On renormalization of Analytic Infinite Derivative (AID) theories

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Instead of introduction

- **Einstein's gravity is not renormalizable**
- **Stelle's 1977 and 1978 papers show that R^2 gravity is renormalizable gravity with the price of a physical (Weyl) ghost**
- **Recall: Ostrogradski statement from 1850 forbids higher derivatives in general. The Weyl tensor already has 2, its square has 4 and constraints do not alleviate the problem.**
- **Good thing: Starobinsky inflation is based on R^2 and works perfectly**

The early Universe formation, which is most likely inflation, is for the time being perhaps the only testbed for testing gravity modifications.

So what?

We start with

$$S = \int d^D x \sqrt{-g} \left(\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right)$$

Here $\mathcal{P}$ and $\mathcal{Q}$ depend on curvatures and $\mathcal{O}$ are operators made of covariant derivatives.

Everywhere the respective dependence is *analytic*.

The most general action to consider

We are looking for the most general action capturing in full generality the properties of a linearized model around *maximally symmetric space-times (MSS)* such that

$$R_{\mu\nu\alpha\beta} = f(x)(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$$

The result is [arxiv.1602.08475]

$$S = \int d^D x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2}$$

$$+\frac{\lambda}{2}\left(R\mathcal{F}(\Box)R + L_{\mu\nu}\mathcal{F}_L(\Box)L^{\mu\nu} + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right) - \Lambda\Bigg)$$

Here $L_{\mu\nu} = R_{\mu\nu} - \frac{1}{D}Rg_{\mu\nu}$ and for any X

$$\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X_n}\Box^n$$

Even more, the derived action can be reduced further!

This is thanks to the Bianchi identities.

Around MSS in $D = 4$ one can fix any of tree functions $\mathcal{F}$ and not only their constant Taylor coefficients. For example we can drop $\mathcal{F}_L$ entirely and remain with

$$S = \int d^4x \sqrt{-g}\left(\frac{M_P^2 R}{2} + \frac{\lambda}{2}\left(R\mathcal{F}(\Box)R + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right) - \Lambda\right)$$

Reduction in other dimensions

Around MSS but in $D \geq 5$ the GB term is not a topological invariant and we are left with

$$S = \int d^4x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} + \frac{\lambda}{2}\Big(R\mathcal{F}(\Box)R + f_{L0}L_{\mu\nu}^2 + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\Big) - \Lambda\Bigg)$$

Still, we are able to drop all higher derivative terms for $L_{\mu\nu}$

Quadratic action around (A)dS with $\bar{R} = 4\Lambda/M_P^2$

The covariant decomposition is

$$h_{\mu\nu} = \frac{2}{M_P^2} h^{\perp}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu + \left(\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4} \frac{2}{M_P^2} \sqrt{\frac{8}{3}} \bar{g}_{\mu\nu} \bar{\Box} \right) B + \frac{1}{4} \frac{2}{M_P^2} \sqrt{\frac{8}{3}} \bar{g}_{\mu\nu} h$$

Here $\bar{\nabla}^\mu h^{\perp}_{\mu\nu} = \bar{g}^{\mu\nu} h^{\perp}_{\mu\nu} = \bar{\nabla}^\mu A_\mu = 0.$

Vector part and $\bar{\nabla}_\mu \bar{\nabla}_\nu B$ **terms go away around MSS.**

Spin-2:

$$S_2 = \frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\; h^{\perp}_{\nu\mu}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} + \frac{\lambda}{M_P^2}\left\{\mathcal{F}_L(\bar{\Box})\left(\bar{\Box} - \frac{\bar{R}}{6}\right) + 2\mathcal{F}_W\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\right\}$$

The Stelle's case corresponds to $\mathcal{F}_L = 0,\ \mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} + \frac{\lambda}{M_P^2} 2\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

This is an obvious second pole which will be the ghost.

Spin-0 (here $\phi \equiv \bar{\Box} B - h$):

$$S_0 = -\frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\ \phi(3\bar{\Box} + \bar{R})\left[\mathcal{S}(\bar{\Box})\right]\phi$$

$$\mathcal{S}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} - \frac{\lambda}{M_P^2}\left\{2\mathcal{F}(\bar{\Box})(3\bar{\Box} + \bar{R}) + \frac{1}{2}\mathcal{F}_L\left(\bar{\Box} + \frac{2}{3}\bar{R}\right)\bar{\Box}\right\}$$

This *is* the ghost in Einstein-Hilbert case, but it is constrained and is not physical.

