Effective field theory approach to scalar-tensor inspiralling

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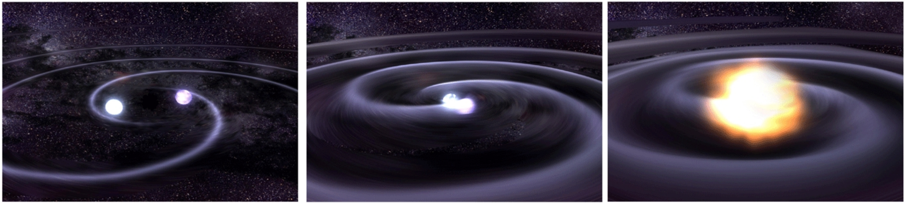
Outline

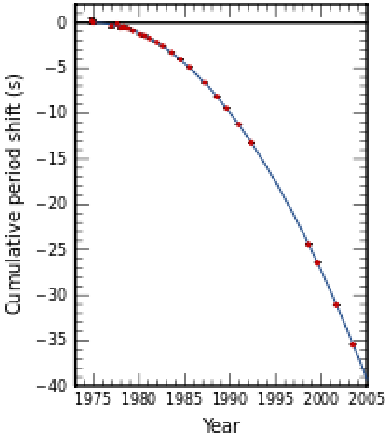
The object under study



Credit : LIGO/Virgo

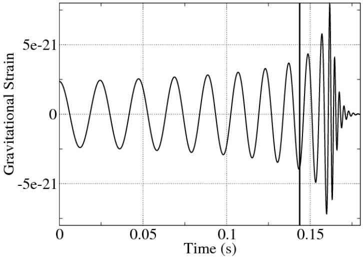
Which observables ?

Prehistoric times



$\dot{E}$

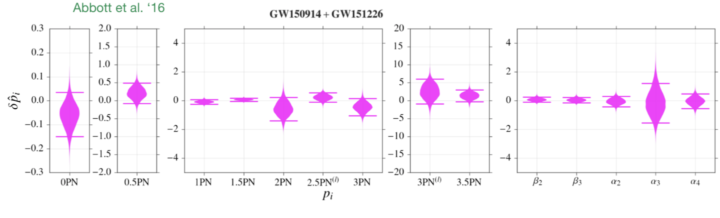
Today



$\phi(t)$

What if GR is not the ultimate theory ?

- Source : Dipolar radiation $\Rightarrow \delta\omega(t)$ (Eardley 1975), PN coefficients



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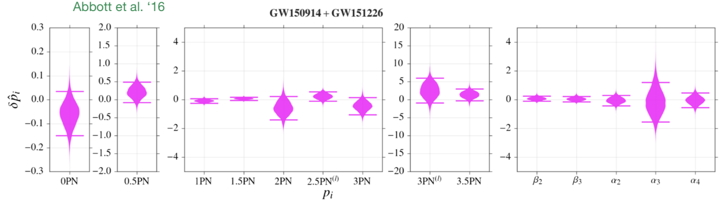
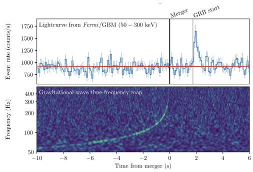


- Propagation :



- Speed $|c_T - c| < 10^{-15}$ LIGO/Virgo 2017

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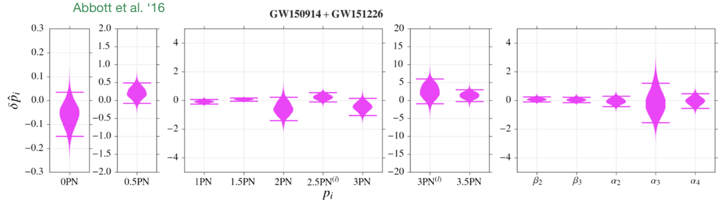

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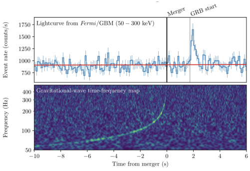

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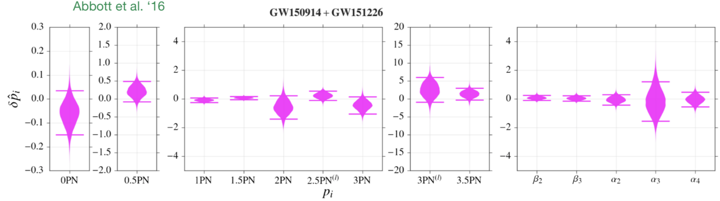

