Stability of Stellar Filaments in Modified Gravity

Speaker: Dr. Zeeshan Yousaf

Assistant Professor
Department of Mathematics
University of the Punjab
Lahore-Pakistan

Hot Topics in Modern Cosmology, XIIth Spontaneous Workshop, 14-19 May, 2018; Cargèse, France.

- What is stability?

- What is $f(R,T,Q)$ gravity?

Stellar Filaments

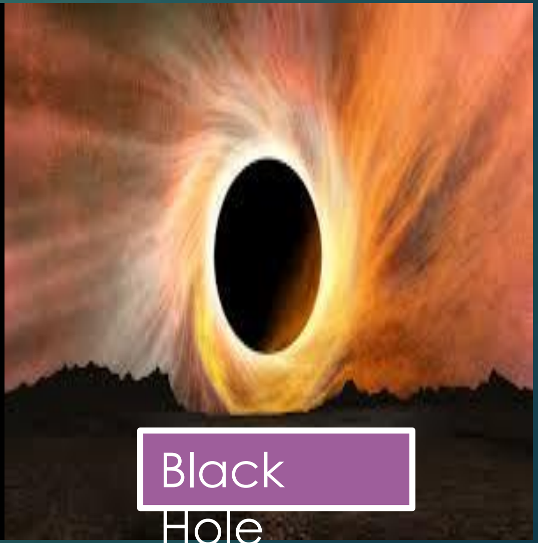
The subject of exploring the cosmic filamentary celestial objects has been a focus of great attention of many astrophysicists. On a large cosmic scale, it has been analyzed that matter is usually configured to make large filaments. These stellar structures have been found to be very clear characteristics of the interstellar medium. They may give rise to galaxies upon contraction. Motivated by several simulations and observational results, the stability analysis of cosmic filaments with more realistic assumptions has received great interest.

S. Chandrasekhar, E. Fermi, Astrophys. J. **118**, 116 (1953); J. Comparetta, A.C. Quillen, Mon. Not. R. Astron. Soc. **414**, 810 (2011)

Compact stars

- Stars in which the density of matter is much greater than in ordinary stars

- In addition to very high density, these objects are characterized by fact that nuclear reactions have completely ceased in their interiors



Stability

- Problem of stability has utmost relevance to structure formation and evolution of self gravitating objects
- Great deal of work has been devoted to this issue since the pioneering work by Chandrasekhar
  - Stable

    $$\Gamma > \frac{4}{3}$$

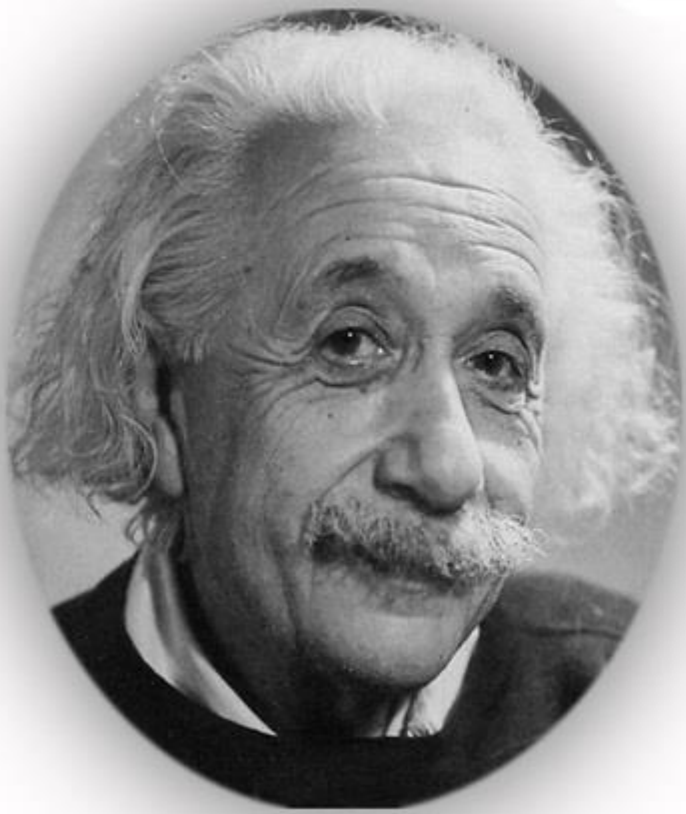
  - Unstable

    $$\Gamma < \frac{4}{3}$$

S. Chandrasekhar, Astrophys. J. 140, 417 (1964)

Albert Einstein

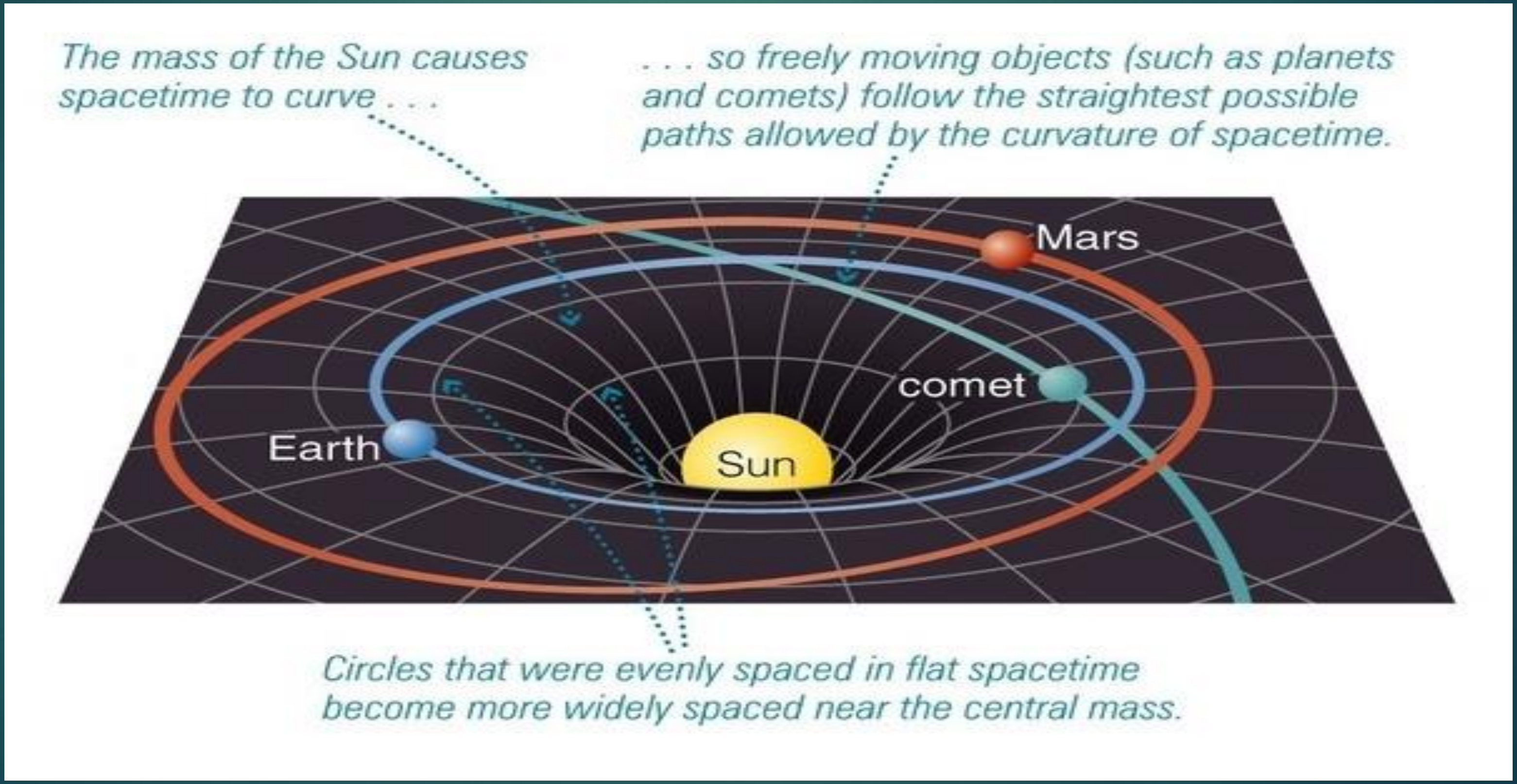
10 Amazing Life Lessons from Albert Einstein:



