Effects of the QCD Equation of State and Lepton Asymmetry on Primordial Gravitational Waves

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Based on:

F. H., J. Schaffner-Bielich, S. Wystub, M. M. Wygas,
arXiv:1904.01046 [hep-ph] (PRD)
and
N. Bernal, F. H., arXiv:1905.XXXXX

CRC-TR 211
Strong-interaction matter
under extreme conditions

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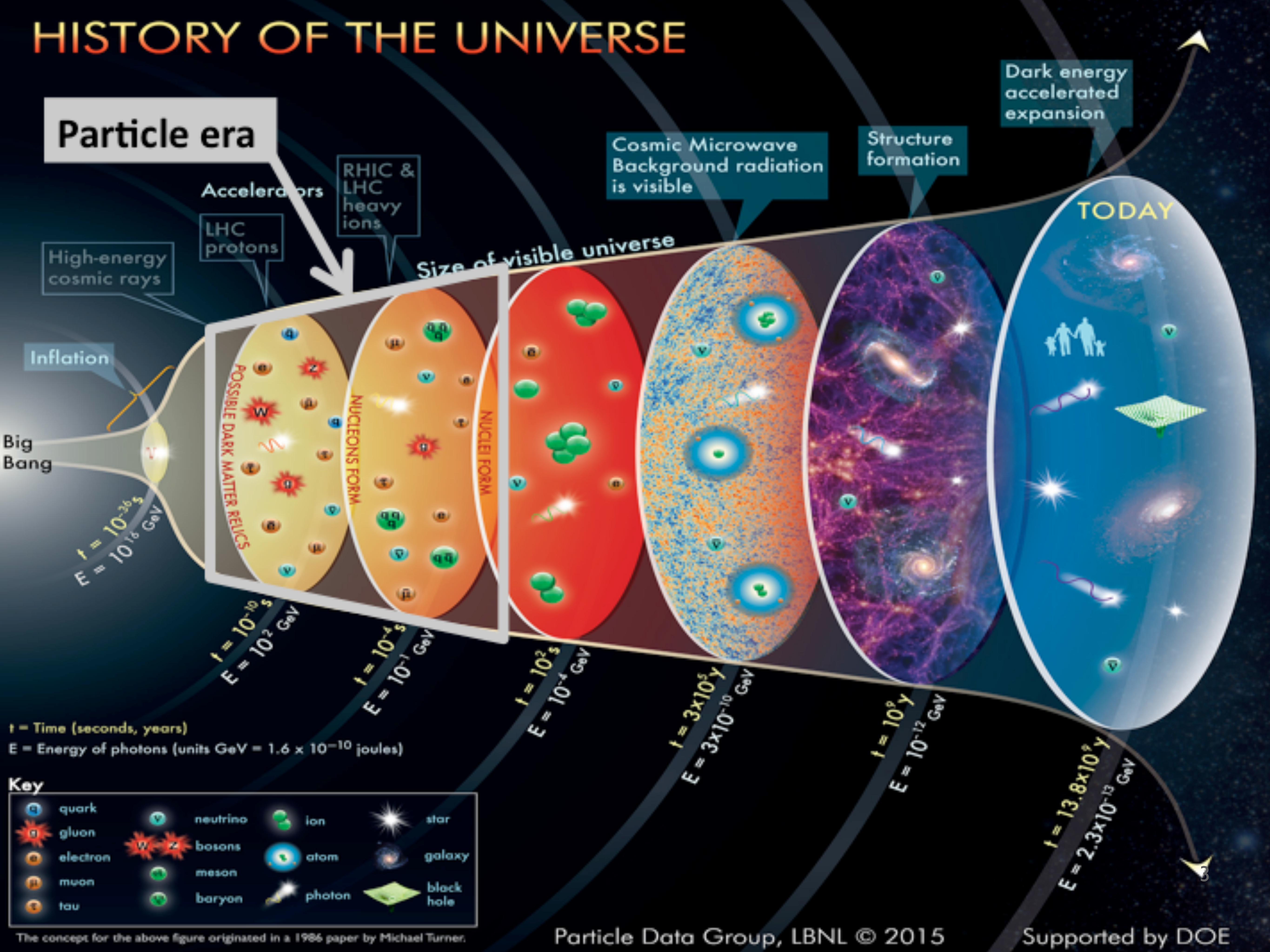
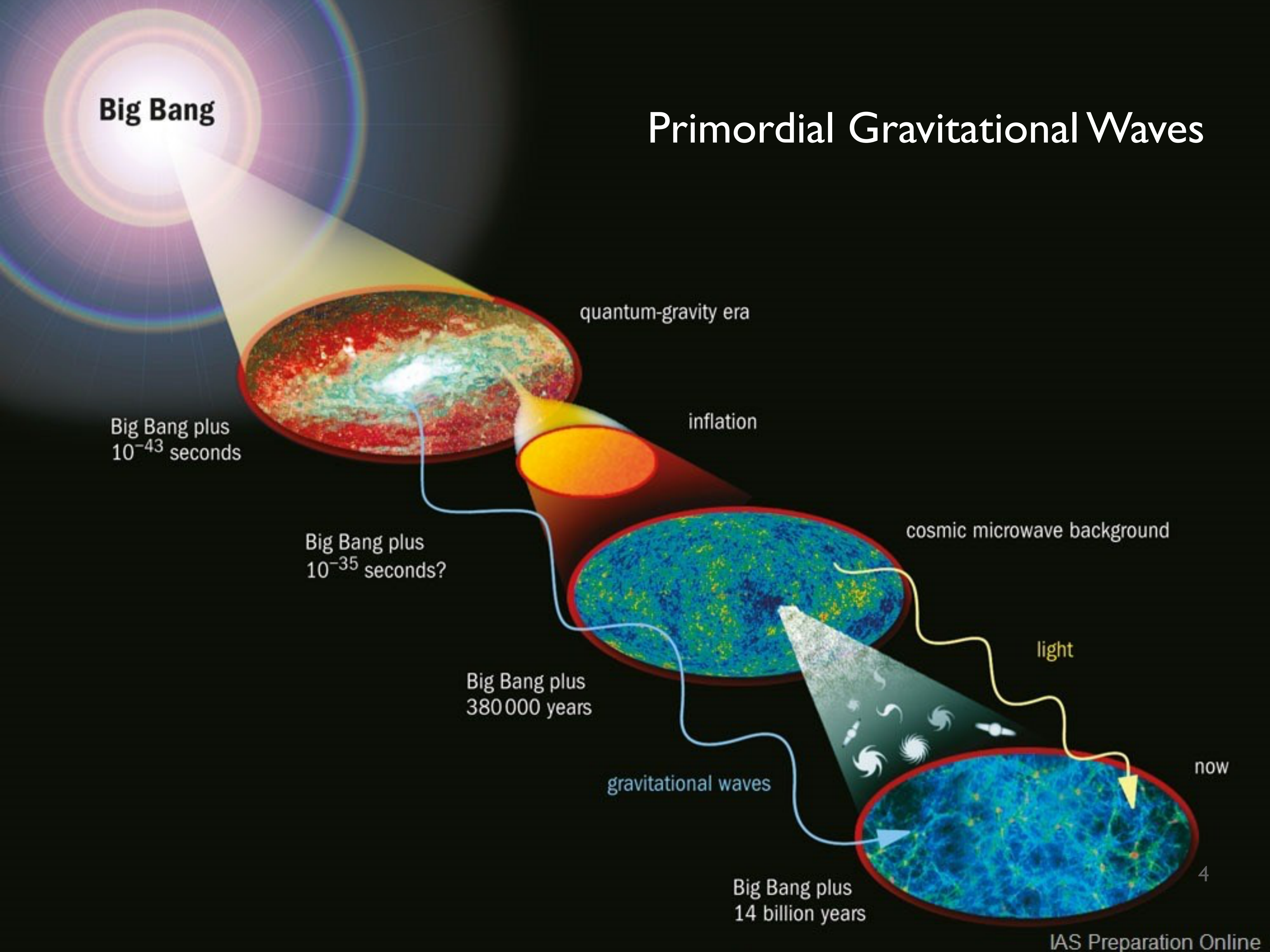
Overview (1st)

- Primordial Gravitational Waves Spectrum
- Thermodynamics of the Standard Model
- Effects of the QCD Equation of State on the PGW
- Non-vanishing Lepton Chemical Potentials and the SM EoS
- Impact of Lepton Asymmetry on the PGW


HISTORY OF THE UNIVERSE
Particle era
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

Primordial Gravitational Waves



Cosmological (Phase) Transitions and the Standard Model

- QCD transition is a smooth crossover transition.
- EW transition in the SM with the measured Higgs mass is also crossover.
- Cosmological transitions only assuming the SM are not first order phase transitions and do not produce any GW.
- They can leave an imprint on the PGW produced after the Big Bang from the inflationary scenario.

