UV-complete gravity as Analytic Infinite Derivative (AID) theory

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Introduction

- **Particle physics benefited from QFT but a particular QFT may have problems**
- **Gravity at least in Einstein-Hilbert form does not have a well defined quantization**
- **Quantization problems usually are non-renormalizability, loss of unitarity, anomalies**
- **Naively and NOT naively quantized gravity has them all**

About strings

- Strings were supposed to resolve at least the issue of renormalization due to the absence of point-like objects
- String Field Theory is the non-perturbative description of strings
- Lowest mass tree level scattering amplitudes were guessed and then proven by Veneziano
- p-adic string model is a scalar field model reproducing the Veneziano amplitudes

p-adic strings in more details

Vladimirov, Volovich, Zelenov; Dragovich, Khrennikov;

Brekke, Freund, Olson, Witten, ...

The Lagrangian is

$$L = -\frac{1}{2}\phi p^{-\Box/m_p^2}\phi + \frac{1}{p+1}\phi^{p+1}$$

In the original form p was supposed to be a prime number and the potential was fixed. From the QFT side the model was proven to be finite.

Further, thinking QFT, the exponent of the Box operator is the crux of this model. The nature of p and the form of the potential can be relaxed.

SFT side picture

Low level action from SFT:

$$L \sim \frac{1}{2}\phi(\Box - m^2)\phi + \frac{\lambda}{4}\left(e^{-\Box/\mathcal{M}^2}\phi\right)^4$$
$$\Rightarrow \frac{1}{2}\varphi(\Box - m^2)e^{+2\Box/\mathcal{M}^2}\varphi + \frac{\lambda}{4}\varphi^4$$

The Lagrangian to understand is

$$S = \int d^D x \left(\frac{1}{2}\varphi\mathcal{F}(\Box)\varphi - \lambda v(\varphi) + \ldots\right)$$

$\mathcal{F}(\Box) = \sum_{n\geq 0} f_n \Box^n$, **i.e. it is an analytic function.**

Thus, exponent is relaxed to $\mathcal{F}(\Box)$ as well.

Renormalizability

In the Fourier space (i.e. momentum space) $\Box \to -k^2$.

As such, the propagator acquires an exponential factor such that

$$\Pi = -\frac{e^{-k^2/\mathcal{M}^2}}{k^2 + m^2}$$

This guarantees convergence of all loop integrals.

In general

$$\Pi = \frac{1}{\mathcal{F}(-k^2)}$$

Unitarity is almost always equivalent to having no ghosts

Terminology in $(-,+,+,+)$ signature:

$$\text{good: } L = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \ldots$$
$$\text{ghost: } L = +\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \ldots$$

Ghosts lead to a very rapid vacuum decay.

Ostrogradski statement says that higher (> 2) derivatives in a Lagrangian are equivalent to the presence of ghosts.

This statement is not absolutely rigorous. There are systems with higher derivatives which have no ghosts.

What $\mathcal{F}(\Box)$ can be?

Weierstrass factorization for an entire function says

$$\mathcal{F}(z) = \prod_i (z - z_i)^{n_i} e^{-\sigma(z)}$$

where z_i are algebraic roots of multiplicity n_i and $\sigma(z)$ is an entire function.

Thus for the propagator we have

$$\Pi = -\frac{e^{\sigma(-k^2)}}{\prod_i (k^2 + m_i^2)}$$

The number of physical states (particles) is equivalent to the number of simple poles.

Given some pole is normal, the next is ghost, then normal, then ghost, etc.

Thus, no ghosts $\Rightarrow$ one pole $\Rightarrow$ one root.

What if $\mathcal{F}(\Box)$ is not entire?

Weierstrass factorization gets modified as

$$\mathcal{F}(z) = \frac{\prod_i (z - z_i)^{n_i}}{\prod_j (z - z_j)^{p_i}} e^{-\sigma(z)}$$

where z_i are algebraic roots of multiplicity n_i, z_j are poles of order p_i, and $\sigma(z)$ is an entire function.

Thus for the propagator we have

$$\Pi = -e^{\sigma(-k^2)} \frac{\prod_j (k^2 + \mu^2)^{p_j}}{\prod_i (k^2 + m_i^2)}$$

Recall: the number of physical states (particles) is equivalent to the number of simple poles.

