Kinetics of supersymmetric relics in R^2 cosmology

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Hot Topics in Modern Cosmology

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Dark Matter (DM)

First indications:

- J. C. Kapteyn, "First Attempt at a Theory of the Arrangement and Motion of the Sidereal System," Astrophys. J. **55** (1922) 302.
- J. H. Oort, "The force exerted by the stellar system in the direction perpendicular to the galactic plain and some related problems", Bull. Astron. Inst. Netherland **6** (1932) 249.
- F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln," Helv. Phys. Acta **6** (1933) 110.

Later confirmation:

- V. C. Rubin and W. K. Ford, "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions," Astrophys. J. **159** (1970) 379.
- Einasto, J., Kaasik, A., Saar, E., "Dynamic Evidence on Massive coronas of galaxies", Nature, **250** (1974) 309.
- J. P. Ostriker, P. J. E. Peebles and A. Yahil, "The size and mass of galaxies, and the mass of the universe," Astrophys. J. **193** (1974) L1.

Many theoretical models have been proposed to describe this elusive form of matter.

DM candidates

Dark matter:

- electrically neutral, since doesn't scatter light
- properties are practically unknown

Possibilities for innumerable particles to be DM candidates.

Very popular candidate: Lightest Supersymmetric Particle (LSP)

- R. Catena and L. Covi, "SUSY dark matter(s)," arXiv:1310.4776 [hep-ph].
- G. B. Gelmini, "The Hunt for Dark Matter," arXiv:1502.01320 [hep-ph].
- T. R. Slatyer, "Indirect Detection of Dark Matter," arXiv:1710.05137 [hep-ph].

SUSY particles:

- Negative results at LHC $\Longrightarrow$ characteristic energy scale higher than 10 TeV.
- The cosmological energy density of LSPs $\boldsymbol{\varrho_{LSP} \sim m_{LSP}^2} \Longrightarrow$ for $m_{LSP} \sim 1$ Tev ϱ_{LSP} is of the order of the observed energy density of the universe.
- For larger masses LSPs would overclose the universe.

These unfortunate circumstances exclude LSPs as DM particles in the conventional cosmology.

Attempts to save SUSY Dark Matter

Modification of the cosmological scenarios of LSP production in such a way that the relic density of heavy LSPs would be significantly suppressed:

- Non-thermal production of heavy relics (G.L. Kane, P. Kumar, B.D. Nelson, B. Zheng, Phys.Rev. D93 (2016) no.6, 063527)
- Assumption: after the freezing of LSP the universe was matter dominated and this epoch transformed into the radiation dominated stage with low reheating temperature (M. Drees, F. Hajkarim, arXiv:1808.05706)
- Earlier: At some early stage the universe might be dominated by primordial black holes which created the necessary amount of entropy to dilute the heavy particle relics (A. D. Dolgov, P. D. Naselsky, I. D. Novikov, arXiv: astro-ph/0009407)

Generalization of R^2 inflation to supergravity:

- S. V. Ketov and A. A. Starobinsky, "Embedding $(R + R^2)$-Inflation into Supergravity," Phys. Rev. D **83** (2011) 063512 [arXiv:1011.0240 [hep-th]].
- S. Ketov and M. Khlopov, "Extending Starobinsky inflationary model in gravity and supergravity," arXiv:1809.09975 [hep-th].

Scenario with superheavy gravitino as a viable candidate for DM particle

- A. Addazi, S.V. Ketov, M.Yu. Khlopov "Gravitino and Polonyi production in supergravity", Eur.Phys.J. **C78** (2018) no.8, 642, [arXiv:1708.05393 [hep-ph]]

Present talk

In $(R + R^2)$-gravity the energy density of LSPs may be much lower
$\Longrightarrow$ it reopens for them the chance to be the dark matter.

