Scalar Fields in Cosmology

Orfeu Bertolami
Departamento de Física e Astronomia
(**http://web.ist.utl.pt/orfeu.bertolami/homeorfeu.html**)

U. PORTO
FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO

Hot Topics in Cosmology
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Yang-Mills theories

Symmetry Group: G $\Psi_f(x) \to \Psi'_f(x) = \exp\left(-i\theta^a(x)t_a\right)\Psi_f(x) \equiv U(x)\Psi_f(x)$

$A^a_\mu t_a = A_\mu \qquad A_\mu \to A'_\mu = U(x)A_\mu U(x)^{-1} + iU(x)^{-1}\partial_\mu U(x)$

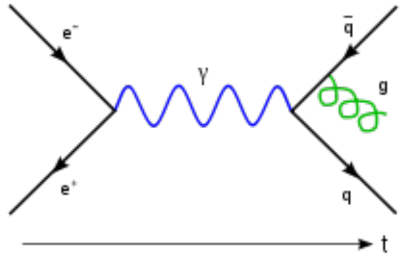
Generators t^a (a=1, …, n; n=dim G) satisfy a Lie algebra: $\left[t_a, t_b\right] = f^{abc}t_c$

Matter fields Ψ_f
Gauge fields A^a_μ ⟶ **Covariant derivative** $D_\mu = \partial_\mu + igA^a_\mu t_a$

Field strength/ Gauge "curvature"

$$\left(D_\mu D_\nu - D_\nu D_\mu\right)\Psi = igt_a F^a_{\mu\nu}\Psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$



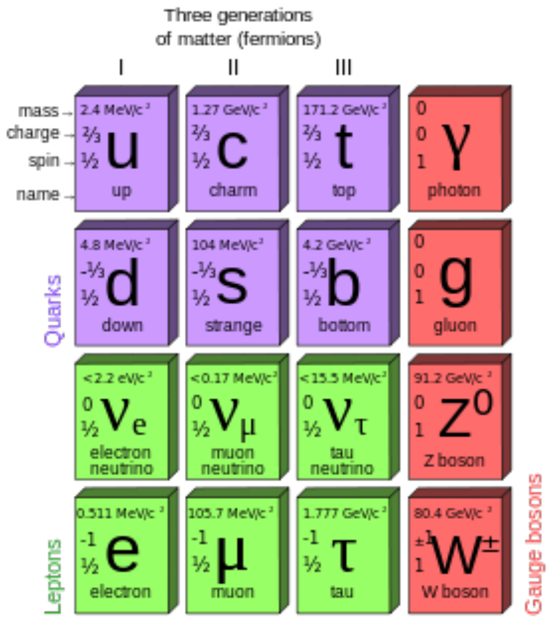
Action $S_{YM-f} = \int [-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_f \overline{\Psi_f}(i\gamma^\mu D_\mu - m_f)\Psi_f]d^4x$

Yang-Mills field eqs. $D_\mu F^{a\mu\nu} = J^{a\nu} \qquad J^{a\mu} = \sum_f \overline{\Psi_f}\gamma^\mu t^a \Psi_f$

Bianchi ids. $D_\mu F^a_{\nu\lambda} + D_\lambda F^a_{\mu\nu} + D_\nu F^a_{\lambda\mu} = 0$

Dirac eqs. $(i\gamma^\mu D_\mu - m_f)\Psi_f = 0$

Yang-Mills theories



Fermions and gauge bosons

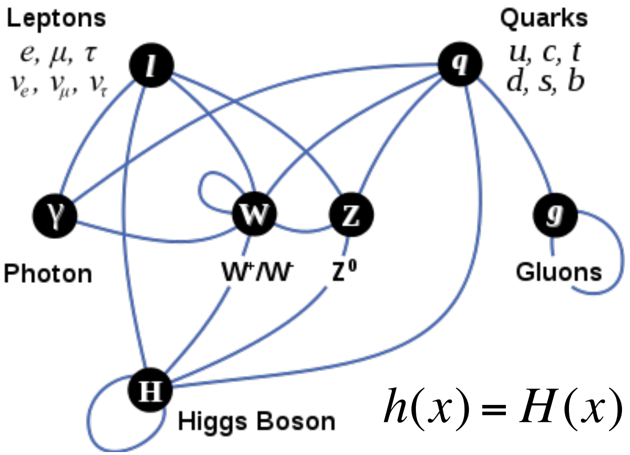
Relevant Gauge Groups:

Electrodynamics G=U(1)
Electroweak theory G=SU(2) X U(1)
Cromodynamics G=SU(3)*
Standard Model G=SU(3) x S(2) x U(1)
Grand Unified Theories G=SU(5), SO(10), E_6, ...
Heterotic String Theory G=E_8 x E_8

Allow for a successful quantization and lead to renormalizable theories!

* **Asymptotic freedom & confinement**

Brout-Englert–Higgs–Guralnik–Hagen–Kibble Mechanism
First Scalar Field Avatar: the Higgs Boson



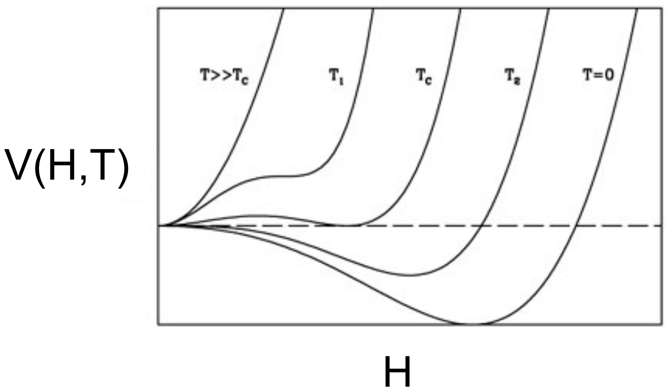
$$h(x) = H(x) - \langle H \rangle \leftarrow$$

Spontaneous symmetry breaking mechanism

↓

$$S_H + S_{H\Psi} = \int d^4x [D_\mu H^+ D^\mu H - m^2 H^+ H + \frac{\lambda}{2}(H^+ H)^2] + \int d^4x \sum_f g_f H \overline{\Psi_f} \Psi_f$$

↓



$\langle H \rangle \neq 0$ → **Non-vanishing vacuum energy**

Min. V(H,T)

$$m_V = g_V \langle H \rangle, m_f = g_f \langle H \rangle$$

↓

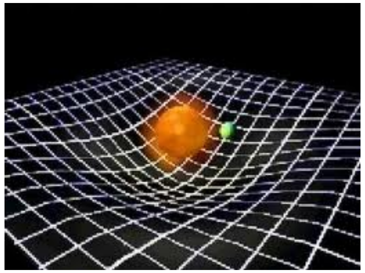
Cosmological constant problem

Higgs field Universal history

$$V(\langle H \rangle, 0) = O(\langle H \rangle^4) \cong O(246 GeV)^4 >> \rho_C \cong (10^{-3} eV)^4$$

General Relativity

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$



Invariance under diffs. → **Space-time curvature**
Matter/Energy

Covariant derivative/ Minimal coupling

$$A^\nu_{;\mu} := D_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu_{\lambda\mu} A^\lambda$$

Torsionless connection $(\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\lambda\nu})$

$$g_{\mu\nu;\lambda} = 0 \longrightarrow \Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\rho} \left[\partial_\nu g_{\rho\lambda} + \partial_\lambda g_{\nu\rho} - \partial_\rho g_{\nu\lambda} \right] \equiv \left\{ {\mu \atop \nu\lambda} \right\}$$

Riemann tensor/ Space-time curvature

$$A_{\nu;\rho\sigma} - A_{\nu;\sigma\rho} = A_\lambda R^\lambda_{\nu\rho\sigma}$$

$$R^\lambda_{\rho\mu\nu} = \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\mu\rho}$$

Action (Λ≠0)

$$S = \int \left[\frac{1}{2\kappa} (R - 2\Lambda) + L_m \right] \sqrt{-g}\, d^4 x \qquad \kappa = \frac{8\pi G}{c^4}$$

Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \qquad T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}$$

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}, R = R^\lambda_\lambda, g := \det(g_{\mu\nu})$$

Fundamental Symmetries

- **CPT symmetry**

$$\left(\frac{\Delta m}{m}\right)_{K^0-\bar{K}^0} < 9\times 10^{-19}$$

- **Equivalence Principle**

$$2\left(\frac{a_1-a_2}{a_1+a_2}\right) = \Delta\left(\frac{M_G}{M_I}\right) < 1.9x10^{-14}$$

New! MICROSCOPE!

