Hot topics in Modern Cosmology XII, Cargèse

Pauli – Zeldovich cancellation of the vacuum energy divergences, auxiliary fields and supersymmetry

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May 14 - May 19, 2018

Based on

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Introduction

- Pauli suggested that the vacuum (zero-point) energies of all existing fermions and bosons compensate each other.
- The vacuum energy of fermions has a negative sign whereas that of bosons has a positive one.
- Such a cancellation indeed takes place in supersymmetric models.
- Zeldovich related vacuum energy to the cosmological constant.
- Covariant regularization of all contributions leads to finite values for both the energy density ε and (negative) pressure p corresponding to a cosmological constant, i.e. connected by the equation of state $p = -\varepsilon$.

- We examined the conditions for the cancellation of the ultraviolet divergences of the vacuum energy to the leading order in $\hbar$, i.e. by considering free theories and neglecting interactions.
- Such conditions are reduced to some sum rules involving the masses of particles present in the model.
- We applied such considerations to observed particles of the Standard Model and also studied the finite part of vacuum energy.
- It is interesting to take interactions into account, at least to the lowest order of perturbation theory.
- We consider only toy models, including particles with spin 1 and spin $1/2$.

Vacuum energy and the balance between the fermion and boson fields

$$\frac{\hbar\omega}{2}, \qquad \omega = \sqrt{k^2c^2 + m^2c^4},$$

$$\varepsilon = \frac{1}{2}\int d^3k\sqrt{k^2+m^2} = 2\pi\int_0^\infty dk k^2\sqrt{k^2+m^2}.$$

$$\varepsilon = 2\pi\int_0^\Lambda dk k^2\sqrt{k^2+m^2}$$

$$= 2\pi m^4\left[\frac{\Lambda}{8m}\left(\frac{2\Lambda^2+1}{m^2}\right)\sqrt{\frac{\Lambda^2}{m^2}+1} - \frac{1}{8}\ln\left(\frac{\Lambda}{m}+\sqrt{\frac{\Lambda^2}{m^2}+1}\right)\right]$$

$$\varepsilon = \frac{\pi}{2}\Lambda^4 + \frac{\pi}{2}\Lambda^2 m^2 + \frac{\pi}{16} m^4 (1 - 4\ln 2) - \frac{\pi}{4} m^4 \ln\frac{\Lambda}{m} + o\left(\frac{m}{\Lambda}\right).$$

The fermion contribution has the opposite sign.
To cancel the quartic ultraviolet divergences proportional to Λ^4, one has to have equal numbers of boson and fermion degrees of freedom:

$$N_B = N_F.$$

The conditions for the cancellation of quadratic and logarithmic divergences are

$$\sum m_S^2 + 3\sum m_V^2 = 2\sum m_F^2,$$

$$\sum m_S^4 + 3\sum m_V^4 = 2\sum m_F^4.$$

The remaining finite part of the vacuum energy density is

$$\varepsilon_{\text{finite}} = \sum m_S^4 \ln m_s + 3 \sum m_V^4 \ln m_V - 2 \sum m_F^4 \ln m_F .$$

The pressure is

$$p = \frac{2\pi}{3} \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m^2}} .$$

$$p = \frac{2\pi}{3} \int_0^\Lambda dk \frac{k^4}{\sqrt{k^2 + m^2}}$$

$$= \frac{2\pi}{3} m^4 \left[\frac{1}{8} \frac{\Lambda}{m} \left(\frac{2\Lambda^2}{m^2} \right) \sqrt{\frac{\Lambda^2}{m^2} + 1} - \frac{\Lambda}{m} \sqrt{\frac{\Lambda^2}{m^2} + 1} \right.$$

$$\left. + \frac{3}{8} \ln \left(\frac{\Lambda}{m} + \sqrt{\frac{\Lambda^2}{m^2} + 1} \right) \right] .$$

$$p = \frac{\pi}{6}\Lambda^4 - \frac{\pi}{6}\Lambda^2 m^2 - \frac{7\pi}{48} m^4 + \frac{\pi}{4}\ln 2 + \frac{\pi}{4} m^4 \ln\frac{\Lambda}{m} + o\left(\frac{m}{\Lambda}\right).$$

The quartic divergence satisfies the equation of state for radiation $p = \frac{1}{3}\varepsilon$, the quadratic divergence - $p = -\frac{1}{3}\varepsilon$ (string gas) the logarithmic divergence behaves as a cosmological constant with $p = -\varepsilon$.
The finite part of the pressure is

$$p_{\rm finite} = -(\sum m_s^4 \ln m_s + 3\sum m_V^4 \ln m_V - 2\sum m_F^4 \ln m_F),$$

which also behaves as a cosmological constant.

