GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Flavour Gauge Bosons at TeV Scale

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CKM UNITARITY PROBLEM

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- Superallowed nuclear beta-decays ($0^+ \to 0^+$ Fermi transitions) determine:

$$G_V = G_F |V_{ud}|$$

  within the SM

$$G_F = G_\mu$$

  then $V_{ud} = 0.97420 \pm 0.0002$;

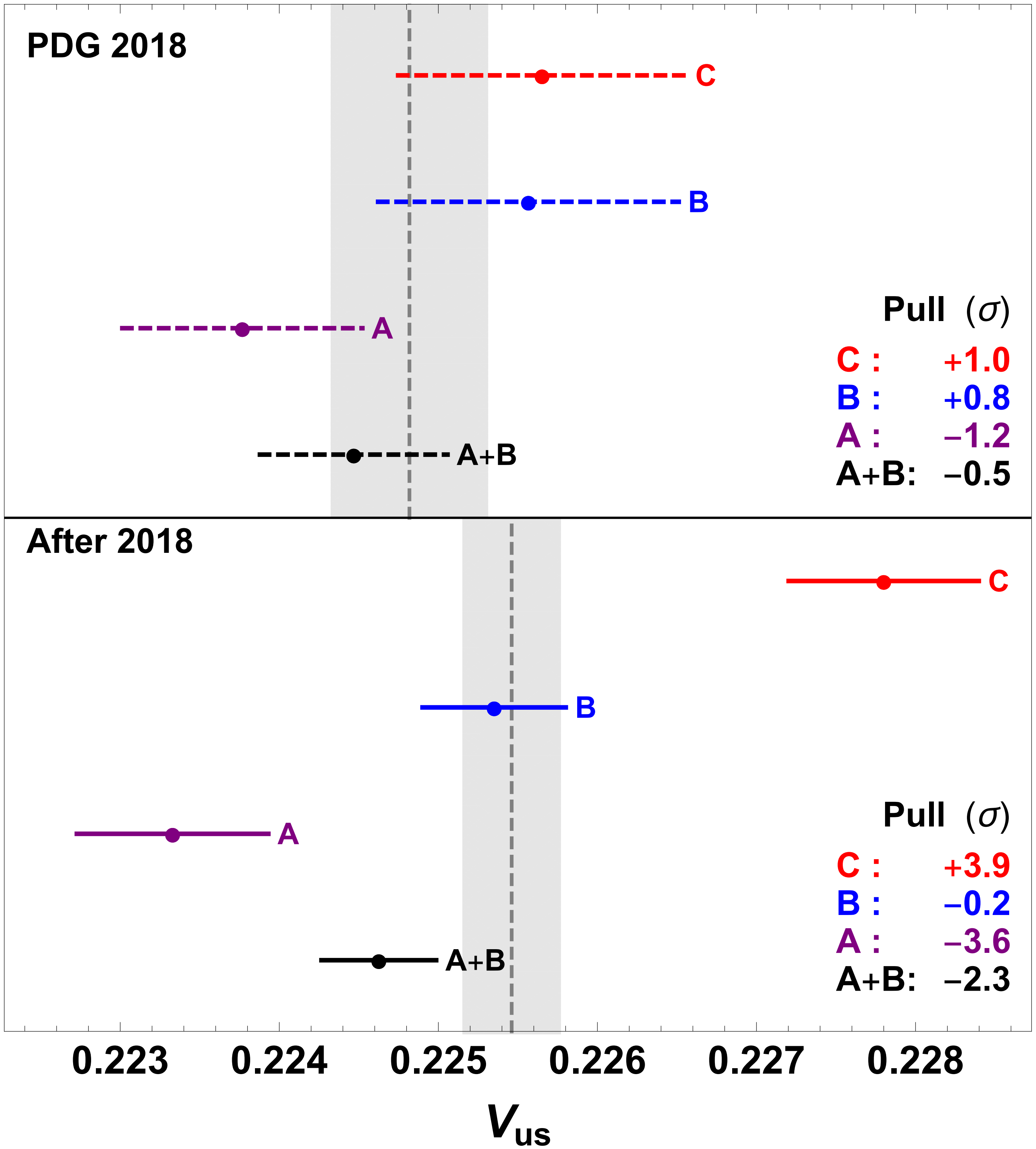
- New computation of radiative corrections with reduced hadronic uncertainty (Seng et al. PRL 2018):

$$|V_{ud}| = \frac{G_V^{\text{exp}}}{G_F(= G_\mu)} = 0.97366(15)$$

- New determinations of the ratio for kaon and pion decay constant $f_{K^\pm}/f_{\pi^\pm}$ (FLAG 2019) and of the form factor relevant for semileptonic decay $f_+(0)$ (Fermilab Lattice and MILC) determining V_{us} and V_{ud} from:

$$f_+(0)|V_{us}| = 0.21654(41) \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(38)$$