Weak Equivalence Principle (WEP)

$$\gamma_1-\gamma_2 < 10^{-28}$$

Polarized GRB + DM
[O.B. & Landim 2018]

Local Lorentz Invariance (LLI)

$$\delta_{Lorentz} \equiv \left|c^2/c_i^2-1\right| \le 1.7\times 10^{-25}$$

Local Position Invariance (LPI)

$$\frac{\Delta\nu}{\nu} = (1+\mu)\frac{U}{c^2} \qquad |\mu| < 2.1\times 10^{-6}$$

Strong Equivalence Principle (SEP)

$$(GR:\gamma=\beta=1;\eta=-1-\beta+2\gamma=0)$$

$$\left[\frac{M_G}{M_I}\right]_{SEP} = 1+\eta\left(\frac{\Omega}{Mc^2}\right) \qquad \eta \le 10^{-5}$$

- **Variation of the fundamental couplings (LPI)**

$$\dot{G}/G = (4\pm 9)\times 10^{-13}\,yr^{-1}$$

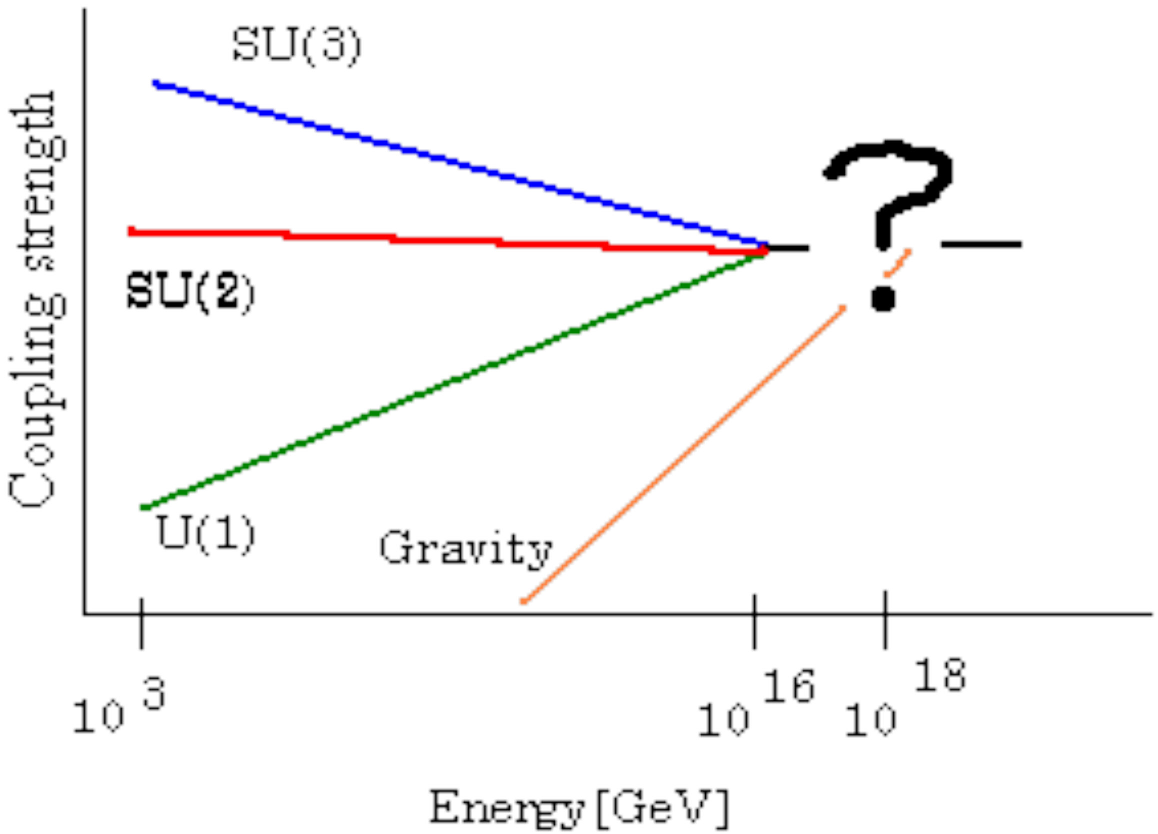
$$\left|\dot{\alpha}_{em}/\alpha_{em}\right| < 4.2\times 10^{-15}\,yr^{-1};\alpha_{em} := \frac{e^2}{\hbar c}$$

Tales of Mystery and Imagination

(Budapest 2012)

- Higgs boson (found!)
- Cosmological constant problem (work in progress for the last 40 years …)
- Violations of Lorentz symmetry and Equivalence Principle (No evidence!)
- Dark matter (Observationally *consensual* & plenty of candidates … Detection)
- Dark energy (Observational tracers)
- Dark energy - dark matter unification and interaction (Observational signatures)
- Variation of fundamental constants? (No evidence!)
- Gravitational Waves (Detected!)
- Black holes (Singularities, Nature, Proliferation … Detected!)
- Pioneer (NO more!) and Flyby anomalies (Evidence has shrank considerably)
- …

Forces of Nature, Unite!



Superstring/M-theory
Second Scalar Field Avatar: the dilaton

- Unification of the existing string theories in the context of M-theory
  - Spectrum of closed string theory contains as zero mass eigenstates:
    - Graviton g_{MN}
    - Dilaton Φ
    - Antisymmetric second-order tensor B_{MN}
- Physics of our 4-dimensional world
  - Require a natural mechanism to fix the value of the dilaton field
  - Drop B_{MN} and introduce fermions $\hat{\psi}$, Yang-Mills fields $\hat{A}_\mu$ with field strength $\hat{F}_{\mu\nu}$
  - Space-time described by the metric $\hat{g}_{\mu\nu}$
  - Effective low-energy four-dimensional action

$$S = \int_M d^4x\sqrt{-\hat{g}}B(\Phi)\left[\frac{1}{\alpha'}[\hat{R} + 4\hat{\nabla}_\mu\hat{\nabla}^\mu\Phi - 4(\hat{\nabla}\Phi)^2] - \frac{k}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - \bar{\hat{\psi}}\gamma^\mu\hat{D}_\mu\hat{\psi} + ...\right]$$

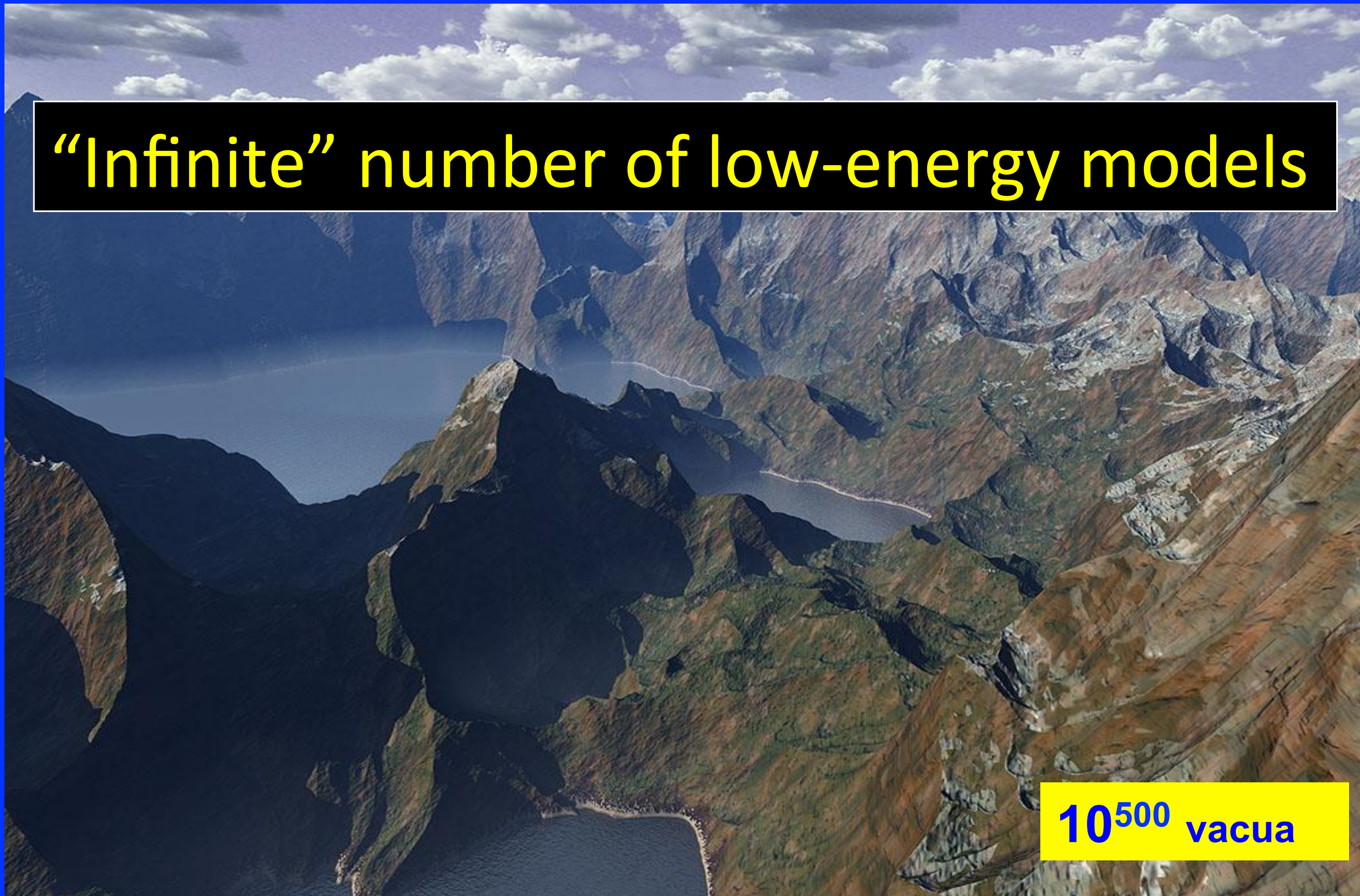
- where $B(\Phi) = e^{-2\Phi} + c_0 + c_1e^{2\Phi} + c_2e^{4\Phi} + ...$
- α' is the inverse of the string tension and k is a gauge group constant
- Constants c_0, c_1, ..., *etc.*, are, in principle, computable

[Damour, Polyakov 1994]

- $4q = 16\pi G = \alpha' / 4$ and a conformal transformation → coupling constants and masses become dilaton-dependent
  - $g^{-2} = k\,B(\phi)$ and $m_A = m_A(B(\phi))$

- Minimal coupling principle: dilaton is driven towards a local minimum of all masses
  - Local maximum of B(ϕ)
  - Mass dependence on the dilaton → particles fall differently → violation of the WEP
    - In the solar system, effect is of order $\Delta a/a \simeq 10^{-16}$
    - Almost within reach of the MICROSCPE mission …
    - Within reach of STEP (Satellite Test of the Equivalence Principle) mission …