- EA, A. D. Dolgov and R. S. Singh, "Dark matter in $R + R^2$ cosmology," JCAP **04** (2019) 014, arXiv:1811.05399 [astro-ph.CO]

Outline:

- Evolution of matter density: $\boldsymbol{R^2}$-gravity versus GR
- LSP density for the scalaron decay into scalars
- Decay into fermions or conformal scalars
- Anomalous decay into gauge bosons

EA, A. D. Dolgov and R. S. Singh, "Distortion of the standard cosmology in $R + R^2$ theory," JCAP **1807** (2018) no.07, 019 [arXiv:1803.01722 [gr-qc]]

EA, A. D. Dolgov and L. Reverberi, "Cosmological evolution in R^2 gravity," JCAP **1202** (2012) 049 [arXiv:1112.4995 [gr-qc]].

Cosmological evolution in R^2 theory: 4 distinct epochs

I. The inflationary stage: the curvature was sufficiently large, and the universe expanded exponentially with slowly decreasing $R(t)$.

II. Scalaron dominated regime:

- $R(t)$ approached zero and started to oscillate around it as

$$R = -\frac{4m_R \cos(m_R t + \theta)}{t}, \quad m_R = 3 \times 10^{13} \text{ GeV}$$

- The Hubble parameter:

$$H = \frac{2}{3t}\left[1 + \sin(m_R t + \theta)\right]$$

The oscillations of $R \Longrightarrow$ Particle production, but the energy density of the produced particles was negligible $\Longrightarrow$ no noticeable impact on the cosmological evolution.

III. The transition period from scalaron domination to domination of the usual (relativistic) matter.

IV. After complete decay of the scalaron we arrive to the conventional cosmology governed by GR.

Scalaron dominated epoch

Parker theorem: gravitational production of massless particles in FLRW-metric is absent in conformally invariant theory.

Massless scalar field with minimal coupling to gravity is not conformally invariant $\Longrightarrow$ The width of the scalaron decay into two scalars (not conformally invariant):

$$\Gamma_s = \frac{m_R^3}{48 m_{Pl}^2}, \qquad \varrho_s = \frac{m_R^3}{120\pi t}$$

The scalaron decay into pair of fermions (conformally invariant):

$$\Gamma_f = \frac{m_R m_f^2}{48 m_{Pl}^2}, \qquad \varrho_f = \frac{m_R m_f^2}{120\pi t}$$

Similar suppression factor proportional to the boson mass squared appears for scalar particles with conformal coupling to gravity.

Much slower decrease of the energy density of matter, which is ensured by the flux of energy from the scalaron decay, than normally for relativistic matter.

- Normally for relativistic matter: $\varrho \sim 1/a^4(t) \sim 1/t^{8/3}$, since $a(t) \sim t^{(2/3)}$ at SD.

The equilibrium condition

The particle reaction rate in the equilibrium at temperature $\boldsymbol{T}$:

$$\Gamma_{scat} \sim \alpha^2 \beta_{scat} T$$

- α is the coupling constant of the particle interactions, typically $\alpha \sim 10^{-2}$
- β_{scat} is the number of scattering channels, $\beta_{scat} \sim 100$

Equilibrium is enforced if

$$\Gamma_{scat} > H \sim 1/t \quad \text{or} \quad \alpha^2 \beta_{scat} T t > 1$$

The energy density of relativistic matter in thermal equilibrium:

$$\varrho_{therm} = \frac{\pi^2 g_*}{30}\, T^4$$

- g_* is the number of relativistic species in the plasma, $g_* \sim 100$.