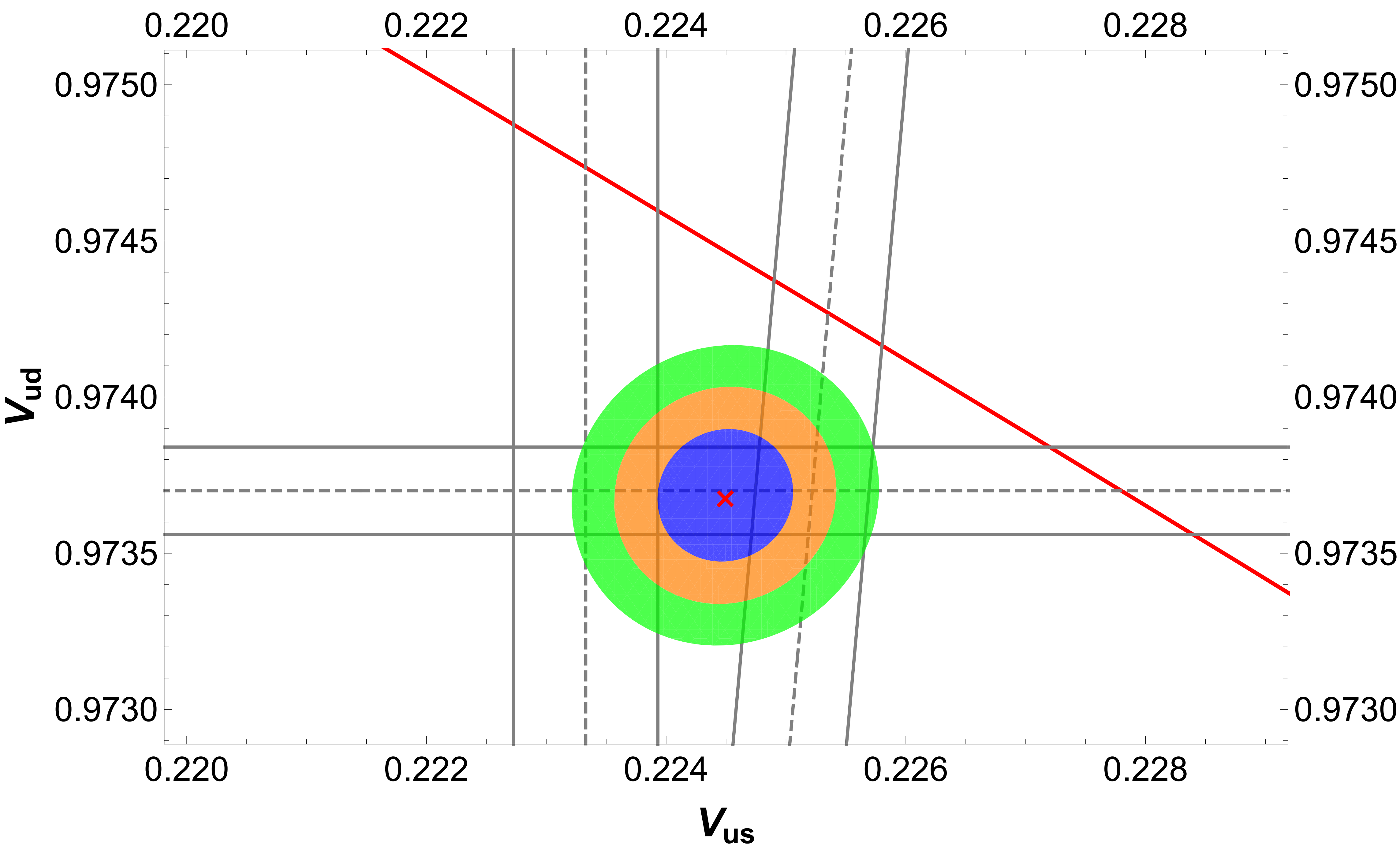
CKM UNITARITY PROBLEM



$$\chi^2_{dof} = 1.7$$

$$\rightarrow \chi^2_{dof} = 13.9$$

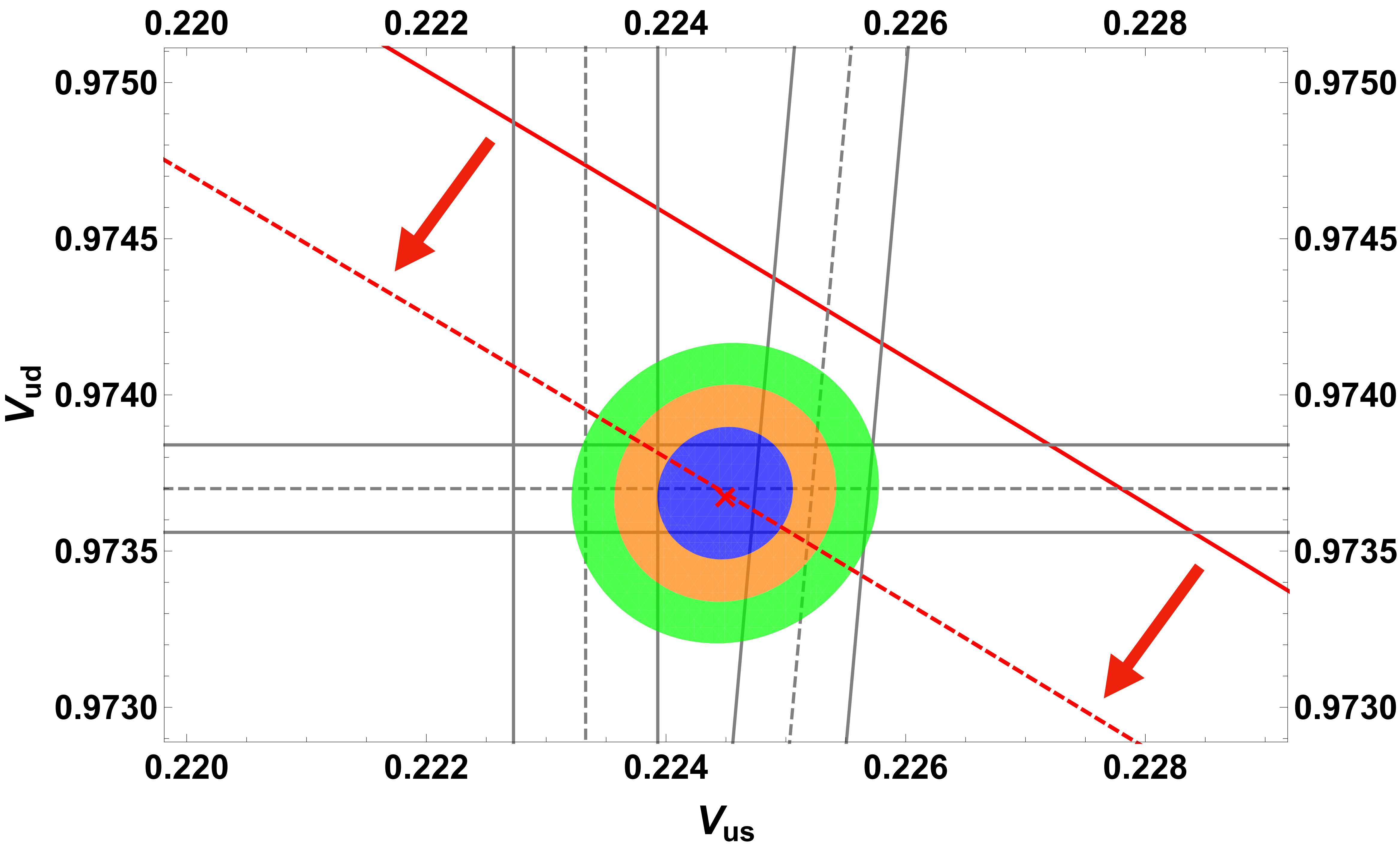
CKM UNITARITY PROBLEM



$4.3\,\sigma$ far from unitarity curve (excluded at 99.998% C.L.)

SOLUTION #1

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 \,, \qquad |V_{ub'}| = 0.04$$

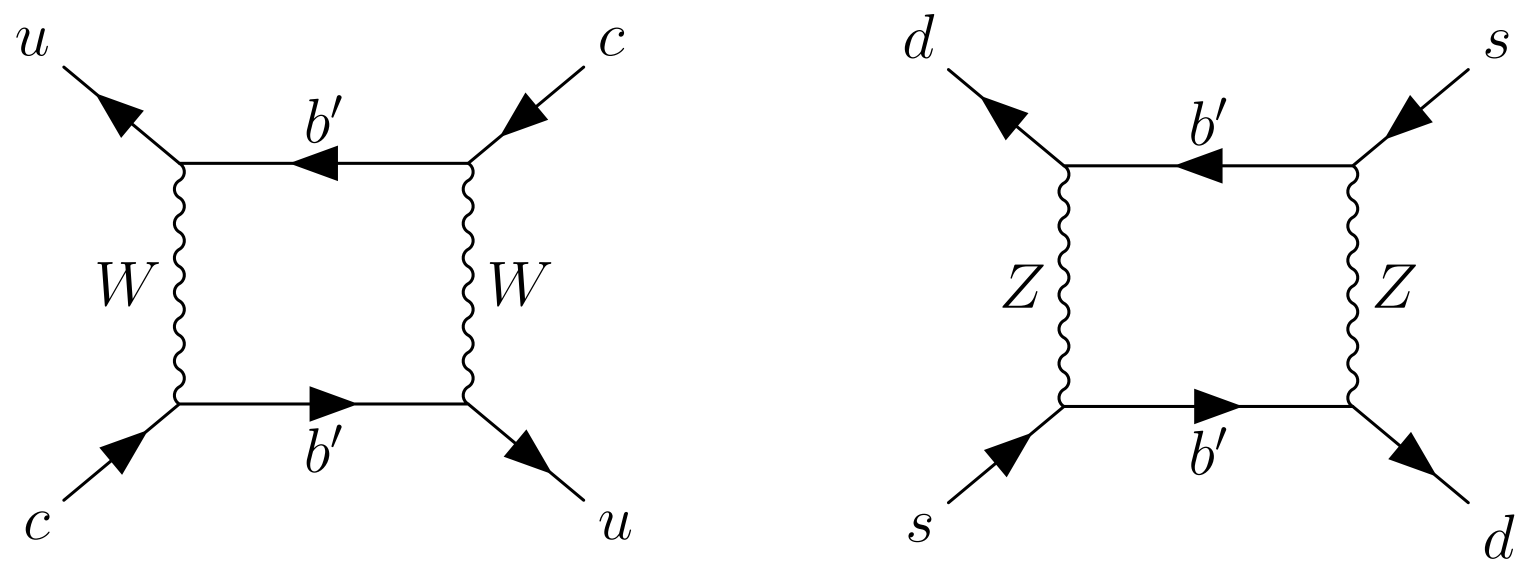


SOLUTION #1

- Vector-like isosinglet down-type quark b'_L, b'_R involved in quark mixing:

$$h_i \phi \overline{Q_{Li}} b'_R + M \overline{b'_L} b'_R$$

- $V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}$



If $V_{ub'} = 0.04$:

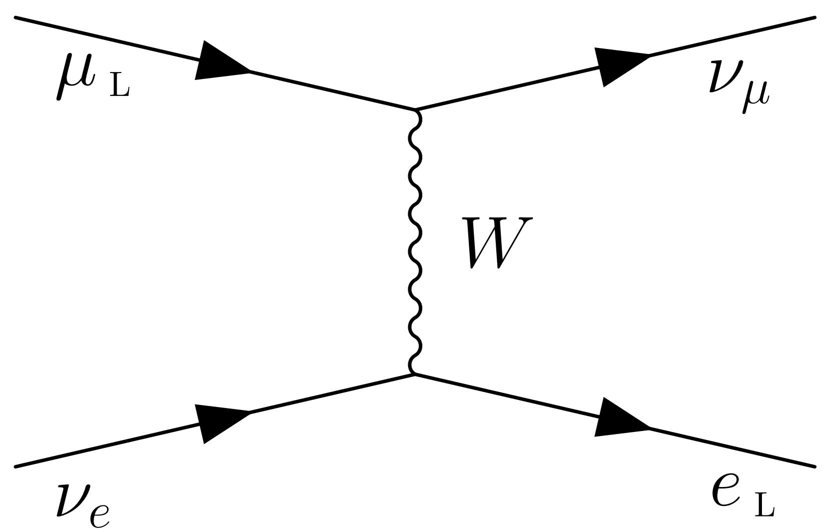
- $\left|\frac{V_{cb'}}{V_{ub'}}\right| \left(\frac{M}{1\mathrm{TeV}}\right) < \frac{1}{3}$ from $D^0 - \bar{D}^0$ mixing (mass splitting).

- $Arg\left(\frac{V_{cb'}}{V_{ub'}}\right) \left|\frac{V_{cb'}}{V_{ub'}}\right| \left(\frac{M}{1\mathrm{TeV}}\right) < \frac{1}{10}$ from $K^0 - \bar{K}^0$ mixing (ϵ parameter).

