# Impact of nonlinearities on modified gravity constraints

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Cargese, May 18, 2018

based on:
S. Casas, M. Kunz, M. Martinelli, V. Pettorino arXiv:1703.01271

# Outline

# Overview

# Why modified gravity?

Modified gravity is usually introduced to explain cosmic acceleration without a cosmological constant.

It relies on modifications of the left hand side of Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

One hopes to solve this way theoretical issues of cosmological constant

- fine tuning
- coincidence problem

# A possible way out from tensions?

Recent observations have shown tensions between low and high redshift measurements.

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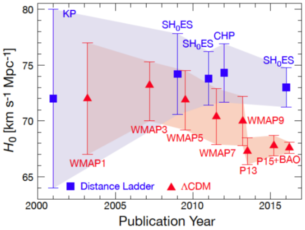
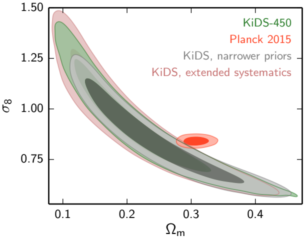



$$T(H_0) \approx 3\sigma$$

Riess et al. 2016




$$T(S_8) \approx 2\sigma$$

Joudaki et al. 2016

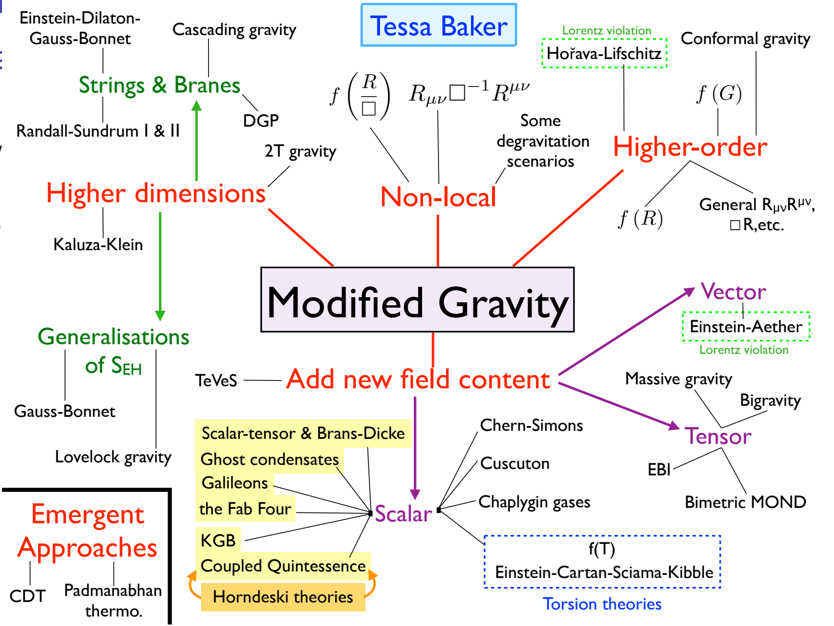
# Searching for modified gravity

If we want to test for deviations from GR we have to describe departures from it:

- specific alternative models
  assume a specific model and test whether or not it better fits the data

# Sea

If w … t:




Bull, Akrami et al. 2015 (arXiv:1512.05356)

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- parametrized deviations
  find peculiar properties of your model and parametrise them

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- parametrized deviations
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GR+$\Lambda$ main characteristics:

$$w = -1 \qquad \Phi = \Psi \qquad k^2\Psi = -4\pi G a^2 \rho\Delta$$

# Parametrized modified gravity

Focusing on the perturbations evolution ($w = -1$), we can encode modifications to GR in 3 functions of time and scale (only two independent)

$$k^2\Psi = -4\pi G a^2 \rho\Delta$$

$$k^2\left[\Phi + \Psi\right] = -8\pi G a^2 \rho\Delta$$

$$\frac{\Phi}{\Psi} = 1$$

# Parametrized modified gravity

Focusing on the perturbations evolution ($w = -1$), we can encode modifications to GR in 3 functions of time and scale (only two independent)

$$k^2\Psi = -4\pi G\mu(a,k)a^2\rho\Delta$$

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$$\frac{\Phi}{\Psi} = \eta(a,k)$$

One obviously has to assume a functional form for these, e.g.