String Landscape Problem



Third Scalar Field Avatar: Scalar-tensor theories of gravity

- **Gravitational coupling strength depends on a scalar field, φ**
  - General action

$$S = \frac{c^3}{4\pi G}\int d^4x\sqrt{-g}\left[\tfrac{1}{4}f(\varphi)R - \tfrac{1}{2}g(\varphi)\partial_\mu\varphi\partial^\mu\varphi + V(\varphi)\right] + \sum_i q_i(\varphi)\mathcal{L}_i$$

  - $f(\varphi)$, $g(\varphi)$, $V(\varphi)$ are generic functions, $q_i(\varphi)$ are coupling functions
  - $\mathcal{L}_i$ is the Lagrangian density of the matter fields
  - Graviton-dilaton system of string/M-theory can be viewed as a scalar-tensor theory

- **Brans-Dicke theory**

[Brans, Dicke 1961]

  - Defined by $f(\varphi) = \varphi$, $g(\varphi) = \omega / \varphi$, a vanishing potential $V(\varphi)$ *and* $q_i(\varphi)=1$
  - Non-canonical kinetic term; φ has a dimension of energy squared
  - ω marks observational deviations from GR, which is recovered in the limit $\omega \to \infty$
  - Satifies Mach's Principle
  - $G \propto \varphi^{-1}$ depends on the matter energy-momentum tensor through the field equations
  - Observational bounds: $|\omega| > 40000$

[Will, gr-qc/0504086]

- **Induced gravity models:**
  - $f(\varphi) = \varphi^2$ and $g(\varphi) = 1/2$
  - Potential V (φ) allows for a spontaneous symmetry breaking
  - Field φ acquires a non-vanishing vacuum expectation value,

$$f(\langle 0|\varphi|0\rangle) = \langle 0|\varphi^2|0\rangle = M_P^2 = G^{-1}$$

  - The cosmological constant Λ is given by interplay of $V(\langle 0|\varphi|0\rangle)$ and all other contributions to the vacuum energy

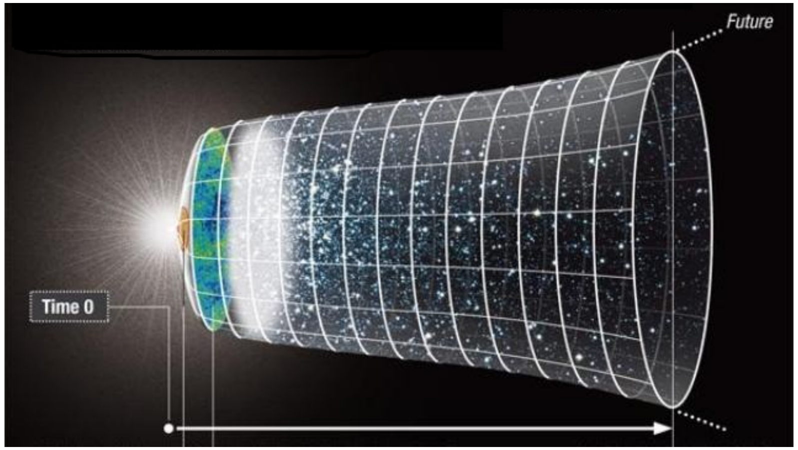
[Fujii 1979]
[Zee 1979]
[Adler 1982]

- **Horndeski gravity (1974) …**

Can be scrutinized with GWs
[Arai, Nishizawa 2017]
[Kopp et al. 2018]

$$S_H = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\Big\{G_2(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + G_{4X}(\phi,X)[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] + G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}(\phi,X)}{6}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]\Big\} + S_m[g]\,,$$

$$X \equiv -\tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi$$



Fourth Scalar Field Avatar: the inflaton

Inflation, an accelerated expansion of the Universe which took place about 10^{-35} secs. after the Big Bang, which accounts for the main observational features of the Universe:

isotropy, homogeneity, horizon, flatness, absence of magnetic monopoles and rotation, and the origin of energy density fluctuations that generated the first galaxies

[Guth 1981, Linde 1982, Albrecht & Steinhardt 1982, …]

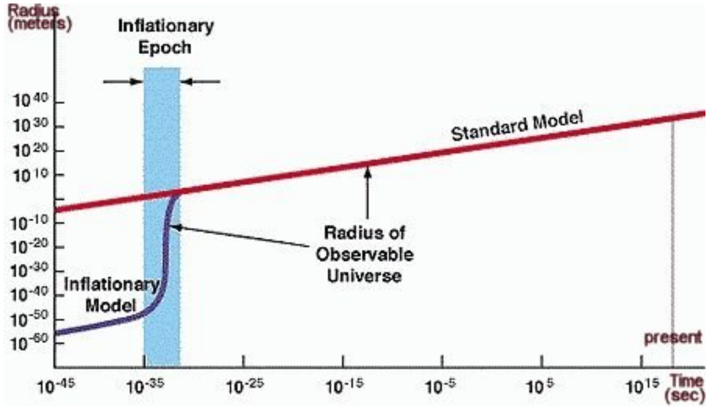
Model: $L = \frac{1}{2}\partial_\mu \varphi \; \partial^\mu \varphi - V(\varphi)$ V(φ) - your favorite …

Quantum fluctuations of the inflaton ➔ Energy density fluctuations + gravitational waves!

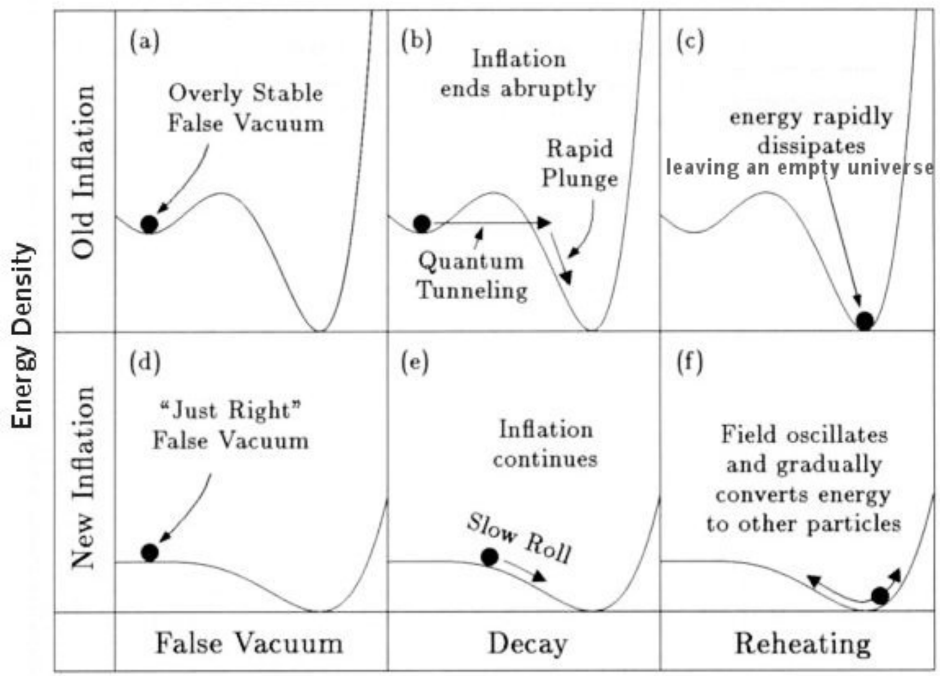
↓

Observed through the Cosmic Microwave Background Radiation (CMBR)

Inflation for voyeurs



$$a(t_f) = a(t_i)\exp\overbrace{\left[\left(\frac{8\pi}{3}\right)^{1/2}\frac{V^{1/2}}{M_P}(t_f - t_i)\right]}^{>\exp 65 \cong 10^{28}}, V \cong 10^{-12} M_P^4$$



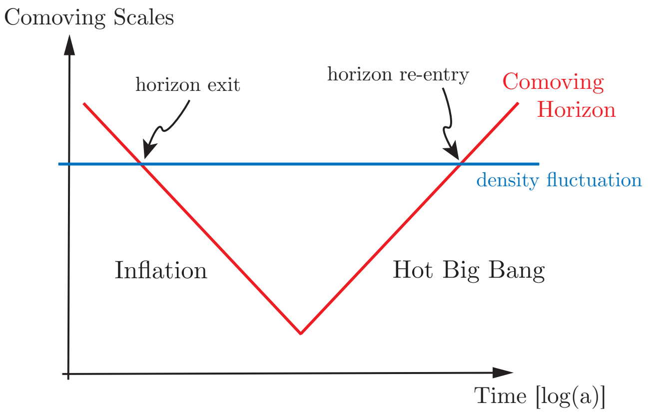
Chiral superfields

GUTs with Higgs field (troublesome)

Supergravity-like (fine)

Inflation and the CMBR

- Simple inflation: $V \gg \dot{\phi}^2$ $(|\ddot{\phi}| \ll |V'|)$ $\longrightarrow$ $a(t) \approx a(0)e^{Ht}\,, \qquad H \approx \text{const}\,.$



$$P_s(k) = A_s(k_\star)\left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1+\frac{1}{2}\alpha_s(k_\star)\ln(k/k_\star)} \qquad n_s - 1 \equiv \frac{d\ln P_s}{d\ln k} \qquad \alpha_s \equiv \frac{dn_s}{d\ln k}$$

$$P_t(k) = A_t(k_\star)\left(\frac{k}{k_\star}\right)^{n_t(k_\star)} \qquad n_t \equiv \frac{d\ln P_t}{d\ln k} \qquad r \equiv \frac{P_t}{P_s}$$