The equilibrium condition

Using equations for ϱ_s, ϱ_f, and ϱ_{therm} we find the equilibriums conditions for the cases of scalaron decay

- into a pair of massless scalars:

$$\left(\alpha^2 \beta_{scat} Tt\right)_s = \frac{\alpha^2 \beta_{scat}}{4\pi^3 g_*} \left(\frac{m_R}{T}\right)^3 \approx 8 \cdot 10^{-7} \left(\frac{m_R}{T}\right)^3 > 1$$

- into a pair of fermions:

$$\left(\alpha^2 \beta_{scat} Tt\right)_f = \frac{\alpha^2 \beta_{scat}}{4\pi^3 g_*} \frac{m_R m_f^2}{T^3} \approx 8 \cdot 10^{-7} \frac{m_R m_f^2}{T^3} > 1$$

For the GR-cosmology the equilibrium is established when:

$$\left(\alpha^2 \beta_{scat} Tt\right)_{GR} = \alpha^2 \beta_{scat} \left(\frac{90}{32\pi^3 g_*}\right)^{1/2} \frac{m_{Pl}}{T} \approx 3 \cdot 10^{-4} \frac{m_{Pl}}{T} > 1$$

Heating temperature: the temperature of the cosmological plasma after complete decay of the scalaron

Can be estimated from the energy density of matter at the time $t_d = 1/\Gamma$.

For the decay into scalars:

$$\varrho_s(t_d = 1/\Gamma_s) = \frac{m_R^3}{120\pi t_d} = \frac{m_R^6}{5760\pi m_{Pl}^2} = \frac{\pi^2}{30} g_* T_{hs}^4$$

The heating temperature for the dominant decay of the scalaron into scalars:

$$T_{hs} \approx \frac{m_R}{(192\pi^3)^{1/4}} \left(\frac{m_R}{m_{Pl}}\right)^{1/2}$$

- For $\boldsymbol{m_R = 3 \times 10^{13}}$ GeV $\boldsymbol{T_{hs} \approx 6 \times 10^8}$ GeV.

The temperature of the universe heating for the scalaron decay into fermions:

$$T_{hf} = \frac{1}{(192\pi^3 g_*)^{1/4}} \left(\frac{m_R}{m_{Pl}}\right)^{1/2} m_f \approx 5.7 \cdot 10^{-5} m_f$$

In both cases the heating temperature is considerably lower than the temperature at which thermal equilibrium is established!

LSP density for the scalaron decay into scalars

The freezing of massive species $X \Longrightarrow$ Lee-Weinberg equation (1977) (Zeldovich, 1965):

$$\dot{n}_X + 3Hn_X = -\langle \sigma_{ann} v \rangle \left(n_X^2 - n_{eq}^2 \right)$$

- n_X is a number density of particles X, $\boldsymbol{v}$ is the center-of-mass velocity
- σ_{ann} is the annihilation cross-section

For annihilation of the non-relativistic particles:

$$\langle \sigma_{ann} v \rangle = \sigma_{ann} v = \frac{\alpha^2 \beta_{ann}}{M_X^2},$$

- M_X is a mass of X-particle, α is a coupling constant, in SUSY theories $\boldsymbol{\alpha \sim 0.01}$
- $\boldsymbol{\beta_{ann}}$ is a numerical parameter $\sim$ the number of annihilation channels, $\boldsymbol{\beta \sim 10}$.

The equilibrium number density of X-particles:

$$n_{eq} = g_s \left(\frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T}$$

where g_s is the number of spin states.

Some comments

An additional term describing X-particle production by $R(t)$ should be included. However, we assume that this channel is suppressed in comparison with inverse annihilation of light particles into $X\bar{X}$-pair.

We do not specify which precisely supersymmetric particle is the lightest (it can be, e.g., sneutrino, neutralino or gravitino). We only need the value of its mass, M_X, and the magnitude of the annihilation cross-section.