Thus, $\mathcal{S}(\bar{\Box})$ *can* have one root to generate one pole and it will be not a ghost.

This would be exactly the scalar mode in a local $f(R)$ gravity.

Physical excitations

Effectively we modify the propagators as follows

$$\Box - m^2 \to \mathcal{G}(\Box)$$

To preserve the physics we demand

$$\mathcal{G}(\Box) = (\Box - m^2)e^{\sigma(\Box)}$$

where $\sigma(\Box)$ must be an *entire* function resulting that the exponent of it has no roots.

We arrange this in our model by virtue of functions $\mathcal{F}$. At this stage we can drop any one of three $\mathcal{F}$-s. The simplest choice is to drop $\mathcal{F}_L$.

UV completeness

Minkowski propagator:

$$\Pi = -\left(\frac{P^{(2)}}{k^2 e^{H_2(-k^2)}} - \frac{P^{(0)}}{2k^2 e^{H_0(-k^2)} \left(1 + \frac{k^2}{M^2}\right)} \right)$$

To guarantee that the QFT machinery works we arrange a polynomial decay of the propagator near infinity. The rate of the decay is our choice.

Recall that we still need the functions $H_{0,2}$ to be entire.

An example of such a function can be, for instance

$$H \sim \Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right)$$

where $p(z)$ is a polynomial.

Beyond 1-loop the powercounting arguments work just like in the higher derivative regularization.

Amplitudes and Cross-sections

Power-counting works because we have chosen the polynomial decay at infinity

Slavnov-Taylor identities work thanks to the presence of the diffeomorphism invariance

Exponential decay of form-factors renders the system to be in the strong-coupling regime. This way amplitudes become divergent for large external momenta.

The ongoing work in progress is to determine conditions on form-factors wich would retain standardly expected behavior of amplitudes.

p-adic reformulation of the non-local gravity

The scalar part of the previous action is equivalent to the following one

$$S = \int d^4x \sqrt{-g}\left(\frac{M_P^2 R}{2}\left(1 + \frac{2}{M_P^2}\psi\right) - \frac{1}{2\lambda}\psi \frac{1}{\mathcal{F}(\Box)}\psi + \dots \right)$$

An important property here is the non-minimal coupling of a scalar field to gravity.

p-adic reformulation of the non-local gravity, continued

The conformal transform $\left(1+\frac{2}{M_P^2}\psi\right)^2 g_{\mu\nu} = \hat{g}_{\mu\nu}$ allows us to completely decouple the gravity and the scalar field

$$S = \int d^4x \sqrt{-\hat{g}} \left(\frac{M_P^2}{2}\hat{R} - \frac{M_P^2}{2}\frac{6}{(M_P^2+2\psi)^2}\hat{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{M_P^4}{2\lambda(M_P^2+2\psi)^2}\psi\mathcal{G}(\mathcal{D})\psi \right)$$

Here

$$\mathcal{G}(\mathcal{D}) = \frac{1}{\mathcal{F}(\mathcal{D})} \text{ and } \mathcal{D} = \left(1+\frac{2}{M_P^2}\psi\right)\hat{\Box} - \frac{2}{M_P^2}\hat{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

Where are p-adic strings?

We carefully extract the quadratic in ψ part of the above derived Lagrangian. The answer is

$$L_\psi = \frac{3}{M_P^2}\psi\hat{\Box}\psi - \frac{1}{\lambda}\psi\mathcal{G}(\hat{\Box})\psi$$

Thus

$$\frac{3}{M_P^2}\hat{\Box} - \frac{1}{\lambda}\mathcal{G}(\hat{\Box}) = (\varepsilon\hat{\Box} - m^2)e^{\sigma(\hat{\Box})}$$

Limiting $\sigma = \Box/\mathcal{M}^2$ and taking $\varepsilon = 0$ we restore the p-adic Lagrangian originally written as

$$L = -\frac{1}{2}\phi p^{-\Box/m_p^2}\phi + \frac{1}{p+1}\phi^{p+1}$$

Conclusions

- **A UV complete and unitary gravity is discussed**
- **It features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **The theory predicts a modified value for r for example**
- **A connection to p-adic strings is maintained**
- **The theory has clear connection to SFT**

Open questions

- More concrete understandning of how form-factors are constrained from the point of view of QFT
- Explicit demonstration of the absence of singular solutions in this model
- Deeper study of inflation and bouncing scenarios in this model
- Derive the graviton action from the SFT in the full rigor

Thank you for listening!