1. Follow Your Curiosity
2. Perseverance is Priceless
3. Focus on the Present
4. The Imagination is Powerful
5. Make Mistakes
6. Live in the Moment
7. Create Value
8. Don't be repetitive
9. Knowledge Comes From Experience
10. Learn the Rules and Then Play Better

General Relativity

- Einstein mandated that the mass of the sun creates a dimple on the spacetime and the planets are following that path



General Relativity

- Einstein field equations

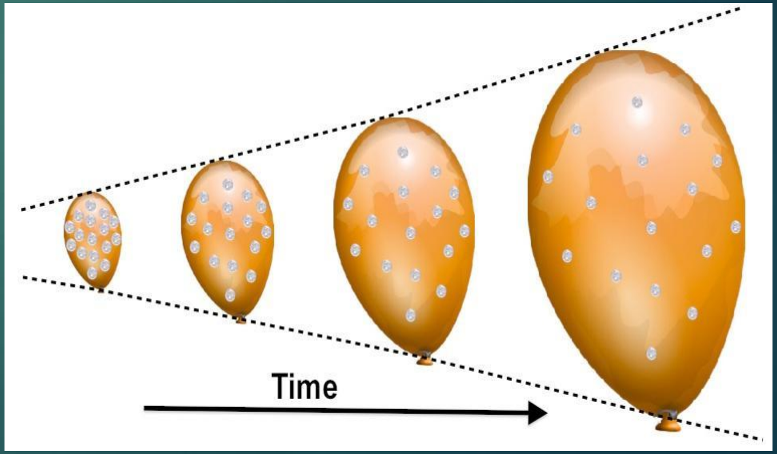
$$R_{\gamma\delta} - \frac{R}{2} g_{\gamma\delta} = G_{\gamma\delta} = \kappa T_{\gamma\delta}$$

- *Matter tells spacetime how to curve*
- *Spacetime curvature tells matter how to move*

Expanding Nature of Universe

- Hubble observed galaxies at different distances
- Concluded that these galaxies were moving away from earth with velocities having direct relation to their distances from earth

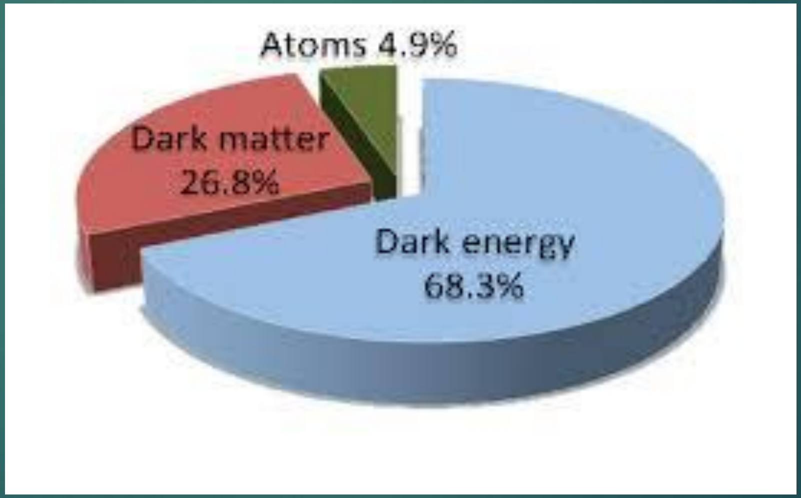
Hubble (1929)



Expansion of universe, balloon analogy:

Current nature of Universe

- Current physical cosmology is dominated by DE era
- Describes the accelerated expanding nature of our Universe
- Estimated Energy budget



P. A. R. Ade et al., Astron. Astrophys, 571, 16 (2014)

Evidence of accelerating expansion of Universe

Is GR explains current cosmic acceleration?

- Although GR is widely accepted and authentic theory with its graceful structures & elegant of its concepts
- Rather disconcerting to note that questions about these Dark sources remained unanswered in this theory

Need of Modifying gravity theories

Motivation

- Proposed to explain Astrophysical and cosmological data on DM & DE

- Preserve the elegant concepts & results of GR at small scales

Modified Theories of GR

- f(R) gravity theory

$$S_{f(R)} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_M$$

- f(R,T) gravity theory

$$S_{f(R,T)} = \frac{1}{2}\int d^4x\sqrt{-g}[f(R,T) + L_M]$$

T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, Phys. Rev. D 84, 024020 (2011)

Modified Theories of GR

- f(R,T,Q) gravity theory

$$S_{f(R,T,Q)} = \frac{1}{2}\int d^4x\sqrt{-g}[f(R,T,Q) + L_M]$$

where

$$Q = R_{\gamma\delta}T^{\gamma\delta}$$

Z. Haghani, T. Harko, F. S. N. Lobo, H. R. Sepangi and S. Shahidi, Phys. Rev. D 88, 044023 (2013)

Our concern!

The field equations in respective theory can be evaluated as

$$T_{\gamma\delta}{}^{\text{eff}} = \frac{1}{(f_R - f_Q\mathcal{L}_M)}\left[(1 + f_T + \frac{1}{2}Rf_Q)T^{(m)}_{\gamma\delta} + \left\{\frac{R}{2}\left(\frac{f}{R} - f_R\right) - f_T\mathcal{L}_M - \frac{1}{2}\right.\right.$$
$$\times\nabla_\mu\nabla_\nu(f_Q T^{\mu\nu})\Big\}\, g_{\gamma\delta} - \frac{1}{2}\Box(f_Q T_{\gamma\delta}) - (g_{\gamma\delta}\Box - \nabla_\gamma\nabla_\delta)f_R - 2f_Q R_{\mu(\gamma}T^{\mu}{}_{\delta)}$$
$$\left.+\nabla_\mu\nabla_{(\gamma}[T^{\mu}{}_{\delta)}f_Q] + 2\left(f_T g^{\mu\nu} + f_Q R^{\mu\nu}\right)\frac{\partial^2\mathcal{L}_M}{\partial g^{\gamma\delta}\partial g^{\mu\nu}}\right]$$

where

$$\Box = g^{\gamma\delta}\nabla_\gamma\nabla_\delta$$

Proposals

- *f(R,T,Q)* gravity involves Lagrangian, demonstrating non-minimal coupling between matter and geometry
- Very good approach to study the current nature of universe
- Provides the results for conserved as well as for non-conserved systems
- Permits to pursue some well-consistent cosmological models

βR(1+αQ) Gravity

- For α = 0, β = 0, the inactive conversion of field is redefined which makes the respective model physically well-consistent.

- Here, we are interested in dealing with the model for β=1 to introduce the consequences of cosmic expansion including Q parametric quantity.