Late time parameterization

$$\mu(a,k) = 1 + E_{11}\Omega_{DE}(a)$$
$$\eta(a,k) = 1 + E_{21}\Omega_{DE}(a)$$

Early time parameterization

$$\mu(a,k) = 1 + E_{11} + E_{12}(1-a)$$
$$\eta(a,k) = 1 + E_{21} + E_{22}(1-a)$$

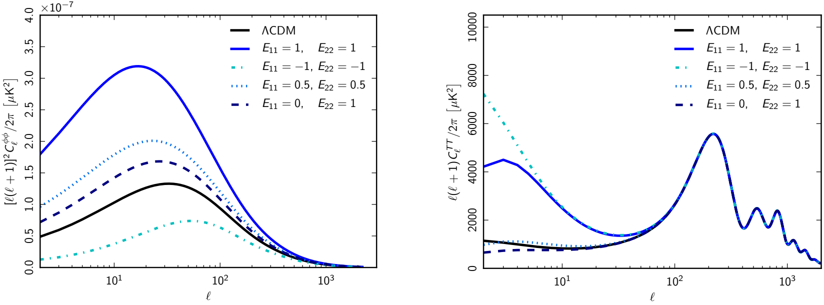
$$\Sigma(a,k) = \frac{\mu(a,k)}{2}\left[1 + \eta(a,k)\right]$$

Planck 2015 results. XIV. Dark energy and modified gravity

# Overview

# Planck constraints

After choosing a parameterization for the MG functions, one can compute the potentials $\Phi$ and $\Psi$ setting the values of the free parameters.
This allows to obtain the behaviour of the cosmological observables when moving away from GR and to constraints this deviations

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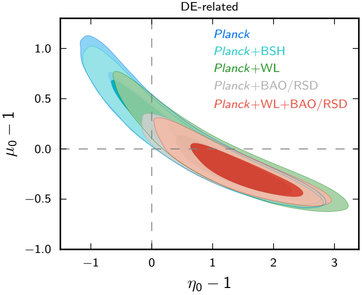


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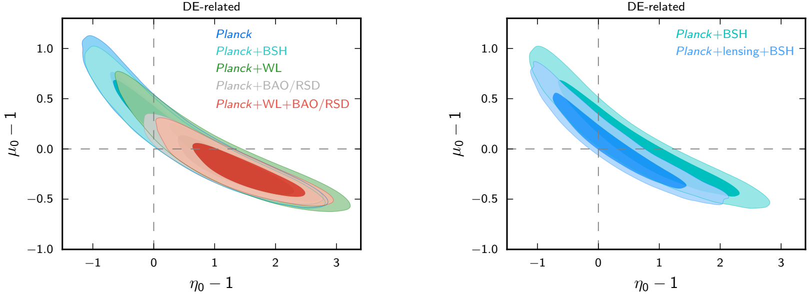


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# Solving tensions

One can then use these parameterizations to attempt solving the tensions between datasets.

Here there is no modification of background expansion, so we are not interested in the $H_0$.

We are changing how matter cluster; this can help our measurements of $S_8$.

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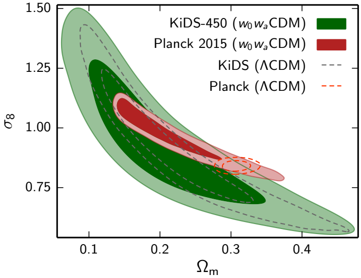


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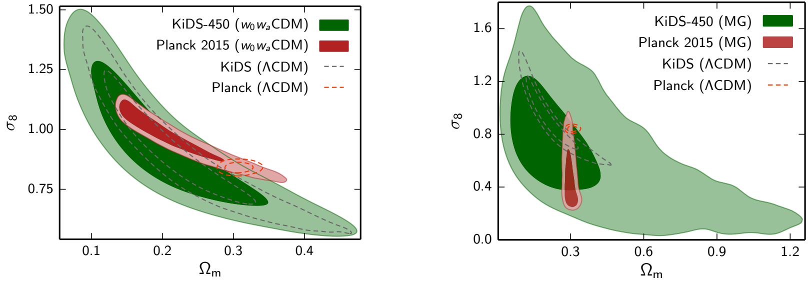