Thus, extra analytic factors in the nominator are irrelevant for physics.

Exorcising ghosts

- In some cases ghosts do not appear, like in $f(R)$ gravity for special parameters.

  This is because the system is constrained.

- There are special field theories which have higher derivatives in the Lagrangian but no more than 2 derivatives act on a field in the equations of motion. For example KGB models, galileons, Horndeski models, ...

  The fine-tuning is required.

- Propagators can be modified and be non-local without changing the physical excitations

$$\Box - m^2 \rightarrow \mathcal{F}(\Box) = (\Box - m^2)e^{\gamma(\Box)}$$

  $\gamma(\Box)$ must be an *entire* function. This guarantees that no extra degrees of freedom appear. Let $\gamma(0) = 0$ to preserve the normalization.

Quantization: what we have?

- **Gravity is not renormalizable**
- **Stelle's 1977 and 1978 papers show that R^2 gravity is renormalizable gravity with the price of a physical (Weyl) ghost**
- **Recall: Ostrogradski statement from 1850 forbids higher derivatives in general. The Weyl tensor already has 2, its square has 4 and constraints do not alleviate the problem.**
- **Good thing: Starobinsky inflation is based on R^2 and works perfectly**

The early Universe formation, which is most likely inflation, is for the time being perhaps the only testbed for testing gravity modifications.

So what?

We start with

$$S = \int d^D x \sqrt{-g} \left(\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right)$$

Here $\mathcal{P}$ and $\mathcal{Q}$ depend on curvatures and $\mathcal{O}$ are operators made of covariant derivatives.

Everywhere the respective dependence is *analytic*.

The most general action to consider

We are looking for the most general action capturing in full generality the properties of a linearized model around *maximally symmetric space-times (MSS)* given $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$.

The result is [arxiv.1602.08475]

$$S = \int d^D x \sqrt{-g} \Bigg(\frac{M_P^2 R}{2} + \frac{\lambda}{2} \Big(R \mathcal{F}(\Box) R + L_{\mu\nu} \mathcal{F}_L(\Box) L^{\mu\nu} + W_{\mu\nu\lambda\sigma} \mathcal{F}_W(\Box) W^{\mu\nu\lambda\sigma} \Big) - \Lambda \Bigg)$$

Here $L_{\mu\nu} = R_{\mu\nu} - \frac{1}{D} R g_{\mu\nu}$ and for any X

$$\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X n} \Box^n$$

How come?

MSS

$$R_{\mu\nu\alpha\beta} = f(x)(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$$

Bianchi identities dictate $f(x) = \frac{R_0}{(D-1)(D-2)}$

Consider term by term

$$R^2 \to h^2, \quad R^3 \to 3h^2R, \ldots$$

$$\partial R^2 \to \partial h^2, \quad \partial R^2 R \to \partial h^2 R, \quad \partial R^3 \to 3\partial h^2 \partial R \to 0, \ldots$$

The same logic applies for $R_{\mu\nu}$ and even simpler for the Weyl tensor as it is zero on an MSS background.

Finally, We can fix any of f_0, f_{L0}, f_{W0} because the Gauss-Bonnet (GB) term is a topological invariant.

Even more, the derived action can be reduced further!

This is thanks to the Bianchi identities.

Around MSS in $D = 4$ one can fix any of tree functions $\mathcal{F}$ and not only their constant Taylor coefficients. For example we can drop $\mathcal{F}_L$ entirely and remain with

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2} + \frac{\lambda}{2}\left(R\mathcal{F}(\Box)R + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right) - \Lambda\right)$$

Reduction in other dimensions

Around MSS but in $D \geq 5$ the GB term is not a topological invariant and we are left with

$$S = \int d^4x \sqrt{-g}\left(\frac{M_P^2 R}{2} + \frac{\lambda}{2}\left(R\mathcal{F}(\Box)R + f_{L0}L_{\mu\nu}^2 + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right) - \Lambda\right)$$

Still, we are able to drop all higher derivative terms for $L_{\mu\nu}$

Quadratic action around (A)dS with $\bar{R} = 4\Lambda/M_P^2$

The covariant decomposition is

$$\begin{aligned} h_{\mu\nu} =& \frac{2}{M_P^2} h^{\perp}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu \\ & + \left(\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4}\frac{2}{M_P^2}\sqrt{\frac{8}{3}} \bar{g}_{\mu\nu} \bar{\Box} \right) B + \frac{1}{4}\frac{2}{M_P^2}\sqrt{\frac{8}{3}} \bar{g}_{\mu\nu} h \end{aligned}$$

Here $\bar{\nabla}^\mu h^{\perp}_{\mu\nu} = \bar{g}^{\mu\nu} h^{\perp}_{\mu\nu} = \bar{\nabla}^\mu A_\mu = 0$.