Laine & Meyer, 2015 Borsanyi et al. 2016 Mazumdar & White 2018

Equations of Tensor Perturbation and Gravitational Waves Relic Density

Evolution equation for gravitational wave amplitude “h”:

$$h^{''}(k,\eta) + 2\mathcal{H}(\eta)h^{'}(k,\eta) + k^2 h(k,\eta) = 0\,, \quad \mathcal{H} = a'/a = aH$$

Wave number

Conformal time

Hubble rate:

$$H^2 = \frac{8\pi G}{3}\rho_{\rm tot}$$

Tensor perturbation polarisation modes:

Transfer function

$$h_\lambda(k,\eta) = h_\lambda^{\rm prim}(k)Y(\eta,k) = \frac{v(k,\eta)}{a(\eta)}$$

Tensor perturbation in the comoving frame:

$$v^{''}(k,\eta) + \left(k^2 - \frac{a^{''}}{a}\right)v(k,\eta) = 0$$

Starobinsky 1979 Guth 1980 Linde 1981 Maggiore 1999

In standard cosmology:

Assuming no phase transition or modified cosmology $\longrightarrow$ Entropy conservation

Friedmann equation:

$$\frac{a''}{a} = \frac{4\pi G}{3} a^2 (\rho_{tot} - 3p_{tot})$$

Trace anomaly:

$$\frac{I(T)}{T^4} = \frac{\rho_{\rm tot} - 3p_{\rm tot}}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p_{\rm tot}}{T^4}\right)_{\mu/T}$$

Horizon scale:

$$k = a(\eta_{\rm hc})H(\eta_{\rm hc})$$

We do not consider the effect of neutrinos and photons damping on the PGW, since we are interested in temperatures above 10 MeV. So there is no source term for GW.

PGW energy density:

$$\rho_{\rm GW}(\eta) = \frac{M_{\rm Pl}^2}{32\pi a(\eta)^2} \left\langle h'_{ij}(k,\mathbf{x}) h^{ij\prime}(k,\mathbf{x}) \right\rangle$$

$$\left\langle h'_{ij}(k,\mathbf{x}) h^{ij\prime}(k,\mathbf{x}) \right\rangle = \int \frac{dk}{k} \mathcal{P}_T(k,\eta)$$

Tensor power spectrum:

$$\mathcal{P}_T(k,\eta) = \frac{k^3}{\pi^2} \sum_\lambda \left\langle |h_\lambda(k,\eta)|^2 \right\rangle = \mathcal{P}_T^{\rm prim}(k) [Y(k,\eta)]^2$$

PGW relic density:

$$\Omega_{\rm GW}(k,\eta) = \frac{\mathcal{P}_T^{\rm prim}(k)}{12 a(\eta)^2 H(\eta)^2} [Y'(k,\eta)]^2$$

At horizon crossing:

$$[Y'(k,\eta)]^2 = k^2[Y(k,\eta)]^2$$

PGW relic density:

$$\Omega_{\rm GW}(k,\eta_0) \propto \Omega_{\rm GW}(k,\eta_{\rm hc}) \propto k^5|v(k,\eta_{\rm hc})|^2$$

It gives roughly a flat spectrum

PGW relic density and the SM equation of state:

$$\Omega_{\rm GW}(k,\eta_0) \propto \rho_{\rm tot}(T_{hc})s_{\rm tot}(T_{\rm hc})^{-4/3}$$

QCD affects the GW background in the frequency range of pulsar timing arrays, e.g. IPTA, SKA, etc.

Schwarz 1997 Watanabe & Komatsu, 2006 Saikawa & Shirai, 2018

Thermodynamics of the Standard Model

Noninteracting part in ideal gas limit

Energy density: $$\rho_{\rm tot}(T,\mu) = \sum_i \frac{g_i}{2\pi^2}\int_{m_i}^{\infty} dE \times E^2\sqrt{E^2 - m_i^2}\left(\frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1}\right)$$

Pressure density: $$p_{\rm tot}(T,\mu) = \sum_i \frac{g_i}{6\pi^2}\int_{m_i}^{\infty} dE \times \left(E^2 - m_i^2\right)^{3/2}\left(\frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1}\right)$$

Entropy density: $$T s_{\rm tot}(T,\mu) = \rho_{\rm tot}(T,\mu) + p_{\rm tot}(T,\mu) - \sum_i \mu_i n_i(T,\mu_i)$$

Number density of each particle:

$$n_i(T,\mu_i) = \frac{g_i}{2\pi^2}\int_{m_i}^{\infty} dE \times E\sqrt{E^2 - m_i^2}\left(\frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1} - \frac{1}{e^{\frac{E+\mu_i}{T}} \pm 1}\right)$$

Interacting part from lattice QCD

$$\chi_{ab} = \frac{\partial^2 p^{QCD}(T,\mu)}{\partial\mu_a \partial\mu_b} = \chi_{ba} \qquad p^{QCD}(T,\mu) = p^{QCD}(T,0) + \frac{1}{2}\mu_a \chi_{ab}(T)\mu_b$$

Susceptibility

Conserved charges: Q, B

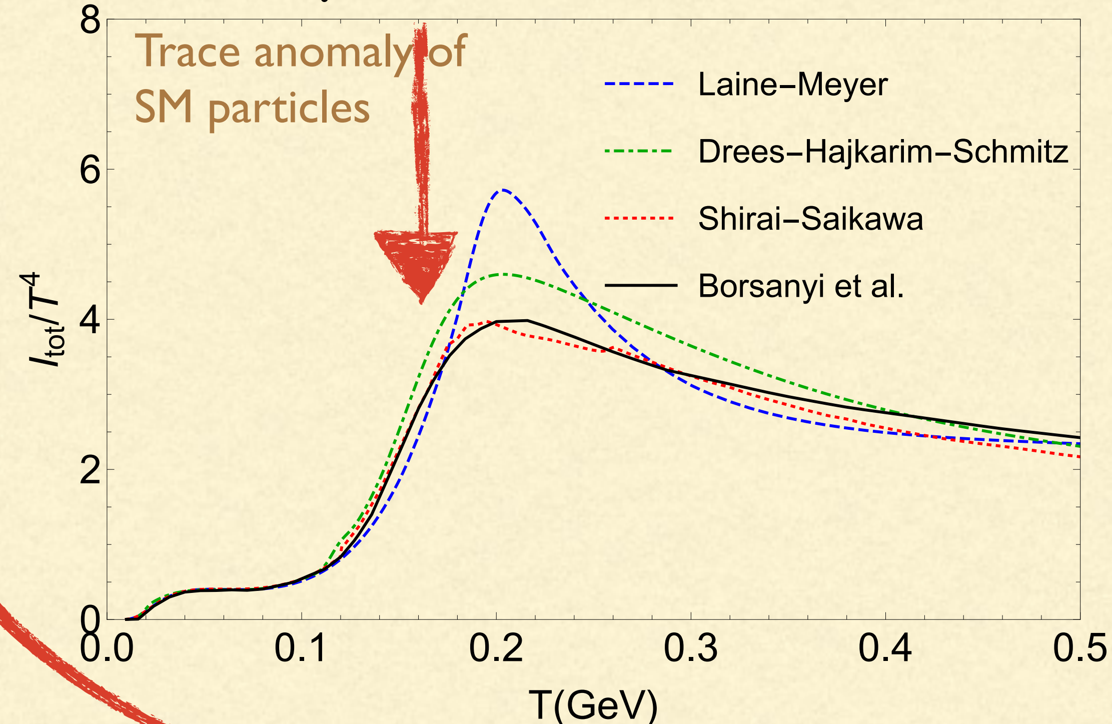
Schwarz & Stuke 2009 Wygas et al. 2018 Laine & Meyer, 2015 Borsanyi et al. 2016