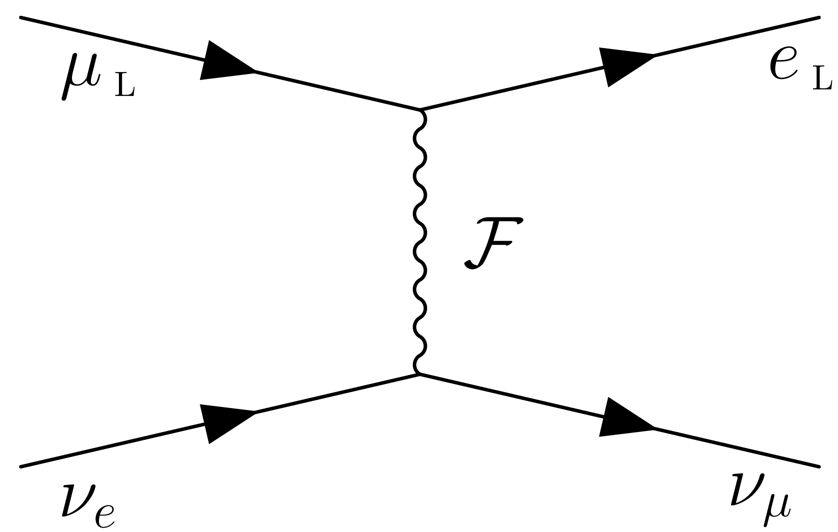
SOLUTION #2

- Suppose the existence of **flavor changing bosons**.

$$-\frac{4G_{\rm F}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e)$$



$$-\frac{4G_{\rm H}}{\sqrt{2}}(\overline{e_L}\gamma^\alpha\mu_L)(\overline{\nu_\mu}\gamma_\alpha\nu_e)$$



- Horizontal interactions give the operator $-\frac{4G_{\rm H}}{\sqrt{2}}(\overline{e_L}\gamma^\alpha\mu_L)(\overline{\nu_\mu}\gamma_\alpha\nu_e)$
- Interference with SM $-\frac{4G_{\rm F}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e)$:

$$\boxed{G_\mu \neq G_F}$$

$$G_\mu = G_F + G_H$$

SOLUTION #2

- Different $\mathbf{G}_{\mu} = \mathbf{G_F} + \mathbf{G_H} = \mathbf{G_F}(\mathbf{1} + \delta_{\mu}) = \mathbf{1} + \mathbf{v}_{\mathrm{w}}^{\mathbf{2}}/\mathbf{v}_{\ell}^{\mathbf{2}}$

- The values of V_{us}, V_{ud} (and corresponding errorbars) are changed by a factor:

$$V_{\mathrm{us}} \to (1+\delta_{\mu})\, V_{\mathrm{us}}\ , \qquad V_{\mathrm{ud}} \to (1+\delta_{\mu})\, V_{\mathrm{ud}}$$

while the ratio is not affected.

- Unitarity recovered:

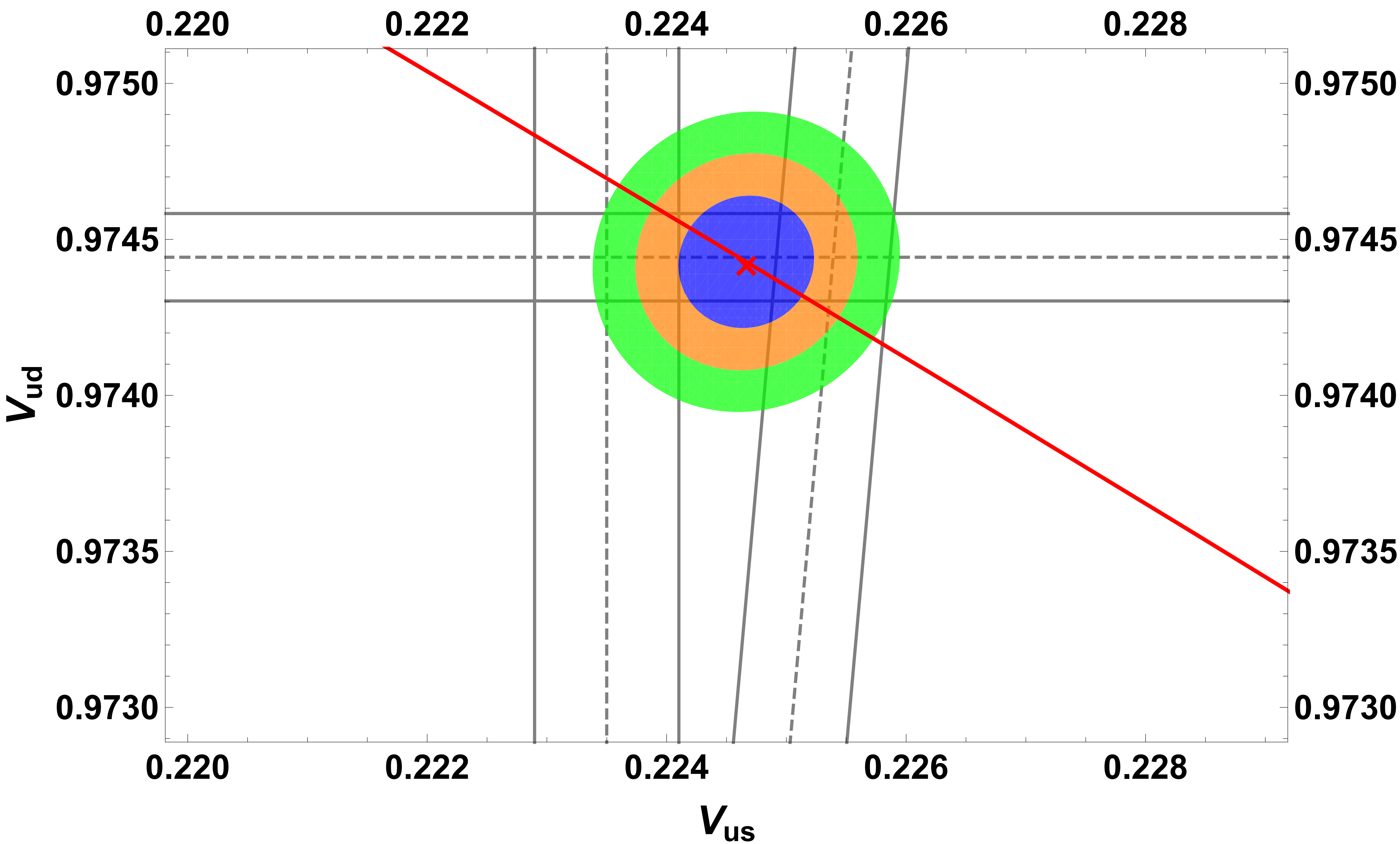
$$\frac{2G_{\mathrm{H}}}{G_{\mathrm{F}}} = 1 - |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \tag{1}$$

- After performing the fit of V_{us}, V_{ud} and their ratio with two parameters V_{us} and δ_{μ} by imposing unitarity, the minimum $\chi^{\mathbf{2}}_{\mathbf{dof}} = \mathbf{3.0}$ is found for $\delta_{\mu} = \mathbf{0.00076}$, or

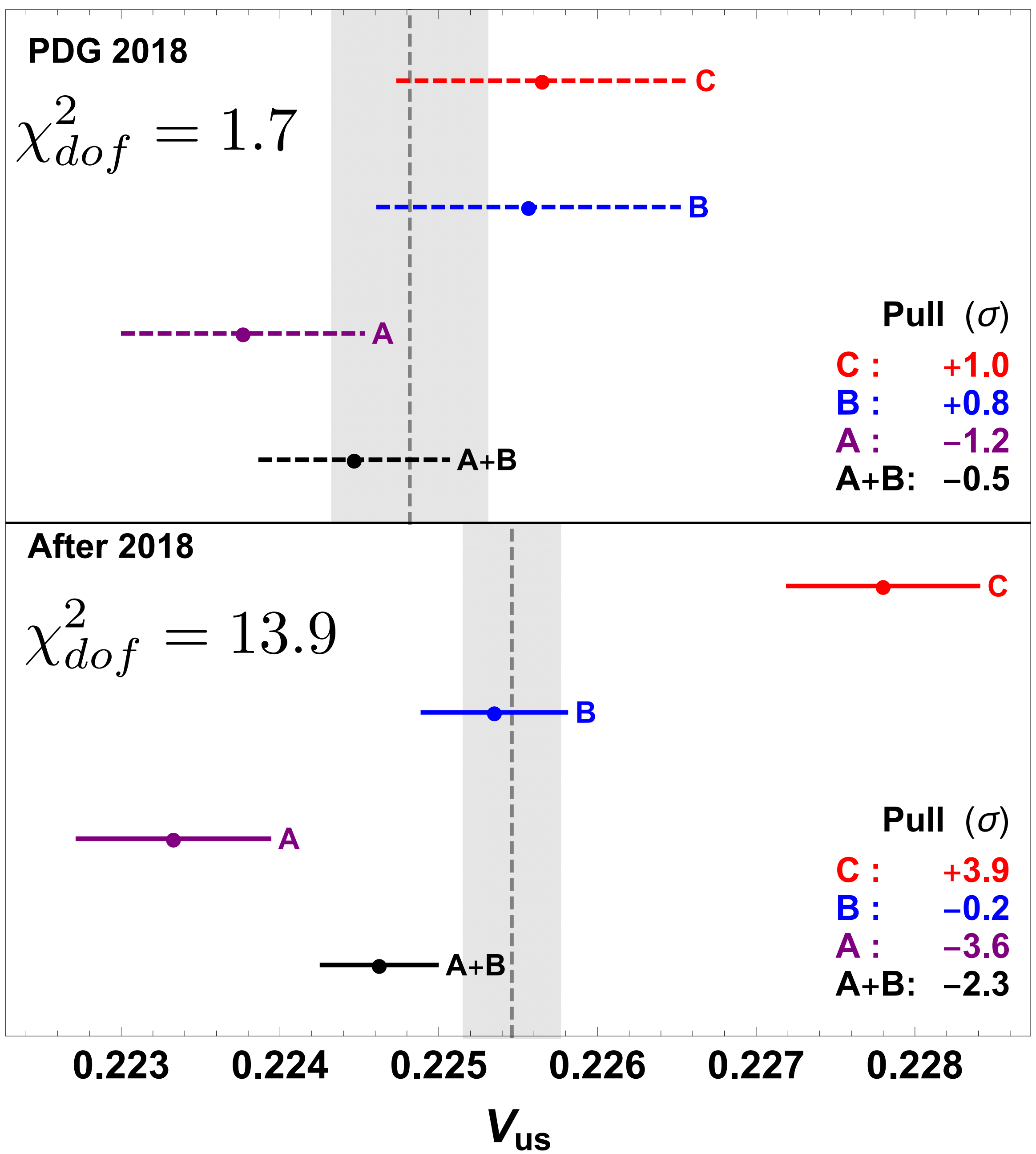
$$\boxed{v_{\ell} = 6.3\ \mathrm{TeV}}$$