Vector part and $\bar{\nabla}_\mu \bar{\nabla}_\nu B$ **terms go away around MSS.**

Spin-2:

$$S_2 = \frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\; h^{\perp}_{\nu\mu}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} + \frac{\lambda}{M_P^2}\left\{\mathcal{F}_L(\bar{\Box})\left(\bar{\Box} - \frac{\bar{R}}{6}\right) + 2\mathcal{F}_W\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\right\}$$

The Stelle's case corresponds to $\mathcal{F}_L = 0,\ \mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} + \frac{\lambda}{M_P^2} 2\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

This is an obvious second pole which will be the ghost.

Spin-0 (here $\phi \equiv \bar{\Box} B - h$):

$$
\begin{aligned}
S_0 &= -\frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\; \phi(3\bar{\Box} + \bar{R})\left[\mathcal{S}(\bar{\Box})\right]\phi \\
\mathcal{S}(\bar{\Box}) &= 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} \\
&\quad - \frac{\lambda}{M_P^2}\left\{2\mathcal{F}(\bar{\Box})(3\bar{\Box} + \bar{R}) + \frac{1}{2}\mathcal{F}_L\left(\bar{\Box} + \frac{2}{3}\bar{R}\right)\bar{\Box}\right\}
\end{aligned}
$$

This *is* the ghost in Einstein-Hilbert case, but it is constrained and is not physical.
Thus, $\mathcal{S}(\bar{\Box})$ *can* have one root to generate one pole and it will be not a ghost.
This would be exactly the scalar mode in a local $f(R)$ gravity.

Physical excitations

Effectively we modify the propagators as follows

$$\Box - m^2 \to \mathcal{G}(\Box)$$

To preserve the physics we demand

$$\mathcal{G}(\Box) = (\Box - m^2)e^{\sigma(\Box)}$$

where $\sigma(\Box)$ must be an *entire* function resulting that the exponent of it has no roots.

We arrange this in our model by virtue of functions $\mathcal{F}$. At this stage we can drop any one of three $\mathcal{F}$-s. The simplest choice is to drop $\mathcal{F}_L$.

Starobinsky inflation in non-local gravity

For any and in most cases only if:

$$\Box R = r_1 R + r_2$$

We have a solution if:

$$\mathcal{F}^{(1)}(r_1) = 0,\ \frac{r_2}{r_1}(\mathcal{F}(r_1) - f_0) = -\frac{M_P^2}{2\lambda} + 3r_1\mathcal{F}(r_1),$$
$$4\Lambda r_1 = -r_2 M_P^2,\ \text{in the case of interest } \Lambda = 0$$

Notice that the we have started with the trace of Einstein equations in a local R^2 gravity.
Saying local gravity we do mean *any* including patological parameters in that local counterpart.

Choice of $\mathcal{F}(\Box)$

We should arrange that the theory is ghost-free meaning that no more than one pole arises in the scalar sector. The new degree of freedom is named scalaron and its mass is denoted as M. A possible form is:

$$\frac{\lambda}{M_P^2}\mathcal{F}(\Box) = -\frac{1}{6\Box}\left[e^{H_0(\Box)}\left(1-\frac{\Box}{M^2}\right)-1\right]$$

The conditions on $\mathcal{F}(\Box)$ imply that $H_0(\Box)$ is an entire function and moreover:

$$r_1 = M^2 \qquad\qquad H_0(r_1) = 0$$

Power spectra

Tensor modes

$$|\delta_h|^2 = \frac{H^2}{2\pi^2\lambda\mathcal{F}_1\bar{R}} e^{2\omega(\bar{R}/6)} \quad \text{where} \quad \mathcal{P}(\bar{\Box}) = e^{2\omega(\bar{\Box})}$$