Effects of the QCD Equation of State on the Primordial Gravitational Waves

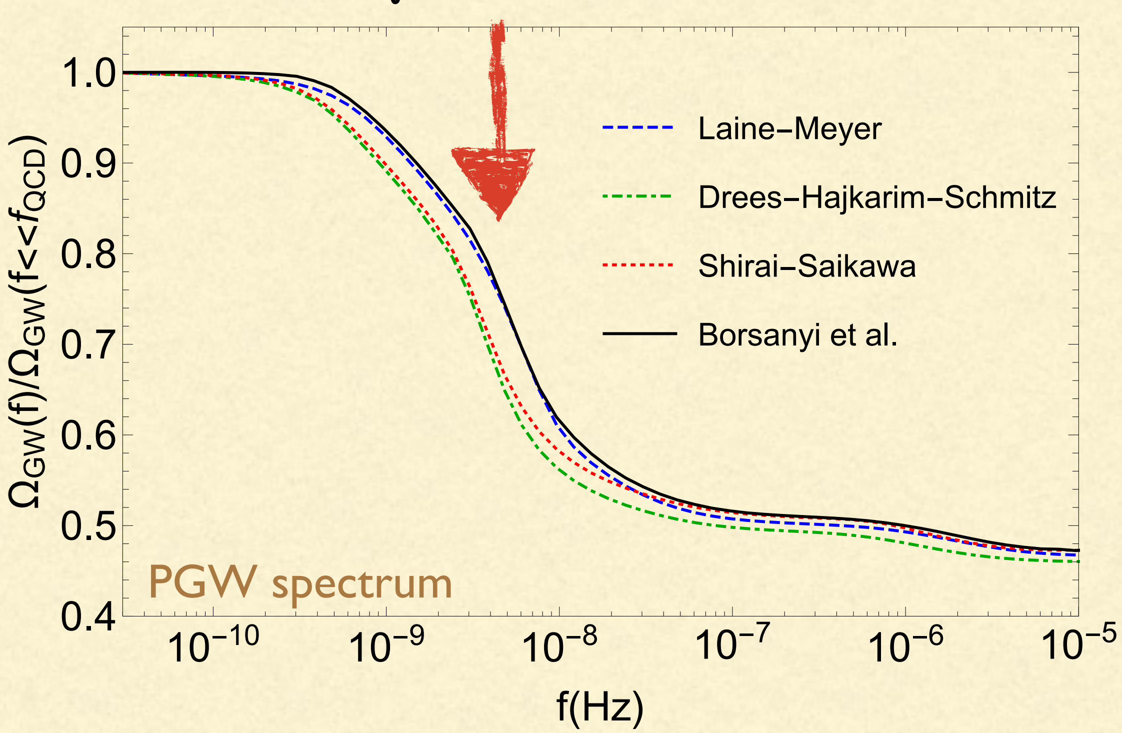
Deviation in the PGW relic due to different QCD equations of state is up to 3.5 %.

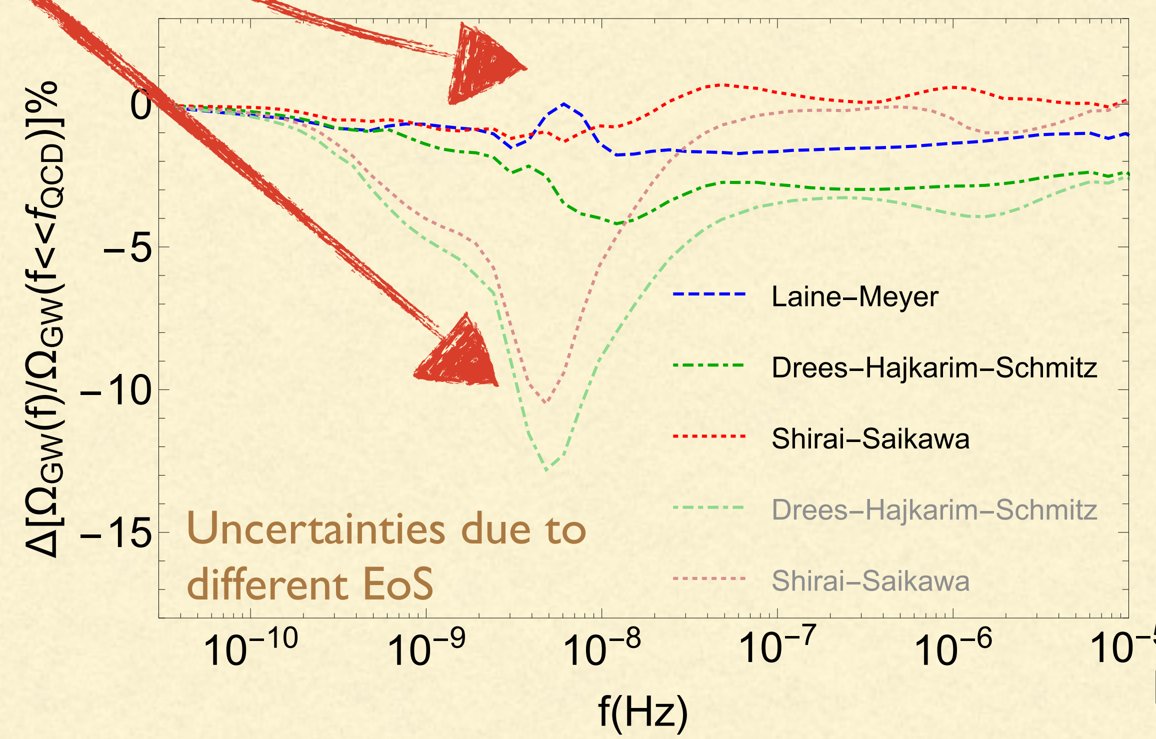
Neutrino decoupling and electron annihilation lead to 10% effects in the predicted relic density of the PGW (frequency shift).

$T_{\rm QCD} \sim 150\ MeV$



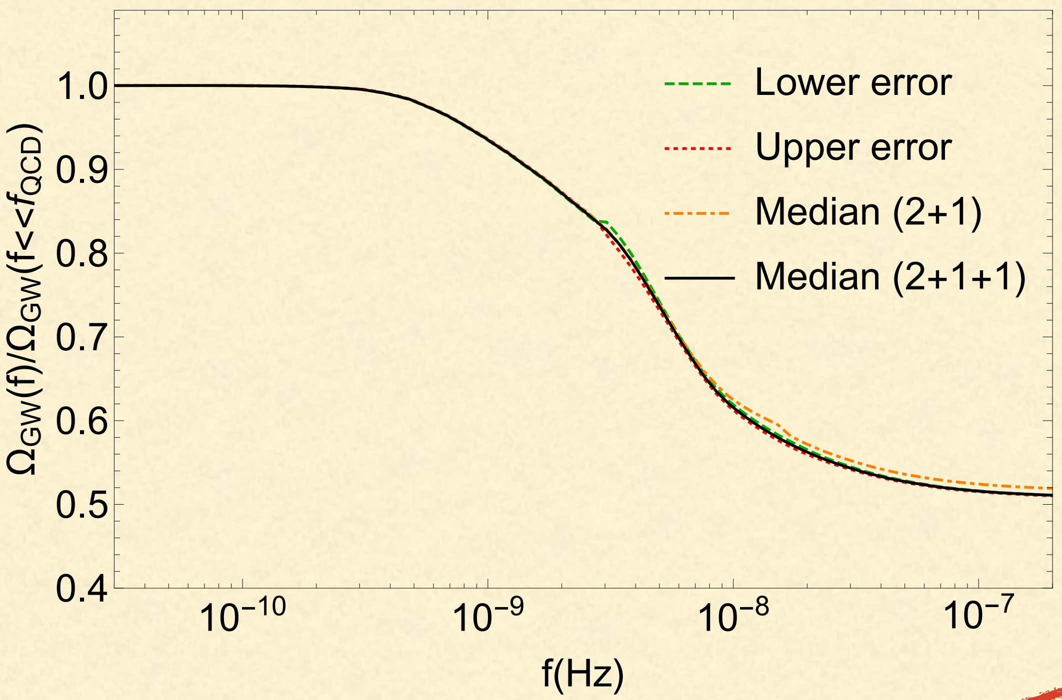
$f_{\rm QCD} \approx 3 \times 10^{-9}\ Hz$