We assume that the plasma is thermalised $\Longrightarrow$ the temperature satisfies the condition

$$\left(\alpha^2 \beta_{scat} T t\right)_s = \frac{\alpha^2 \beta_{scat}}{4\pi^3 g_*} \left(\frac{m_R}{T}\right)^3 \approx 8 \cdot 10^{-7} \left(\frac{m_R}{T}\right)^3 > 1$$

GR cosmology $\Longleftrightarrow$ R^2 cosmology (decay in scalars)

Equating the energy densities ϱ_{GR} and ϱ_s to the energy density of relativistic plasma with temperature T, we obtain:

$$\varrho_{GR} = \frac{3m_{Pl}^2}{32\pi t^2} = \frac{\pi^2 g_* T^4}{30} \qquad\qquad \varrho_s = \frac{m_R^3}{120\pi t} = \frac{\pi^2 g_* T^4}{30}$$

Connection of the temperature with time:

$$(tT^2)_{GR} = \left(\frac{90}{32\pi^3 g_*}\right)^{1/2} m_{Pl} = const \qquad (tT^4)_s = \frac{m_R^3}{4\pi^3 g_*} = const$$

Correspondingly

$$\left(\frac{\dot T}{T}\right)_{GR} = -\frac{1}{2t} \qquad\qquad \left(\frac{\dot T}{T}\right)_s = -\frac{1}{4t}$$

Kinetic equation

New function $\boldsymbol{f}$:

$$n_X = n_{in} \left(\frac{a_{in}}{a} \right)^3 f$$

- n_{in} is the value of X-particle density at $a = a_{in}$ and $T_{in} = M_X$.

The X-particles can be considered as relativistic and thus

$$n_{in} = 0.12 g_s T_{in}^3 = 0.12 g_s M_X^3$$

NB: The final result does not depend upon $\boldsymbol{n_{in}}$ and $\boldsymbol{T_{in}}$.

With new variable $\boldsymbol{x = M_X / T}$ we arrive to the equations:

$$\mathbf{GR}: \qquad \frac{df}{dx} = -\sigma v \, m_{Pl} M_X \left(\frac{45}{4\pi^3 g_*} \right)^{1/2} \left(\frac{a_{in}}{a} \right)^3 \frac{n_{in}}{T^3} \frac{(f^2 - f_{eq}^2)}{x^2}$$

$$\boldsymbol{R}^2: \qquad \frac{df}{dx} = -\sigma v \frac{m_R^3}{\pi^3 g_* M_X} \left(\frac{a_{in}}{a} \right)^3 \frac{n_{in}}{T^3} \left(f^2 - f_{eq}^2 \right)$$

The products $(a_{in}^3 n_{in})/(a^3 T^3)$

GR-regime:

- ***aT ≈ const*** up to the corrections due to the heating of plasma by the massive particles annihilation when the temperature drops below their masses.
- The number density of X-particles is normalized to the photon density, though it should be normalized to the entropy density which is conserved in the comoving volume.
- Up to this factor the coefficient $\boldsymbol{(a_{in}^3 n_{in})/(a^3 T^3)}$ can be taken as unity.

In R^2 theory $\boldsymbol{T \sim t^{-1/4}}$, $\boldsymbol{a \sim t^{2/3}}$, and $\boldsymbol{a^3 T^3 \sim 1/T^5}$, so:

$$\left(\frac{a_{in}}{a(t)}\right)^3 = \left(\frac{t_{in}}{t}\right)^2 = \left(\frac{T_{in}}{T}\right)^{-8} = \frac{1}{x^8}, \quad \frac{n_{in}}{T^3} = 0.12 g_s \left(\frac{M_X}{T}\right)^3 = 0.12 g_s x^3$$

and

$$\boldsymbol{\frac{a_{in}^3\, n_{in}}{a^3 T^3} = \frac{0.12\, g_s}{x^5}}$$

Evolution of X-particles in the scalaron dominated regime

$$\frac{df}{dx} = -\frac{0.12 g_s \alpha^2 \beta_{ann}}{\pi^3 g_*} \left(\frac{m_R}{M_X}\right)^3 \frac{f^2 - f_{eq}^2}{x^5} \equiv -Q_s \frac{f^2 - f_{eq}^2}{x^5}$$