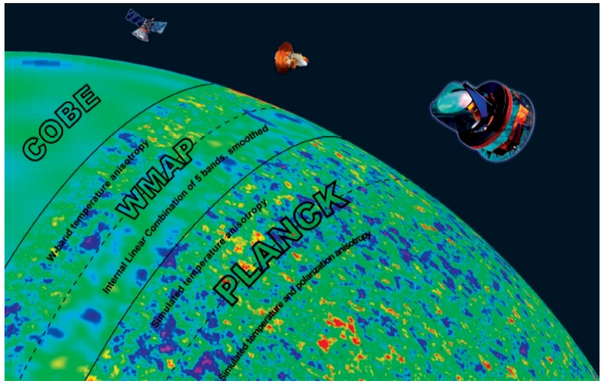
- Slow-roll predictions: $\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_{\rm pl}^2}{2}\frac{\dot{\phi}^2}{H^2} \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad |\eta| \approx M_{\rm pl}^2\left|\frac{V''}{V}\right|$

$$P_s(k) = \frac{1}{24\pi^2 M_{\rm pl}^4}\frac{V}{\epsilon}\bigg|_{k=aH}\,, \qquad n_s - 1 = 2\eta - 6\epsilon$$

$$P_t(k) = \frac{2}{3\pi^2}\frac{V}{M_{\rm pl}^4}\bigg|_{k=aH}\,, \qquad n_t = -2\epsilon\,, \qquad r = 16\epsilon$$

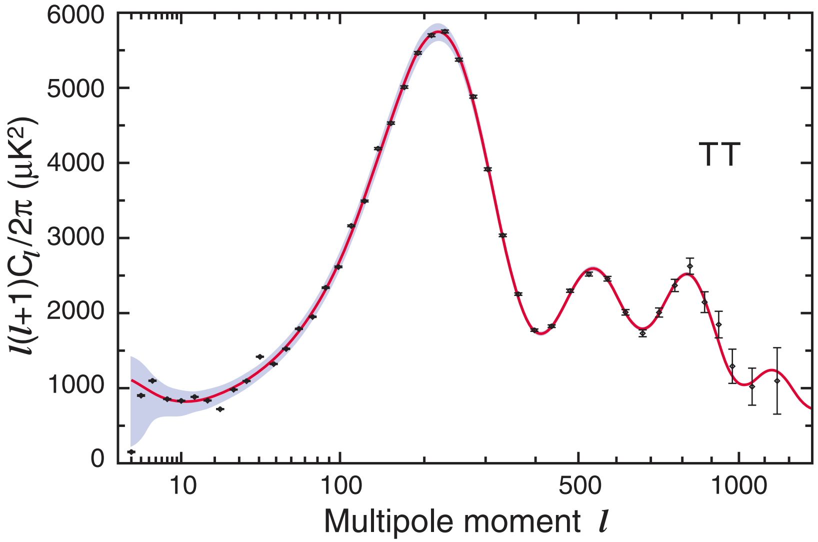
CMBR

(Last surface of matter-radiation scattering, circa 376 Kys after BB)



➔

WMAP
9 year data

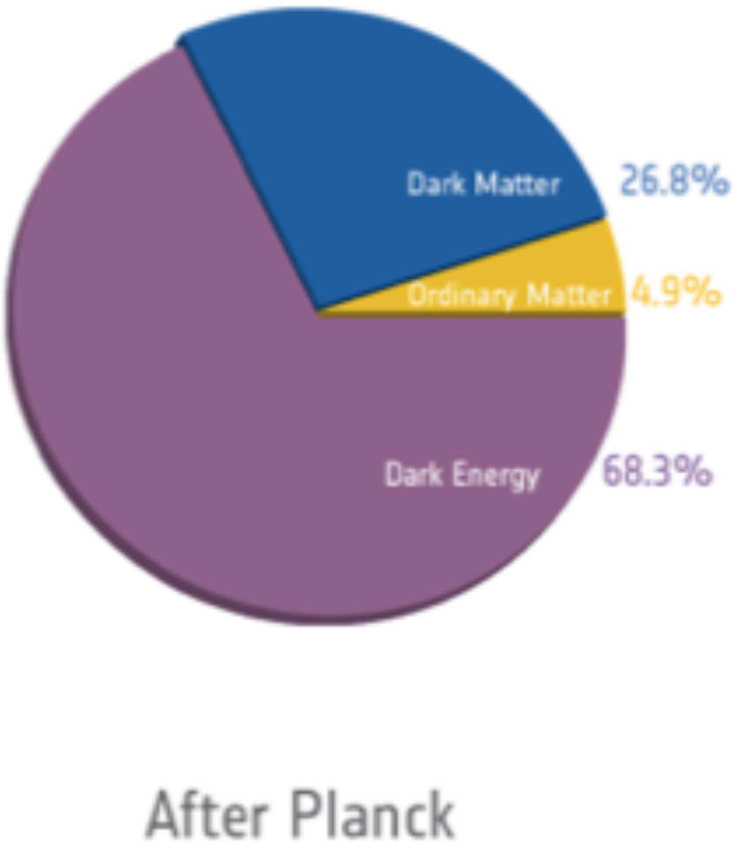


Label	Definition	Physical Origin	Value
Ω_b	Baryon Fraction	Baryogenesis	0.0456 ± 0.0015
Ω_{CDM}	Dark Matter Fraction	TeV-Scale Physics (?)	0.228 ± 0.013
Ω_Λ	Cosmological Constant	Unknown	0.726 ± 0.015
τ	Optical Depth	First Stars	0.084 ± 0.016
h	Hubble Parameter	Cosmological Epoch	0.705 ± 0.013
A_s	Scalar Amplitude	Inflation	$(2.445 \pm 0.096) \times 10^{-9}$
n_s	Scalar Index	Inflation	0.960 ± 0.013

$H=100\ h\ \text{kms}^{-1}\text{Mpc}^{-1}$

The Universe

Has more matter and less dark energy



$$\Omega_b h^2 = 0.02205 \pm 0.00028$$

$$\Omega_c h^2 = 0.1199 \pm 0.0027$$

$$n_s = 0.9603 \pm 0.0073$$

$$\ln(10^{10} A_s) = 3.089 \pm 0.025$$

$$100\theta = 1.04131 \pm 0.00063$$

$$H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Age = 13.81± 0.05 billion years

Consistent with spatial flatness to % level

Inflation with Planck: a survey of some exotic models, Gomes, O.B., Rosa, PRD (2018)

Dark Energy

- **Evidence:**

  Dimming of type Ia Supernovae *z > 0.35* (about a thousand of them now)

  Accelerated expansion (negative deceleration parameter): $q_0 \equiv -\frac{\ddot{a}a}{\dot{a}^2} \leq -0.47$

  [Perlmutter et al. 1998; Riess et al. 1998, …]

- **Homogeneous and isotropic expanding geometry**

  Driven by the vacuum energy density Ω_Λ and matter density Ω_M

  Equation of state: $p = w\rho \qquad w \leq 1$

- **Friedmann and Raychaudhuri equations imply:** $q_0 = \frac{1}{2}(3w+1)\Omega_m - \Omega_\Lambda$

  $q_0 < 0$ suggests an invisible smooth energy distribution

- **Candidates:**

  Cosmological constant, quintessence (scalar field), more complex equations of state, etc.

Fifth Scalar Field Avatar: Quintessence

Varying vacuum energy models

[Bronstein 1933; O.B. 1986; Ratra, Peebles 1988; Wetterich 1988; …]

- $V=V_0 \exp(-\lambda\phi)$ [Ratra, Peebles 1988; Wetterich 1988; Ferreira, Joyce 1998]
- $V=V_0 \phi^{-\alpha}, \alpha > 0$ [Ratra, Peebles 1988]
- $V=V_0 \phi^{-\alpha} \exp(\lambda\phi^2), \alpha > 0$ [Brax, Martin 1999, 2000]
- $V=V_0 [\exp(M_p / \phi) - 1]$ [Zlatev, Wang, Steinhardt 1999]
- $V=V_0 (\cosh \lambda\phi - 1)^p$ [Sahni, Wang 2000]
- $V=V_0 \sinh^{-\alpha}(\lambda\phi)$ [Sahni, Starobinsky 2000; Urena-López, Matos 2000]
- $V=V_0 [\exp(\beta\phi) + \exp(\gamma\phi)]$ [Barreiro, Copeland, Nunes 2000]

- Scalar-Tensor Theories of Gravity

[Uzan 1999; Amendola 1999; O.B., Martins 2000; Fujii 2000; ...]

- $V=V_0 \exp(-\lambda\phi) [A + (\phi - B)^2]$ [Albrecht, Skordis 2000]
- $V=V_0 \exp(-\lambda\phi) [a + (\phi - \phi_0)^2 + b(\psi - \psi_0)^2 + c\,\phi(\psi - \psi_0)^2 + d\,\psi(\phi - \phi_0)^2]$

[Bento, O.B., Santos 2002]

Sixth Scalar Field Avatar:
the Generalized Chaplygin gas model

- **Unified model for Dark Energy and Dark Matter**

Generalized Born-Infeld Action

$$\mathcal{L} = -A^{\frac{1}{1+\alpha}} \left(1 - (g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu})^{\frac{\alpha+1}{2\alpha}}\right)^{\frac{\alpha}{\alpha+1}}$$

$\alpha = 1$: d-brane

→

Generalized Chaplygin gas

$$p = -\frac{A}{\rho^{\alpha}}$$

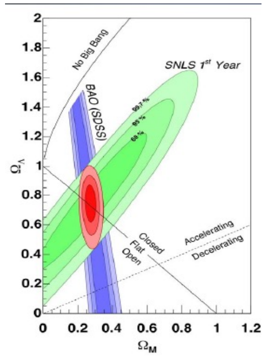
$\alpha = 1$: Chaplygin gas

↓

$p \ll \rho$	→	$p = \alpha\rho$	→	$p = -\rho$
Dust		$\alpha = 1$: stiff matter		De Sitter

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$$

[Kamenshchik, Moschella, Pasquier 2001]
[Bilíc, Tupper, Viollier 2002]
[Bento, O.B., Sen 2002]