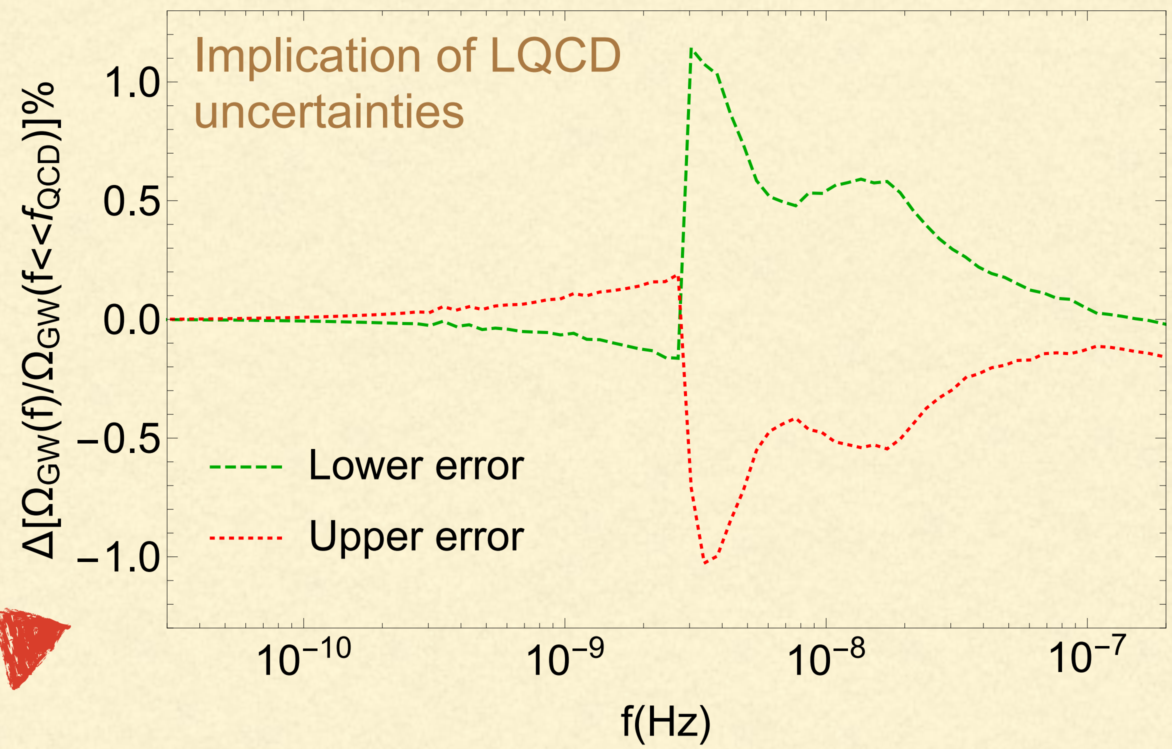




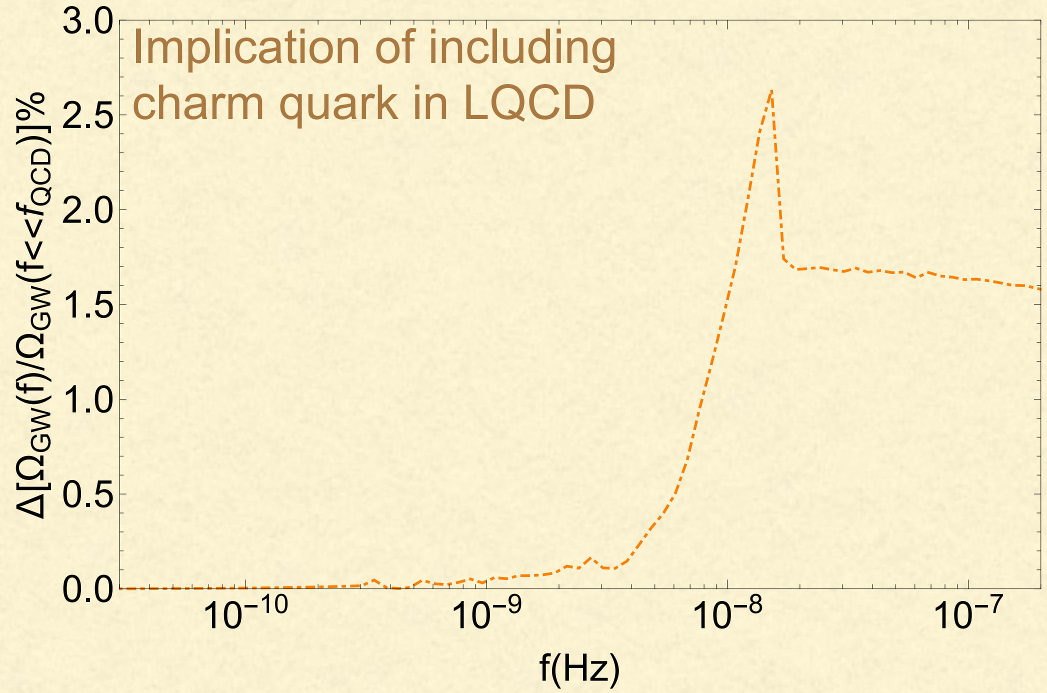
Laine & Meyer, 2015 Drees, Hajkarim, Schmitz, 2015 Saikawa & Shirai, 2018 Borsanyi et al. 2016

Charm Quark and Lattice Uncertainties Imprints on the PGW





Effect of lattice QCD error bars on the PGW energy spectrum is up to ~1%.
Effect of including lattice QCD data with charm will be up to ~2.5%.



Vanishing and Nonvanishing Lepton Asymmetry & PGW

Assuming nonvanishing lepton asymmetry leads to nonzero lepton chemical potentials in cosmology. Using lattice QCD equation of state for nonvanishing chemical potentials the evolution of SM chemical potentials are possible.

Constraint equations for finding the evolution of SM chemical potentials:

I) Constraint on baryon asymmetry:

$$b = \frac{n_B}{s} \approx 8 \times 10^{-11}$$

$$bs(T, \mu) = \sum_i b_i n_i(T, \mu_i)$$

II) Constraint on lepton asymmetry:

$$l = \frac{n_L}{s} \lesssim 0.012$$

$$l_f s(T, \mu) = n_f(T, \mu_f) + n_{\nu_f}(T, \mu_{\nu_f}) \, , f = e, \mu, \tau$$

III) Charge neutrality of the universe:

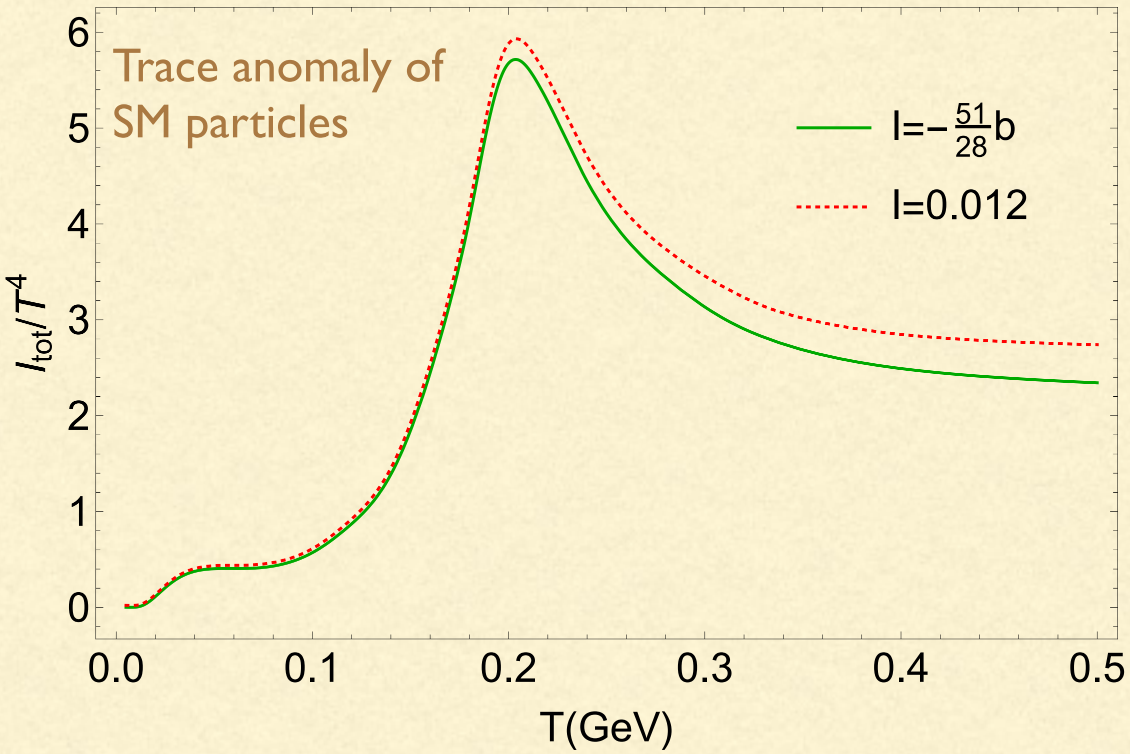
$$0 = \sum_i q_i n_i(T, \mu_i)$$

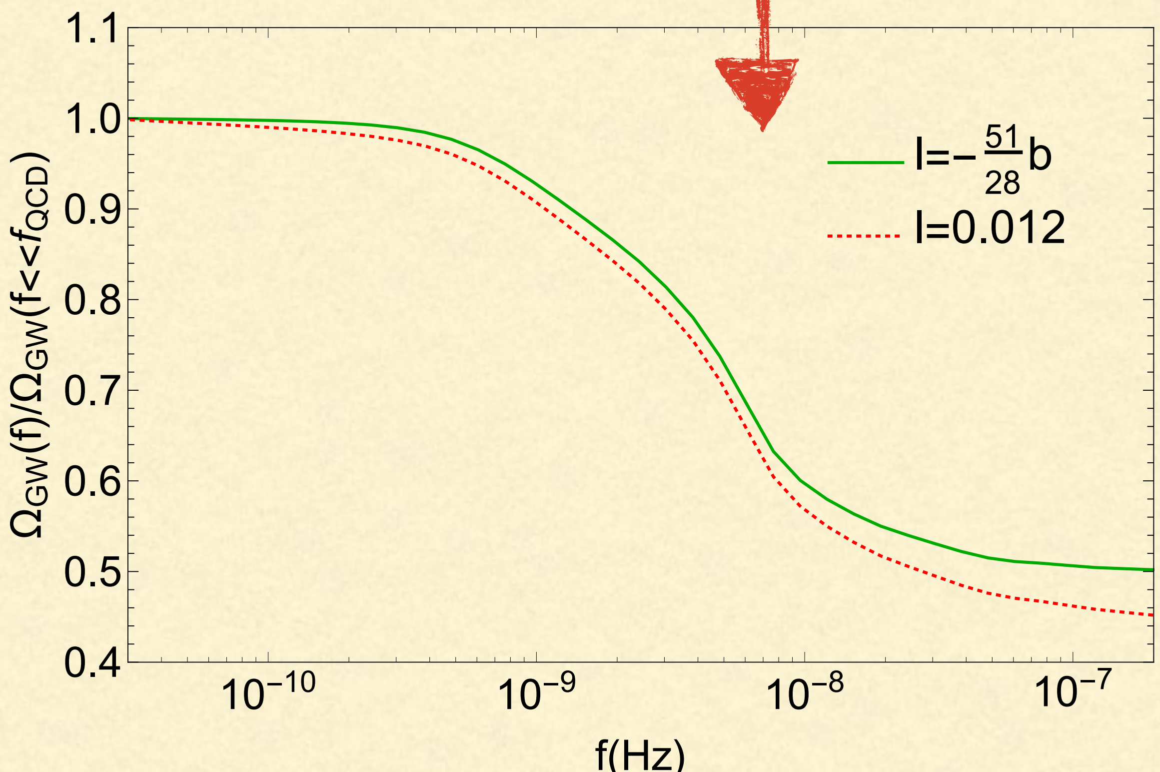
Schwarz & Stuke 2009 Wygas et al. 2018

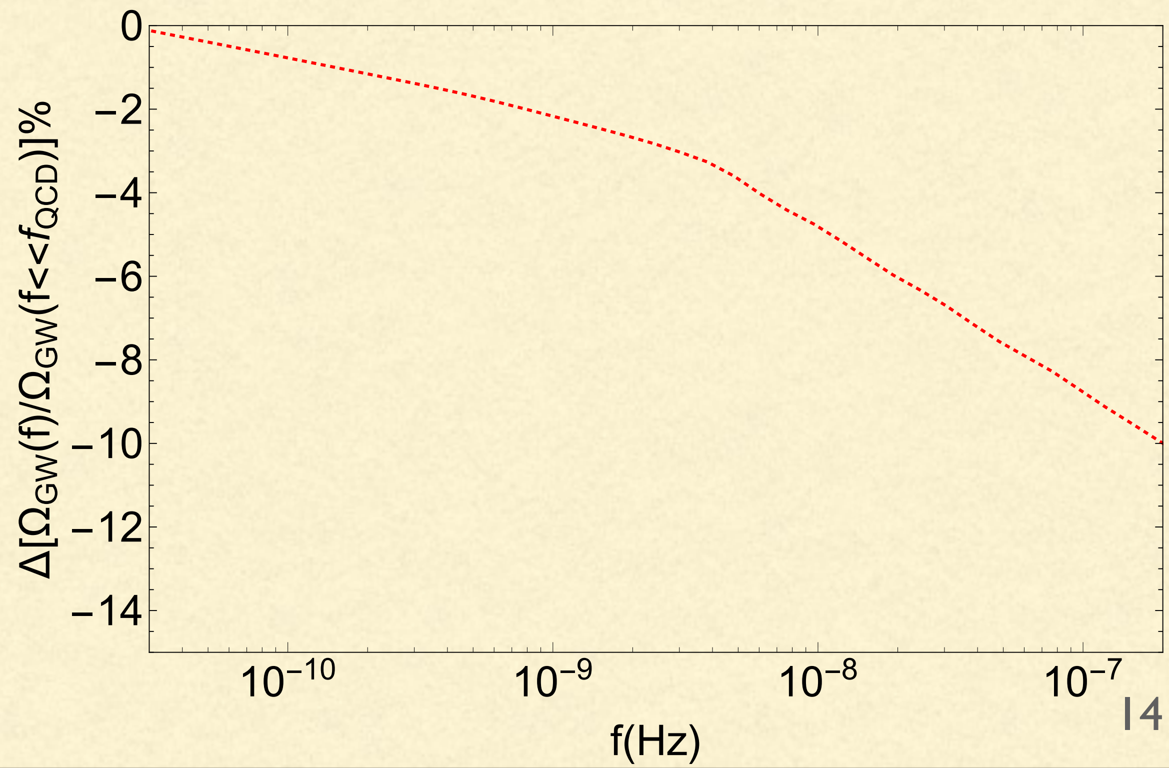
Vanishing and Nonvanishing Lepton Asymmetry & PGW

Non-vanishing lepton asymmetry leads to non-vanishing lepton chemical potentials, which affect the cosmic trajectory, equation of state and the PGW.
Wygas et al. 2018

Large lepton asymmetry can affect the PGW spectrum up to ~10%. Effects might be larger allowing for lepton flavour asymmetries.







Primordial Gravitational Waves in Standard and Non-standard Cosmologies

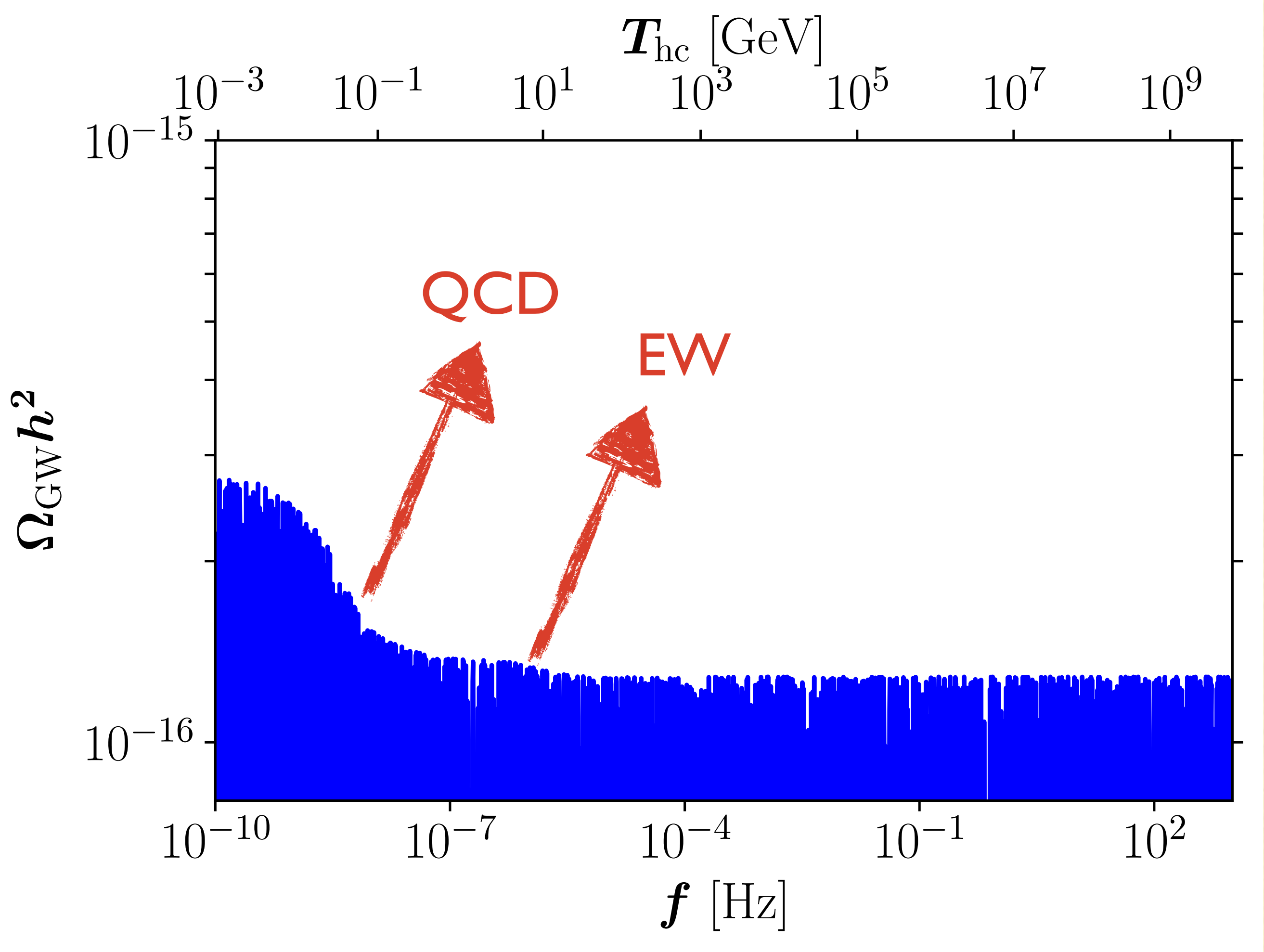
Overview (2nd)