Scalar modes (actually $\mathcal{R} = \Psi + \frac{H}{\dot{\bar{R}}}\delta R_{GI}$**)**

$$|\delta_{\mathcal{R}}|^2 \approx \frac{H^6}{16\pi^2\dot{H}^2}\frac{1}{3\lambda\mathcal{F}_1\bar{R}}$$

Tensor to scalar ratio r

$$r = \frac{2|\delta_h|^2}{|\delta_{\mathcal{R}}|^2} = 48\frac{\dot{H}^2}{H^4}e^{2\omega(\bar{R}/6)}$$

All quantities here are at the horizon crossing $k = Ha$.

Analogously

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{2\epsilon_1} \;\Rightarrow\; r = 48\epsilon_1^2 e^{2\omega(\bar{R}/6)} = \frac{12}{N^2}e^{2\omega(\bar{R}/6)}$$

We have gained an extra factor $e^{2\omega(\bar{R}/6)}$ compared to the local R^2 inflation.

UV completeness

Minkowski propagator:

$$\Pi = -\left(\frac{P^{(2)}}{k^2 e^{H_2(-k^2)}} - \frac{P^{(0)}}{2k^2 e^{H_0(-k^2)}\left(1+\frac{k^2}{M^2}\right)}\right)$$

To guarantee that the QFT machinery works we arrange a polynomial decay of the propagator near infinity. The rate of the decay is our choice.

Recall that we still need the functions $H_{0,2}$ to be entire.

An example of such a function can be, for instance

$$H \sim \Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right)$$

where $p(z)$ is a polynomial.

Beyond 1-loop the powercounting arguments work just like in the higher derivative regularization.

p-adic reformulation of the non-local gravity

The scalar part of the previous action is equivalent to the following one

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2 R}{2} \left(1 + \frac{2}{M_P^2} \psi \right) - \frac{1}{2\lambda} \psi \frac{1}{\mathcal{F}(\Box)} \psi + \dots \right)$$

An important property here is the non-minimal coupling of a scalar field to gravity.

p-adic reformulation of the non-local gravity, continued

The conformal transform $\left(1+\frac{2}{M_P^2}\psi\right)^2 g_{\mu\nu} = \hat{g}_{\mu\nu}$ allows us to completely decouple the gravity and the scalar field

$$S = \int d^4x \sqrt{-\hat{g}} \left(\frac{M_P^2}{2}\hat{R} - \frac{M_P^2}{2}\frac{6}{(M_P^2+2\psi)^2}\hat{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{M_P^4}{2\lambda(M_P^2+2\psi)^2}\psi\mathcal{G}(\mathcal{D})\psi \right)$$

Here

$$\mathcal{G}(\mathcal{D}) = \frac{1}{\mathcal{F}(\mathcal{D})} \text{ and } \mathcal{D} = \left(1+\frac{2}{M_P^2}\psi\right)\hat{\Box} - \frac{2}{M_P^2}\hat{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

Where are p-adic strings?

We carefully extract the quadratic in ψ part of the above derived Lagrangian. The answer is

$$L_\psi = \frac{3}{M_P^2}\psi\hat{\Box}\psi - \frac{1}{\lambda}\psi\mathcal{G}(\hat{\Box})\psi$$

Thus

$$\frac{3}{M_P^2}\hat{\Box} - \frac{1}{\lambda}\mathcal{G}(\hat{\Box}) = (\varepsilon\hat{\Box} - m^2)e^{\sigma(\hat{\Box})}$$

Limiting $\sigma = \Box/\mathcal{M}^2$ and taking $\varepsilon = 0$ we restore the p-adic Lagrangian.

Conclusions

- **A UV complete and unitary gravity is presented**
- **Starobinsky inflation is natively embedded in this model**
- **The theory predicts a modified value for r**
- **A connection to p-adic strings is maintained**
- **Few words about SFT in this story, Cutkosky rules, etc. are in order**

Open questions

- **Deeper study of the full Starobinsky model embedded in this non-local setup.**

- **Explicit demonstration of the absence of singular solutions in this model**

- **This theory does not a priori prohibits a coexistence of a bounce and inflation. The question is to find such a configuration**

- **Derive the graviton action from the SFT in the full rigor.**

Thank you for listening!