Running masses and anomalous mass dimensions

If we include the interactions, the masses begin their running.

$$\sum \gamma_{m\mathrm{S}} = 2 \sum \gamma_{m\mathrm{F}},$$

where γ_m is the mass anomalous dimension:

$$\gamma_m \equiv \mu \frac{\partial m^2}{\partial \mu},$$

μ is the renormalization mass parameter.

$$\sum m_S^2 \gamma_{m\mathrm{S}} = 2 \sum m_F^2 \gamma_{m\mathrm{F}}.$$

Our treatment of the anomalous mass dimensions in the presence of quadratic divergences is based on the approach presented in

I. Jack, D. R. T. Jones, Quadratic Divergences and Dimensional Regularization, Nucl. Phys. B 342, 127 (1990),

which uses the version of renormalization group formalism connected with dimensional regularization

G. 't Hooft, Dimensional regularization and the renormalization group, Nucl. Phys. B 61, 455 (1973).

Model, including a Dirac spinor with a mass M and a scalar field with a mass m.

$$L = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{m^2\phi^2}{2} - \frac{\lambda\phi^4}{4!} + i\bar{\psi}\gamma^\mu\partial_\mu\psi - M\bar{\psi}\psi - g\bar{\psi}\psi\phi.$$

$$\gamma_M = \frac{3g^2M}{16\pi^2}.$$

The calculation of the analogous quantity for the scalar field is more complicated because the mass renormalization in this case includes quadratic divergences. These divergences arise as poles at the spacetime dimensionality $d = 2$:

$$\gamma_m = \frac{\lambda m^2}{16\pi^2} - \frac{3g^2M^2}{2\pi^2} + \frac{g^2m^2}{4\pi^2} - \frac{\lambda\mu^2}{4\pi} + \frac{2g^2\mu^2}{\pi}.$$

The terms, including μ^2 are responsible for the quadratic divergences.

The interaction between a pseudoscalar and a fermion is described by :

$$L = -h\bar{\psi}\gamma^5\psi\chi.$$

$$(\gamma^5)^2 = -1,\ \gamma^5\hat{k}\gamma^5 = \hat{k}.$$

The contribution of the this interaction to the anomalous mass dimension of the fermion field is

$$\gamma_M = \frac{h^2 M}{16\pi^2}.$$

The contribution of the fermion loop to the anomalous mass dimension of the pseudoscalar field χ is

$$\gamma_{m_\chi} = -\frac{h^2 M^2}{2\pi^2} + \frac{h^2 m^2}{4\pi^2} + \frac{2h^2\mu^2}{\pi}.$$

Contribution of potential terms into the vacuum energy and the auxiliary fields

We should consider not only the mass sum rules, but also the potential terms. The contributions of the potential terms to the vacuum energy density have the following structure

$$E_{\rm pot} = \frac{\langle 0| T(V \exp(i \int d^4x L_{\rm int}))|0\rangle}{\langle 0| T \exp(i \int d^4x L_{\rm int})|0\rangle}.$$

For the term

$$V = \lambda \phi^4,$$

$$E_1 = -3\lambda I^2,$$

$$I = \int \frac{dk}{k^2 - m^2}.$$

The contribution of the Yukawa interaction term is given by the structure

$$E_2 = \langle 0| T(g\bar{\psi}\psi\phi \times (-ig)\bar{\psi}\psi\phi|0\rangle.$$

This contribution (for the case of a Majorana spinor) is

$$E_2 = 2g^2 \int \frac{\mathrm{Tr}[(\hat{p} + \hat{k} + M)(\hat{k} + M]}{[(p+k)^2 - M^2][k^2 - M^2][p^2 - m^2]}.$$

A simple calculation shows that for the case of the Wess-Zumino model, when $m = M$ and there are well-known relations between the coupling constants, the quartic divergences present in the contributions shown above do not cancel each other. The point is that the number of fermion degrees of freedom is doubled off shell. To compensate this effect, we should introduce the auxiliary scalar fields as is done in supersymmetric models.

A simple example shows that this exactly gives the doubling of the leading contribution to vacuum energy.

