The luminosity distance - redshift relation in cosmologies beyond FLRW

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Background and motivation

- Standard Cosmological model - ΛCDM
- Type Ia supernovae allow you to probe the geometry of the Universe
- Data is usually fitted with a $d_L(z)$ relation based on FLRW cosmology
- Departures from FLRW due to:
  - inhomogeneities from cosmic structure
  - alternative cosmological models

  $\implies$ Need an improved formula for d_L!
- Main idea: use data from type Ia supernovae and an improved $d_L(z)$ formula to analyse alternative models and possible deviations from FLRW

Main papers:

- arXiv:1802.06550v2
- arXiv:1803.08766

Cosmography:

- arXiv:gr-qc/0411131

van Vleck determinant:

- arXiv:hep-th/9303020

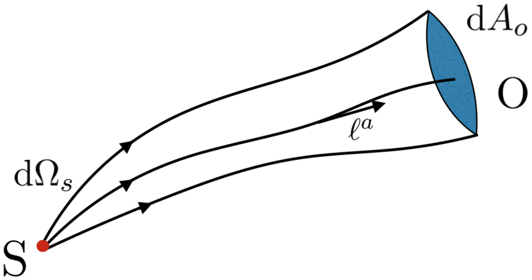
CFLRW cosmologies:

- arXiv:1502.02758

Other approaches:

- arXiv:astro-ph/0511183v5
- arXiv:1209.4326v2

The luminosity distance and the redshift

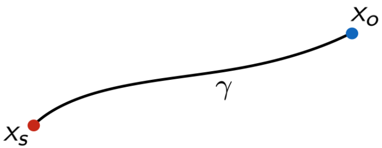


L_s - intrinsic luminosity of S
F_o - measured flux at O

$$d_L := \sqrt{\frac{L_s}{4\pi F_o}} = (1+z)\sqrt{\frac{\mathrm{d}A_o}{\mathrm{d}\Omega_s}}$$

$$1 + z := \frac{\omega_s}{\omega_o} = \frac{(g_{\mu\nu} U^\mu \ell^\nu)_s}{(g_{\mu\nu} U^\mu \ell^\nu)_o}$$

The van Vleck determinant



Definition: $$\Delta_{vV}(x_s, x_o) := -\frac{\det\{\nabla_\mu^{x_s}\nabla_\nu^{x_o}\sigma_\gamma(x_s, x_o)\}}{\sqrt{g(x_s)g(x_o)}}$$

$$\sigma_\gamma(x_s, x_o) := \pm\frac{1}{2}[s_\gamma(x_s, x_o)]^2$$ geodesic interval

$$s_\gamma(x_s, x_o) = \begin{cases} \int_\gamma \mathrm{d}\tau & \text{timelike} \\ \int_\gamma \mathrm{d}s & \text{spacelike} \\ 0 & \text{null} \end{cases}$$

The van Vleck determinant and the luminosity distance

Evolution of van Vleck determinant along the null geodesic congruence:

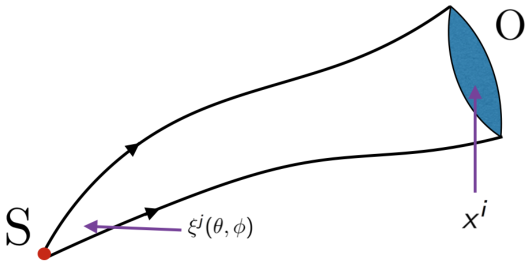
$$\frac{d\Delta_{vV}}{d\lambda} = \left(\frac{2}{\lambda - \lambda_s} - \theta \right) \Delta_{vV}$$

The expansion: $$\theta = \frac{1}{dA/d\Omega_s} \frac{d(dA/d\Omega_s)}{d\lambda}$$

Solution: $$\Delta_{vV}(\lambda_o, \lambda_s) = \frac{(\lambda_o - \lambda_s)^2}{(dA_o/d\Omega_s)}$$

$$\Longrightarrow \; d_L = (1+z)\,(\lambda_o - \lambda_s)\,\Delta_{vV}^{-1/2}$$

The Jacobi determinant



$$J^i_j = \frac{\partial x^i}{\partial \xi^j} \qquad \text{Jacobi map}$$

$$\frac{dA_o}{d\Omega_s} = |\det J| \qquad \text{Jacobi determinant}$$

$$\Longrightarrow\ d_L = (1+z)\sqrt{|\det J|}$$

So far

$$d_L = (1+z)\,(\lambda_o - \lambda_s)\,\Delta_{vV}^{-1/2}$$
$$= (1+z)|\det J|^{\frac{1}{2}}$$

$$|\det J| = (\lambda_o - \lambda_s)^2 \Delta_{vV}^{-1}$$

$$F_o = \frac{L_s}{4\pi d_L^2} = \frac{L_s \Delta_{vV}}{4\pi(1+z)^2(\lambda_o - \lambda_s)^2}$$

$\Delta_{vV} > 1$ geodesic focussing

$\Delta_{vV} < 1$ geodesic defocussing

FLRW: luminosity distance and redshift

$$ds^2_{FLRW} = a^2(\eta)\left\{-d\eta^2 + \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right]\right\}$$

$$1 + z = (1+z_c)(1+z_D) = \frac{a_o}{a_s}\frac{\gamma_s(1-\vec{v}_s.\hat{n})}{\gamma_o(1-\vec{v}_o.\hat{n})}$$

$$d_L = a_o(1+z)\frac{\sin\left(\sqrt{k}\Delta\eta\right)}{\sqrt{k}}$$

3 cases:

- $k = 0$ $\quad d_L = a_o(1+z)\Delta\eta$
- $k = 1$ $\quad d_L = a_o(1+z)\sin(\Delta\eta)$
- $k = -1$ $\quad d_L = a_o(1+z)\sinh(\Delta\eta)$

Simple cosmography

- Generally, express $\Delta\eta$ as a power series of z_c:

$$\Delta\eta = p(z_c) \qquad \text{with} \qquad p(0) = 0$$

- Cosmographic expansion ($z_D = 0$, $k = 0$ case):

$$d_L(z) = \frac{1}{H_o}\left\{z + \frac{1}{2}\left[1 - q_o\right]z^2 - \frac{1}{6}\left[1 - q_o - 3q_o^2 + j_o\right]z^3 + O(z^4)\right\}$$

- Cosmographic parameters:

$$H = \frac{\dot{a}}{a} \qquad q = -\frac{1}{H^2}\frac{\ddot{a}}{a} \qquad j = \frac{1}{H^3}\frac{\dddot{a}}{a}$$

- From observational $d_L(z)$ curve one can constrain H_o, q_o, j_o.
- Want to investigate deviations from simple FLRW and simple cosmography.

Deviations from FLRW

- Nonperturbative vs. perturbative
- Conformally FLRW

$$ds^2_{CFLRW} = f^2(x)\left\{-d\eta^2 + \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right]\right\}$$

- Linearly perturbed FLRW, $k = 0$

$$\mathrm{d}s^2 = a^2(\eta)\Big[-(1+2\Psi(x))\mathrm{d}\eta^2 + (1+2\Phi(x))\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\Big]$$

Conformally FLRW (CFLRW)

$$ds^2_{CFLRW} = f^2(x)\left\{-d\eta^2 + \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]\right\}$$

The only way to introduce nonperturbative deviations from FLRW without distorting the CMB

$$f(x) =: \exp[\theta(x)]a(\eta) \qquad < e^{\theta} >_{spatial} = 1 \qquad \Delta\theta_{LS} \leq 10^{-5}$$

$$1 + z = \frac{f_o}{f_s} = \exp(\theta_o - \theta_s)\frac{a_o}{a_s} = (1 + z_{loc})(1 + z_c)$$

$$d_L = f_o(1+z)\frac{\sin(\sqrt{k}\Delta\eta)}{\sqrt{k}}$$

Cosmography: $\quad \Delta\eta = p(z_c)$

Perturbed FLRW: the luminosity distance

2 scalar mode perturbations $\Psi, \Phi << 1$, Newtonian gauge, $k = 0$:

$$\mathrm{d}s^2 = a^2(\eta)\Big[-(1+2\Psi(\vec{x},\eta))\mathrm{d}\eta^2 + (1+2\Phi(\vec{x},\eta))\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\Big]$$

$$\begin{aligned} d_L = \frac{a_o^2}{a_s}\Big[&(\eta_o - \eta_s) + 2\Phi_o(\eta_o-\eta_s) - \Psi_s(\eta_o-\eta_s) \\ &+ (\eta_o-\eta_s)\int_{\eta_s}^{\eta_o}\vec{\nabla}\xi\cdot\hat{n}\mathrm{d}\eta + \int_{\eta_s}^{\eta_o}\xi\mathrm{d}\eta + \int_{\eta_s}^{\eta_o}(\eta_o-\eta)(\vec{\nabla}\xi\cdot\hat{n})\mathrm{d}\eta \\ &-\frac{1}{2}\int_{\eta_s}^{\eta_o}(\eta_o-\eta)(\nabla^2\xi - n^i n^j\xi_{,ij})(\eta-\eta_s)\mathrm{d}\eta\Big] \end{aligned}$$

where $\xi = \Psi - \Phi$.

Perturbed FLRW: the redshift

$$1 + z = \frac{a_o}{a_s}\left(1 + \Psi_o - \Psi_s - \int_{\eta_s}^{\eta_o} \partial_\eta \xi \mathrm{d}\eta\right)$$

A product of 3 contributions:

1. $1 + z_c = \frac{a_o}{a_s}$
2. $1 + z_{loc} = \sqrt{\frac{1+2\Psi_o}{1+2\Psi_s}} \approx 1 + \Psi_o - \Psi_s$
3. $1 + z_{ISW} = 1 - \int_{\eta_s}^{\eta_o} \partial_\eta \xi \mathrm{d}\eta$

One can also add the Doppler redshift $1 + z_D = \frac{\gamma_s(1-\vec{v}_s.\hat{n})}{\gamma_o(1-\vec{v}_o.\hat{n})}$:

$$1 + z = (1 + z_D)(1 + z_c)(1 + z_{gr})(1 + z_{ISW})$$

Summary and future work

So far:

- Need an improved formula for d_L to test alternative models
- Derived such a formula for CFLRW and linearly perturbed FLRW

Future:

- Within a particular model derive a $d_L(z)$ relation (ideally as a cosmographic series)
- Use data from supernovae to constrain the parameter space of this model
- Look at the power spectrum of d_L at a fixed z or at the correlation of d_L at different z along a single line of sight

Thank you for your attention!