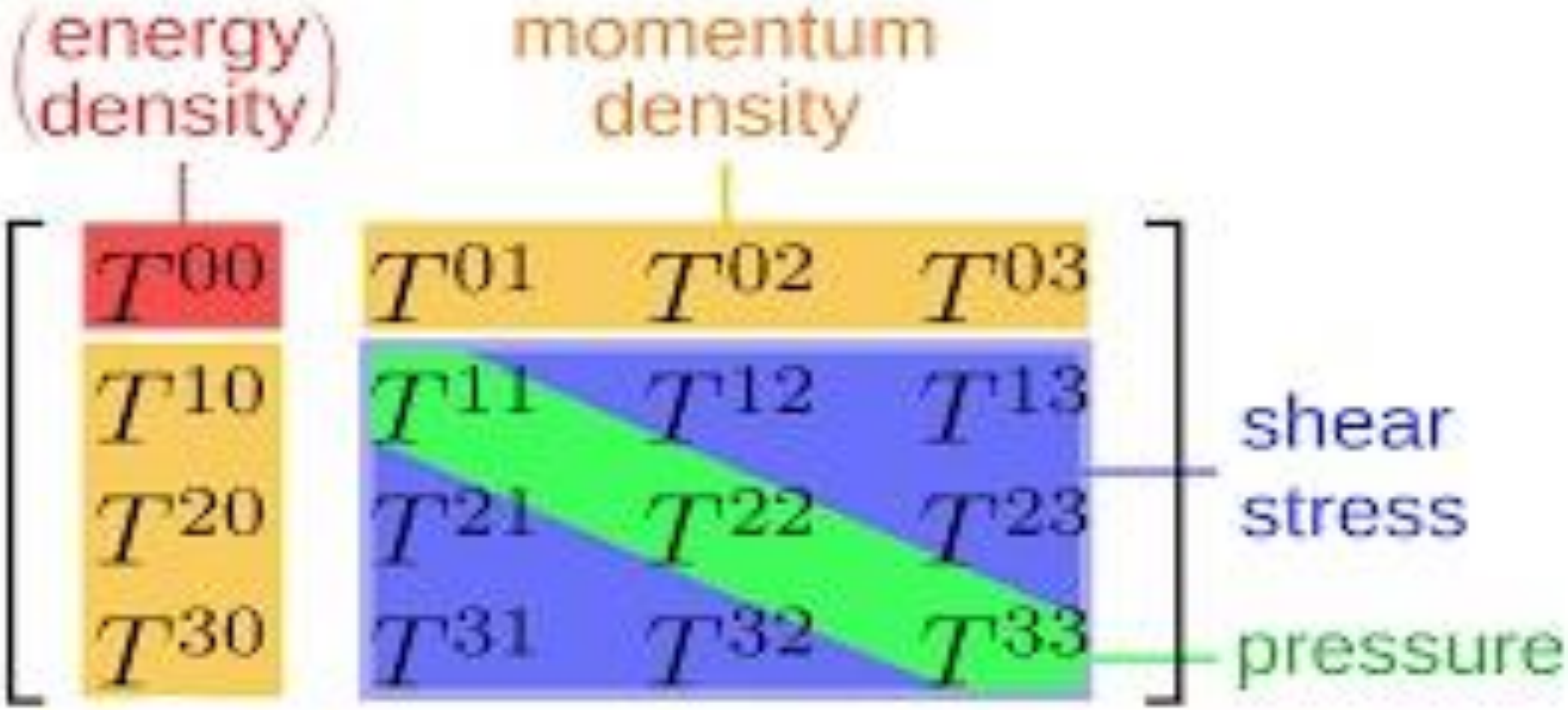
I. Ayuso, J. B. Jiménez and Á. de la Cruz-Dombriz, Phys. Rev. D 91, 104003 (2015)

Paramount concepts related to the designed work

Space-time

- Space: where things happen
- Time: when things happen
- Space-time continuum: mathematical model, interlinked Space & time into single continuum

Source of gravitational field equations



Perturbation Strategy

- mathematical method for computing approximate solution to problem of celestial objects that are initially at equilibrium
- shows that fluid contents have only radial dependence, such quantities are identified with subscript zero
- As time passes, system evolved and subjected to oscillatory motion and the time dependence involves

$$Y(t,r) = y_o(r) + \epsilon\omega(t)y(r), \quad Z(t,r) = z_0(r) + \epsilon\bar{z}(t,r)$$

L. Herrera, G. Le Denmat and N. O. Santos, Gen. Relativ. Gravit. 44, 1143(2012)

Stiffness Parameter

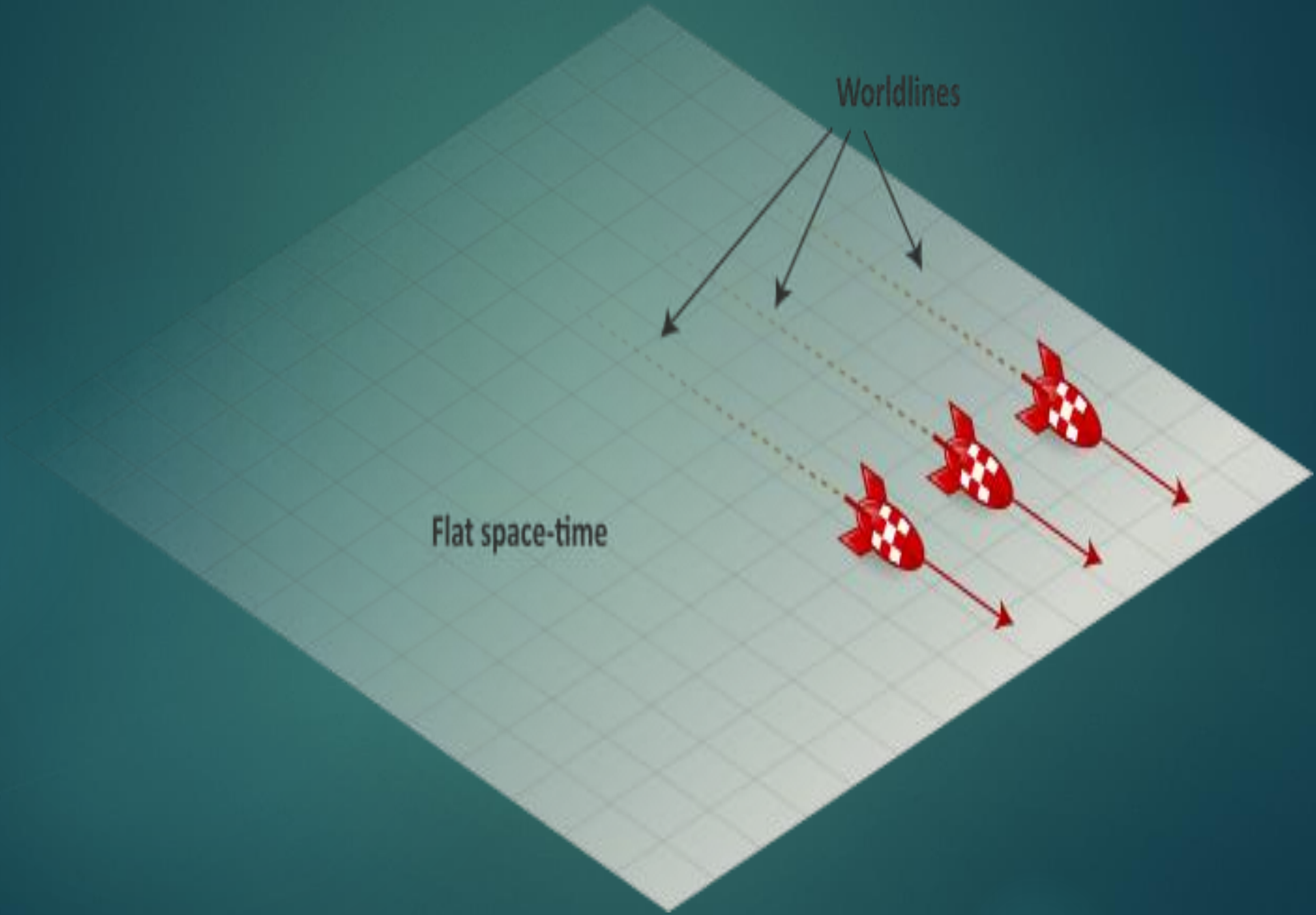
- Supports the relationship between perturbed configuration of energy density and fluid pressure