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{F^2}{2} + hF\phi^2.$$

On shell this theory is equivalent to the theory where the auxiliary field F is excluded by means of the equation of motion

$$F + h\phi^2 = 0$$

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}h^2\phi^4.$$

$$E_{\mathrm{vacuum}} = -\frac{3}{2}h^2 I^2.$$

In the model with the auxiliary field

$$E_{\mathrm{vacuum}} = \langle 0|T(-hF\phi^2 \times (ih)F\phi^2|0\rangle = -3h^2I^2.$$

The propagator of the auxiliary field in the massless theory is given by

$$\langle 0|T(FF)|0\rangle = i.$$

The result is doubled because of the effective doubling of the number of degrees of freedom.

The requirement of the explicit account of auxiliary fields arises only in the diagrams possessing quartic ultraviolet divergences and including only boson propagators, because their contribution is proportional to the number of degrees of freedom present off shell in the model under consideration. This fact gives us a practical recipe: when one calculates vacuum energy contribution of the scalar field diagrams, having the shape of "eight", one should multiply it by the factor 2.

In the action the term $\frac{F^2}{2}$ is also present. One can consider this term as a part of the kinetic energy. The contribution of this term into vacuum energy is

$$\left\langle 0 \middle| T\left(-\frac{F^2}{2} \times (ih)F\phi^2|0 \times (ih)F\phi^2|0 \times \frac{1}{2}\right) \middle| 0 \right\rangle$$
$$= +\frac{3}{2}h^2 I^2.$$

Summing the kientic and potential energies we reproduce the correct result. The results for vacuum energy in the model with an auxiliary field and in the model , where the auxiliary field is eliminated, coincide. However, the expressions for the contributions of the potential energy and of the kinetic energy do not coincide separately. A similar effect can be observed in the supersymmetric Wess-Zumino model.

A model with one Majorana and two scalar fields

$$H_{\rm int} = \lambda_1 A^4 + \lambda_2 B^4 + \lambda_3 A^2 B^2$$
$$+g_1 \bar{\psi}\psi A + g_2 \bar{\psi}\psi B$$
$$+mh_1 A^3 + mh_2 B^3 + mh_3 A^2 B + mh_4 AB^2.$$

The cancellation of the tadpole diagrams for A and B requires

$$3h_1 + h_4 = 4g_1,$$
$$3h_2 + h_3 = 4g_2.$$

The self-energy operator for the propagator of the field A obtains the contributions from the vertex A^4, from the vertex A^2B^2 and from the pair of vertexes $\bar{\psi}\psi A$, A^3, A^2B and AB^2. The contributions of two quartic vertexes are both proportional to the integral I. The corresponding coefficients are $12\lambda_1$ and $2\lambda_3$.

The contribution of the fermion loop is

$$C_1 = -2g_1^2 \mathrm{Tr} \int \frac{\mathrm{Tr}((\hat{p}+\hat{k}+m)(\hat{k}+m)}{[(p+k)^2-m^2][k^2-m^2]} dk,$$

$$C_1 = -8g_1^2 \int \frac{k^2+kp+m^2}{[(p+k)^2-m^2][k^2-m^2]} dk$$

$$= -4g_1^2 \int \frac{(k^2-m^2)+((k+p)^2-m^2)-p^2+4m^2}{[(p+k)^2-m^2][k^2-m^2]} dk$$

$$= -8g_1^2 I + (4p^2-16m^2) g_1^2 K,$$

$$K = \int \frac{dk}{[(p+k)^2-m^2][k^2-m^2]}.$$

Quadratic divergences present in the integral I should be canceled because such divergences do not arise in the self-energy correction to the fermion propagator.

$$12\lambda_1 + 2\lambda_3 - 8g_1^2 = 0,$$

$$12\lambda_2 + 2\lambda_3 - 8g_2^2 = 0.$$

The contribution of the pairs of the triple scalar vertices is

$$C_2 = (18h_1^2 + 4h_3^2 + 2h_4^2)m^2 K.$$

We find similar expressions for the one-loop contributions to the propagators of the second scalar field and of the Majorana fermion.

$$18h_1^2 + 4h_3^2 + 2h_4^2 - 12g_1^2 = 12(g_1^2 + g_2^2),$$

$$18h_2^2 + 4h_4^2 + 2h_3^2 - 12g_2^2 = 12(g_1^2 + g_2^2).$$

The contribution of the potential term to vacuum energy.
The contribution of the quartic terms is

$$E_1 = (3\lambda_1 + 3\lambda_2 + \lambda_3)I^2.$$

The contribution coming from the two scalar-fermion vertices is

$$E_2 = -8\int \frac{\mathrm{Tr}((\hat{p} + \hat{k} + m)(\hat{k} + m))}{[(p+k)^2 - m^2][k^2 - m^2][p^2 - m^2]}dkdp$$
$$= -4(g_1^2 + g_2^2)I^2 - 12m^2(g_1^2 + g_2^2)L,$$

$$L = \int \frac{dkdp}{[(p+k)^2 - m^2][k^2 - m^2][p^2 - m^2]}.$$

The contribution to vacuum energy of the triple scalar interactions is

$$E_3 = m^2(6h_1^2 + 6h_2^2 + 2h_3^2 + 2h_4^2)L.$$

We can now observe that

$$2E_1 + E_2 + E_3 = 0.$$

The coefficient 2 in front of the term E_1 is introduced to take into account the fact that the number of boson and fermion degrees of freedom should be equal also off shell.