Dark Energy - Dark Matter Unification: Generalized Chaplygin Gas Model

- **CMBR Constraints** [Bento, O. B., Sen 2003, 2004; Amendola et al. 2004, Barreiro, O.B., Torres 2008]
- **SNe Ia** [O. B., Sen, Sen, Silva 2004; Bento, O.B., Santos, Sen 2005]
- **Gravitational Lensing** [Silva, O. B. 2003]
- **Structure Formation** *

  [Sandvik, Tegmark, Zaldarriaga, Waga 2004; Bento, O. B., Sen 2004; Avelino et al. 2004; Bilic, Tupper, Viollier 2005; …]
- **Gamma-ray bursts** [O. B., Silva 2006, Barreiro, O.B., Torres 2010]
- **Cosmic topology** [Bento, O. B., Rebouças, Silva 2006]
- **Inflation** [O.B., Duvvuri 2006]
- **Coupling with electromagnetic coupling** [Bento, O.B., Torres 2007]
- **Coupling with neutrinos** [Bernardini, O.B. 2007, 2008, 2010]

Background tests: $\alpha \leq 0.35, \quad 0.8 \leq A_s \leq 0.9 \qquad A_s \equiv \frac{A}{\rho_{Ch0}^{1+\alpha}}$

Structure formation and BAO: $\alpha \leq 0.2$

Model 2

- **Real scalar field φ:**

$$L = \frac{1}{2}\partial_{\mu}\varphi\ \partial^{\mu}\varphi - V(\varphi)$$

$$V(\varphi) = V_0 A^{1/\alpha+1}[\cosh^{2/\alpha+1}\left[\frac{(\alpha+1)\varphi}{2}\right] + \cosh^{-2\alpha/\alpha+1}\left[\frac{(\alpha+1)\varphi}{2}\right]]$$

[O.B., Sen, Sen, Silva, MNRAS 2004]

Hamiltonian formulation

[Bernardini, O.B., Annals Physics 2013]

Dark Matter

- **Evidence:**

  Flatness of the rotation curve of galaxies
  Large scale structure
  Gravitational lensing
  N-body simulations and comparison with observations
  Merging galaxy cluster 1E 0657-56
  Massive Clusters Collision Cl 0024+17
  Dark core of the cluster A520

- **Cold Dark Matter (CDM) Model**

  Weakly interacting non-relativistic massive particle at decoupling

- **Candidates:**

  Neutralinos (SUSY WIMPS), axions, scalar fields, self-interacting scalar particles (adamastor particle), *etc.*

7th & 8th Scalar Field Avatars: Dark Energy – Dark Matter Interaction

[O.B., Carrilho, Páramos, Phys. Rev. D 86, 2012]

Two-scalar field model: $$\mathcal{L}_d = -\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) - V(\phi,\chi)$$

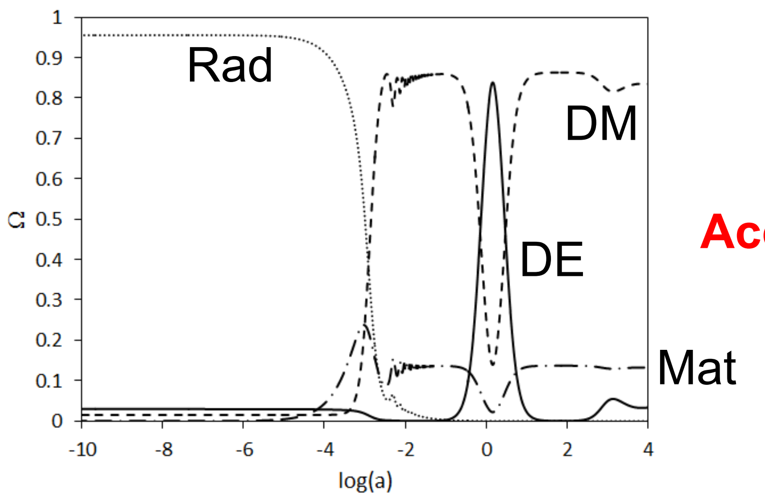
$$V(\phi,\chi) = e^{-\lambda\phi}\left(A + (\phi-\phi_0)^2\right) + e^{-\bar{\lambda}\phi}\tilde{P}(\phi)\chi^2 + \frac{1}{2}m^2\chi^2$$

ϕ - dark energy χ - dark matter $$\tilde{P}(\phi) = B + C\phi + D\phi^2$$

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = Q \ , \ \dot{\rho}_{dm} + 3H\rho_{dm} = -Q = \frac{1}{2}\dot{\phi}\frac{1}{M^2(\phi)}\frac{\partial M^2(\phi)}{\partial\phi}\rho_{dm}$$

$$M^2(\phi) = m^2 + 2\tilde{P}(\phi)e^{-\lambda\phi}$$

Results:



Accelerated phase can be transient!

Nineth Scalar Field Avatar: Chameleon Field

- **Coupling to matter depends on a scalar field ϕ**
  - **Action**

$$S=\int d^4x\sqrt{-g}\Big(\frac{1}{2}M_{\rm Pl}^2R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)-\frac{1}{4}e^{\beta_\gamma\phi/M_{\rm Pl}}F^{\mu\nu}F_{\mu\nu}+\mathscr{L}_{\rm m}(e^{2\beta_{\rm m}\phi/M_{\rm Pl}}g_{\mu\nu},\psi^i_{\rm m})\Big)$$

[Khoury, Weltman 2004]

  - ***$M_{Pl}=G^{-1/2}/8\pi=2.43 \times 10^{18}$ GeV, $\beta_i=O(1)$***
  - **$\mathcal{L}_m$ is the Lagrangian density of the matter fields**
  - **Potential:** $V(\phi)=M_\Lambda^4 e^{\kappa\left(\frac{\phi}{M_\Lambda}\right)^N}\approx M_\Lambda^4\left[1+\kappa\left(\frac{\phi}{M_\Lambda}\right)^N\right]$
  - **Effective potential:** $V_{\rm eff}(\phi,\vec{x})=V(\phi)+e^{\frac{\beta_{\rm m}\phi}{M_{\rm Pl}}}\rho_{\rm m}(\vec{x})+e^{\frac{\beta_\gamma\phi}{M_{\rm Pl}}}\rho_\gamma(\vec{x})$
  - **Can evade fifth force constraints through dependence on the energy density**
  - **Photon-coupled chameleons could be detected through laser experiments and radio frequency cavities …** [Rybka et al. 2010]
  - **Consistent with MICROSCOPE results?**

Tenth Scalar Field Avatar:
Scalar field dark matter -- Higgs: the Adamastor particle (I)

Motivation: "cuspy core" problem [Spergel, Steinhardt 2000]

Model:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g}{4!}\phi^4 + g' v \phi^2 h$$

[Bento, O.B., Rosenfeld, Teodoro 2000]

Higgs decay width

$$\Gamma(h \to \phi\phi) = 5.23 \left(\frac{m_h}{115 \text{ GeV}}\right)^{-1} g'^2 \text{ GeV}$$

[Bento, O.B., Rosenfeld 2001]

Unified model for dark energy – dark matter: $g' \Phi^2 H^2$

[O.B., Rosenfeld 2008]

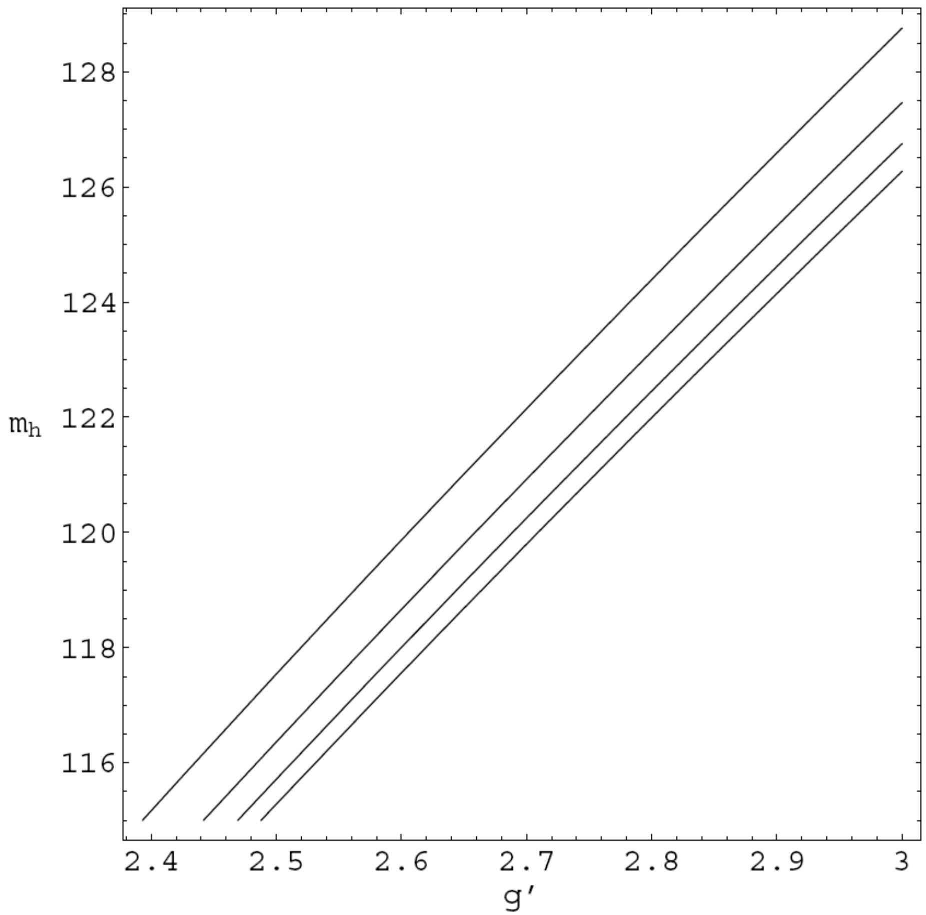


FIG. 2. Contour of $\Omega_\phi h^2 = 0.3$ as a function of m_h (in GeV) and g', for $m_\phi = 0.5$ GeV (top), 1.0, 1.5 and 2 GeV (bottom).

