References:
CT: 1710.06394
ADM: 1604.01396, 1610.00553, 1711.09893
COV: 1406.10091
BH: 1711.01992

"Landscape of Cosmological Models"

Today I will show you a method we developed (in collaboration with the cosmology group at the University of Oxford) to construct parametrized cosmological models, such that they encompass and describe a variety of theories at once. Along the way, I will show some examples and applications of this method, and discuss the advantages and disadvantages of this approach. Let me start first with a short intro.

I. Intro

* Small Scales (Solar system) → General Relativity (GR ✓) is very accurate and we feel very confident about its validity in this regime. This is not just because it fits well observational data, but also because it has been shown that a number of possible modifications to GR can be tested and constrained, and they have been found to be highly suppressed. Therefore, we have not just checked that GR works but also that no other model works. We have in some way falsified GR, and that is why we feel very confident about it.

* Large Scales (cosmological scales) → We describe the evolution of the universe as a whole with the ΛCDM model (based on GR).
  - ↳ Agrees with observational data so far.
  - ↳ theory: unsatisfied because it assumes that 95% of energy content of universe comes from unknown source. I will be focusing on DE: constant? why small? origin?

We certainly see that the model with use at small and large scales are not at the same foot. Here I would like to take and approach similar to the one taken at small scales, and not simply check that ΛCDM fits data, but also compare with alternative models, and hopefully in the future try to constrain these modifications and see if they are viable or maybe highly suppressed (as it happend with the small scales). Therefore, we would like to try to falsify the ΛCDM model. For this reason I will continue now talking about modified gravity theories.

II. Modified Gravity

People have proposed modifying GR at large scales to try to give a more natural and alternative explanation to dark energy. There are different ways of modifying gravity, but the most common one is by adding a new DOF or field, which would evolve dynamically in time and lead to a late time accelerated expansion of the universe.

I would like to mention some examples of modified gravity:

* Examples

- Scalar: Galileons ($\phi_{,\mu}\phi^{,\mu}\Box\phi$; $\nabla_\mu\phi\nabla^\mu\phi\Box\phi$), Horndeski ($G(\phi,X)R$; $\cancel{G_{\mu\nu}\nabla^\mu\nabla^\nu\phi}$), Beyond H ($\Box\phi\,\phi_{,\mu\nu}\phi^{,\mu}\phi^{,\nu}$) (simplest)
- Vector: Generalised Proca ($m^2A_\mu A^\mu$, $G_{\mu\nu}\nabla^\mu A^\nu$), Einstein-Aether ($A_\mu A^\mu=-1$) ($\nabla A\nabla A$)
- Tensor: Massive gravity

Here I show some theories but there are many more. Once one theory is proposed, we must proceed to check if it does what is supposed to do. Let me discuss then in general the analysis of these models.

* Analysis → bkgd ✓ (MG models can easily fit well obs data)
  ↳ Perts (evolution & distribution of LSS & CMB anisotropies)
  these theories can highly differ in the evolution of perturbations, and many times we run into issues, which is why there is no competitive model to ΛCDM so far. Let me mention some of these issues.
  - ↳ Instabilities: e.g. perturbations run away exponentially fast at early times and thus they won't fit data but also break perturbative approach. Common problem with massive gravity.
  - ↳ Observations: models may not fit well data and I want to mention one particular observation that recently got a great deal of attention, which is the recent LIGO/Virgo detection of GW & EM counterpart from a NS binary merger. The almost simultaneous arrivals of signals showed that GW propagate at the speed of light: $|c_T/c - 1| \lesssim 10^{-15}$. This highly affected the viability of some MG because in the same way that EM speed changes in the presence of a medium, in many cases GW speed changes in the presence of additional DoFs that act as a medium. In our paper (CT ref) we found that large subsets of Horndeski, BH, GP & EA are now highly disfavoured and all Galileons, because they modify the propagation speed of GW.
  - ↳ Arbitrarily close to GR: difficult to test. Nonetheless, there are still models with particular signatures that could be tested in the future.

Now the question is how to overcome these issues in a more efficient way, instead of studying models 1 by 1 and then discovering something wrong with them. In order to do that I will take a more practical approach in which we parametrize cosmological & phenomenological models.

III. Parametrized Cosmology

Let me start by mentioning some features of this approach.

* Large scales: model valid only for cosmological scales, and not full non-linear gravity theory, as opposed to the previously mentioned examples.
* Parametrize deviation from ΛCDM: different parameters will describe a different modification, and will be associated to different non-linear gravity theories. The hope is to constrain these parameters in the future & see if there is any viable modification or not.
* Method:
  there are different ways to characterize deviations from ΛCDM. In our case:
  - ↳ bkgd arbitrary: consider a H&I bkgd with arbitrary time evolution, which in practice will be assumed to fit data.
  - ↳ parametrize linear perts: we don't worry about the bkgd and only focus on analyzing the evolution of perts which are the ones that bring the issues previously discussed.
    - ↳ Actions (EFT): we work at the level of the action and write a parametrized quadratic action for linear cosmological perturbations, and each parameter is responsable for a different interaction term in this action. In order to construct this action we follow a similar approach to the standard EFT: write most general action compatible with symmetries & set of fields (most MG have some symmetry)
  - ↳ 3 steps: (eg. ST)
    1. Field content, bkgd + linear perts
    2. Gral quadractic action + free params, max derivatives
    3. Impose gauge symmetries by using Noether identities

Unfortunately I don't have time to discuss the Noether id in detail but I want to higlight that they are the most important ingredient of the method as they allow you to systematically impose any set of symmetries on any fields.