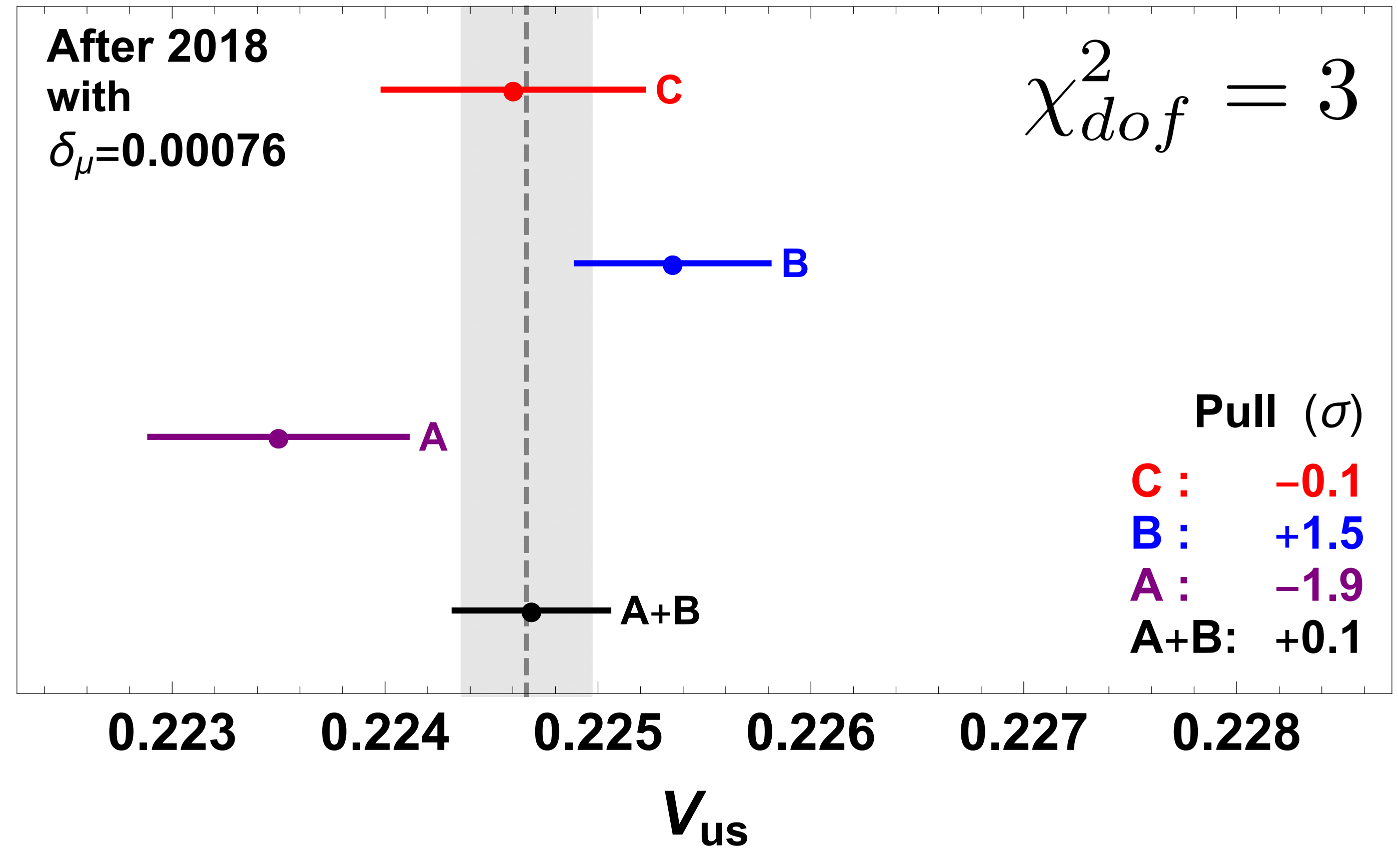
SOLUTION #2

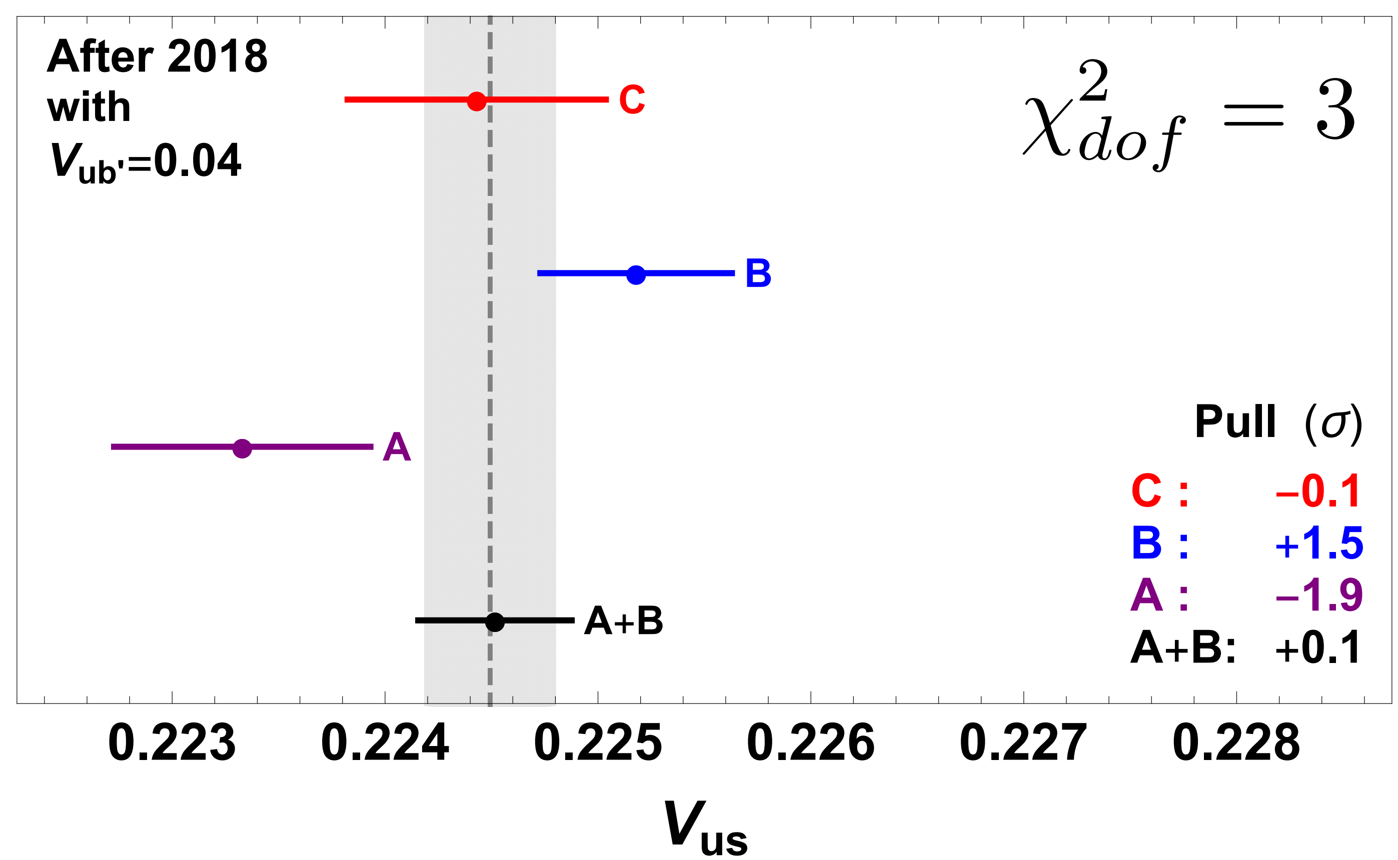
$$\delta_\mu = 0.000\,76 \quad \longrightarrow \quad v_\ell = 6.3\ \mathrm{TeV}$$