- Constraining the Tensor Tilt and Tensor to Scalar Perturbation Ratio by GW Experiments in the Standard Cosmology
- Nonstandard Cosmological Scenarios and the PGW
  Orfeu Bertolami & Elena Arbuzova talks
- Constraining Nonstandard Cosmologies with GW Experiments

Scale independent tensor power spectrum and the scale of inflation:

$$\mathcal{P}_T(k) = \frac{2}{3\pi^2}\frac{V_{\rm inf}}{M_{Pl}^4}$$

$$V_{\rm inf}^{1/4} = 1.5 \times 10^{16}\ GeV$$



Tensor power spectrum and its scale dependence:

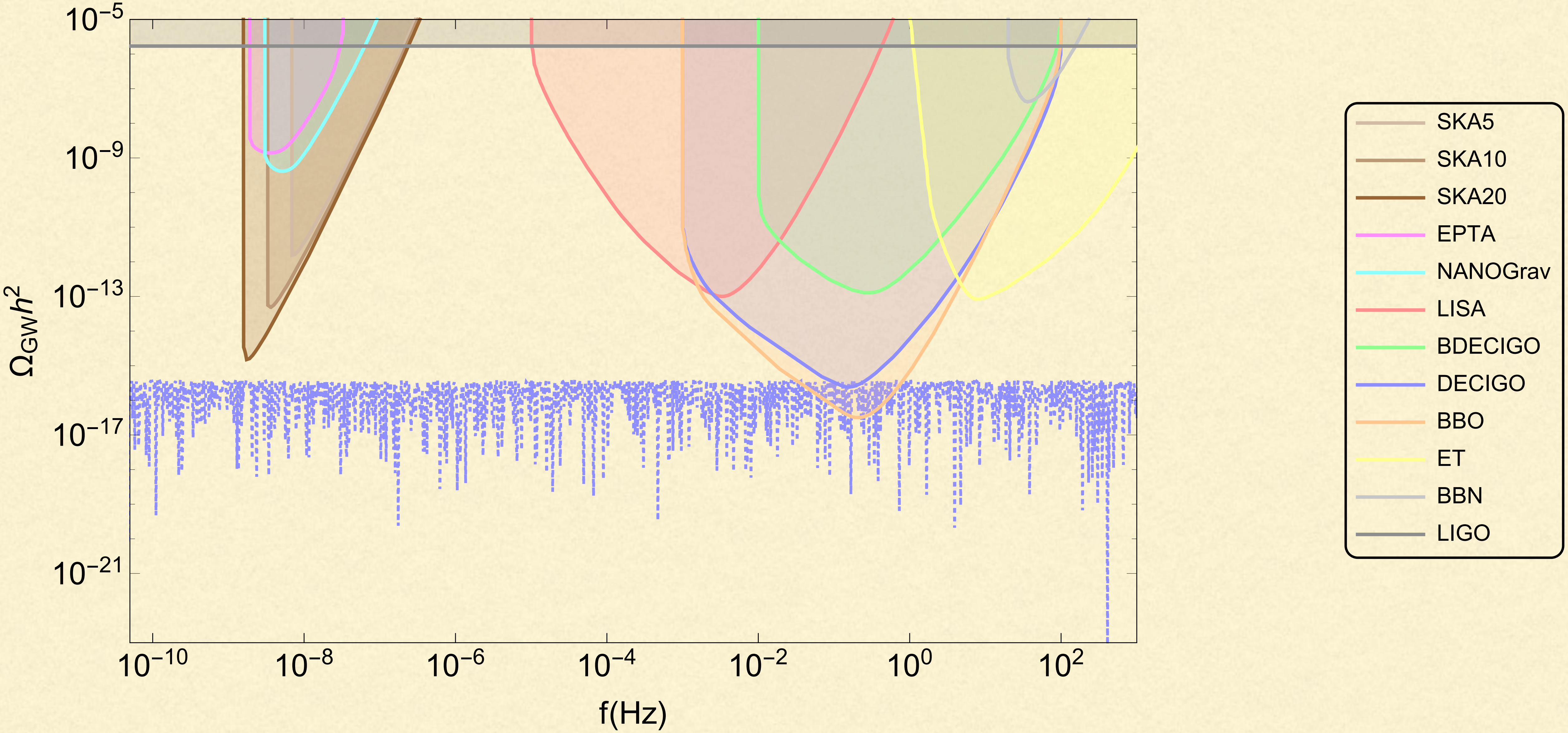
$$\mathcal{P}_T(k) = A_T \left(\frac{k}{\tilde{k}}\right)^{n_T}$$

Tensor tilt

Pivot scale

Tensor to scalar perturbation ratio:

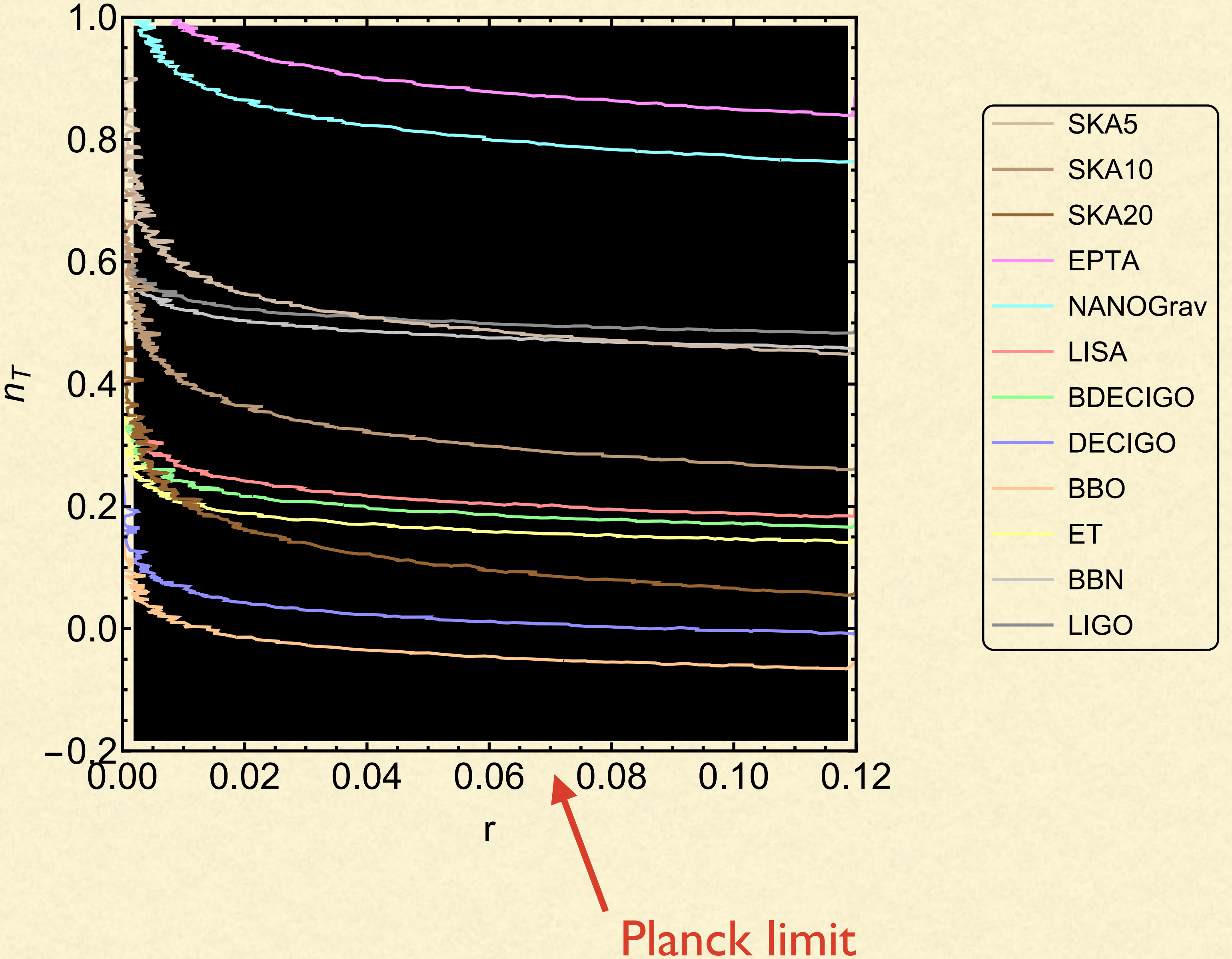
$$r \equiv \frac{A_T}{A_S}$$



10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
$\Omega_{GW}h^2$
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
f(Hz)
SKA5
SKA10
SKA20
EPTA
NANOGrav
LISA
BDECIGO
DECIGO
BBO
ET
BBN
LIGO