$$\bar{P}_i = \Gamma_1 \frac{P_{i0}}{\mu_0 + P_{i0}} \bar{\mu}$$

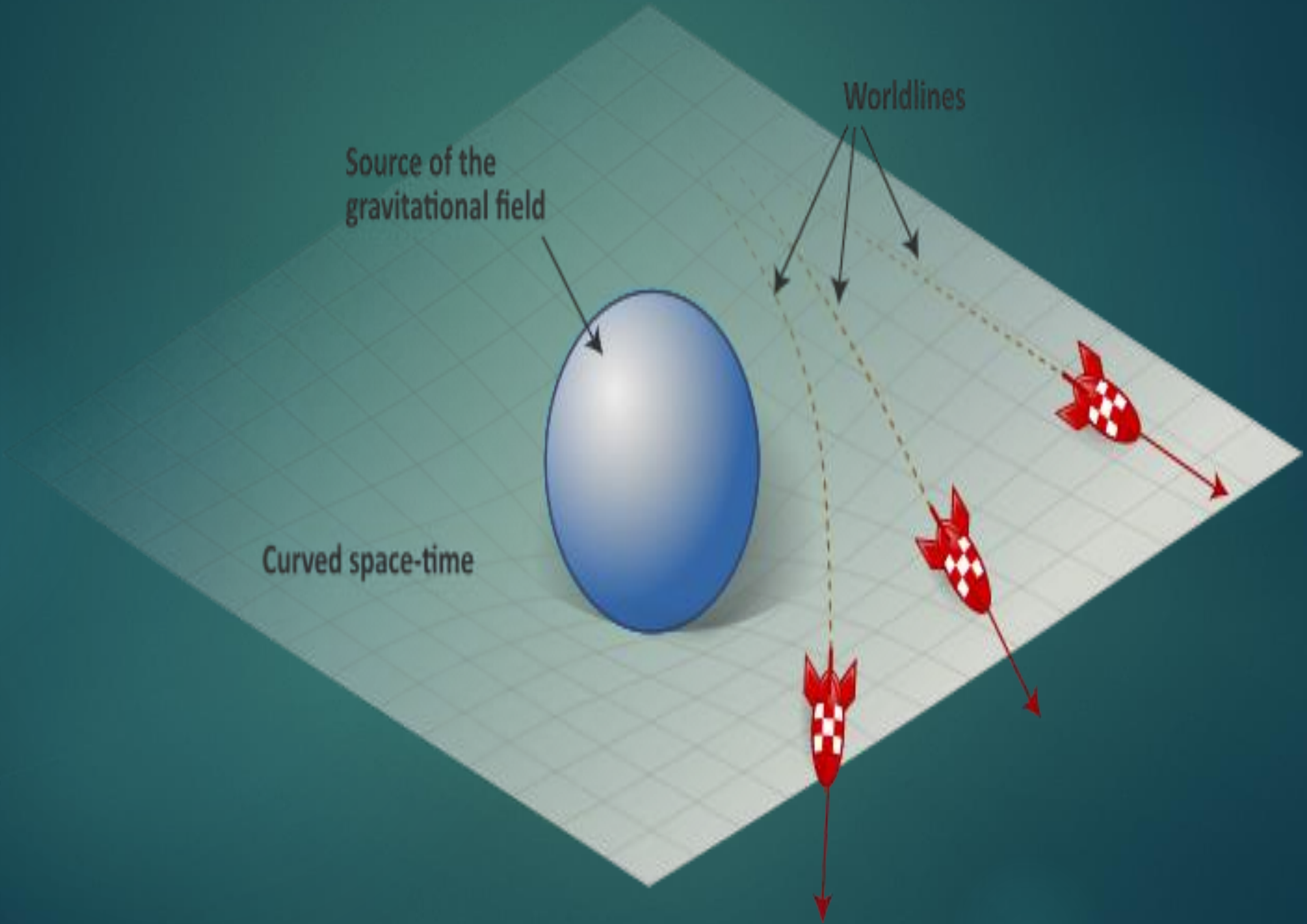
- Has remarkable role in stability analysis as its value define the range of stability

B. K. Harrison, K. S. Throne, M. Wakano and J. A. Wheeler, *Gravitation Theory and Gravitational Collapse* (University of Chicago press, 1965).

Newtonian approximation



post Newtonian approximation



IOP Publishing — Classical and Quantum Gravity

Class. Quantum Grav. **34** (2017) 145002 (19pp) https://doi.org/10.1088/1361-6382/aa73b9

Stability analysis of stellar radiating filaments

Z Yousaf, M Zaeem-ul-Haq Bhatti and Ume Farwa

Department of Mathematics, University of the Punjab, Quaid-i-Azam Campus, Lahore-54590, Pakistan

Plan of Work

- Consider the anisotropic dissipative fluid within the cylindrical configuration, which collapses under the condition of non-zero expansion scalar

- Construct the field equations and conservation laws in the perspective of this gravity

- Apply perturbation technique to analyze the behavior of *f(R,T,Q)* model on the evolution of dissipative system

- Check the role of adiabatic index in the formulations of instability regions, also explored the instability constraints with N approximations

The configuration of our relativistic celestial system to be cylindrical whose spacetime is

$$ds^2_- = -A^2(t,r)(dt^2 - dr^2) + B^2(t,r)dz^2 + C^2(t,r)d\phi^2.$$

Our cylindrical geometry is filled with anisotropic non-radiating collapsing fluid distribution

$$T_{\lambda\sigma} = (P_r + \mu)V_\lambda V_\sigma + P_r g_{\lambda\sigma} + q(L_\lambda V_\sigma + L_\sigma V_\lambda) - K_\lambda K_\sigma(P_r - P_\phi) - S_\lambda S_\sigma(P_r - P_z)$$

where

$$V_\lambda = -A\delta^0_\lambda, K_\lambda = C\delta^3_\lambda, \; L_\lambda = A\delta^1_\lambda \text{ and } S_\lambda = B\delta^2_\lambda$$

$$V^\lambda V_\lambda = -1, \; K^\lambda K\lambda = 1 = S^\lambda S_\lambda, \; V^\lambda K_\lambda = V^\lambda S_\lambda = K^\lambda S_\lambda = 0$$