CKM UNITARITY

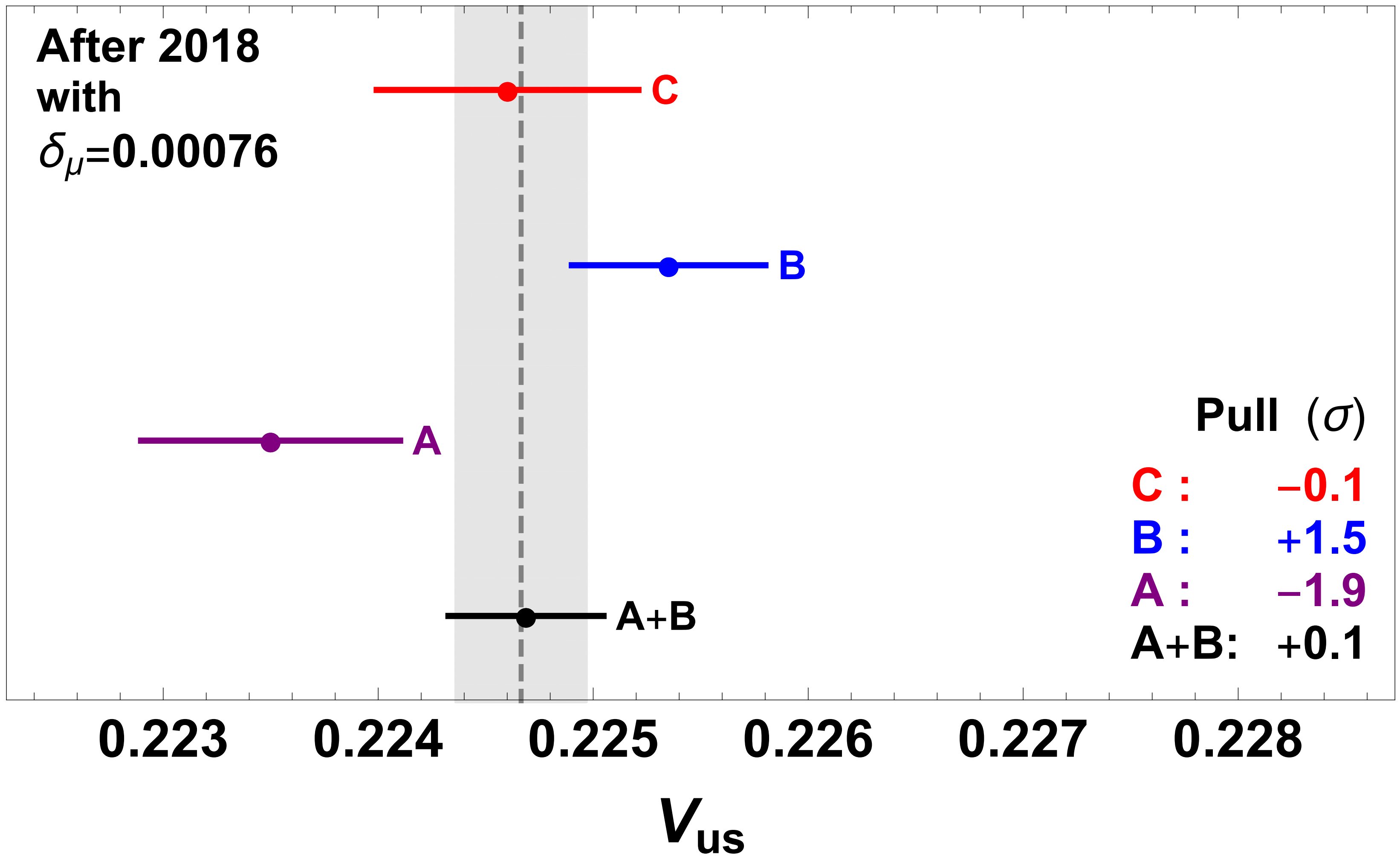






CKM UNITARITY

Focus on



$$\chi^2_{dof} = 3$$

$$v_\ell = 6.3 \text{ TeV}$$

MORE ABOUT SOLUTION #2

Something not explained in the SM:

- Replication of fermion **families**;
- inter-family **mass hierarchy** (**Yukawa hierarchy**);
- weak **mixing pattern**: small angles for quarks, large angles for neutrinos;
- neutrino masses: very small (seesaw?), mass hierarchy yet unknown.

Hierarchy between quarks and CKM angles parametrized by $\epsilon \sim 1/20$:

$$m_d : m_s : m_b \sim \epsilon^2 : \epsilon : 1 \qquad\qquad m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1$$

$$\sin\theta^q_{12} \sim \sqrt{\epsilon} \sim 4\epsilon; \quad \sin\theta^q_{23} \sim \epsilon; \quad \sin\theta^q_{13} \sim \epsilon^2$$

Hierarchy between charged leptons parametrized by same $\epsilon \sim 1/20$:

$$m_e : m_\mu : m_\tau \sim k^{-1}\epsilon^2 : k\epsilon : k$$

$k \simeq 3$ (factor $O(1)$).

Technically natural: SM tolerates Yukawa hierarchy but cannot explain it.

STANDARD MODEL $\otimes$ FAMILY SYMMETRY

- A gauge **family symmetry** can be introduced and **mass hierarchy** between families can be related to the **hierarchy of the symmetry breaking**.
- Family symmetry should **not allow fermions to have mass** until this symmetry is spontaneously broken. So it should not be **vector-like**: L and R fermions should transform differently under family symmetry.

Maximal family symmetry could be a chiral symmetry:

$$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$$

$$q_{Li} \sim 3_q \qquad u_R^j \sim \bar{3}_u \qquad d_R^\alpha \sim \bar{3}_d \qquad l_{L\beta} \sim 3_l \qquad e_R^k \sim \bar{3}_e$$

In grand unification SU(5) is reduced to

$$U(3)^2 = U(3)_l \times U(3)_e$$

$$(q, u^c, e^c)_i = (10, 3_e)$$
$$(l, d^c)_\alpha = (\bar{5}, 3_l)$$

$SU(3)_f$ gauged, U(1) global

SSB & FERMION MASSES

$$\boxed{SU(3)_l \times SU(3)_e}$$

$$\eta^{\alpha}_{(n)} \sim \bar{3}_l \qquad SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{3}$$

$n = 1, 2, 3$

$$\xi^{i}_{(n)} \sim \bar{3}_e \qquad SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$n = 1, 2, 3$

Then mass ratios, $V_{\rm CKM}$, U_{PMNS} are obtained.

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$n = 1, 2, 3$

Then mass ratios, V_{CKM}, U_{PMNS} are obtained.

Effective Operators for Fermion Masses

$$\frac{\xi^i \xi^j}{M^2}\bar{\phi}\bar{u}_j q_i + \frac{\xi^i \eta^\alpha}{M^2}\phi \bar{d}_\alpha q_i + \frac{\xi^k \eta^\beta}{M^2}\phi \bar{e}_k l_\beta + \frac{\eta^\alpha \eta^\beta}{M^3}\bar{\phi}\bar{\phi} l_\alpha l_\beta + h.c.$$

$$m^{(e)} \sim \begin{pmatrix} O(\epsilon^2 \epsilon_{\mathrm{L}}^2) & O(\epsilon \epsilon_{\mathrm{L}}^2) & O(\epsilon_{\mathrm{L}}^2) \\ O(\epsilon^2 \epsilon_{\mathrm{L}}) & O(\epsilon \epsilon_{\mathrm{L}}) & O(\epsilon_{\mathrm{L}}) \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{v_3 u_3}{M^2} v_{EW}$$

SSB & FERMION MASSES

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$n = 1, 2, 3$

Then mass ratios and mixing angles are obtained:

Effective Operators for Fermion Masses

$$\frac{\xi^i \xi^j}{M^2} \bar{\phi} \bar{u}_j q_i + \frac{\xi^i \eta^\alpha}{M^2} \phi \bar{d}_\alpha q_i + \frac{\xi^k \eta^\beta}{M^2} \phi \bar{e}_k l_\beta + \frac{\eta^\alpha \eta^\beta}{M^3} \bar{\phi} \bar{\phi} l_\alpha l_\beta + h.c.$$

$$m^{(e)} \sim \begin{pmatrix} O(\epsilon^2 \epsilon_{\mathrm{L}}^2) & O(\epsilon \epsilon_{\mathrm{L}}^2) & O(\epsilon_{\mathrm{L}}^2) \\ O(\epsilon^2 \epsilon_{\mathrm{L}}) & O(\epsilon \epsilon_{\mathrm{L}}) & O(\epsilon_{\mathrm{L}}) \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{v_3 u_3}{M^2} v_{EW}$$

GAUGE BOSON MASSES

$$\boxed{SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}}$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(e)}$$

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1) $$M^2_{\theta_{4,5,6,7}} = \frac{g^2 v_3^2}{2}$$

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$$M^2_{\theta_{3,8}} = \frac{g^2}{2}(\theta_8, \theta_3)\begin{pmatrix} \frac{4}{3}v_3^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & v_2^2 \end{pmatrix}\begin{pmatrix} \theta_8 \\ \theta_3 \end{pmatrix}$$

GAUGE BOSON MASSES

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$$u_{1,2,3} \sim v_3$$

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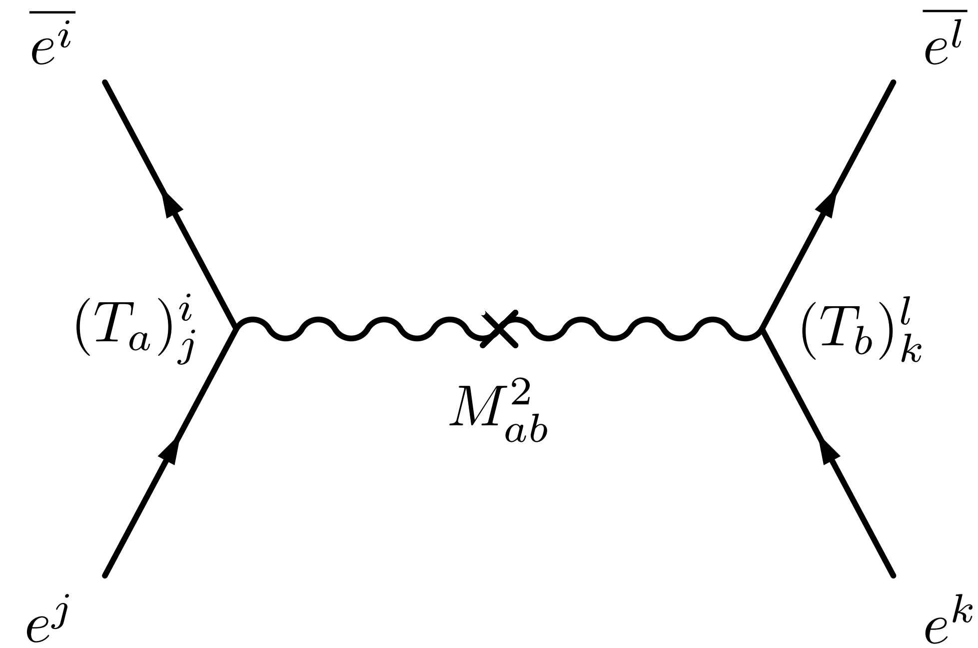
$$u_{1,2,3} \sim v_3 \qquad v_2 \gtrsim ?$$

FLAVOR CHANGING NEUTRAL CURRENTS

- Yukawa couplings, and photon/Z couplings ($V^\dagger V = 1$), are diagonal in mass basis: **no flavour changing neutral currents at tree level in the SM**.