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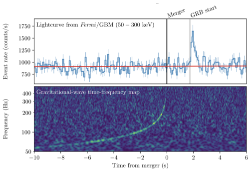
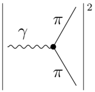
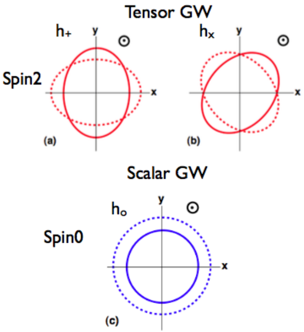


- Speed $|c_T - c| < 10^{-15}$ LIGO/Virgo 2017
- Damping $\Rightarrow d_L^{GW} \neq d_L^{EM}$ Deffayet, Menou 07 ; Amendola et al 17, etc
- Decay of gravitons Creminelly, Lewandoski, Tambalo, Vernizzi 18



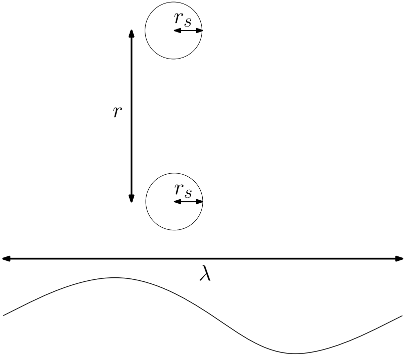
What if GR is not the ultimate theory ?

- Detection : Additional polarization



Hardly detectable, low amplitude and needs at least 3 GW detectors !

Some orders of magnitude



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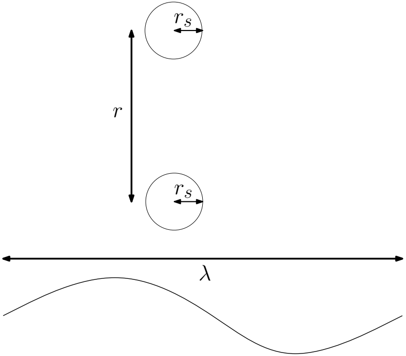

- VIRGO/LIGO band : 10Hz - 1kHz
- $r(10\text{Hz}) \sim 300$ km $\rightarrow r(1\text{kHz}) \sim 14$ km
- $v(10\text{Hz}) \sim 0.06 \rightarrow v(1\text{kHz}) \sim 0.3$
- $t \sim 5$ min.
- $\Phi = 2\int_{t_i}^{t_f} \omega(t)dt \sim 4 \times 10^4$ rad
  $\Rightarrow \Phi$ to 3PN !

Perturbative solution of the EOM

Conventional PN calculations :

$$\Box h^{\mu\nu} = -16\pi G T^{\mu\nu} \Rightarrow \text{Solve for } h^{\mu\nu} \Rightarrow \text{Plug back in the action}$$

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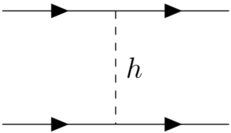
EFT approach :

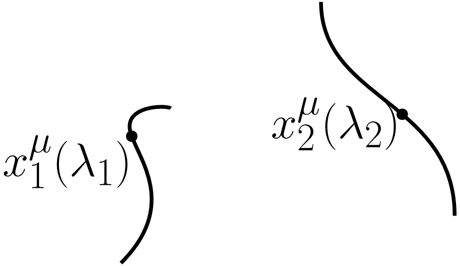
$$e^{iS_{\text{ef}}} = \int \mathcal{D}[h_{\mu\nu}] e^{iS}$$



Goldberger and Rothstein (2006)
Porto (2006)
+ many developments...

Outline

Invariances of the system



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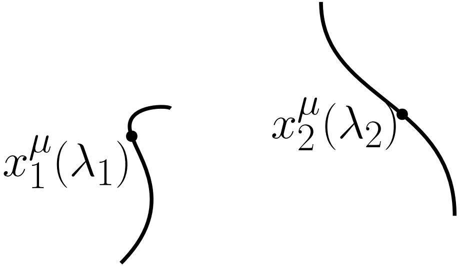

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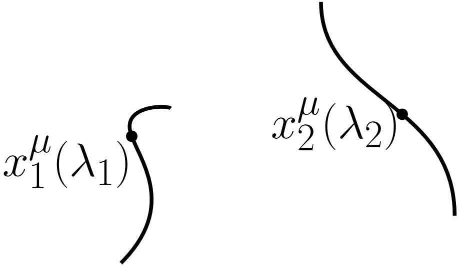