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# Overview

# $\Lambda$CDM non linear regime

Weak Lensing observations give measurements of $\delta$ at small scales.
When getting to small scales, we can't assume $\delta << 1$ and approximate the evolution equations at linear order. Thus, we can't compute $\delta(k, z)$ or $P(k, z)$ for $k \gtrsim 0.5\mathrm{Mpc}^{-1}$.
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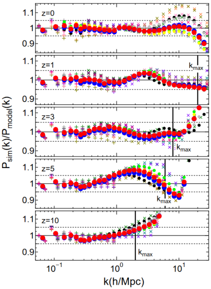


Takahashi et al. (2012)

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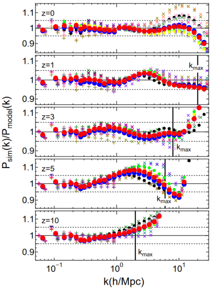


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Nbody simulations allow to obtain $\delta(k, z)$ at small scales. Fitting the results one can obtain phenomenological relations providing non linear corrections to linear $P(k, z)$.

This can be done within the models for which Nbody simulations are available.

# Importance of non linear scales

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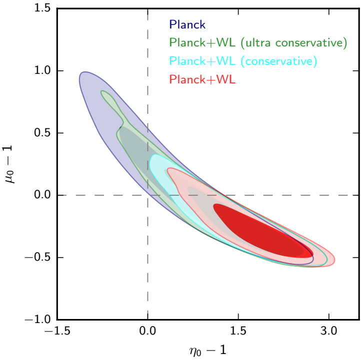
In MG the tension is completely removed, but this is due only to the loss of constraining power (less data points)

# An example




(this is not a Planck plot)

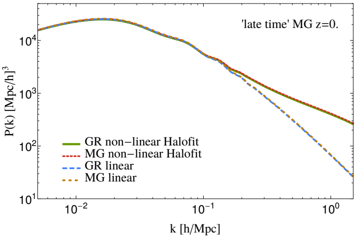
Changing the scale at which the data are cut, significantly impacts constraints.

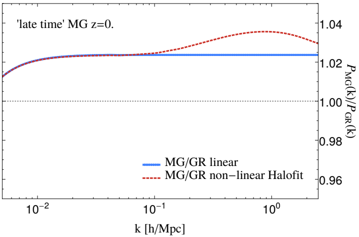
Without any cut CMB+WL show a significant deviation from ΛCDM!

Is this a real detection or is it only due to the fact we are considering non linear MG evolution at all scales with ΛCDM based corrections?

# Non linear corrections in MG

If we avoid cutting out the non linear scales in MG models, we effectively use the $\Lambda$CDM fitting function




We are making 2 assumptions:

- fitting functions from Nbody simulations are valid also in MG
- modifications of gravity act at all scales

While checking the first assumption would require Nbody simulations in MG, we can at least improve on the second.

# Screening of modified gravity

We know GR works at small scales (solar system, galaxy): the evolution of $\delta$ can't be modified in the same way at the smallest scales, where our MG has to reduce to GR.

We need to assume the existence of screening, acting at small scales.

It is reasonable to think that the non-linear power spectrum will have to match GR at sufficiently small scales, while at large scales it is modified.

Without having a specific model in mind, it remains arbitrary how the interpolation between the small scale regime and the large scale regime is done.

# Screening prescription for non linear scales

We use a phenomenological approach, previously used for $f(R)$, interpolating between the non linear MG power spectrum ($P_{HMG}$) and the standard GR one ($P_{HGR}$)

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$$P_{\mathrm{nlHS}}(k,z) = \frac{P_{HMG}(k,z) + c_{nl} S_L^2(k,z) P_{HGR}(k,z)}{1 + c_{nl} S_L^2(k,z)}$$

$$S_L^2(k,z) = \left[ \frac{k^3}{2\pi^2} P_{LMG}(k,z) \right]^s$$

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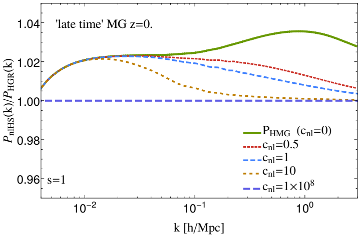
$$S_L^2(k,z) = \left[\frac{k^3}{2\pi^2} P_{LMG}(k,z)\right]^s$$

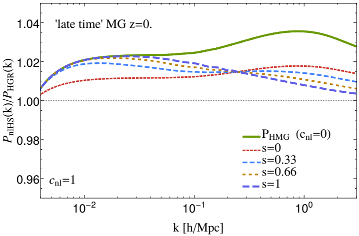
$c_{nl}$ and $s$ are the two parameters describing how MG reduces to GR at small scales. If we have a screening model and Nbody simulations, these can be fixed to some values. If not these will be free parameters to constrain with data.