Here in principle:

$$\mathcal{L}_{eff} = -\frac{g_e^2}{2} J_a^{(e)\dagger} (\mathbf{M}_{\theta_e}^{-2})_{ab} J_b^{(e)}$$



$$J_a^{(e)\mu} = \frac{1}{2} \begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}_R \gamma^\mu V_R^{(e)\dagger} \lambda_a^* V_R^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R$$

BUT FLAVOR CHANGING CAN ALWAYS BE CONTROLLED!

FLAVOR CHANGING NEUTRAL CURRENTS

In $\mathbf{SU(2)_e}$ gauge symmetry limit ($v_3 \gg v_2$):

- $SU(2)_e$ gauge bosons have equal masses;
- there are no FCNC thanks to **CUSTODIAL SYMMETRY**, no matter if two families are mixed:

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\frac{1}{4v_2^2}(\overline{\mathbf{e}_\mathrm{R}}\tau^{a*}\gamma^\mu \mathbf{e}_\mathrm{R})(\overline{\mathbf{e}_\mathrm{R}}\tau^{a*}\gamma_\mu \mathbf{e}_\mathrm{R}) \\
&= -\frac{1}{4v_2^2}(\overline{e_{\mathrm{R}}}_1\gamma_\mu e^1 + \overline{e_{\mathrm{R}}}_2\gamma_\mu e^2_\mathrm{R})^2 = \\
&= -\frac{1}{4v_2^2}(\begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}\gamma^\mu V^{(e)\dagger}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix})^2_R
\end{aligned}
$$

NO MIXING WITH 3rd FAMILY $\longrightarrow$ NO FCNC.

- Then constraints on masses are proportional to violation of custodial symmetry (corrections of order $\epsilon = v_2/v_3$):

$$
\mathcal{L}_{eff} = -\frac{1}{4v_2^2}(J_{(2)})^2 - \frac{1}{4v_3^2}(J_3 + \sqrt{3}J_8)^2 - \frac{1}{v_3^2}\sum_{a=4}^{7}(J_a)^2
$$

FCNC & COMPOSITENESS LIMITS

Compositeness limits:

$$\mathcal{L}_C = \pm \frac{g^2}{(1+\delta_{ef})\Lambda_{RR}^2} \bar{e_R}\gamma_\mu e_R \bar{f_R}\gamma^\mu f_R$$

$$\frac{g^2}{4\pi} = 1$$

$$\Lambda_{RR}^-(eeee) > 10.2\,\mathrm{TeV}$$
$$\Lambda_{RR}^-(ee\mu\mu) > 9.1\,\mathrm{TeV}$$
$$\Lambda_{RR}^-(ee\tau\tau) > 5.5\,\mathrm{TeV}$$

$$\boxed{v_2 > 2\,\mathrm{TeV}} \rightarrow v_3 > 40\mathrm{TeV}$$

LFV mode	Exp. $\Gamma_i/\Gamma_\mu(\Gamma_\tau)$	Main contribution to $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$	Predicted value of $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$
$\mu \to eee$	$< 1.0 \cdot 10^{-12}$	$\frac{1}{8}\left(\frac{v_{\mathrm{EW}}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu} + V_{2e}^* V_{2\mu}\epsilon^2\right\rvert^2$	$\leq 1.1 \cdot 10^{-13}\left(\frac{2\,\mathrm{TeV}}{v_2}\right)^4 \epsilon_{20}^4\tilde{\epsilon}_{20}^2$
$\tau^- \to \mu^- e^+ e^-$	$< 1.8 \cdot 10^{-8}$	$\frac{1}{4}\left(\frac{v_{\mathrm{EW}}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\mathrm{w}}}{\Gamma_\tau}$	$= 6.2 \cdot 10^{-9}\left(\frac{2\,\mathrm{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\tau \to \mu\mu\mu$	$< 2.1 \cdot 10^{-8}$	$\frac{1}{8}\left(\frac{v_{\mathrm{EW}}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\mathrm{w}}}{\Gamma_\tau}$	$= 3.1 \cdot 10^{-9}\left(\frac{2\,\mathrm{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\mu \to e\gamma$	$< 4.2 \cdot 10^{-13}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\mathrm{EW}}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu}\right\rvert^2$	$= 3.1 \cdot 10^{-15}\left(\frac{2\,\mathrm{TeV}}{v_2}\right)^4 \epsilon_{20}^4\tilde{\epsilon}_{20}^2$
$\tau \to \mu\gamma$	$< 4.4 \cdot 10^{-8}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\mathrm{EW}}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\mathrm{w}}}{\Gamma_\tau}$	$= 8.7 \cdot 10^{-11}\left(\frac{2\,\mathrm{TeV}}{v_2}\right)^4 \epsilon_{20}^2$

LEFT-HANDED LEPTONS

There is an interesting possibility for left-handed sector. Considering $\mathbf{SU(3)}_\ell$ gauge symmetry, if a symmetry between flavons η holds and

$$u_3 = u_2 = u_1$$

then

- Gauge bosons have equal masses
- They do not mix
- From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2}(\overline{\mathbf{e}_\mathrm{L}}\,\lambda^a\,\gamma^\mu\mathbf{e}_\mathrm{L})(\overline{\mathbf{e}_\mathrm{L}}\,\lambda^a\,\gamma_\mu\mathbf{e}_\mathrm{L}) = -\frac{1}{6v_2^2}(\overline{\mathbf{e}_\mathrm{L}}\,\mathbb{I}\,\gamma_\mu\mathbf{e}_\mathrm{L})^2$$

That is **no FCNC**.

CONCLUSIONS

- Flavor symmetry leads to a possible solution for the CKM unitarity problem.
- Horizontal gauge bosons can be **as light as few TeV**, without contradiction with the experimental limits (flavour changing processes with leptons, compositeness limits, K-mesons system, B-mesons-system, etc.).
- In order to cancel $SU(3)^3$ anomalies an interesting possibility is to introduce the mirror twins. Mirror matter is a candidate for dark matter.
- The flavor gauge bosons are messengers between the two sectors and so not only they are a portal for direct detection, but also they can mediate new phenomena such as muonium disappearance (conversion into mirror muonium), kaon disappearance, pion disappearance.

backup

BACKUP

	CKM 3	CKM 4	CKM+$\mathcal{F}$
C	0.22780(60)	0.22443(61)	0.22460(61)
B	0.22535(45)	0.22518(45)	0.22535(45)
A	0.22333(60)	0.22333(60)	0.22350(60)
$\overline{A+B}$	0.22463(36)	0.0.22452(36)	0.22469(36)
$\overline{A+B+C}$	0.22546(31)	0.22449(31)	0.22467(31)
	$\chi^2 = 27.71$	$\chi^2 = 6.1$	$\chi^2 = 6.1$

TRIANGLE ANOMALIES

$$SU(3)_\ell \times SU(3)_e \times SU(3)_Q \times SU(3)_u \times SU(3)_d$$

$$\ell_L \sim 3_\ell,\ e_R \sim 3_e,\ Q_L \sim 3_Q,\ u_R \sim 3_u,\ d_R \sim 3_d$$

- In order to cancel $SU(3)^3$ anomalies for each triplet another triplet (SM singlet) with opposite chirality is needed.

- An interesting possibility is to introduce the mirror twins with opposite chirality and analogous representation of mirror SM gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$:

$$\ell'_R \sim 3_\ell,\ e'_L \sim 3_e,\ Q'_R \sim 3_Q,\ u'_L \sim 3_u,\ d'_L \sim 3_d$$

- Couplings with flavons:

$$\frac{g_{in}\xi_n^\alpha}{M}(\phi\overline{\ell_{Li}}e_{R\alpha} + \phi'\overline{\ell'_{Ri}}e'_{L\alpha}) + h.c.$$

COSMOLOGICAL IMPLICATIONS

- Mirror matter is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms.
- The flavor gauge bosons are messengers between the two sectors and so a portal for direct detection.
- $T'/T < 0.2 \div 0.3$ from CMB and large scale structures.
- Freeze-out temperature of horizontal interactions between the two sectors should not exceed $T_d \simeq (v_2/2)^{\frac{4}{3}} \times 130$ MeV. Or $v'_{\rm EW} \gg v_{\rm EW}$.
- For neutrinos

$$\frac{Y_\nu^{ij}}{\mathcal{M}}(\phi\phi l_{Li}^T C l_{Lj} + \phi'\phi' l'^T_{Ri} C l'_{Rj}) + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}}\phi\phi'\overline{l_{Li}} l'_{Rj} + h.c.$$

the last operator gives COLEPTOGENESIS.