The $f(R, T, Q)$ field equations

$$\overset{\text{eff}}{\mu} = \frac{1}{f_R - f_Q \mathcal{L}_M}\left[\mathcal{L}_M f_T - \frac{1}{2}(f - R f_R) + \mu\chi_1 + \dot{\mu}\chi_2 + \frac{\ddot{\mu}}{2A^2} f_Q + \frac{\mu''}{2A^2} f_Q \right.$$

$$+ \mu'\chi_3 + \frac{P_r''}{2A^2} f_Q + P_r\chi_4 + P_r'\left\{\frac{f_Q'}{A^2} - \frac{5A'}{2A^3} f_Q\right\} - \frac{f_Q}{2A^2 B}(\dot{P}_z \dot{B} + P_z' B')$$

$$- \dot{P}_r \frac{\dot{A}}{A^3} f_Q + P_z\chi_5 + P_\phi\chi_6 - \frac{f_Q}{2A^2}\left(\dot{P}_\phi \frac{\dot{C}}{C} - P_\phi' \frac{C'}{C}\right) - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$$

$$\left. - \frac{f_R'}{A^2}\left(\frac{A'}{A} - \frac{B'}{B} - \frac{C'}{C}\right) + \frac{f_R''}{A^2}\right],$$

$$\overset{\text{eff}}{P_r} = \frac{1}{f_R - f_Q \mathcal{L}_M}\left[\frac{1}{2}(f - R f_R) - \mathcal{L}_M f_T + \frac{\ddot{f}_R}{A^2}\psi_1 + \dot{\mu}\left(\frac{5\dot{A}}{2A^3} f_Q - \frac{\dot{f}_Q}{A^2}\right)\right.$$

$$- f_R'\psi_2 + P_r\chi_7 - \frac{\ddot{\mu}}{2A^2} f_Q + \mu\chi_8 + \frac{\mu' A'}{2A^3} f_Q - \frac{\ddot{P}_r}{2A^2} f_Q + P_z\chi_9 + P_\phi\chi_{10}$$

$$\left. + \frac{f_Q}{2A^2}\left\{\dot{P}_z \frac{\dot{B}}{B} - P_z' \frac{B'}{B} + \dot{P}_\phi \frac{\dot{C}}{C} - P_\phi' \frac{C'}{C}\right\} + \dot{P}_r\chi_{11} + P_r'\chi_{12}\right],$$

$$\overset{\text{eff}}{P_z} = \frac{1}{f_R - f_Q \mathcal{L}_M}\left[\frac{1}{2}(f - R f_R) - \mathcal{L}_M f_T + \dot{\mu}\chi_{14} + \mu\chi_{13} - \frac{\ddot{\mu}}{2A^2} f_Q + \frac{\mu' A'}{2A^3} f_Q \right.$$

$$+ P_r\chi_{15} + \frac{\dot{A}\dot{P}_r}{2A^3} f_Q + P_r'\left(\frac{5A'}{2A^3} f_Q - \frac{f_Q'}{A^2}\right) - \frac{P_r''}{2A^2} f_Q + P_z\chi_{16} + P_z'\chi_{17} - \dot{P}_z$$

$$\left. \times \chi_{18} - \frac{\ddot{P}_z}{2A^2} f_Q + \frac{P_z''}{2A^2} f_Q + P_\phi\chi_{19} + \frac{f_Q}{2A^2}\left(\dot{P}_\phi \frac{\dot{C}}{C} - P_\phi' \frac{C'}{C}\right) + \psi_3\right],$$

$$\overset{\text{eff}}{P_\phi} = \frac{1}{f_R - f_Q\mathcal{L}_M}\left[\frac{1}{2}(f - Rf_R) - \mathcal{L}_M f_T + \dot{\mu}\chi_{14} + \mu\chi_{13} - \frac{\ddot{\mu}}{2A^2}f_Q + \frac{\mu' A'}{2A^3}f_Q \right.$$
$$+ P_r\chi_{15} + \frac{\dot{A}\dot{P}_r}{2A^3}f_Q + P_r'\left(\frac{5A'}{2A^3}f_Q - \frac{f_Q'}{A^2}\right) - \frac{P_r''}{2A^2}f_Q + P_z\chi_{20} + P_\phi'\chi_{23} + \dot{P}_\phi$$
$$\left. \times\chi_{22} - \frac{\ddot{P}_\phi}{2A^2}f_Q + \frac{P_\phi''}{2A^2}f_Q + P_\phi\chi_{21} + \frac{f_Q}{2A^2}\left(\dot{P}_z\frac{\dot{B}}{B} - P_z'\frac{B'}{B}\right) + \psi_4\right],$$

$$\overset{\text{eff}}{q} = \frac{1}{f_R - f_Q\mathcal{L}_M}\left[q''\frac{f_Q}{2A^2} - \ddot{q}\frac{f_Q}{2A^2} - \dot{q}\left\{\frac{f_Q}{2A^2}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{4\dot{A}}{A}\right) + \frac{\dot{f}_Q}{A^2}\right\} + q \right.$$
$$\times\frac{f_Q}{A^2}\left\{\frac{A'^2}{A^2} - \frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A} + \frac{A''}{A} - \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{A'}{A}\left(\frac{B'}{B} + \frac{C'}{C}\right) - \frac{\dot{f}_Q}{A^2}\right.$$
$$\times\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{2B} + \frac{\dot{C}}{2C}\right) + \frac{f_Q'}{2A^2}\left(\frac{B'}{B} + \frac{C'}{C} + \frac{4A'}{A}\right) - \frac{\ddot{f}_Q}{2A^2} - \frac{f_Q''}{2A^2}$$
$$\left. + \left(1 + f_T - \frac{3}{2}Rf_Q\right)\right\} + q'\left\{\frac{f_Q''}{A^2} + \frac{f_Q}{A^2}\left(\frac{B'}{B} + \frac{C'}{2C} + \frac{2A'}{A}\right) - \frac{\dot{f}_R{}'}{A^2} + \frac{1}{A^3}\right.$$
$$\left.\left. \times\left(\dot{A}f_R' + A'\dot{f}_R\right)\right\}\right]$$

The corresponding expansion scalar can be given as follows

$$\Theta = \frac{1}{A}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$$

The shear tensor has been found to be

$$\sigma_{\lambda\sigma} = \sigma_s\left(S_\lambda S_\sigma - \frac{h_{\lambda\sigma}}{3}\right) + \sigma_k\left(K_\lambda K_\sigma - \frac{h_{\lambda\sigma}}{3}\right)$$

where σ_s and σ_k are shear scalars, given by

$$\sigma_s = -\frac{1}{A}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right), \quad \sigma_k = -\frac{1}{A}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)$$

The value of the corresponding Ricci scalar is

$$R = \frac{2}{A^2}\left[\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)-\left(\frac{A''}{A}+\frac{B''}{B}+\frac{C''}{C}\right)+\frac{1}{A^2}(A'^2-\dot{A}^2)+\frac{1}{BC}(\dot{B}\dot{C}-B'C')\right]$$

After applying perturbations schemes, it turns out that

$$\omega = \omega(t) = -\exp(\xi t)$$

where

$$\xi^2 = -\left(\frac{a}{A_o}+\frac{b}{B_o}+\frac{c}{C_o}\right)^{-1}\left[\frac{a''}{A_o}-\frac{aA_o''}{A_o}+\frac{B_o'}{B_o}\left(\frac{c}{C_o}\right)'+\frac{C_o'}{C_o}\left(\frac{b}{B_o}\right)'\right]$$

