counting renormalisable and unitary Lifshitz–like theories



[Colombo, AEG, Sotiriou'15]

Extending Hořava gravity

An uninvited guest

- Minimal theory: $y = 1$ and $z = 3$. Therefore, the action is $z = 3$ Hořava action, plus the new terms with 2 spatial and 2 time derivatives:

$$\mathcal{L}_\times = D_i K_{jk} D_l K_{mn} M^{ijklmn} + 2\left(\sigma_1 \mathcal{A}_i \mathcal{A}^i + \sigma_2 \mathcal{A}_i D^i K + \sigma_3 \mathcal{A}_i D_j K^{ij}\right) + \cdots$$

$$\left[M^{ijklmn} \equiv \gamma_1\, g^{ij} g^{lm} g^{kn} + \gamma_2\, g^{il} g^{jm} g^{kn} + \gamma_3\, g^{il} g^{jk} g^{mn} + \gamma_4\, g^{ij} g^{kl} g^{mn}\right]$$

$$\mathcal{A}_i \equiv \pounds_n a_i = \frac{1}{2N}\left(\dot{a}_i - N^j D_j a_i - a_j D_i N^j\right)$$

- σ_1 term spoils the game. Lapse N (which was elliptical in standard HG) now becomes dynamical! The theory now has two scalar gravitons.
- The new dof $\sim \partial_i \delta N$ is a massive mode with

$$m^2 = -\frac{4\, M_*^2\, \alpha}{\sigma_1}$$

- Unitarity demands $\alpha > 0$ and $\sigma_1 > 0$, so the new mode either is a ghost, or has tachyonic instability. [Coates, Colombo, AEG, Sotiriou '16]
- Confirmed by Hamiltonian analysis [Klusoň '16]

End of Part Two

- Low energy percolation of LV from UV can be controlled by the separation of M_* and M_p, but requires fine tuning for Hořava gravity. We found extensions where the suppression realised naturally, but with an unstable extra mode.

Can we fix Hořava gravity? [short answer: possibly]

- Imposing projectability condition on $N = N(t)$ solves the extra mode problem. Good for matter coupling, but perturbative control lost in IR (although see [Izumi, Mukohyama'11; AEG, Mukohyama, Wang'11])
- Other extensions? Dropping parity, we can avoid introducing a new dof, while improving the vector propagator. This is not enough to completely remove the divergence, but the necessary tuning becomes milder. [AEG, in progress]
- UV complete Æ?

Final message: the issue of controlling LV in matter is not specific to HG, but a major challenge for any LV gravity.

Backup slides

Is the $M_* < M_{SC}$ assumption necessary?

Why do we try to hide the strong coupling? Typical answer:

Potential renormalisability of Hořava gravity relies on power counting, and thus perturbative expansion. Strong coupling spoils it.

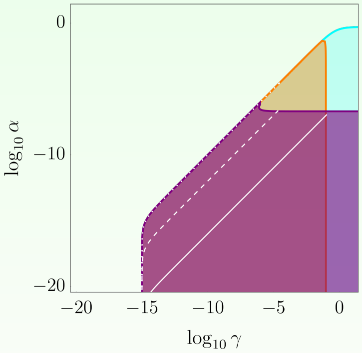
Does strong coupling imply loss of predictivity?

- Strong coupling at *intermediate scale*. Theory can still be weakly coupled in UV

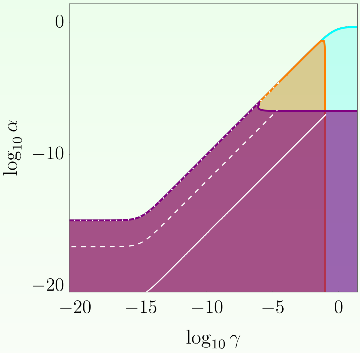
  e.g. [AEG, Mukohyama'11]

- If theory renormalisable, even in the SC regime, infinite # of coefficents in perturbative expansion will depend on finite # of parameters. $\Rightarrow$ SC does not imply loss of predictivity!
- This argument not verified as it requires non-perturbative tools/analyses
- SC might accelerate the flow to IR fixed point
  (Classical screening shown in:
  [Izumi, Mukohyama'11; AEG, Mukohyama, Wang'11])

$\beta = \pm 10^{-15}$ surface



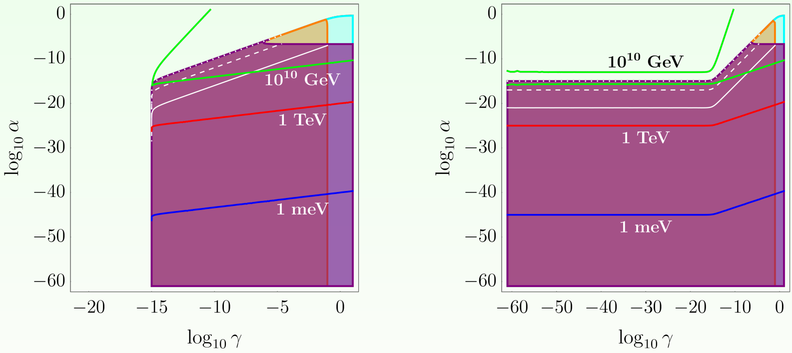
$\beta = -10^{-15}$



$\beta = 10^{-15}$

for $\beta \gg \alpha, \gamma \implies c_S^2 \simeq \frac{\beta}{\alpha}$, i.e. independent of γ

$\beta = \pm 10^{-15}$ surface with M_{SC}



$\beta = -10^{-15}$ $\qquad$ $\beta = 10^{-15}$