TRIANGLE ANOMALIES 2

- As an example, for $SU(3)_e$, mixed triangle anomaly $U(1) \times SU(3)_e^2$ must be cancelled. New leptons

$$\mathcal{E}_{L\alpha} \sim (1, -2, 3_e; X), \qquad \mathcal{E}_{Ri} \sim (1, -2, 1; X)$$

and for mirror parity

$$\mathcal{E}'_{R\alpha} \sim (1, -2', 3_e; X), \qquad \mathcal{E}'_{Li} \sim (1, -2', 1; X)$$

cancel the mixed triangle

$$U(1) \times SU(3)_e^2,\ U(1)_X \times SU(3)_e^2,\ U(1) \times U(1)_X^2,\ U(1)_X \times U(1)^2$$

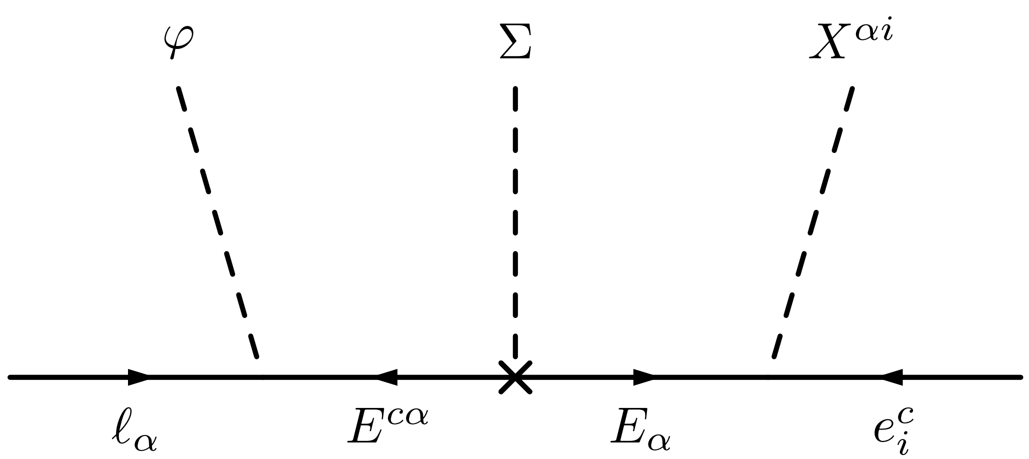
- Masses from Yukawa couplings

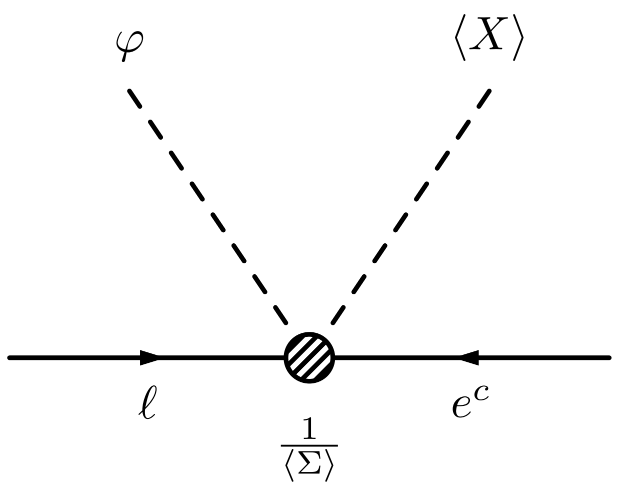
$$y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Ri}} \mathcal{E}_{L\alpha} + y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Li}}' \mathcal{E}'_{R\alpha} + h.c.$$

- The lightest has mass $O(100)$ GeV. If $U(1)_X$ is unbroken, then it is stable. Current experimental lower limit on charged new leptons is 102.6 GeV.

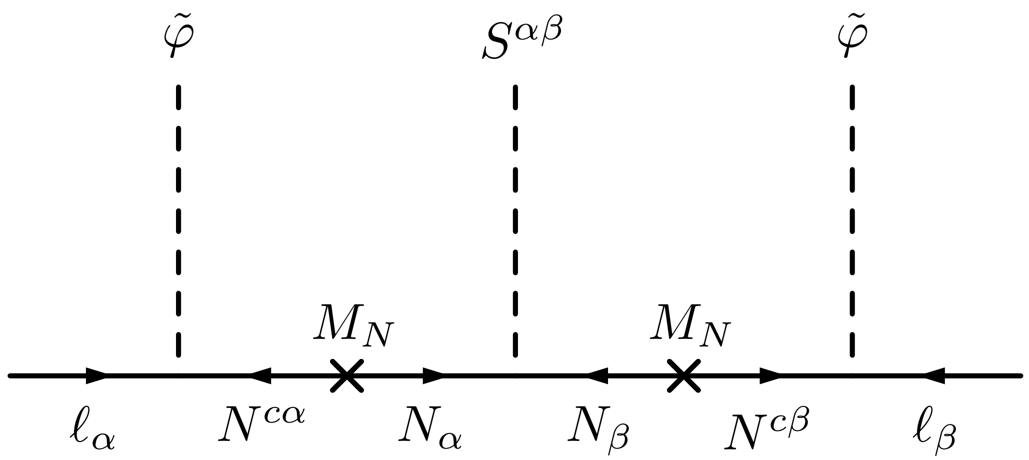
EFFECTIVE OPERATORS FOR FERMION MASSES

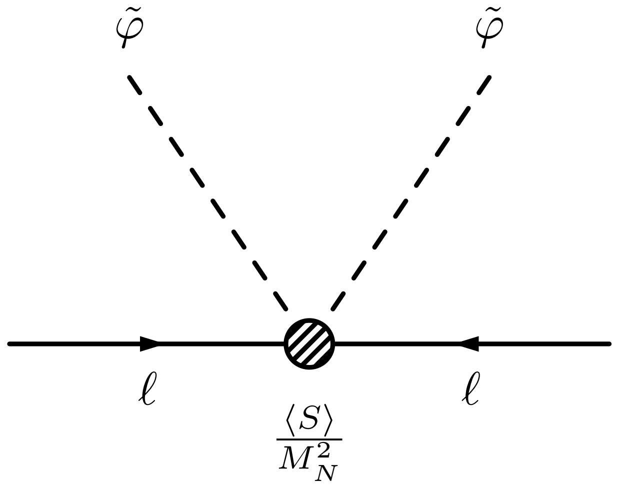
$$X_e^{\beta k} \sim (\bar{3}_l, \bar{3}_e) \qquad X_d^{i\alpha} \sim (\bar{3}_q, \bar{3}_d) \qquad X_u^{ij} \sim (\bar{3}_q, \bar{3}_u)$$





$$S_{\alpha\beta} \sim 6_l$$





$$\mathcal{L} = \frac{\lambda_u}{M_U} X_u^{ij} \bar{\phi} \bar{u}_j q_i + \frac{\lambda_d}{M_D} X_d^{i\alpha} \phi \bar{d}_\alpha q_i + \frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta + \lambda_\nu \mathcal{M}^{-1}_{\alpha\beta} \bar{\phi} \bar{\phi} l_\alpha l_\beta + h.c.$$

SSB

$$\boxed{SU(3)_l \times SU(3)_e}$$

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{2} - \frac{1}{3}$$

$$\eta^{\alpha}_{(n)} \sim \bar{3}_l \qquad \langle \eta_{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ u_3 \end{pmatrix} \quad \langle \eta_{(2)} \rangle = \begin{pmatrix} 0 \\ u_2 \\ 0 \end{pmatrix} \quad \langle \eta_{(1)} \rangle = \begin{pmatrix} u_1 \\ 0 \\ 0 \end{pmatrix}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\xi^{i}_{(n)} \sim \bar{3}_e \qquad \langle \xi_{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix} \quad \langle \xi_{(2)} \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix} \quad \langle \xi_{(1)} \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}$$

- inducing VEV to X_e and S via couplings

$$\mu_X^* \xi_{(l)} X^\dagger \eta_{(n)} + \text{h.c.} \qquad \langle X \rangle \sim \langle X^{33} \rangle \cdot \begin{pmatrix} O(\epsilon^2 \epsilon_{\mathrm{L}}^2) & O(\epsilon \epsilon_{\mathrm{L}}^2) & O(\epsilon_{\mathrm{L}}^2) \\ O(\epsilon^2 \epsilon_{\mathrm{L}}) & O(\epsilon \epsilon_{\mathrm{L}}) & O(\epsilon_{\mathrm{L}}) \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix}$$

FERMION MASSES

$$\boxed{SU(3)_l \times SU(3)_e}$$

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{2} - \frac{1}{3}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

From Yukawa coupling:

$$\frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta \longrightarrow \qquad m^{(e)\dagger} \propto \langle X_e \rangle \simeq \begin{pmatrix} O(\epsilon^2) & 0 & 0 \\ O(\epsilon^2) & O(\epsilon) & 0 \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{|\mu_X| v_3 u_3}{M_X^2}$$

$$V_{\mathrm{L}}^{(e)\dagger} m^{(e)} V_{\mathrm{R}}^{(e)} = m_{\mathrm{diag}}^{(e)} \qquad \overline{\mathbf{e}_{\mathrm{R}}} \mathbf{m}^{(\mathbf{e})\dagger} \mathbf{e}_{\mathrm{L}} = (\overline{\mathbf{e}_{\mathrm{R}}} V_{\mathrm{R}}^{(e)})(V_{\mathrm{R}}^{(e)\dagger} \mathbf{m}^{(\mathbf{e})\dagger} V_{\mathrm{L}}^{(e)})(V_{\mathrm{L}}^{(e)\dagger} \mathbf{e}_{\mathrm{L}})$$

$$\begin{pmatrix} e^1 \\ e^2 \\ e^3 \end{pmatrix}_{\mathrm{R}} = V_R^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathrm{R}} = \begin{pmatrix} V_{1e} & V_{1\mu} & V_{1\tau} \\ V_{2e} & V_{2\mu} & V_{2\tau} \\ V_{3e} & V_{3\mu} & V_{3\tau} \end{pmatrix}_{\mathrm{R}}^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathrm{R}}$$