The divergence of the stress energy tensor in *f(R,T,Q)* theory provides

$$\nabla^{\lambda}T_{\lambda\sigma} = \frac{2}{Rf_Q + 2f_T + 1}\Big[\nabla_{\sigma}(L_m f_T) + \nabla_{\sigma}(f_Q R^{\pi\lambda}T_{\pi\sigma}) - \frac{1}{2}(f_T g_{\pi\rho} + f_Q R_{\pi\rho}) \times \nabla_{\sigma}T^{\pi\rho} - G_{\lambda\sigma}\nabla^{\lambda}(f_Q L_m)\Big],$$

For our cylindrically symmetric system, we have

$$D_t \overset{\text{eff}}{\mu} + \Theta\left[\overset{\text{eff}}{\mu} + \frac{1}{3}\left(\overset{\text{eff}}{P_r} + \overset{\text{eff}}{P_z} + \overset{\text{eff}}{P_\phi}\right)\right] + \nabla \overset{\text{eff}}{q} + \frac{1}{3}\left(\overset{\text{eff}}{P_z} - \overset{\text{eff}}{P_r}\right)(2\sigma_s - \sigma_k) + \frac{1}{3}\left(\overset{\text{eff}}{P_\phi} - \overset{\text{eff}}{P_r}\right)(2\sigma_k - \sigma_s) + \overset{\text{eff}}{q}\left[2a + \frac{1}{A}\left(\frac{B'}{B} + \frac{C'}{C}\right)\right] + Z_1 = 0,$$

$$\nabla \overset{\text{eff}}{P_r} - \frac{1}{A}\left[\left(\overset{\text{eff}}{P_z} - \overset{\text{eff}}{P_r}\right)\frac{B'}{B} + D_t \overset{\text{eff}}{q} + \left(\overset{\text{eff}}{P_\phi} - \overset{\text{eff}}{P_r}\right)\frac{C'}{C}\right] + \left(\overset{\text{eff}}{\mu} + \overset{\text{eff}}{P_r}\right) \times a - \frac{1}{3}(\sigma_s - 4\Theta + \sigma_k)\overset{\text{eff}}{q} + Z_2 = 0,$$

The perturbation of *f(R, T,Q)* model takes the from

$$f = R_o(1+\alpha Q_o) + \epsilon\omega(t)[d + \alpha(dQ_o + R_o g)]$$

Static distributions of *f(R,T,Q)* field equations

$$\mu_o^{\mathrm{eff}} = \frac{1}{1+\alpha(Q_o+R_o\mu_o)}\left[\mu_o'\chi_{3o} + \mu_o\chi_{1o} + P_{ro}\chi_{4o} + P_{\phi o}\chi_{6o} + P_{zo}\chi_{5o} + \frac{\alpha}{A_o^2}\right.$$
$$\left.\times(Q_o'' + R_o'P_{r0}') - \alpha Q_o'\psi_{2o} + \frac{\alpha R_o}{2A_o^2}\left(\mu_o'' + P_{ro}'' + P_{zo}'\frac{B_o'}{B_o} + P_{\phi o}'\frac{C_o'}{C_o} - 5P_{ro}' \times \frac{A_o'}{A_o}\right) - \frac{\beta}{2}Q_o\right]$$

$$P_{ro}^{\mathrm{eff}} = \frac{1}{1+\alpha(Q_o+R_o\mu_o)}\left[P_{ro}'\chi_{12o} + \mu_o\chi_{8o} + P_{ro}\chi_{7o} + P_{\phi o}\chi_{10o} + P_{zo}\chi_{9o} - \alpha\right.$$
$$\left.\times Q_o'\psi_{2o} + \frac{\alpha R_o}{2A_o^2}\left(\mu_o'\frac{A_o'}{2A_o} - P_{zo}'\frac{B_o'}{B_o} - P_{\phi o}'\frac{C_o'}{C_o}\right) + \frac{\beta}{2}Q_o\right],$$

$$P_{zo}^{\mathrm{eff}} = \frac{1}{1+\alpha(Q_o+R_o\mu_o)}\left[P_{ro}\chi_{15o} + \mu_o\chi_{13o} + P_{zo}\chi_{16o} + P_{zo}'\chi_{17o} + \chi_{14o}P_{\phi o} + \psi_{3o}\right.$$
$$\left.+\frac{\alpha R_o}{2A_o^2}\left(\mu_o'\frac{A_o'}{A_o} + 5P_{ro}'\frac{A_o'}{A_o} - 2P_{ro}'\frac{R_o'}{R_o} - P_{ro}'' + P_{zo}'' - P_{\phi o}'\frac{C_o'}{C_o}\right)\right],$$

and

$$P_{\phi o}^{\text{eff}} = \frac{1}{1+\alpha(Q_o+R_o\mu_o)}\left[\chi_{13o}\mu_o + P_{ro}\chi_{15o} + \chi_{20o}P_{zo} + \chi_{23o}P'_{\phi o} + P_{\phi o}\chi_{21o} + \psi_{4o} + \frac{\alpha R_o}{2A_o^2}\left(\mu'_o\frac{A'_o}{A_o} + 5P'_{ro}\frac{A'_o}{A_o} - 2P'_{ro}\frac{R'_o}{R_o} - P''_{ro} + P''_{\phi o} - P'_{zo}\frac{B'_o}{B_o}\right)\right],$$

JUNCTION CONDITIONS

- **What are they?**

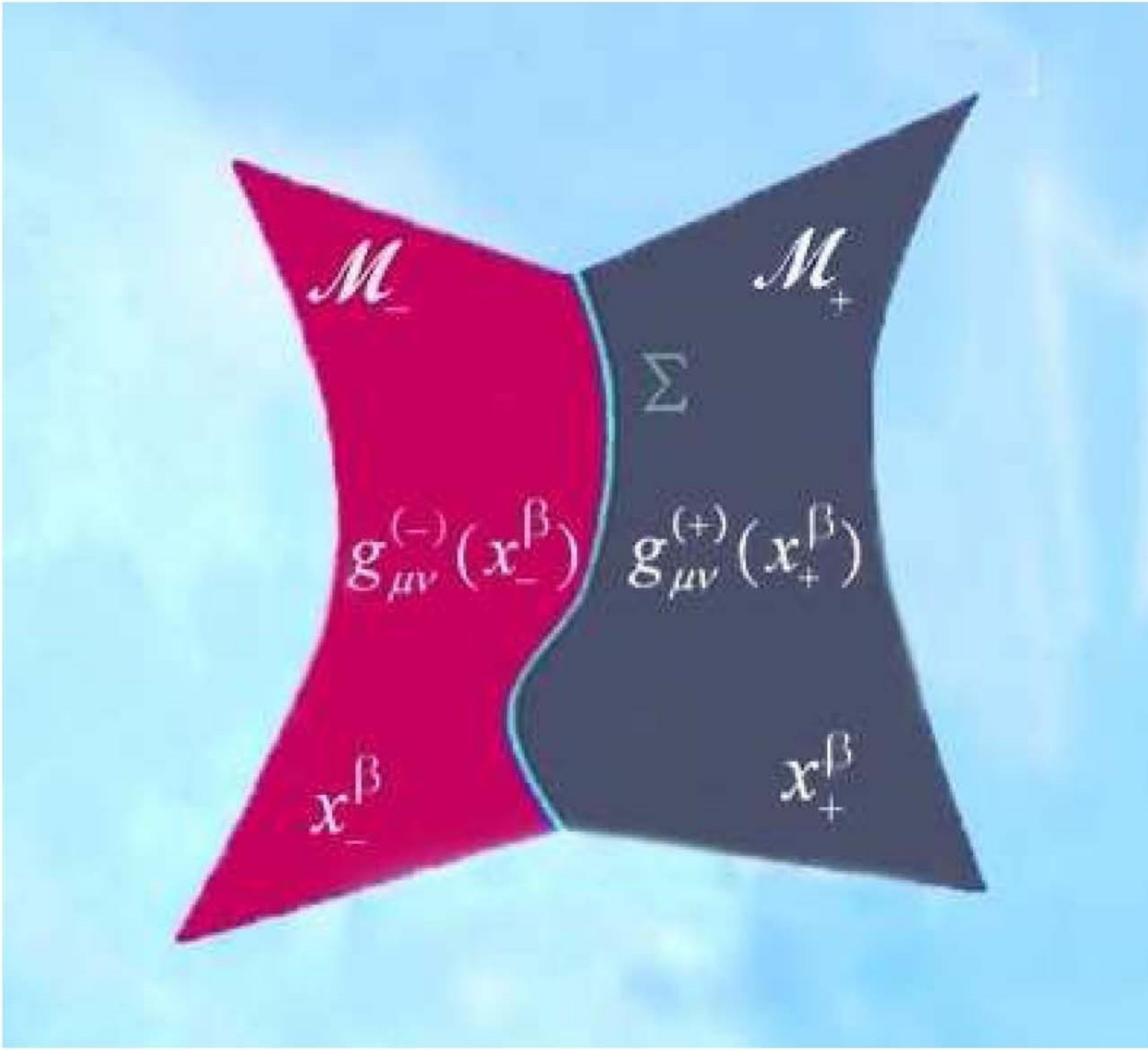
  They join two distinct solutions of the field equations into one across the surface of discontinuity.