The coefficient Q_s is normally huge $\Longrightarrow$ initially the solution is close to the equilibrium one:

$$f = f_{eq}(1+\delta) \quad with \quad \delta = -\frac{x^5}{2Q_s f_{eq}^2}\frac{df_{eq}}{dx} \approx \frac{x^5}{2Q_s f_{eq}}$$

The equilibrium solution:

$$f_{eq} = \frac{1}{0.12}\left(\frac{x}{2\pi}\right)^{3/2} e^{-x} x^5, \qquad \delta = \frac{x^5}{2Q_s f_{eq}} = \frac{0.06}{Q_s}\left(\frac{x}{2\pi}\right)^{-3/2} e^{x}$$

This solution is valid till $\boldsymbol{\delta}$ remains small, $\delta \leq 1$.

The freezing temperature

The deviation from equilibrium becomes of order of unity, or $\delta = 1$, at the so-called freezing temperature T_{fr} or at x_{fr}:

$$x_{fr} \approx \ln Q_s + \frac{3}{2}\ln(\ln Q_s) - \frac{3}{2}\ln(2\pi) + \ln 0.06 \approx \ln Q_s + \frac{3}{2}\ln(\ln Q_s) - 5.7$$

Since $Q_s \gg 1$, then x_{fr} is also large, typically $x_{fr} \sim (10 - 100)$ depending upon the interaction strength.

After $\boldsymbol{x}$ becomes larger than $\boldsymbol{x_{fr}}$ $\Longrightarrow$ the kinetic equation with the initial condition $f = f_{fr}$ at $x = x_{fr}$ is simply integrated $\Longrightarrow$ the asymptotic result at $\boldsymbol{x \to \infty}$:

$$f(x) = \frac{f_{fr}}{1 + \frac{Q_s f_{fr}}{4}\left(\frac{1}{x_{fr}^4} - \frac{1}{x^4}\right)} \to \frac{4x_{fr}^4}{Q_s} = f_{fin}$$

Thus $\boldsymbol{f}$ tends to a constant value $\boldsymbol{f_{fin}}$, when $\boldsymbol{x \gg x_{fr}}$ and $Q_s f_{fr}/(4x_{fr}^4) > 1$

Number densities of X-particles/number density of photons

Since the density of X-particles is given by

$$n_X = n_{in} \left(\frac{a_{in}}{a}\right)^3 f, \quad n_{in} = 0.12 g_s T_{in}^3 = 0.12 g_s M_X^3, \quad \left(\frac{a_{in}}{a}\right)^3 = \frac{1}{x^8}$$

its ratio to the number density of photons, $n_\gamma = 0.24 T^3$, is:

$$\frac{n_X}{n_\gamma} = \frac{g_s}{2} \frac{f}{x^5}, \quad x = \frac{M_X}{T}$$

This ratio:

- Drops down strongly, as $\mathbf{1/x^5}$, in contrast to the analogous ratio in GR.
- This decrease is induced by the rise of the density of relativistic species created by the scalaron decay.

This drop continues till $\boldsymbol{\Gamma t \sim 1}$, when scalaron field disappears and the cosmology returns to the usual GR one. It happens at the temperature

$$\boldsymbol{T_{hs} \approx \frac{m_R}{(192\pi^3)^{1/4}} \left(\frac{m_R}{m_{Pl}}\right)^{1/2} \approx 6 \times 10^8} \ \text{GeV}.$$

which should be compared with $\boldsymbol{T_{eq} \approx 2 \times 10^{-3} m_R \approx 6 \times 10^{10}}$ GeV.