FCNC

$$\mathcal{L}_{eff} = -\frac{1}{4v_2^2}\left(\begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}\gamma^\mu V^{(e)\dagger}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V^{(e)}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}\right)^2_R$$
$$-\frac{1}{4v_3^2}(J_3+\sqrt{3}J_8)^2 - \frac{1}{v_3^2}\sum_{a=4}^{7}(J_a)^2$$

LFV mode	Experimental limit	Contribution to $-4\frac{G_{\mu/\tau l l' l''}}{\sqrt{2}}$ at L.O. from: $-\frac{1}{4v_2^2}J^{(e)}_{(2)}J^{(e)}_{(2)}$	Heavy bosons
$\mu \to eee$	$\frac{1}{\sqrt{2}}\frac{\lvert G_{\mu eee}\rvert}{G_F} < 10^{-6}$	$\frac{V^*_{3e}V_{3\mu}}{2v_2^2}$	$-\frac{(V^*_{1e}V_{1\mu}-V^*_{2e}V_{2\mu})\epsilon^2}{4v_2^2}$
$\mu^- e^+ \to \mu^+ e^-$	$\frac{\lvert G_{\mu e\mu e}\rvert}{G_F} < 3\cdot 10^{-3}$	$\frac{-(V^*_{3e}V_{3\mu})^2}{4v_2^2}$	$-\frac{(V^*_{1e}V_{1\mu}-V^*_{2e}V_{2\mu})^2\epsilon^2}{16v_2^2}$
$\tau^- \to \mu^- e^+ e^-$	$\frac{\lvert G_{\tau\mu ee}\rvert}{G_F} < 1.34\cdot 10^{-4}$	$\frac{V^*_{3\mu}V_{3\tau}}{2v_2^2}$	$\frac{(V^*_{2\mu}V_{2\tau}-V^*_{3\mu}V_{3\tau})\epsilon^2}{4v_2^2}$
$\tau \to \mu\mu\mu$	$\frac{1}{\sqrt{2}}\frac{\lvert G_{\tau\mu\mu\mu}\rvert}{G_F} < 1.45\cdot 10^{-4}$	$\frac{V^*_{3\mu}V_{3\tau}}{2v_2^2}$	$-\frac{V^*_{3\mu}V_{3\tau}\epsilon^2}{v_2^2}$
$\tau \to eee$	$\frac{1}{\sqrt{2}}\frac{\lvert G_{\tau eee}\rvert}{G_F} < 1.64\cdot 10^{-4}$	$\frac{V^*_{3e}V_{3\tau}}{2v_2^2}$	$-\frac{(V^*_{1e}V_{1\tau}-V^*_{2e}V_{2\tau}+V^*_{3e}V_{3\tau})\epsilon^2}{4v_2^2}$
$\tau \to e^-\mu^-\mu^+$	$\frac{\lvert G_{\tau e\mu\mu}\rvert}{G_F} < 1.64\cdot 10^{-4}$	$\frac{V^*_{3e}V_{3\tau}}{2v_2^2}$	$\frac{(3V^*_{3e}V_{3\tau}+V^*_{2e}V_{2\tau}-V^*_{1e}V_{1\tau})\epsilon^2}{4v_2^2} - \frac{(V^*_{3\mu}V_{1\mu}+V^*_{3\mu}V^*_{2e})\epsilon^2}{v_2^2}$

FCNC & COMPOSITENESS LIMITS

LFV mode	Operator	Experimental limit	Main contribution	Constraint
$\mu \to eee$	$-\frac{4G_{\mu eee}}{\sqrt{2}}\overline{e_R}\gamma^\mu \mu_R \overline{e_R}\gamma^\mu e_R$	$\Gamma_{\mu eee}/\Gamma_\mu < 1.0 \cdot 10^{-12}$	$-\frac{4G_{\mu eee}}{\sqrt{2}} \simeq -\frac{\epsilon^3}{2v_2^2}$	$v_2 > 1.16$ TeV
$\tau^- \to \mu^- e^+ e^-$	$-\frac{4G_{\tau\mu ee}}{\sqrt{2}}\overline{\mu_R}\gamma^\mu \tau_R \overline{e_R}\gamma^\mu e_R$	$\Gamma_{\tau\mu ee}/\Gamma_\tau < 1.8 \cdot 10^{-8}$	$-\frac{4G_{\tau\mu ee}}{\sqrt{2}} \simeq -\frac{\epsilon}{2v_2^2}$	$v_2 > 1.5$ TeV
$\mu \to e\gamma$	$-\mu_{\mu e}(\overline{e_L}\frac{\sigma^{\delta\rho}}{2}\mu_R + \overline{e_R}\frac{\sigma^{\delta\rho}}{2}\mu_L)F_{\delta\rho}$	$\Gamma_{\mu e\gamma}/\Gamma_\mu < 4.2 \cdot 10^{-13}$	$-\mu_{\mu e} \simeq -\frac{e}{8\pi^2}\frac{\epsilon^3 m_\mu}{v_2^2}$	$v_2 > 1$ TeV
$\tau \to \mu\gamma$	$-\mu_{\tau\mu}(\overline{\mu_L}\frac{\sigma^{\delta\rho}}{2}\tau_R + \overline{\mu_R}\frac{\sigma^{\delta\rho}}{2}\tau_L)F_{\delta\rho}$	$\Gamma_{\tau e\gamma}/\Gamma_\tau < 4.4 \cdot 10^{-8}$	$-\mu_{\tau\mu} \simeq -\frac{e}{8\pi^2}\frac{\epsilon m_\tau}{v_2^2}$	$v_2 > 0.7$ TeV

Compositeness limits:

$$\mathcal{L}_C = \pm \frac{g^2}{(1+\delta_{ef})\Lambda_{RR}^2}\bar{e_R}\gamma_\mu e_R \bar{f_R}\gamma^\mu f_R$$

$$\frac{g^2}{4\pi} = 1$$

$$\Lambda_{RR}^-(eeee) > 10.2\,\text{TeV}$$
$$\Lambda_{RR}^-(ee\mu\mu) > 9.1\,\text{TeV}$$
$$\Lambda_{RR}^-(ee\tau\tau) > 5.5\,\text{TeV}$$

$$\boxed{v_2 > 2\,\text{TeV}} \quad \longrightarrow \quad v_3 > 40\text{TeV}$$

STANDARD MODEL

- Weak eigenstates are not mass eigenstates;
- fermion mass matrices

$$m_{ij}^{(f)} = Y_{ij}^{f} v_{\mathrm{EW}}$$

$v_{\mathrm{EW}} = 174$ GeV, can be diagonalized $V_L^{(f)\dagger} m^{(f)} V_R^{(f)} = m_{\mathrm{diag}}^{(f)}$;

- all masses proportional to Higgs VEV;
- fermion mixing in charged currents is

$$V_{\mathrm{CKM}} = V_L^{(u)\dagger} V_L^{(d)} \qquad U_{\mathrm{PMNS}} = V_L^{(\nu)\dagger} V_L^{(e)};$$

- Yukawa couplings, and photon/Z couplings ($V^\dagger V = 1$), are diagonal in mass basis: **no flavour changing neutral currents at tree level**;
- all flavour changing and CP-violation is originated from loop diagrams;
- no mixing in the right particles sector (unless right W bosons exist).

BACKUP

$$\Gamma(K^+ \to \pi^+ \mu^- e^+)/\Gamma_{\text{total}} < 5.2 \cdot 10^{-10}$$

$$v_2 > 30\,\text{TeV}$$

$$M_{12} = \frac{G_F^2 m_W^2}{4\pi^2} \tilde{F}^* \langle K^0 | (\overline{d_L} \gamma^\mu s_L)(\overline{d_L} \gamma_\mu s_L) | \bar{K}^0 \rangle =$$

$$= -\frac{G_F^2 m_W^2}{12\pi^2} \tilde{F}^* f_K^2 m_K B_K e^{i(\xi_s - \xi_d - \xi)}$$

$$\mathcal{L}_{K\bar{K}\text{SM}} = \frac{-G_F^2 M_W^2}{4\pi^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) \tilde{F} + h.c. =$$

$$= -\frac{1 - i\alpha_{SM}}{\Lambda_{SM}^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) + h.c.$$

$$\sim -\frac{1 - i10^{-2}}{(1,6 \cdot 10^6 \text{GeV})^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) + h.c.$$

$$\left| \frac{1 - i\alpha_{ds}^{(d)}}{\Lambda_{ds}^{(d)2}} \right| = \left| \frac{(V_{3d}^{(d)} V_{3s}^{(d)*})^2}{2v_2^2} \right| =$$

$$= X_d \left| \frac{(V_{Ltd} V_{Lts}^*)^2}{2v_2^2} \right|$$

$$\approx \frac{(\epsilon^3)^2}{2v_2^2}$$