- **Why do we need them?**

  -Gravitational Collapse.

  -Describing stars, galaxies, etc.

  -Black Hole Formation.



What conditions must be put on $g^-_{\mu\nu}$ and $g^+_{\mu\nu}$ such that $g^-_{\mu\nu} \cup g^+_{\mu\nu}$ forms a valid solution of the Einstein field equations?

The one of the most adaptive conditions are the Darmois junctions conditions which state that

1. The continuity of the line elements over the hypersurface, i.e.,

$$\left(ds_{-}^{2}\right)_{\Sigma} = \left(ds_{+}^{2}\right)_{\Sigma} = \left(ds^{2}\right)_{\Sigma}$$

This is called the continuity of the first fundamental form.

2. The continuity of the extrinsic curvatures over the hypersurface, i.e.,

$$[K_{ij}] = K_{ij}^{+} - K_{ij}^{-} = 0, \quad (i, j = 0, 2, 3)$$

This is called the continuity of the second fundamental form.

We consider that we consider Einstein-Rosen bridge to the geometry exterior to the hypersurface

$$ds_+^2 = -e^{2(\gamma-\upsilon)}(d\nu^2 - d\rho^2) + e^{-2\upsilon}\rho^2 d\phi^2 + e^{2\upsilon}dz^2,$$

where γ and υ and are the functions of ρ and ν. The corresponding vacuum fields equations are

$$\rho(\upsilon_\nu^2 + \upsilon_\rho^2) = \frac{\tilde{f} - \tilde{R}\tilde{f}_R}{2\tilde{f}_R} e^{2(\gamma-\upsilon)}$$

$$2\rho\upsilon_\nu\upsilon_\rho = \gamma_\nu.$$

and

$$\upsilon_{\nu\nu} - \frac{\upsilon_\rho}{\rho} - \upsilon_{\rho\rho} = \frac{e^{2(\gamma-\upsilon)}}{4\rho}\left(\frac{\tilde{f} - \tilde{R}\tilde{f}_R}{\tilde{f}_R}\right)\left\{\rho e^{-4\upsilon} + \frac{e^{2\gamma}}{\rho}\right\}$$

For smooth matching between exterior and interior geometries, we shall make use of Darmois matching conditions. We consider a timelike hypersurface, for which we impose *r* = constant. In this context, the first fundamental form provides

$$d\tau \overset{\Sigma}{=} e^{2\gamma-2\upsilon}\sqrt{1-\left(\frac{d\rho}{d\nu}\right)^2}\,d\nu = A dt$$

and

$$C \overset{\Sigma}{=} e^{\upsilon}, \quad e^{\upsilon} = \frac{1}{\rho}$$

with

$$1 - \left(\frac{d\rho}{d\nu}\right)^2 > 0$$

The second fundamental form gives us the following set of equations

$$e^{2\gamma-2\upsilon}[\nu_{\tau\tau}\rho_\tau - \rho_{\tau\tau}\nu_\tau - \{\rho_\tau(\upsilon_\nu - \gamma_\nu)\}(\rho_\tau^2 - \nu_\tau^2) + \nu_\tau(\gamma_\rho - \upsilon_\rho)] \triangleq -\frac{A'}{A^2},$$

and

$$e^{2\upsilon}(\nu_\tau\upsilon_\rho + \rho_\tau\upsilon_\nu) \triangleq \frac{BB'}{A}, \quad e^{-2\upsilon}\rho^2\left(\nu_\tau\upsilon_\rho + \rho_\tau\upsilon_\nu - \frac{\nu_\tau}{\rho}\right) \triangleq -\frac{CC'}{A}$$

After using field equations, these provide

$$\overset{\text{eff}}{P_r} \overset{\Sigma}{=} \frac{A(BC)_{,0}}{(BC)'}\overset{\text{eff}}{q}.$$

After applying perturbation scheme, the first dynamical equation provides

$$\dot{\bar{\mu}}^{\mathrm{eff}} = -\dot{\omega}\eta.$$

Its integration gives

$$\bar{\mu}^{\mathrm{eff}} = -\eta\omega.$$

where

$$\eta = \mu_o^{\mathrm{eff}}\left(\frac{b}{B_o} + \frac{a}{A_o} + \frac{c}{C_o}\right) + P_{ro}^{\mathrm{eff}}\frac{a}{A_o} + P_{zo}^{\mathrm{eff}}\frac{b}{B_o} + \frac{c}{C_o}P_{\phi o}^{\mathrm{eff}}$$
$$+ S\left(2\frac{A_o'}{A_o} + \frac{C_o'}{C_o} + \frac{B_o'}{B_o}\right) + A_o Z_3,$$

After applying perturbation scheme, the second dynamical equation provides

$$\frac{1}{A_o}\left[\dot{\bar{q}}^{\mathrm{eff}}+\bar{P}_r^{\mathrm{eff}\,\prime}+\frac{\omega a}{A_o}\left\{\left(P_{zo}^{\mathrm{eff}}-P_{ro}^{\mathrm{eff}}\right)\frac{B_o'}{B_o}+\left(P_{\phi o}^{\mathrm{eff}}-P_{ro}^{\mathrm{eff}}\right)\frac{C_o'}{C_o}+\left(P_{ro}^{\mathrm{eff}}+\mu_o^{\mathrm{eff}}\right)\right.\right.$$

$$\left.\times\left(\frac{a'}{a}-\frac{2A_o'}{A_o}\right)\right\}+\omega\left\{\left(\left(P_{ro}^{\mathrm{eff}}-P_{\phi o}^{\mathrm{eff}}\right)\left(\frac{c}{C_o}\right)'+P_{ro}^{\mathrm{eff}}-P_{zo}^{\mathrm{eff}}\right)\left(\frac{b}{B_o}\right)'+\frac{B_o'}{B_o}\right.$$

$$\left.\left.\times\left(\bar{P}_r^{\mathrm{eff}}-\bar{P}_z^{\mathrm{eff}}\right)+\left(\bar{\mu}^{\mathrm{eff}}+\bar{P}_r^{\mathrm{eff}}\right)\frac{A_o'}{A_o}-\left(\bar{P}_\phi^{\mathrm{eff}}+\bar{P}_r^{\mathrm{eff}}\right)\frac{C_o'}{C_o}\right\}+P_{ro}^{\mathrm{eff}\,\prime}\right]+\omega Z_2=0,$$