The present day ratio

n_X/n_γ at the present time can be estimated at $T = T_{hs}$ and we find:

$$\left(\frac{n_X}{n_\gamma}\right)_{now} = \frac{2g_s x_{fr}^4}{Q_s x_d^5} = \frac{\pi^3 g_* x_{fr}^4}{0.06(192\pi^3)^{5/4}\alpha^2\beta_{ann}} \left(\frac{m_R}{M_X}\right)^2 \left(\frac{m_R}{m_{Pl}}\right)^{5/2}$$

For $g_* = 100$, $\alpha = 0,01$, $\beta_{ann} = 10$, $m_R = 3 \times 10^{13}$ GeV, and $n_\gamma = 412/\text{cm}^3$
the present day energy density of the X-particles is:

$$\varrho_X = M_X n_\gamma f_{fin} \approx 1.7 \times 10^8 \left(\frac{10^{10}\text{Gev}}{M_X}\right) \text{keV}/\text{cm}^3$$

The observed energy density of dark matter: $\boldsymbol{\varrho_{DM} \approx 1}$ keV/cm^3.

- X-particles must have huge mass, $M_X \gg m_R$, to make reasonable dark matter density.
- However, if $\boldsymbol{M_X > m_R}$, the decay of the scalaron into $X\bar{X}$-channel would be strongly suppressed $\Longrightarrow$ such LSP with the mass slightly larger than $\boldsymbol{m_R}$ could successfully make the cosmological dark matter.

Scalaron decay into fermions or conformal scalars

If the bosons are coupled to curvature as $\xi R \phi^2$ with $\xi = 1/6 \Longrightarrow$
they are conformally invariant $\Longrightarrow$ are not produced if their mass is zero.

- The probability of production of both bosons and fermions $\sim \boldsymbol{m^2_{particle}}$
- In what follows we confine ourselves to consideration of fermions only.

The width of the scalaron decay into a pair of fermions is:

$$\Gamma_f = \frac{m_R m_f^2}{48 m_{Pl}^2}$$

The largest contribution into the cosmological energy density at scalaron dominated regime is presented by the decay into heaviest fermion species.

We assume:

- The mass of the LSP is considerably smaller than the masses of the other decay products, $\boldsymbol{m_X < m_f}$, at least as $\boldsymbol{m_X \lesssim 0.1 m_f}$.
- the direct production of X-particles by $\boldsymbol{R(t)}$ can be neglected.

In such a case LSPs are dominantly produced by the secondary reactions in the plasma, which was created by the scalaron production of heavier particles.

Kinetic equation for freezing of fermionic species

$$\frac{df}{dx} = -\frac{\alpha^2 \beta_{ann}}{\pi^3 g_*} \frac{n_{in}\, m_R m_f^2}{m_X^6} \frac{f^2 - f_{eq}^2}{x^5} = -Q_f \frac{f^2 - f_{eq}^2}{x^5}$$

$n_{in} = 0.09 g_s m_X^3$ is the initial number density of X-particles at $T \sim m_X$.

The asymptotic solution of this equation for large $x \gg x_{fr}$ with the initial condition $f(x_{fr}) = f_{fr}$ (in complete analogy with scalars):

$$\boldsymbol{f(x) = \frac{f_{fr}}{1 + \frac{Q_f f_{fr}}{4}\left(\frac{1}{x_{fr}^4} - \frac{1}{x^4}\right)} \to \frac{4x_{fr}^4}{Q_f} = f_{fin}}$$

Here the freezing temperature is defined by

$$x_{fr} \approx \ln Q_f + \frac{3}{2}\ln(\ln Q_f) - \frac{3}{2}\ln(2\pi) + \ln 0.045 \approx \ln Q_f + \frac{3}{2}\ln(\ln Q_f) - 5.86$$

and the so called frozen value of $\boldsymbol{f}$ is equal to

$$f_{fr} = x_{fr}^5 / Q_f$$

Number density of X-particles

The frozen number density of X-particles, taken at $T = T_{fr} = m_X / x_{fr}$ is

$$n_{Xfr} = \frac{\pi^3 g_*}{\alpha^2 \beta_{ann} \ln^3 Q_f} \frac{m_X^6}{m_R m_f^2}$$

After freezing, the X-particles number density remains const. in comoving vol.

$$n_X = n_{Xfr} \left(\frac{a_{fr}}{a} \right)^3 = n_{Xfr} \left(\frac{t_{fr}}{t} \right)^2 = n_{Xfr} \left(\frac{x_{fr}}{x} \right)^8 ,$$

- a_{fr} is the value of the cosmological scale factor at the moment of freezing
- we used the expansion law $\boldsymbol{a \sim t^{2/3}}$ and the relation between time and temperature $\boldsymbol{tT^4 = const}$.