Toy example: quadratic vector field in Minkowski with $U(1)$ symmetry $A^\alpha \to A^\alpha + \partial^\alpha \epsilon$
② $S_A = \int [c_1 \partial_\alpha A^\beta \partial^\alpha A_\beta + c_3 \partial_\alpha A^\beta \partial_\beta A^\alpha + m^2 A^\alpha A_\alpha]$; ③ impose $\delta_\epsilon S_A = 0 = 2\int [(c_1+c_3)\partial^2 \partial^\beta A_\beta - m^2 \partial^\alpha A_\alpha]\epsilon$
$\Rightarrow c_1 + c_3 = 0$; $m = 0$ $\Rightarrow$ Maxwell.

I say this because in standard EFT (and other similar approaches to cosmology) people follow only steps ① and ②. In step ② they immediately write down all the gauge-invariant possible interaction terms, but you need this previous knowledge on the symmetries in order to do that. With the Noether identities there is no need for this previous knowledge, and no risk of missing terms.

I also want to highlight that the method is more general than for cosmology, as it can be applied to any background (refer to ADM paper, covariant paper, & BH paper).

Let me now continue with the cosmological examples. As you can see, the end product will be the most general quadratic action for perts around given bkgd, compatible with given symmetries & nº of derivatives.

* Examples: 2 derivatives, diffeomorphism invariant, H&I bkgd (more general cases can be found on ADM paper).
↳ Scalars: $\{ \alpha_M$ (or M), $\alpha_K, \alpha_T, \alpha_B \}$
 ↳ Dimensionless 4 params αs, $\alpha = 0 \Rightarrow$ GR
 ↳ $\alpha(t)$ arbitrary
 ↳ they appear in the action and characterize different interaction terms, and we can give them a meaning according to what they do. I won't write down the entire action but I will give you a small example. Action for GW (tensor perturbations):

$$S_T^{(2)} = \int d^4x \frac{M^2(t)}{2} \left[\dot{h}_A^2 - (1+\alpha_T(t)) (\vec{\nabla} h_A)^2 \right]$$

where h_A is amplitude of GW, A is polarization index $A = \{+, \times\}$.
This is a subset of metric perturbations in which the only free parameters are $M(t)$ and $\alpha(t)$.
We can see that $M(t)$ is effective Planck mass and α_M describe its running in time, whereas α_T modifies GW speed.

↳ Vector: → Gral: $\{\alpha_M (\text{or } M), \alpha_K, \alpha_T, \alpha_C, \alpha_D, \alpha_A, \alpha_V, \alpha_{VS}\}$
↳ EA ($A_\mu A^\mu = -1$): $\{\alpha_M (\text{or } M), \alpha_K, \alpha_T\}$

(these params characterize the most gral vector-tensor actions)
I want to notice that here we also have a parameter α_T (same as ST theories). As I was saying, it is quite common to have modified GW speeds in MG theories.

↳ Tensor: $\{\alpha_M (\text{or } M), \alpha_S, \alpha_{SI}, \alpha_{TI}\}$
(Potential interactions)

Generalization with derivative interactions can be found in our paper (reference). there is no α_T because there is a massive graviton and whole dispersion relation is modified.
these are the parameters that characterize the evolution of linear cosmological perturbations in diffeomorphism invariant theories with this field content.
We applied the method separately for each family of theories, but then we went one step beyond and unified them all at the level of the equations of motion (reference) therefore, this set of eqns describe a large portion of the landscape of cosmological models usually considered in the literature. the eqns were given in a form that can be readily implemented in numerical codes to compare with observational data in the future.

Now that we have these general parametrized models, let me discuss then how their analysis proceeds, and go back to the issues I mentioned at the beginning of the talk.

* Analysis $\to$ bkgd ✓ (arbitrary but chosen to fit observations)
  - Perturbations
    - Instabilities (negative kinetic energy).
      We went through models and find kinetic energy of physical DoFs as function of free parameters and impose positivity.
      E.g. Scalar-tensor $\longrightarrow 3\alpha_B^2 + 2\alpha_K > 0$
      E-A $\to \alpha_K > 0$
      (other more complicated expressions found in our paper).
      We are then easily left with a stable subset of models.
    - Observations: in order to impose agreement with cosmological observations we need to numerically scan parameter space and compare observable outputs. This is not so straightforward, but it has been done by other groups for scalar-tensor theories.
      But let me go back to the GW speed ~~dealer~~ constraint. That can easily be satisfied for ST & VT theories by simply imposing $\alpha_T = 0$. For TT theories it is not so straightforward, but we did it for specific case in GT paper.

So far we have seen how to more efficiently reduce the space of models to a viable one, without going back and forth. Let us discuss the third problem now:

- Arbitrarily close to GR: we still have that problem ($\alpha_T = 0 \Rightarrow$ GR), and I would like to leave you with two questions that will help us find ~~an~~ a solution.
  - How well to measure αs? How close to 0? $|\alpha| \lesssim 10^{-4}$? (comparison with PPN parameters).
  - How much complexity? (how many parameters we will add, 10?)