The consistency conditions on the constants h_1, h_2, h_3 and h_4:

$$18h_1^2 - 27h_2^2 + 13h_3^2 + 2h_4^2 - 36h_1h_4 - 18h_2h_3 = 0,$$

$$18h_2^2 - 27h_1^2 + 13h_4^2 + 2h_3^2 - 36h_2h_3 - 18h_1h_4 = 0.$$

These equations are homogeneous in h_1, h_2, h_3 and h_4. We can fix $h_1 = 1$ and change the value of h_2. We shall have a system of two quadratic equations for h_3 and h_4. It is equivalent to one quartic equation. We present some numerical solutions.

$$h_2 = 1,$$
$$h_3 = h_4 \approx 3.8$$
or
$$h_3 = h_4 \approx -0.2.$$

$$h_2 = 0.9$$
$$h_3 \approx 3.56, \;\; h_4 \approx 3.58$$
or
$$h_3 \approx -0.46, \;\; h_4 \approx 0.17.$$

$$h_2 = 10/9$$
$$h_3 \approx 3.98, \quad h_4 \approx 3.96$$
or
$$h_3 \approx 0.2, \quad h_4 \approx -0.5.$$

$$h_2 = 1/2$$
$$h_3 \approx 2.8, \quad h_4 \approx 2.9$$
or
$$h_3 \approx -1.2, \quad h_4 \approx 1.2.$$

$$h_2 = 2$$
$$h_3 \approx 5.8, \quad h_4 \approx 5.6$$
or
$$h_3 \approx 2.5, \quad h_4 \approx -2.4.$$

$$h_2 = 1/10$$

$$h_3 \approx 2.09, \quad h_4 \approx 2.26$$

or

$$h_3 \approx -1.8, \quad h_4 \approx 1.9.$$

$$h_2 = 10$$

$$h_3 \approx 22, \quad h_4 \approx 21$$

or

$$h_3 \approx 19, \quad h_4 \approx -18.$$

Model with a Majorana field, a scalar field and a pseudoscalar field

A toy model where the field B is a pseudoscalar. In this case

$$h_2 = h_3 = 0$$

the interaction between the pseudoscalar and the fermion is described by the Lagrangian

$$g_2 \bar{\psi} \gamma_5 \psi B.$$

We have only one condition for the tadpole cancellation for the scalar field A. The conditions for the cancellation of quadratic divergences in the propagators of the scalar and pseudoscalar fields are the same as before.

Requiring that the running of three masses are the same:

$$-12g_1^2 + 18h_1^2 + 2h_4^2 = 4h_4^2 + 4g_2^2$$

$$4h_4^2 + 4g_2^2 = 12g_1^2 - 4g_2^2.$$

From these two equations we obtain

$$g_1 = \pm h_1.$$

The only consistent option is

$$g_1 = h_1$$

$$h_4 = g_1,$$

$$g_1^2 = g_2^2$$

$$\lambda_1 = \lambda_2.$$

For the case with one Majorana field, one scalar and one pseudoscalar we have less freedom in the choice of the coupling constants than in the case of two scalar fields and one Majorana field, but this choice is still broader than that in the Wess-Zumino model.

Models with non-degenerate masses

The simplest models of this kind are those which include a certain number of "triplets" of the types described before, i.e. with degenerate masses inside any triplet and with coupling constants (again, describing interactions within a triplet) which satisfy the relations obtained above.

If we introduce interactions between different triplets with different masses, then the coupling constants should satisfy some constraints.

Concluding remarks

- We have studied the Pauli-Zeldovich mechanism for the cancellation of ultraviolet divergences in vacuum energy which is associated with the fact that bosons and fermions produce contributions to it having opposite signs.
- We have taken interactions up to the lowest order of perturbation theory into account.
- We have constructed a number of simple toy models having particles with spin 0 and spin $1/2$, wherein masses of the particles are equal while interactions can be quite non-trivial.
- To make calculations simpler and more transparent, it was found useful to introduce some auxiliary fields.