The energy density of the relativistic particles drops in the course of expansion from the moment of X-freezing as:

$$\varrho^{rel} = \varrho^{rel}_{fr} \left(\frac{t_{fr}}{t} \right) = \varrho^{rel}_{fr} \left(\frac{x_{fr}}{x} \right)^4 , \qquad \varrho^{rel}_{fr} = \frac{\pi^2 g_*}{30} T_{fr}^4$$

where $\boldsymbol{\varrho^{rel}_{fr}}$ is the energy density of relativistic matter at the moment of X-freezing.

The number density of relativistic particles

$$n^{rel} \approx \frac{\varrho^{rel}}{3T} = n^{rel}_{fr}\left(\frac{x_{fr}}{x}\right)^3, \qquad n^{rel}_{fr} \approx \frac{\pi^2 g_* T^3_{fr}}{90}$$

Correspondingly

$$\frac{n_X}{n^{rel}} = \frac{90\pi}{\alpha^2\beta_{ann}}\frac{m_X^3}{m_R m_f^2}\left(\frac{x_{fr}}{x}\right)^5$$

This ratio would evolve in this way as a function of $\boldsymbol{x}$ till the complete decay of the scalaron at $\boldsymbol{T = T_{hf}}$

$$T_{hf} = \frac{1}{(192\pi^3 g_*)^{1/4}}\left(\frac{m_R}{m_{Pl}}\right)^{1/2} m_f \approx 5.7\cdot 10^{-5} m_f$$

Ultimately :

$$\boldsymbol{\left(\frac{n_X}{n^{rel}}\right)_h = \frac{90\pi(\ln(Q_f))^5}{\alpha^2\beta_{ann}(192\pi^3 g_*)^{5/4}}\left(\frac{m_R}{m_{Pl}}\right)^{5/2}\frac{m_f^3}{m_R m_X^2}}$$

Later on at GR stage this ratio does not change much, decreasing only due to the heating of the plasma by the massive particle annihilation.

The contemporary energy density of X-particle

$$\varrho_X = m_X n_\gamma \left(\frac{n_X}{n_{rel}}\right)_h = 7 \cdot 10^{-9} \frac{m_f^3}{m_X m_R}\,\mathrm{cm}^{-3}$$

- $n_\gamma \approx 412/\mathrm{cm}^3$, $\alpha = 0.01$, $\beta_{ann} = 10$, $g_* = 100$
- If we take $m_f = 10^5$ GeV and $m_X = 10^4$ GeV, then $\varrho_X \ll \varrho_{DM}$.
- For the chosen values of the parameters $Q_f \approx 1.7 \cdot 10^4$, and $\ln Q_f \approx 10$.

This energy density should be close to the energy density of the cosmological dark matter, $\varrho_{DM} \approx 1$ keV/cm^3.