Zhao (2013) - Hu and Sawicki (2007)

# Effect of screening prescription

The effect of this approach is to effectively mimic the GR $P(k)$ at very small scales




The way we reduce to GR at small scales depends on the choice of $c_{nl}$ and $s$.

# Overview

# Fisher Matrix for LSS

We applied this prescription to forecast of future constraints from LSS experiments (Euclid, SKA...) using the Fisher matrix approach.
These will measure with great accuracy Weak Lensing ang Galaxy Clustering observables.

$$F_{ij}^{GC} = \frac{V_{\rm survey}}{8\pi^2}\int_{-1}^{+1} \mathrm{d}\mu \int_{k_{\rm min}}^{k_{\rm max}} \mathrm{d}k\, k^2 \frac{\partial \ln P_{\rm obs}(k,\mu,z)}{\partial\theta_i}\frac{\partial \ln P_{\rm obs}(k,\mu,z)}{\partial\theta_j}$$
$$\left[\frac{n(z)P_{\rm obs}(k,\mu,z)}{n(z)P_{\rm obs}(k,\mu,z)+1}\right]^2$$

$$F_{\alpha\beta}^{WL} = f_{\rm sky}\sum_{\ell}^{\ell_{\rm max}}\sum_{i,j,k,l}\frac{(2\ell+1)\Delta\ell}{2}\frac{\partial C_{ij}(\ell)}{\partial\theta_\alpha}{\rm Cov}_{jk}^{-1}\frac{\partial C_{kl}(\ell)}{\partial\theta_\beta}{\rm Cov}_{li}^{-1}$$

# Fiducial Cosmology

In order to use the Fisher matrix approach we need to assume a fiducial cosmology.
We use the values of Planck 2015 for the standard cosmological parameters.
We model the modified gravity functions $\mu(z,k)$, $\eta(z,k)$ in 3 ways:

- Late time parameterization:

$$\mu(z) = 1 + E_{11}\Omega_{DE}(z)$$

- Early time parameterization:

$$\mu(z) = 1 + E_{11} + E_{12}(z/(1+z))$$

- binned reconstruction (5 $z$ bins):

$$\mu(z) = \mu(z_1) + \sum_{i=1}^{N-1} \frac{\mu(z_{i+1}) - \mu(z_i)}{2}\left[1 + \tanh\left(s\frac{z - z_{i+1}}{z_{i+1} - z_i}\right)\right]$$

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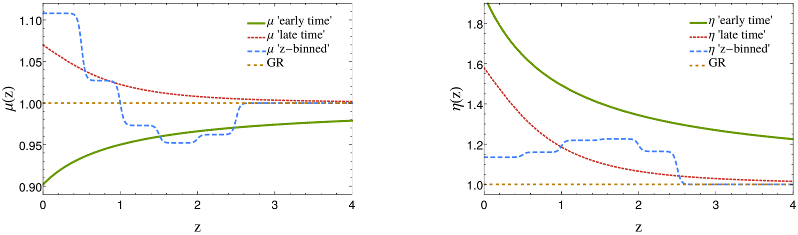


Fiducial values of the MG free parameters are obtained from Planck 2015 data constraints.

# Forecasted constraints on MG

Using the specifications of the Euclid satellite to build the Fisher matrix, we can test how the inclusion of non linear scales(with and without the screening prescription) will impact the constraints.