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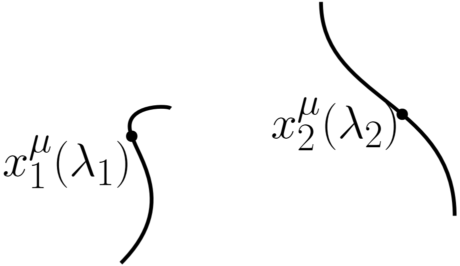

Fluctuating field

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Small parameter

$\frac{Gm}{r} \sim v^2$

- $x^\mu \to x'^\mu(x) \Rightarrow$ Use R and $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
- $\lambda_a \to \lambda'_a(\lambda_a) \Rightarrow$ Use $d\tau = \sqrt{g_{\mu\nu}dx^\mu dx^\nu} = dt\sqrt{1 - v^2 - h_{\mu\nu}v^\mu v^\nu}$

Point particles action

$$S = S_{grav} + S_{pp,1} + S_{pp,2}$$

$$S_{grav} = \frac{m_P^2}{2} \int d^4x \sqrt{-g} R$$

$$S_{pp,a} = - m_a \int d\tau_a$$

Point particles action

$$S = S_{grav} + S_{pp,1} + S_{pp,2}$$

$$S_{grav} = \frac{m_P^2}{2}\int d^4x\sqrt{-g}R - \boxed{\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$$

$$S_{pp,a} = -\,m_a\int d\tau_a + \boxed{a\frac{m_a}{m_P}\int d\tau_a\phi + b\frac{m_a}{m_P^2}\int d\tau_a\phi^2} + \ldots$$

'Quantum' gravity

Integrate out fluctuating fields :

$$e^{iS_{\mathrm{ef}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathcal{D}[h_{\mu\nu}]\mathcal{D}[\phi]e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu},\phi]}$$

S_{ef} contains the dynamics of the point-particles only

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S_{ef} contains the dynamics of the point-particles only

$$\Re(S_{\text{ef}}) = \int dtL[\mathbf{x}_a, \mathbf{v}_a]$$ Conservative dynamics

and

$$\Im(S_{ef}) = \frac{T}{2}\int dEd\Omega \frac{d^2\Gamma}{dEd\Omega}$$ Dissipative dynamics

Outline

v^0 (or Newtonian) Lagrangian

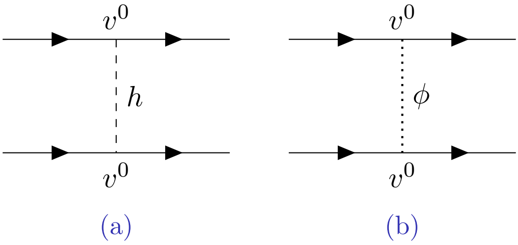
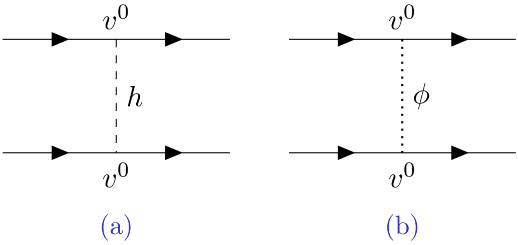


Figure: Feynman diagrams contributing to the Newtonian potential

v^0 (or Newtonian) Lagrangian



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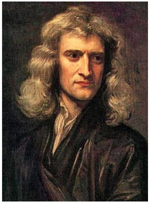

$$L_{v^0} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{G_N m_1 m_2}{|\mathbf{x_1}(t) - \mathbf{x_2}(t)|}(1 + 2a^2)$$

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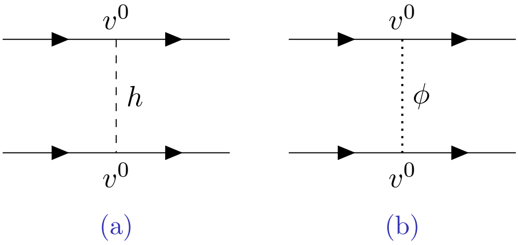

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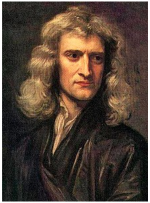

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$$\tilde{G} = G_N(1 + 2a^2)$$

v^2 (or EIH) Lagrangian

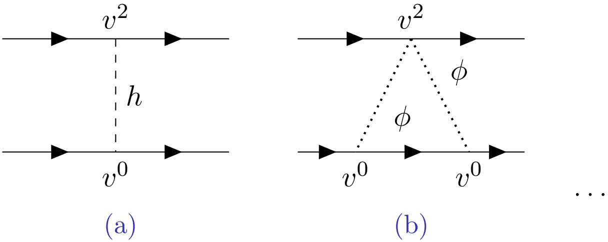
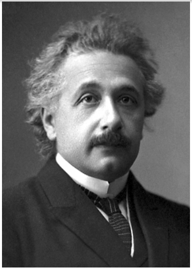