[Bento, O.B., Rosenfeld 2001]

Tenth Scalar Field Avatar:
Scalar field dark matter and the Higgs Field (II)

[O.B., Cosme, Rosa, Scalar field dark matter and the Higgs field, PL B759 (2016) 1- 8]

- **Our proposal:** Higgs-portal DM candidate - in particular, an **oscillating scalar field (SFDM) as a DM candidate**, coupled to the **Higgs:**

$$-\mathcal{L}_{int} = g^2\,|\Phi|^2|\mathcal{H}|^2$$

Coupling constant — SFDM — Higgs

- ϕ acquires **mass through the Higgs**, at the Electroweak phase transition;

- We have studied the dynamics and phenomenology of this DM candidate without self-interactions (**arxiv:1603.06242**) and taking into account the effect of self-interactions (**arxiv:1709.09674**).

[O.B., Cosme, Rosa, Scalar field dark matter and the Higgs field, PL B759 (2016) 1- 8]

- Oscillating scalar field, ϕ, as DM candidate: $\phi(t) \simeq \frac{\phi_i}{a(t)^{\frac{3}{2}}} \cos(m_\phi t + \delta_\phi)$
- ϕ acquires mass through the Higgs mechanism, after the electroweak phase transition (**EWPT**), at $T_{EW} \sim 100\ GeV$;
- Equation of motion: $\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\, \phi = 0$
- **Before** the **EWPT**: $\boldsymbol{H > m_\phi}$ ⇒ Overdamped regime. No oscillations.
- **After** the **EWPT**: $\boldsymbol{H < m_\phi}$ ⇒ Underdamped regime. The field oscillates.

Main results:

- In this model, we found a **lower bound** for the DM mass, that **takes into account** all the **cosmological constraints** for DM:

$$m_\phi \gtrsim 10^{-6} - 10^{-5}\ eV$$

$$|h|^4 \frac{\phi^2}{M^2}$$

- $m_\phi \sim \frac{v^2}{M_P} \sim 10^{-5}\ eV$ obtained through either non-renormalizable interactions between ϕ and the Higgs field or through a warped extra-dimension model.

Tenth Scalar Field Avatar:
Scalar field dark matter and the Higgs Field (III)

[Cosme, Rosa, O.B., Scalar filed dark matter with spontaneous symmetry-breaking and the 3.5 KeV line, PL B781 (2018) 649]

$$-\mathcal{L}_{int} = -g^2\,|\Phi|^2|\mathcal{H}|^2 + \boxed{\lambda_\phi|\Phi|^4} + V(\mathcal{H}) + \xi R|\Phi|^2$$

$\boldsymbol{g}$ - SFDM-Higgs coupling , $\boldsymbol{\lambda_\phi}$ - SFDM self-coupling, $\boldsymbol{\xi}$ – non-minimal coupling (NMC), $\boldsymbol{R}$ – Ricci Scalar; $\boldsymbol{\Phi} = \frac{\phi}{2}$

- **Before** the **EWPT**: **Quartic** term **dominates** ⇒ Field oscillates around the origin behaving like **dark radiation**;
- The **SFDM** acquires a **vacuum expectation value** (vev) at the EWPT;
- **After** the **EWPT**: **Quadratic** term **dominates** ⇒ Field oscillates around its vev behaving like **non-relativistic matter**.

Tenth Scalar Field Avatar:
Scalar field dark matter and the Higgs Field (III)

[Cosme, Rosa, O.B., Scalar filed dark matter with spontaneous symmetry-breaking and the 3.5 KeV line, PL B781 (2018) 649]

Main results:

Decay of a DM particle with $\boldsymbol{m \simeq 7\ \mathrm{keV}}$ and $\boldsymbol{\tau \sim (6-9) \times 10^{27}}$ **sec** can explain the line observed in the Galactic Center, Andromeda and Perseus.

- Our candidate: for $\boldsymbol{m_\phi \simeq 7\ \mathrm{keV}}$, $\boldsymbol{\tau_\phi \simeq 7 \times 10^{27}}$ **sec** $\Rightarrow$ **Meets** the requirements.

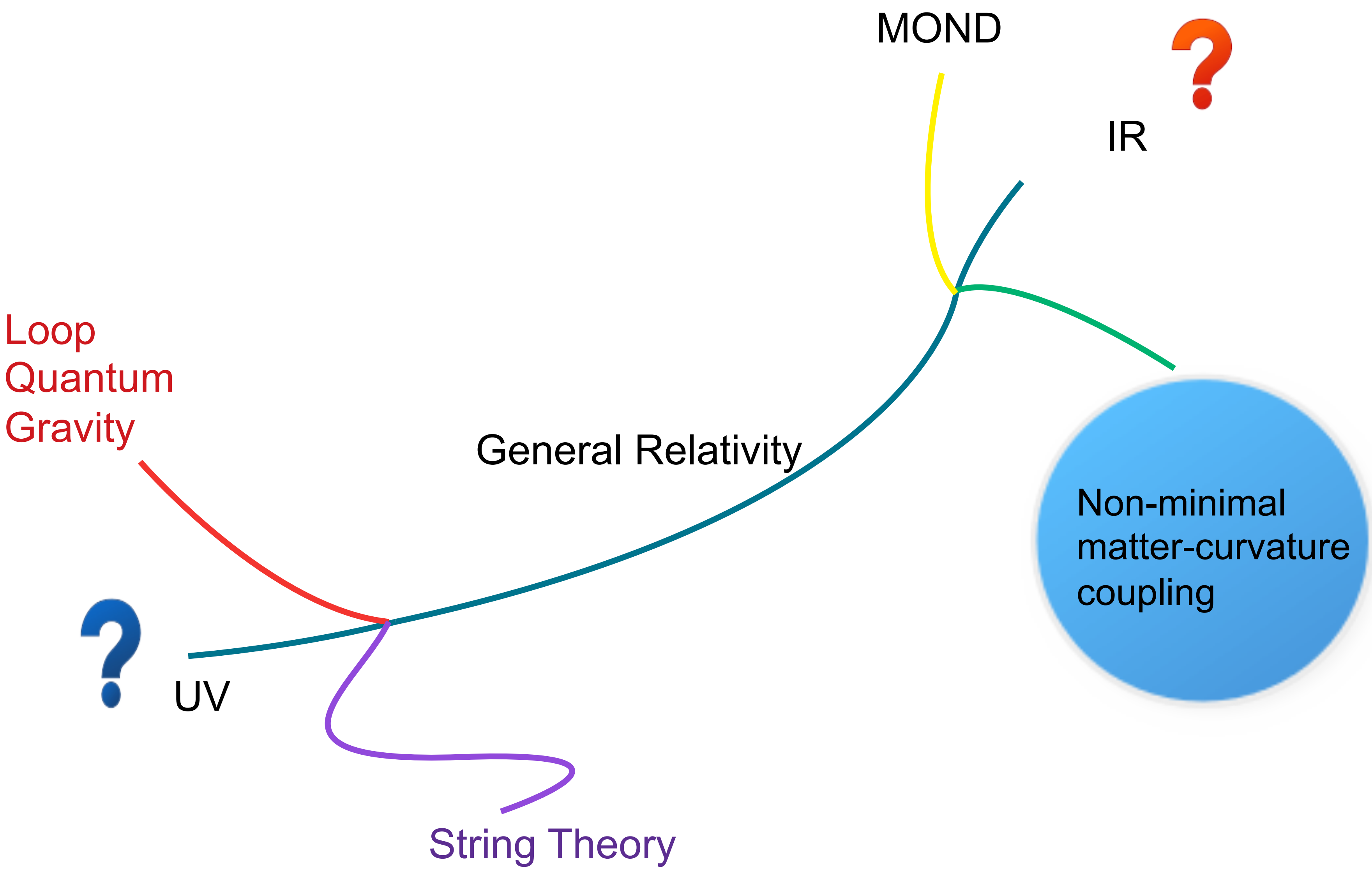
<u>Conclusion</u>: This **oscillating scalar field coupled to the Higgs boson** is a viable DM candidate that **can explain** the observed **3.5 keV X-ray line**, with **just one** free parameter: its **mass**.

- If mass is not acquired by the Higgs mechanism, the scalar field remains a good candidate for DM, but the dependence of parameters is quite involved and no striking observational feature is found

[Cosme, Rosa, O. B., Scale-invariant scalar filed dark matter through the Higgs Portal, JHEP 1805, 129 (2018)]

What if … General Relativity is not the theory?

[OB, 2011]



Alternative theory of gravity with non-minimal coupling between matter and curvature

↙

Dark Gravity!