Late time parameterization

| **Euclid** (Redbook) | $\Omega_c$ | $\Omega_b$ | $n_s$ | $\ell\mathcal{A}_s$ | $h$ | $\mu$ | $\eta$ | $\Sigma$ | MG FoM |
|---|---|---|---|---|---|---|---|---|---|
| Fiducial | 0.254 | 0.048 | 0.969 | 3.060 | 0.682 | 1.042 | 1.719 | 1.416 | relative |
| **GC(lin)** | 1.9% | 6.4% | 3% | 2.8% | 4.5% | 17.1% | 1030% | 641% | 0 |
| **GC(nl-HS)** | 0.9% | 2.5% | 1.3% | 0.8% | 1.7% | 1.7% | 475% | 291% | 2.9 |
| **GC(nl-HS)**+*Planck* | 0.7% | 0.6% | 0.3% | 0.2% | 0.3% | 1.7% | 16.8% | 10.3% | 6.3 |
| **WL(lin)** | 7.8% | 25.7% | 9.9% | 10.3% | 19.1% | 58.2% | 106% | 9.3% | 3.2 |
| **WL(nl-HS)** | 6.3% | 20.7% | 4.6% | 5.8% | 13.8% | 23.3% | 40.9% | 4.6% | 4.5 |
| **WL(nl-HS)**+*Planck* | 2.1% | 1.1% | 0.4% | 0.7% | 0.7% | 11.8% | 21.8% | 2.8% | 5.7 |
| **GC+WL(lin)** | 1.8% | 5.9% | 2.8% | 2.3% | 4.2% | 7.1% | 10.6% | 2% | 6.6 |
| **GC+WL(lin)**+*Planck* | 1.0% | 0.7% | 0.4% | 0.4% | 0.4% | 6.2% | 9.8% | 1.5% | 7.0 |
| **GC+WL(nl-HS)** | 0.8% | 2.2% | 0.8% | 0.7% | 1.5% | 1.6% | 2.4% | 1.0% | 8.8 |
| **GC+WL(nl-HS)**+*Planck* | 0.7% | 0.6% | 0.2% | 0.2% | 0.3% | 1.6% | 2.4% | 0.9% | 8.9 |
| **GC+WL(nl-Halofit)**+*Planck* | 0.6% | 0.5% | 0.2% | 0.2% | 0.2% | 0.8% | 1.7% | 0.8% | 9.6 |

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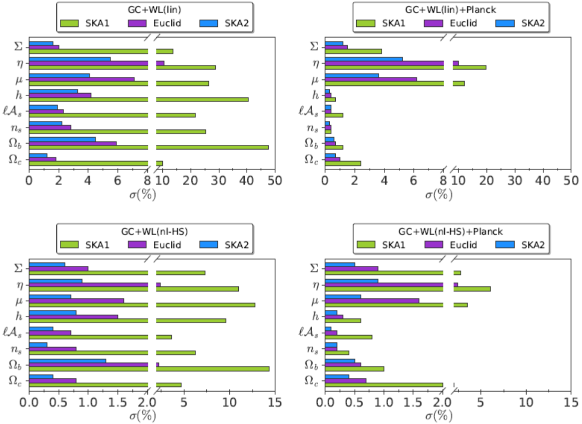

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Binned reconstruction

| **Euclid** (Redbook) | $\ell\mathcal{A}_s$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\mu_5$ | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ | $\eta_5$ | MG FoM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Fiducial | 3.057 | 1.108 | 1.027 | 0.973 | 0.952 | 0.962 | 1.135 | 1.160 | 1.219 | 1.226 | 1.164 | relative |
| **GC (lin)** | 160% | 119% | 159% | 183% | 450% | 1470% | 509% | 570% | 586% | 728% | 3390% | 0 |
| **GC (nl-HS)** | 0.8% | 7.0% | 6.7% | 10.9% | 27.4% | 41.1% | 20% | 24.3% | 19.9% | 38.2% | 930% | 19 |
| **WL (lin)** | 640% | 165% | 2210% | 4150% | 13100% | 22500% | 2840% | 3140% | 8020% | 29300% | 39000% | -27 |
| **WL (nl-HS)** | 7.3% | 188% | 255% | 419% | 222% | 206% | 330% | 488% | 775% | 8300% | 9380% | -10 |
| **GC+WL (lin)** | 11.3% | 5.8% | 10% | 19.2% | 282% | 469% | 7.9% | 9.6% | 16.1% | 276% | 2520% | 12 |
| **GC+WL+*Planck* (lin)** | 1.1% | 3.4% | 4.8% | 7.8% | 9.3% | 13.1% | 6.2% | 7.7% | 9.1% | 12.7% | 23.6% | 27 |
| **GC+WL (nl-HS)** | 0.8% | 2.2% | 3.3% | 8.2% | 24.8% | 34.1% | 3.6% | 5.1% | 8.1% | 25.4% | 812% | 24 |
| **GC+WL+*Planck* (nl-HS)** | 0.3% | 1.8% | 2.5% | 5.8% | 7.8% | 10.3% | 3.2% | 4.1% | 5.9% | 9.6% | 19.5% | 33 |
| **GC+WL+*Planck* (nl-Halofit)** | 0.4% | 2.0% | 2.4% | 5.1% | 7.4% | 10.2% | 3.5% | 4.1% | 5.8% | 9.2% | 18.9% | 33 |