The 01 field equation provides

$$\bar{q}^{\text{eff}} = S\dot{\omega}.$$

The static form of Ricci invariant is

$$R_o = -\frac{2}{A_o}\left[\frac{B_o''}{B_o} + \frac{A_o''}{A_o} + \frac{C_o''}{C_o} + \frac{B_o' C_o'}{B_o C_o} - \frac{A_o'^2}{A_o}\right]$$

Substituting the value of $\bar{\mu}^{\mathrm{eff}}$ in EoS

$$\overset{\mathrm{eff}}{\bar{P}_r} = -\Gamma_1 \frac{\overset{\mathrm{eff}}{P_{ro}}\eta\omega}{(\overset{\mathrm{eff}}{P_{ro}} + \overset{\mathrm{eff}}{\mu_o})}, \quad \overset{\mathrm{eff}}{\bar{P}_z} = -\Gamma_1 \frac{\overset{\mathrm{eff}}{P_{zo}}\eta\omega}{(\overset{\mathrm{eff}}{P_{zo}} + \overset{\mathrm{eff}}{\mu_o})}, \quad \overset{\mathrm{eff}}{\bar{P}_\phi} = -\Gamma_1 \frac{\overset{\mathrm{eff}}{P_{\phi o}}\eta\omega}{(\overset{\mathrm{eff}}{P_{\phi o}} + \overset{\mathrm{eff}}{\mu_o})}$$

Substituting the values of perturbed configuration of matter variables in non- static part of second dynamical equation, the resulting **collapse equation** is

$$\Gamma_1 \left[\frac{\overset{\mathrm{eff}}{P_{ro}}}{\overset{\mathrm{eff}}{\mu_o} + \overset{\mathrm{eff}}{P_{ro}}} \left\{ \eta \left\{ \left(\frac{B_o'}{B_o} + \frac{C_o'}{C_o} - \frac{A_o'}{A_o} \right) + \frac{(\overset{\mathrm{eff}'}{\mu_o} + \overset{\mathrm{eff}}{P'_{ro}})}{(\overset{\mathrm{eff}}{\mu_o} + \overset{\mathrm{eff}}{P_{ro}})} \right\} - \eta' \right\} - \eta \right.$$

$$\left. \times \frac{\overset{\mathrm{eff}}{P'_{ro}}}{\overset{\mathrm{eff}}{\mu_o} + \overset{\mathrm{eff}}{P_{ro}}} + \frac{\overset{\mathrm{eff}}{P_{zo}}}{\overset{\mathrm{eff}}{\mu_o} + \overset{\mathrm{eff}}{P_{zo}}} \eta \frac{B_o'}{B_o} + \frac{\overset{\mathrm{eff}}{P_{\phi o}}}{\overset{\mathrm{eff}}{\mu_o} + \overset{\mathrm{eff}}{P_{\phi o}}} \eta \frac{C_o'}{C_o} \right] + \xi^2 S = -\frac{a}{A_o} \overset{\mathrm{eff}}{P'_{ro}} + \overset{\mathrm{eff}}{P_{ro}}$$

$$\times \left\{ \frac{aB_o'}{A_o B_o} + \frac{a'}{a} - \frac{C_o'}{C_o} - \frac{2A_o'}{A_o} - \left(\frac{c}{C_o} \right)' - \left(\frac{b}{B_o} \right)' \right\} + \overset{\mathrm{eff}}{P_{zo}} \left\{ \left(\frac{b}{B_o} \right)' - \frac{aB_o'}{A_o B_o} \right\}$$

$$+ \overset{\mathrm{eff}}{\mu_o} \left(\frac{a'}{a} - \frac{2A_o'}{A_o} \right) + \overset{\mathrm{eff}}{P_{\phi o}} \left\{ \left(\frac{c}{C_o} \right)' - \frac{C_o'}{C_o} \right\} + A_o Z_2 + \frac{\eta A_o'}{A_o},$$

Newtonian Approximation

We take

$$\mu_0 \gg P_{i0}, \quad A_0 = 1, \quad B_0 = 1.$$

Under these constraints, the collapse equation takes the form

$$\left[\overset{\text{eff}}{\mu_o}\left(a + b + \frac{c}{C_o}\right)\right]\Gamma_1 = \overset{\text{eff}}{\mu_o}(a'/a) + S\xi_N^2 + \Pi + Z_2$$

where

$$\Pi = \left(\overset{\text{eff}}{P_{\phi o}} + \overset{\text{eff}}{P_{ro}}\right)\frac{C_o'}{C_o} + b'\left(\overset{\text{eff}}{P_{zo}} - \overset{\text{eff}}{P_{ro}}\right) + \left(\frac{c}{C_o}\right)'\left\{-\left(\overset{\text{eff}}{P_{\phi o}} + \overset{\text{eff}}{P_{ro}}\right)\right\} + a\left(-\overset{\text{eff}'}{P_{ro}}\right)$$

Newtonian approximation

Spherical celestial object will be in equilibrium, whenever
it satisfies

$$\Gamma_1 = \frac{|\mu_o^{\text{eff}}(a'/a) + \Pi + S\xi_N^2 + Z_2|}{|\mu_o^{\text{eff}}\left(a + \frac{c}{C_o} + b\right)|}$$

while for the system to enter in the instability window, the system needs to obey

$$\Gamma_1 < \frac{|\mu_o^{\text{eff}}(a'/a) + S\xi_N^2 + \Pi + Z_2|}{|\mu_o^{\text{eff}}\left(a + \frac{c}{C_o} + b\right)|}$$

Analysis

- The term Π constitutes pressure anisotropic effects. One can see that Π tends to produce increments in the instability regions of the self-gravitating cylindrical systems, whence Π > 0. For this purpose, we need to impose some conditions, i.e.,

$$P_{zo}^{\text{eff}} > P_{ro}^{\text{eff}}, \quad |P_{\phi o}^{\text{eff}} + P_{ro}^{\text{eff}}|$$

with

$$|P_{ro}^{\text{eff}'}|$$

The pressure anisotropy would have opposite effects and would decrease the regions of instabilities of the collapsing cosmic filaments, if Π < 0. Alternatively, if

$$P_{zo}^{\text{eff}} < P_{ro}^{\text{eff}}, \quad |P_{\phi o}^{\text{eff}} + P_{ro}^{\text{eff}}|$$

with

$$|P_{ro}^{\text{eff}'}|$$

- The quantity $\xi_N^2 S$ comes from the (01) component of the $f(R,T,Q)$ field equations and contributes the influences of heat radiations in the instability constraint. This term tends to increase the instability limits against gravitation implosion, thereby making our system less stable.

- The terms Zi s correspond to dark source terms induced from $f(R, T, Q)$ gravity. These terms are inducing DE effects in the instability constraints and are creating anti gravity effects due to their non-attractive nature, thereby producing stability against gravitational collapse.


Thank You