[O.B., Böhmer, Harko, Lobo PRD 2007]

Action:

$$S = \int \left[\frac{1}{2} f_1(R) + f_2(R)\mathcal{L}_m\right] \sqrt{-g} d^4x \,,$$

Field equations:

$$(f_1' + 2\mathcal{L}_m f_2')R_{\mu\nu} - \frac{1}{2} f_1 g_{\mu\nu} - \Delta_{\mu\nu}(f_1' + 2\mathcal{L}_m f_2') = f_2 T_{\mu\nu}$$

$$\Delta_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \nabla_\beta$$

Effective energy-momentum tensor non-conservation:

$$\nabla^\mu T_{\mu\nu} = \frac{f_2'}{f_2} \left[g_{\mu\nu}\mathcal{L}_m - T_{\mu\nu}\right] \nabla^\mu R \,.$$

Eq. motion test particle:
(Perfect fluid)

$$u^\nu \nabla_\nu u^\lambda = \frac{1}{\epsilon + p} \left(\frac{f_2'}{f_2}(\mathcal{L}_m + p)\nabla_\nu R + \nabla_\nu p\right) h^{\nu\lambda} \equiv f^\lambda \,.$$

f(R) theory of gravity with non-minimal curvature-matter coupling (IV)

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

$$f_2(R) = 1 + \left(\frac{R}{R_0}\right)^n$$

- Stellar stability [O.B., Páramos, Phys. Rev. D 77 (2008)]
- On the non-trivial gravitational coupling to matter
  [O.B., Páramos, Class. Quant. Grav. 25 (2008)]
- Non-minimal coupling of perfect fluids to curvature
  [O.B., Lobo, Páramos, Phys. Rev. D 78 (2008)]
- Non-minimal curvature-matter couplings in modified gravity (Review)
  [O.B., Páramos, Harko, Lobo, arXiv:0811.2876 [gr-qc]]
- A New source for a braneworld cosmological constant from a modified gravity model in the bulk
  [O.B., Carvalho, Laia, Nucl. Phys. B 807 (2009)]
- Energy Conditions and Stability in f(R) theories of gravity with non-minimal coupling to matter
  [O.B., Sequeira, Phys. Rev. B 79 (2009)]
- Mimicking dark matter through a non-minimal gravitational coupling with matter
  [O.B., Páramos, JCAP 1003, 009 (2010)]
- Accelerated expansion from a non-minimal gravitational coupling to matter
  [O.B., Frazão, Páramos, Phys. Rev. D 81 (2010)]
- Reheating via a generalized non-minimal coupling of curvature to matter
  [O.B., Frazão, Páramos, Phys. Rev. D 83 (2011)]
- Mimicking the cosmological constant: Constant curvature spherical solutions in non-minimally coupled model [O.B., Páramos, Phys. Rev. D 84 (2011)]
- Mimicking dark matter in clusters through a non-minimal gravitational coupling with matter: the case of the Abell cluster A586 [O.B., Frazão, Páramos, Phys. Rev. D 86 (2012)]

- On the dynamics of perfect fluids in non-minimally coupled gravity [O.B., Martins, Phys. Rev. D 85 (2012)]
- Traversable Wormholes and Time Machines in non-minimally coupled curvature-matter f(R) theories [O.B., Ferreira, Phys. Rev. D 85 (2012)]
- Solar System constraints to nonminimally coupled gravity [O.B., March, Páramos, Phys. Rev. D 88 (2013)]
- Cosmological perturbations in theories with non-minimal coupling between curvature and matter [O.B., Frazão, Páramos, JCAP 1305 (2013)]
- Minimal extension of General Relativity: alternative gravity model with non-minimal coupling between matter and curvature (Review) [O.B., Páramos, Int. J. Geom. Meth. Mod. Phys. 11 (2014)]
- The Layzer-Irvine equation in theories with non-minimal coupling between and matter and curvature [O.B., Gomes, JCAP 1409 (2014)]
- Modified Friedmann Equation from Nonminimally Coupled Theories of Gravity [O.B., Páramos, Phys. Rev. D 89 (2014)]
- Black hole solutions of gravity theories with non-minimal coupling between matter and curvature [O.B., Cadoni, Porru, Class. Quant. Grav. 32 (2015)]
- Viability of nominimally coupled f(R) gravity [O.B., Páramos, Gen. Rel. Grav. 48 (2016)]
- 1/c expansion on nonminimally coupled curvature-matter gravity models and constraints from planetary preceaaion [March, Páramos, O.B., Dell'Agnello, Phys. Rev. D 95 (2017)]
- Inflation in non-minimal matter-curvature coupling theories [Gomes, Rosa, O.B., JCAP 1706 (2017)]
- Gravitational waves in theories with a non.minimal curvature-matter coupling [O.B., Gomes, Lobo, Eur. Phys. C 78 (2018)]
- Constraining a nonminimally coupled curvature-matter model with ocean experiments [March, O.B., Muccino, Baptista, Dell'Agnello, arXiv: 1904.12789]

Gravitational waves in theories with a non-minimal curvature-matter coupling

[O.B., Gomes, Lobo, Euro Phys. J. C78 (2018)]

Action:

$$S = \int \left[\frac{1}{2} f_1(R) + f_2(R)\mathcal{L}_m\right]\sqrt{-g}d^4x\ ,$$

Linearised field eqs. around a Minkowsky background:

$$(F_1 + 2F_2\mathcal{L}_m)\,\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}F_1\delta R - \frac{1}{2}h_{\mu\nu}f_1$$
$$-\left[\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\right](\delta f' + \delta h') = f_2\delta T_{\mu\nu} + F_2 T_{\mu\nu}\delta R$$

$$\delta f \equiv (F_1 - 2F_2\mathcal{L}_m + F_2 T)\,\delta R\ ,$$
$$\delta f' \equiv (F_1' + 2F_2'\mathcal{L}_m)\,\delta R\ ,$$
$$\delta h \equiv f_2\delta T\ ,$$
$$\delta h' \equiv 2F_2\delta\mathcal{L}_m\ .$$

$$3\Box(\delta f' + \delta h') = -R_0(\delta f' + \delta h') + \delta f + \delta h$$

R_0 = 0 for Minskowski

Cosmological constant as a source:

$$\Box\left(h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h\right) = \frac{f_1 - 2f_2\Lambda}{F_1 - 2F_2\Lambda}h_{\mu\nu}$$

Scalar mode absorbed in this gauge choice

Dispersion relation:

$$h_{\mu\nu} = A^{+}e^{ik_\alpha x^\alpha}e^{+}_{\mu\nu} + A^{\times}e^{ik_\alpha x^\alpha}e^{\times}_{\mu\nu}$$

$$k_\alpha k^\alpha \equiv \omega^2 - k^2 = \frac{f_1 - 2f_2\Lambda}{F_1 - 2F_2\Lambda}$$

Gravitational waves in theories with a non-minimal curvature-matter coupling

[O.B., Gomes, Lobo, Euro Phys. J. C78 (2018)]

Propagating scalar logitudinal mode: $\Omega \equiv \frac{\delta f'}{F_1 - 2F_2\Lambda} = \frac{F_1' - 2F_2'\Lambda}{F_1 - 2F_2\Lambda}\delta R$

Newman-Penrose formalism (full theory):

$$\Psi_0 \equiv C_{kmkm} = R_{kmkm}$$
$$\Psi_1 \equiv C_{klkm} = R_{klkm} - \frac{R_{km}}{2}$$
$$\Psi_2 \equiv C_{km\bar{m}l} = R_{km\bar{m}l} + \frac{R}{12}$$
$$\Psi_3 \equiv C_{kl\bar{m}l} = R_{kl\bar{m}l} + \frac{R_{l\bar{m}}}{2}$$
$$\Psi_4 \equiv C_{l\bar{m}l\bar{m}} = R_{l\bar{m}l\bar{m}}$$
$$\Phi_{00} \equiv \frac{R_{kk}}{2}$$
$$\Phi_{11} \equiv \frac{R_{kl} + R_{m\bar{m}}}{4}$$
$$\Phi_{22} \equiv \frac{R_{ll}}{2}$$
$$\Phi_{01} \equiv \frac{R_{km}}{2} = \Phi_{10}^* \equiv \left(\frac{R_{k\bar{m}}}{2}\right)^*$$
$$\Phi_{02} \equiv \frac{R_{mm}}{2} = \Phi_{20}^* \equiv \left(\frac{R_{\bar{m}\bar{m}}}{2}\right)^*$$
$$\Phi_{12} \equiv \frac{R_{lm}}{2} = \Phi_{21}^* \equiv \left(\frac{R_{l\bar{m}}}{2}\right)^*$$
$$\tilde{\Lambda} \equiv \frac{R}{24} ,$$

NP quantities built from the decomposition of the Weyl tensor in terms of irreducible parts: Riemann tensor, Ricci tensor and scalar curvature.
In GR, only Ψ_4 is non-vanishing → polarisations + and x

$$\begin{aligned} C_{\kappa\lambda\mu\nu} &= R_{\kappa\lambda\mu\nu} - g_{\kappa[\mu}R_{\nu]\lambda} + g_{\lambda[\mu}R_{\nu]\kappa} - \frac{1}{3}g_{\kappa[\nu}g_{\mu]\lambda}R \\ &= R_{\kappa\lambda\mu\nu} + \frac{1}{2}\left(g_{\kappa\nu}R_{\mu\lambda} - g_{\kappa\mu}R_{\nu\lambda} + g_{\lambda\mu}R_{\nu\kappa} - g_{\lambda\nu}R_{\mu\kappa}\right) \\ &\quad + \frac{1}{6}\left(g_{\kappa\mu}g_{\nu\lambda} - g_{\kappa\nu}g_{\mu\lambda}\right)R \end{aligned}$$

In NMC with a C.C. other scalar, vector and tensor modes are also possible (Φ_{00},Φ_{11},Φ_{00},R are nonzero), but a complete characterisation can only be carried out once the full solution is known (required for the Ψ_i).

Dark Energy fluid as a source:

$$\Box\left(h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h\right) = \frac{f_1 - 2f_2\rho}{F_1 - 2F_2\rho}h_{\mu\nu}$$

Scalar modes:

$$\omega_f \equiv \frac{\delta f'}{F_1 - 2F_2\rho} = \frac{F_1' - 2F_2'\rho}{F_1 - 2F_2\rho}\delta R$$

$$\omega_h \equiv \frac{\delta h'}{F_1 - 2F_2\rho} = \frac{-2F_2}{F_1 - 2F_2\rho}\delta\rho$$

A further scalar mode is found due to fluctuactions of the matter Lagrangian (at linear level!)

Extra modes may be detected in future measurements with more precise data is available

It might allow to distinguish between GR and other models!