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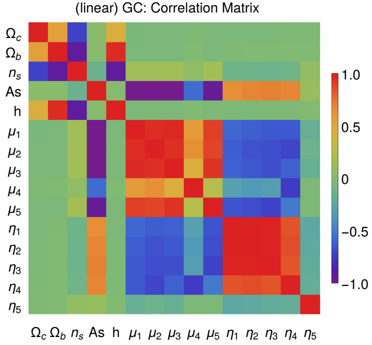


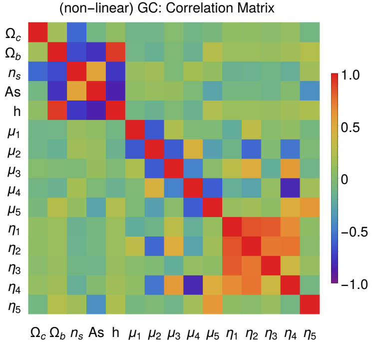

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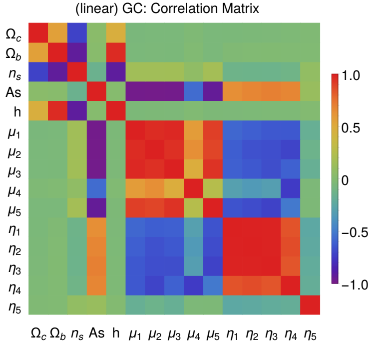


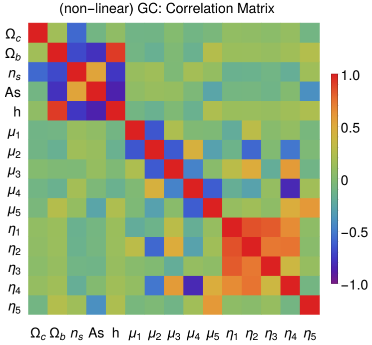


Warning: analysis is done using $c_{nl} = s = 1$ for the non linear interpolation

# Impact of non linear parameters

In order to assess the impact of the choice of these parameters on the results, we included them as free parameters and marginalized over them when obtaining bounds on MG functions.

| **Euclid** (Redbook) | $\Omega_c$ | $\Omega_b$ | $n_s$ | $\ell\mathcal{A}_s$ | $h$ | $\mu$ | $\eta$ | $\Sigma$ | $c_{\rm nl}$ | $s$ | MG FoM |
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| **GC(nl-HS)** | 1.0% | 2.8% | 1.3% | 1.1% | 2.0% | 1.7% | 784% | 480% | 372% | 236% | 2.4 |
| **WL(nl-HS)** | 6.5% | 25% | 8.3% | 9.1% | 19% | 25% | 46% | 6.0% | 1680% | 899% | 4.2 |
| **GC+WL(nl-HS)** | 1% | 2.8% | 1.2% | 1% | 1.9% | 1.6% | 2.6% | 1.2% | 333% | 166% | 8.5 |

Results are stable even introducing these extra parameters in the GC+WL case, while there is less stability in the single probe cases.

# Overview

# Take home message

- There is theoretical and observational motivation to test modified gravity
- Given the amount of available model, testing parametrized departures from GR might be more efficient
- Currently available data allow to constrain these parameterizations, but LSS observables are limited by our knowledge of non linear evolution in theories that are not $\Lambda$CDM.
- Future LSS observations will greatly improve our measurements and possibly detect or rule out some possible MG models
- Since methods to compute non linear evolution in MG are not available (yet), phenomenological approaches can at least guarantee we are including the requirement of a return to GR at small scales.
- Finding a reliable approach for non linearities is crucial to fully exploit the constraining power of future LSS surveys.