Scale Dependent Power Spectrum and Constraints on
the tensor tilt and tensor to scalar perturbation ratio

By fixing the scalar perturbation amplitude from Planck data:



Nonstandard cosmologies and the PGW

- The history of universe before big bang nucleosynthesis is unknown.
- UV completion theories predict nonstandard cosmologies (by new scalar fields) beyond the standard radiation dominated era before BBN and after inflationary epoch.
- The production mechanism of dark matter in the early universe is unknown. No hints for DM produced from the standard radiation dominated scenario!

Laine & Meyer, 2015   Borsanyi et al. 2016

Friedmann equations in nonstandard cosmology:

Equation of state parameter for the dominant component

$$\frac{d\rho_\phi}{dt} + 3(1+\omega_\phi) H \rho_\phi = -\Gamma_\phi \rho_\phi$$

$$\frac{ds_R}{dt} + 3\,H\,s_R = +\frac{\Gamma_\phi\,\rho_\phi}{T}$$

$$H^2 = \frac{\rho_\phi + \rho_R + \rho_m + \rho_\Lambda}{3\,M_{Pl}^2}$$

Temperature at which ϕ decays:

$$T_{\rm dec}^4 = \frac{90}{\pi^2\,g_\star(T_{\rm dec})}\,M_{Pl}^2\,\Gamma_\phi^2$$

Decay width

SM degrees of freedom

Ratio of initial densities:

$$\xi \equiv \left.\frac{\rho_\phi}{\rho_R}\right|_{T=T_{\rm max}}$$

Initial value for temperature in the numerical calculation (not physical only to cover all possible cases):

$$T_{\rm max} = 10^{17}\ GeV$$

Tashiro & Shiba & Sasaki 2003 Nakayama & Takahashi 2010 Durrer & Hessenkamp 2011 Ramberg & Visinelli 2019 D'Earmo & Schmitz 2019

Hubble rate during radiation domination:

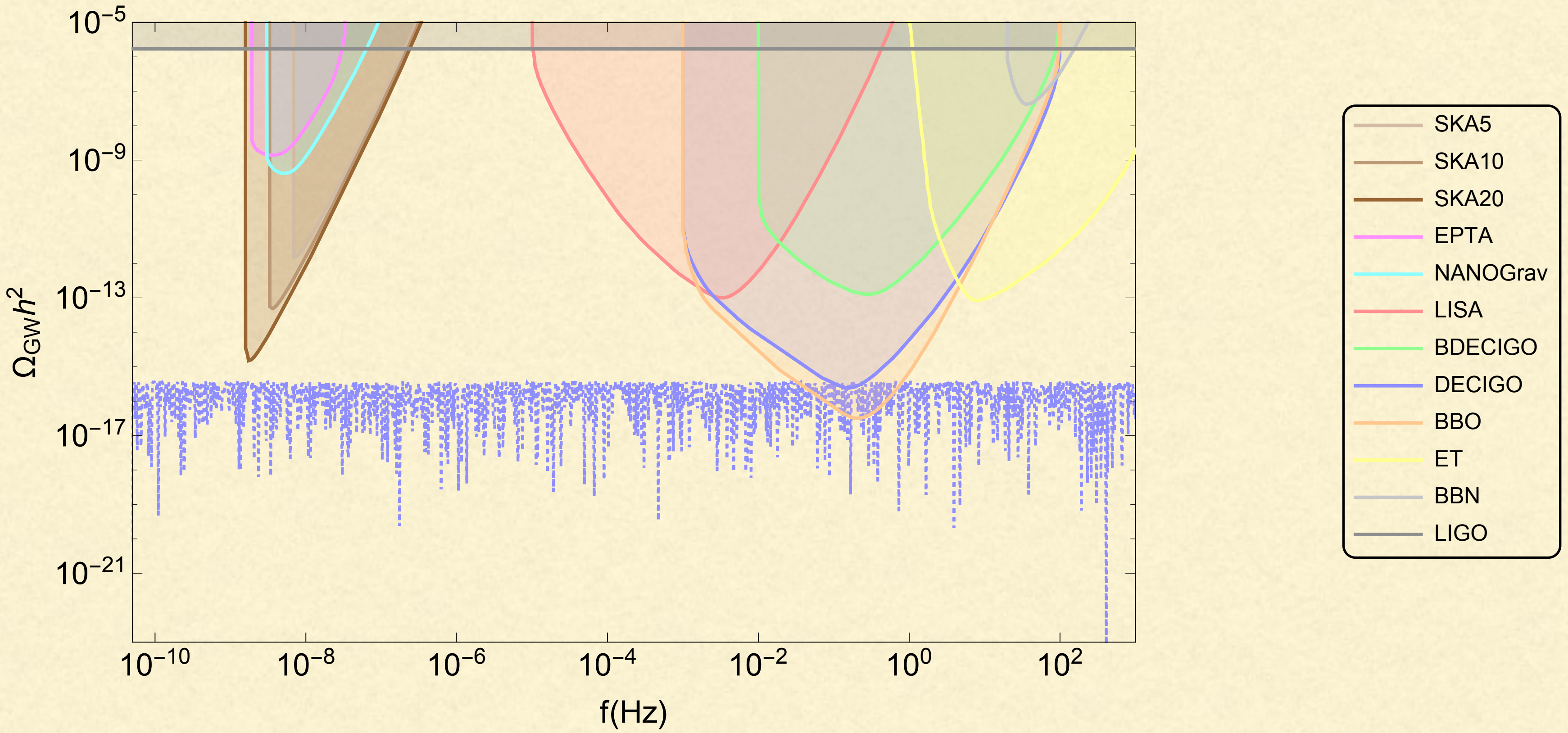
$$H_R \propto a^{-2}$$

Hubble rate in ϕ domination:

$$H_\phi \propto a^{-\frac{3}{2}(1+\omega_\phi)}$$

The PGW relic for modes come inside the horizon during ϕ domination: $\Omega_{GW} \propto k^{-2\frac{1-3\omega_\phi}{1+3\omega_\phi}}$

$$\omega_\phi = 1/3, \xi = 10^{35}$$



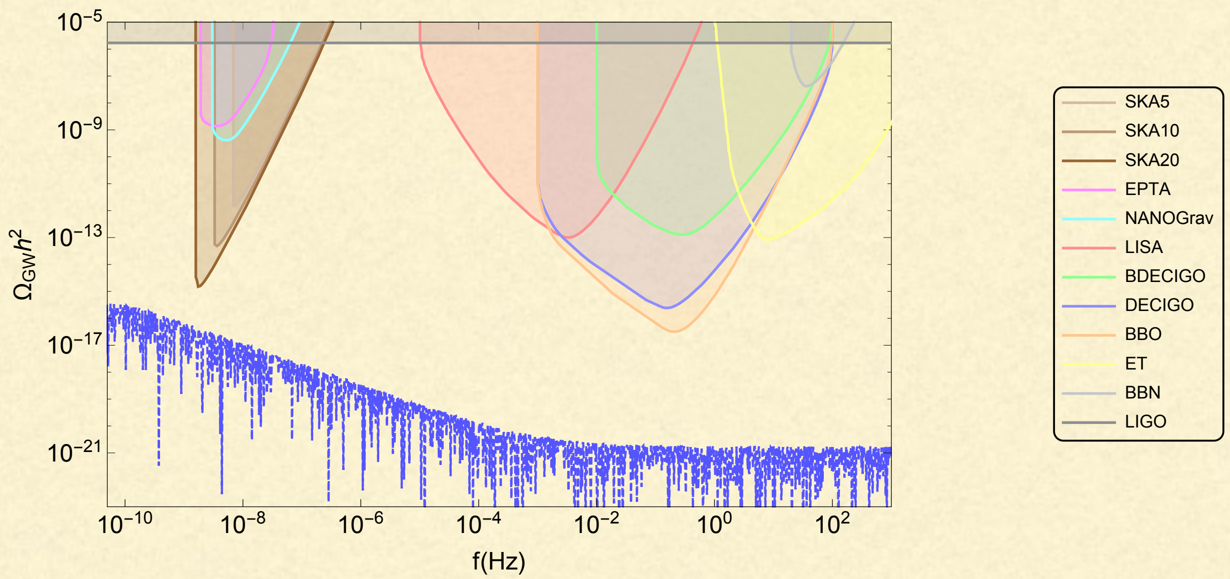
$$T_{\text{dec}} = 10\ MeV$$

The minimum value of ratios to have a modified radiation density due to nonstandard cosmological scenario ($\rho_R \propto a^{-4} \to a^{-\frac{3}{2}(1+\omega_\phi)}$):

$$\xi_{\text{min}} \approx \left[\left(\frac{g_\star(T_{\text{max}})}{g_\star(T_{\text{dec}})} \right)^{\frac{1}{4}} \frac{T_{\text{max}}}{T_{\text{dec}}} \right]^{3\omega_\phi - 1}$$