It can be easily achieved with $m_X \sim 10^6$ GeV and $m_f \sim 10^7$ GeV:

$$\varrho_X = 0.23 \left(\frac{m_f}{10^7\,\mathrm{GeV}}\right)^3 \left(\frac{10^6\,\mathrm{GeV}}{m_X}\right) \frac{\mathrm{keV}}{\mathrm{cm}^3}$$

Anomalous decay into gauge bosons

The coupling of the massless gauge bosons to gravity is determined by the anomaly in the trace of energy-momentum tensor of the gauge fields:

$$T^{\mu}_{\mu} = \frac{\beta^2 \alpha}{8\pi} G_{\mu\nu} G^{\mu\nu}$$

- α is the fine structure constant, $G^{\mu\nu}$ is the gauge field strength
- β is the first coefficient in perturbative expansion of the beta-function

Due to this anomaly massless gauge bosons are efficiently produced in cosmology:

- A. D. Dolgov: "Massless Particle Production By Conformal Plane Gravitation Field" (in Russian), Pisma Zh. Eksp. Teor. Fiz. **32** (1980) 673; "Conformal Anomaly and the Production of Massless Particles by a Conformally Flat Metric," Sov. Phys. JETP **54** (1981) 223; "Breaking of conformal invariance and electromagnetic field generation in the universe," Phys. Rev. D **48** (1993) 2499 [hep-ph/9301280].

In particular, this coupling leads to the decay of the curvature $R(t)$ into gauge bosons.

Particle production due to conformal anomaly

The decay width of particle production by curvature due to conformal anomaly

- D. Gorbunov and A. Tokareva, JCAP **1312** (2013) 021 [arXiv:1212.4466]

is to be compared with the decay widths into minimally coupled massless scalars and fermions:

$$\Gamma_{anom} = \frac{\beta\alpha^2 N}{96\pi^2}\,\frac{m_R^3}{m_{Pl}^2} \qquad \Longleftrightarrow \qquad \Gamma_s = \frac{m_R^3}{48 m_{Pl}^2} \qquad \Longleftrightarrow \qquad \Gamma_f = \frac{m_R m_f^2}{48 m_{Pl}^2}$$

Γ_{anom} is suppressed by factor $(\beta\alpha^2 N)/(2\pi^2)$ in comparison with Γ_s, but still is much larger than the decay width into fermions Γ_f with e.g. mass $m_f \sim 10^5$GeV.

Another route of escape: $N = 4$ supersymmetry for which beta-function vanishes and the conformal anomaly is absent.

- M. F. Sohnius, "Introducing Supersymmetry," Phys. Rept. **128** (1985) 39.

$N = 4$ SUSY

$N = 4$ super Yang-Mills theories:

- are believed to be unrealistic because they do not allow to introduce chiral fermions, even if the symmetry is broken spontaneously.

Though spontaneous symmetry breaking is the most appealing way to deal with the theories with broken symmetries, it is not obligatory!

- The symmetry can be broken explicitly.
- It is possible to break the symmetry "by hand" introducing different masses to particles in the same multiplet. This would allow to construct a phenomenologically acceptable model.
- Since the symmetry is broken by mass, the theory would remain renormalizable.

At higher energies, much larger than the particle masses, it would behave as $\boldsymbol{N = 4}$ super Yang-Mills theory and at this energy scale the trace anomaly would vanish.

$N = 1$ and $N = 2$ SUSY

$N = 1$ and $N = 2$ supersymmetric theories:

- phenomenologically acceptable and possess the so called conformal window $\Longrightarrow$ trace anomaly vanishes with a certain set of the multiplets.

Review:

- M. Chaichian, W. F. Chen and C. Montonen, "New superconformal field theories in four-dimensions and N=1 duality," Phys. Rept. **346** (2001) 89 [hep-th/0007240].

The conformal coupling of scalar fields to gravity postulated above indeed breaks supersymmetry but supersymmetry is broken anyhow and this kind of breaking does not lead to revival of the conformal anomaly.

Gravitational corrections to the trace anomaly $\Longrightarrow$
contribution $\sim R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$, $R_{\mu\nu}R^{\mu\nu}$, ...
This contribution does not lead to production of gauge bosons.

Higher loop gravitational corrections, even if result in gauge boson production, are strongly suppressed.

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- The search for such dark matter particles in low background experiments looks presently more feasible. If they are discovered, it would be an interesting confirmation of R^2 inflationary model.

The END

Thank You for Your Attention