Some “Vector” Thoughts …

Weyl Gravity

[H. Weyl (1918)]
[P- Dirac (1973)]

Weyl Gravity was proposed as an attempt to unify GR with Electromagnetism:

$$D_\lambda g_{\mu\nu} = A_\lambda g_{\mu\nu} \ , \qquad \nabla_\mu g_{\mu\nu} = 0 \quad in\ GR$$

Where the covariant derivative is built from the generalised connection:

$$\bar{\Gamma}^\rho_{\mu\nu} := \left\{{}^{\rho}_{\mu\nu}\right\} - \tfrac{1}{2}\delta^\rho_\mu A_\nu - \tfrac{1}{2}\delta^\rho_\nu A_\mu + \tfrac{1}{2} g_{\mu\nu} A^\rho$$

The Ricci tensor reads:

$$\bar{R}_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} A_\mu A_\nu + \frac{1}{2} g_{\mu\nu} \left(\nabla_\lambda - A_\lambda\right) A^\lambda + F_{\mu\nu} + \nabla_{(\mu} A_{\nu)}$$

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

And its trace:

$$\bar{R} = R + 3\nabla_\lambda A^\lambda - \frac{3}{2} A_\lambda A^\lambda := R + \bar{\bar{R}}$$

Nonminimally coupled Weyl Gravity

[C. Gomes, O.B., 1812.04976 [gr-qc]]

We start from the action:

$$S = \int \left(\kappa f_1(\bar{R}) + f_2(\bar{R})\mathcal{L}\right) \sqrt{-g}\, d^4x$$

Variation with respect to the vector field: $\nabla_\lambda \bar{\Theta} = -A_\lambda \bar{\Theta}$

where $\bar{\Theta} := F_1(\bar{R}) + (F_2(\bar{R})/\kappa)\mathcal{L}$ and $F_i := df_i/d\bar{R}$.

The metric field equations:

$$\left[R_{\mu\nu} + \bar{\bar{R}}_{(\mu\nu)}\right] \bar{\Theta} - \frac{1}{2} g_{\mu\nu} f_1 = \frac{f_2}{2\kappa} T_{\mu\nu}$$

The order of the PDEs is lowered, thus avoiding some well known instabilities!

Does a cosmological constant arise from the model?

[Einstein (1919)9]
[Kaloper, Padilla (2014)]
[O.B., Páramos (2017)]
[Gomes, O.B.,1812.04976]

From the contracted Bianchi identities:

$$\frac{1}{4}\nabla^\nu R = \nabla_\mu\left(R^{\mu\nu} - \frac{1}{4}g^{\mu\nu}R\right)$$

$$= \frac{1}{4}\nabla^\nu R + \nabla_\mu\left(\frac{1}{\bar{\Theta}}\right)\left[-(R^{\mu\nu}+B^{\mu\nu})\bar{\Theta} - \frac{1}{2}g^{\mu\nu}f_1 + \frac{f_2}{2}T^{\mu\nu} + \Box^{\mu\nu}\bar{\Theta}\right] +$$

$$+\frac{1}{4}\nabla^\nu\left[R + B - 2f_1 - \frac{f_2}{2\bar{\Theta}} + \frac{3}{\bar{\Theta}}\Box\bar{\Theta}\right]$$

$$\Longleftrightarrow \quad \nabla^\nu\left[R + B - 2f_1 - \frac{f_2}{2\bar{\Theta}} + \frac{3}{\bar{\Theta}}\Box\bar{\Theta}\right] = 0$$

where $B^{\mu\nu} := \frac{3}{2}A^\mu A^\nu + \frac{3}{2}g^{\mu\nu}\left(\nabla_\lambda - A_\lambda\right)A^\lambda$

Conserved quantity: trace of the field equations

Hence, no integration constant arises from the model

From the generalised contracted Bianchi identities:

$$\frac{1}{4}\mathcal{D}_\mu\left(g^{\mu\nu}\bar{R}\right) = \mathcal{D}_\mu\left[\bar{R}^{(\mu\nu)} - \frac{1}{4}g^{\mu\nu}\bar{R}\right] = \mathcal{D}_\mu\left[\frac{f_2}{2\bar{\Theta}}\left(T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T\right)\right] =$$

$$= \mathcal{D}_\mu\left(\frac{f_2}{2\bar{\Theta}}\right)T^{\mu\nu} + \left[\mathcal{D}_\mu\bar{R}^{(\mu\nu)} - \frac{1}{2}\mathcal{D}_\mu\left(g^{\mu\nu}\frac{f_1}{\bar{\Theta}}\right) - \mathcal{D}_\mu\left(\frac{f_2}{2\bar{\Theta}}\right)T^{\mu\nu}\right] - \frac{1}{4}\mathcal{D}_\mu\left(g^{\mu\nu}\frac{f_2}{2\bar{\Theta}}T\right)$$

$$\Longleftrightarrow \quad \mathcal{D}_\mu\left(\bar{R}^{(\mu\nu)} - \frac{1}{4}g^{\mu\nu}\bar{R}\right) + \frac{1}{4}\mathcal{D}_\mu\left[g^{\mu\nu}\left(-\frac{f_2}{2\bar{\Theta}}T - 2\frac{f_1}{\bar{\Theta}}\right)\right] = 0$$

$$\Longleftrightarrow \quad \frac{1}{4}\mathcal{D}_\mu\left[g^{\mu\nu}\left(\bar{R} - \frac{f_2}{2\bar{\Theta}}T - 2\frac{f_1}{\bar{\Theta}}\right)\right] = 0\ ,$$

where $\mathcal{D}_\mu := (D_\mu + 2A_\mu)$

Space Form Behaviour

A pseudo-Riemannian manifold admit a space form behaviour iff:

$$\bar{R}_{abcd} = K(g_{ac}g_{db} - g_{ad}g_{cb}) \implies \bar{R}_{bd} = 3Kg_{db} \implies \bar{R} = 12K$$

Combining the field equations with their trace for the vacuum, Λ_0 :

$$\frac{2f_1 - \bar{R}F_1}{2f_2 - \bar{R}F_2} = 2\Lambda_0$$

In general, we could have K=K(t), which cannot be identified with the constant matter vacuum, Λ_0 .

In order to proceed one needs to find a form for the vector field.

The Weyl vector field

A natural ansatz for the vector field: **[Bento, OB., Moniz, Mourão, Sá (1993)]**

$$A_0 = \xi(t) \ , \ A_i = \chi(t)\delta_i^a L_a$$

which admits invariance under spatial rotations, SO(3) transformations, with generators L_a , and is consistent with homogeneity and isotropy

We further assume a constant scalar curvature: $R = K_0 := \Lambda$

Two main cases can be studied:

$$\xi = \xi_0 = const. \longrightarrow 12K(t) = \left(12\Lambda + \frac{3}{2}\xi_0\right) - 6\xi_0\left(\chi_0 e^{-2\xi_0(t-t_0)}\right) - \frac{9}{2}\left(\chi_0 e^{-2\xi_0(t-t_0)}\right)^2$$

$$\xi = \chi \longrightarrow 12K(t) = 12\Lambda + 3\partial^0\left(\frac{\chi_0}{1+\chi_0(t-t_0)}\right) - 3\left(\frac{\chi_0}{1+\chi_0(t-t_0)}\right)^2 = 12\Lambda$$

Variation: Dynamic Vector Field

[R. Baptista, O.B., to appear]

$$S[\Psi_r, A_\mu, g^{\mu\nu}] = \frac{1}{c}\int_M \left(\frac{c^4}{16\pi G} f_1(\bar{R}) + (1 + f_2(\bar{R}))\mathcal{L}[\Psi_r, A_\mu, g^{\mu\nu}]\right)\sqrt{|g|}\,\mathrm{d}^4 r$$

$$\mathcal{L}[\Psi_r, A_\mu, g^{\mu\nu}] = \mathcal{L}_m(\Psi_r, \nabla_\mu \Psi_r, g^{\mu\nu}) + \mathcal{L}^{(V)}_{\mathrm{YM}}[A_\mu, g^{\mu\nu}]$$

$$\mathcal{L}^{(V)}_{\mathrm{YM}}[A_\mu, g^{\mu\nu}] = -\frac{1}{4\mu_A}\mathrm{tr}\left\{F_{\mu\nu}F^{\mu\nu}\right\} - V[A]$$

- SO(3) gauge field in a Robertson-Walker spacetime
- …

Another Variation

[O.B., Bessa, Páramos, PRD (2016)]

- Can inflation by driven by vector fields?
- Yes … , but needs a dominating Λ [Ford, PRD (1989)]
- No! SO(3) Gauge Field

[Bento, O.B., Moniz, Mourão, Sá, CQG (1993)]

- Yes! But ... only if nonmimimally coupled to gravity!

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2k^2} + \mathcal{L} \right)$$

$$\mathcal{L} = \frac{1}{8e^2} Tr[F_{\mu\nu} F^{\mu\nu}] + \frac{1}{2} m^2 Tr[A_\mu A^\mu] + \frac{1}{3} \alpha R A_\mu A^\mu + \beta R_{\mu\nu} A^\mu A^\nu$$

Inflationary fixed points

↓

$\alpha \neq \beta$

Conclusions

- The discovery of the Higgs field provides yet another proof that gauge symmetries are realized in Nature and that the vacua do not fully share these symmetries

- Moreover, it confirms, up to the tested energy scales, the existence of at least one fundamental scalar field

- Should one generalize the gauge principle and consider the so-called Grand Unified Theories (GUTs)? Should one consider them in the context supergravity (gauge generalization of supersymmetry), string theory, etc?

- Does gravity have a scalar component?

- Is the inflaton a scalar field in the context of more fundamental theories? It cannot be the minimally coupled Higgs field of GUTs. It can be a chiral superfield of some supergravity models, even though not the moduli fields of string theory ... Some scalar field models with plateau-like potential models are degenerate with the Starobinsky model

- Is there an underlying scalar field associated to dark matter? What about dark energy? What about a putative interaction between them?

- ...