Modulus or matter domination like scenario

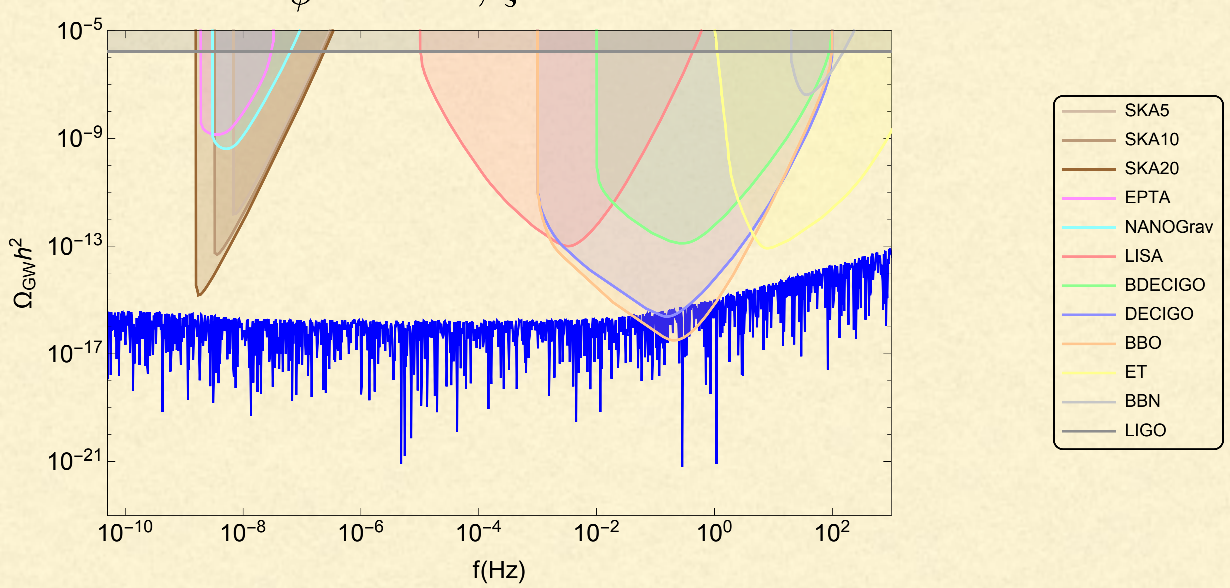
$$\omega_\phi = 0.15, \xi = 10^{-6}$$



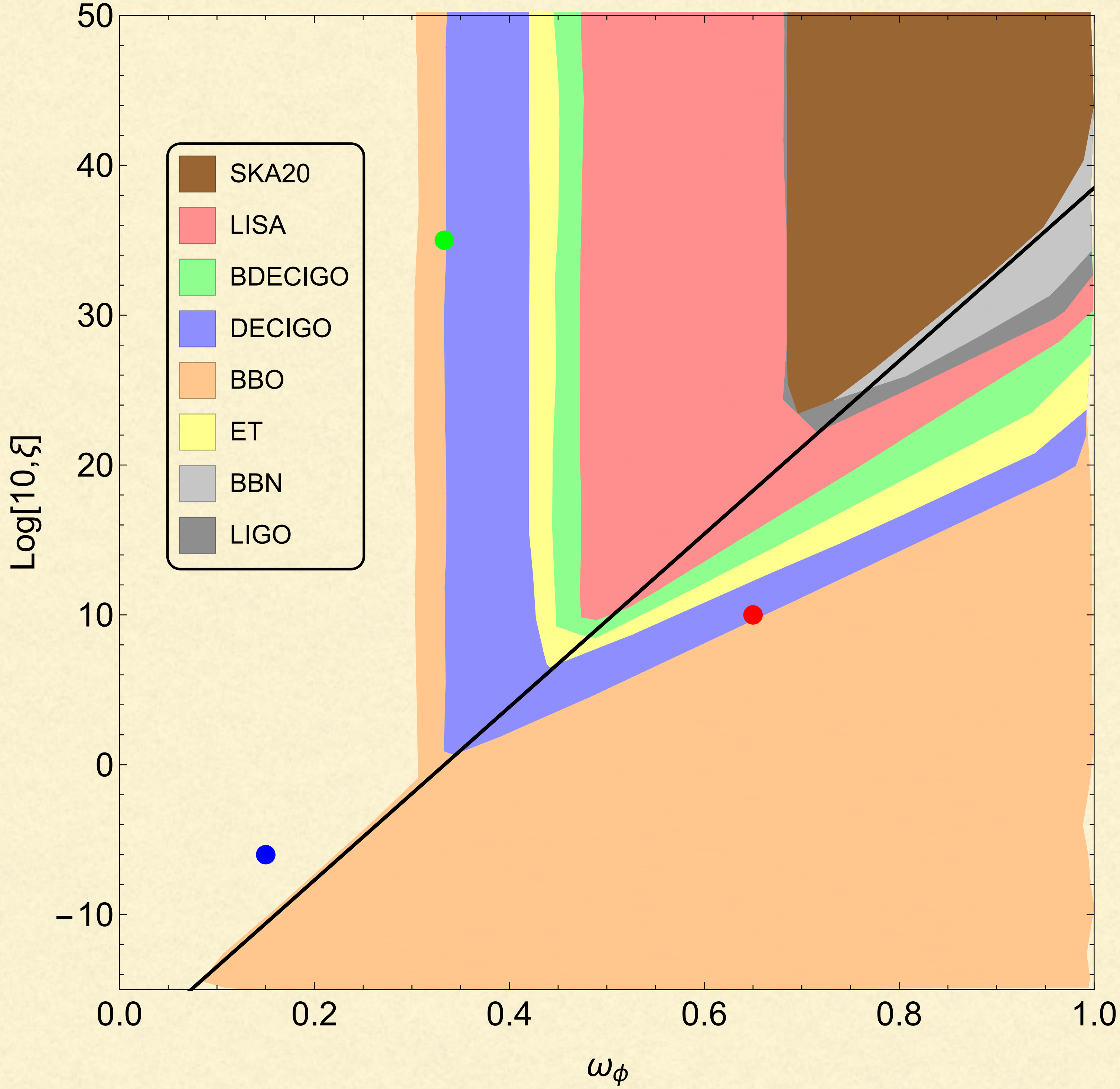
$$T_{\text{dec}} = 10\ MeV$$

Kination domination like scenario

$$\omega_\phi = 0.65, \xi = 10^{10}$$



Scanning over the parameter space of $[\omega_\phi, \xi]$ for $T_{\rm dec} = 10\ MeV$:



Remarks

- (1st part) Nonvanishing lepton chemical potentials might lead to a large enough isospin chemical potential which causes the formation of pion condensate regime in the early universe around the QCD era. If this happens the effect on the PGW will be different from the standard and nonvanishing lepton asymmetry cases. Brandt et al. 2018

- (2nd part) The curvature power spectrum might also affect the PGW in nonstandard cosmological scenarios as a source for tensor perturbations.

Conclusions

- QCD equations of state affect the PGW up to a few percent.

- Pulsar Timing Arrays can observe such effect in near future if the PGW relic is around $\Omega_{GW}h^2 \approx 10^{-16}$.

- Nonvanishing lepton asymmetry can leave an imprint on the PGW up to 10% in comparison to the Vanishing case which is an indirect signature of lepton asymmetry in the early universe.

- GW experiments can constrain the parameter space of nonstandard cosmology regimes dominated before BBN.

- They can indirectly measure the regimes might lead to nonthermal production of dark matter which can help us to understand the DM production in the early universe and give an indirect hint for DM mass and cross section.

Thanks for your attention!