Figure: Some Feynman diagrams contributing to the v^2 Lagrangian

v^2 (or EIH) Lagrangian



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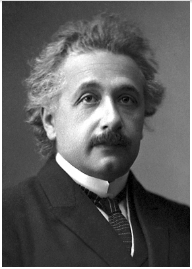

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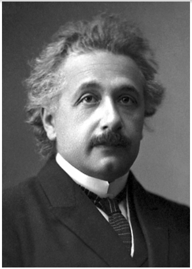

$$L_{EIH} = \frac{1}{8}\sum_a m_a v_a^4 + \frac{\tilde{G}m_1m_2}{2|\mathbf{x}_{12}|}\left[(v_1^2+v_2^2) - 3\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{(\mathbf{v}_1\cdot\mathbf{x}_{12})(\mathbf{v}_2\cdot\mathbf{x}_{12})}{|\mathbf{x}_{12}|^2} + 2\gamma(\mathbf{v}_1-\mathbf{v}_2)^2\right] - \frac{\tilde{G}^2m_1m_2(m_1+m_2)}{2|\mathbf{x}_{12}|^2}(2\beta-1)$$

Renormalization of the mass

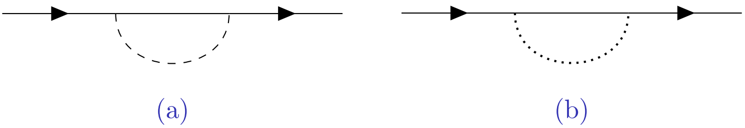

Figure: Diagrams contributing to the mass renormalization.

Renormalization of the mass

(a) (b)

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$$-m_{\rm bare} \int dt \quad \to \quad -(m_{\rm bare} + E(\Lambda)) \int dt$$

Renormalization of the mass

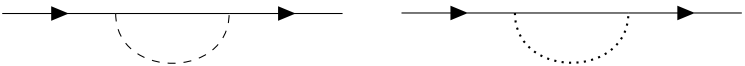

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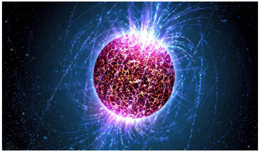
Figure: Diagrams contributing to the mass renormalization.

$$-m_{\text{bare}} \int dt \quad \rightarrow \quad -(m_{\text{bare}} + E(\Lambda)) \int dt$$

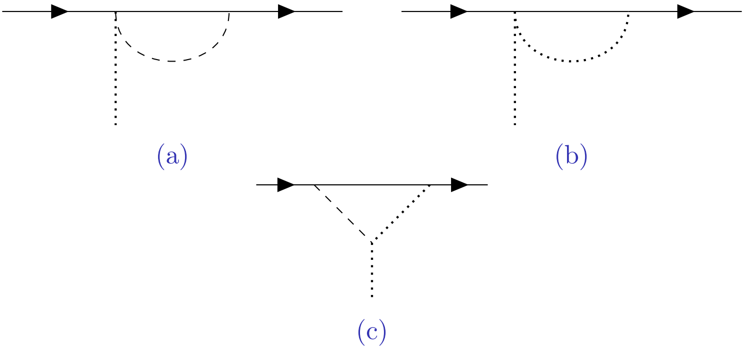
$$E(\Lambda) = -\frac{\tilde{G}}{2} \int d^3x d^3y \frac{\rho(\mathbf{x})\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}$$

$E = 0$



$E \neq 0$

Renormalization of the charge



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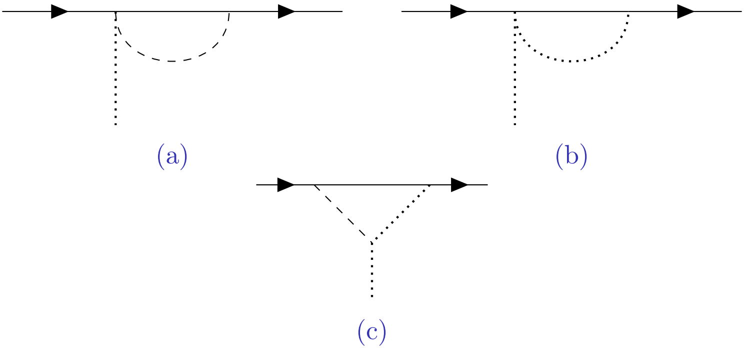

Figure: Diagrams contributing to the charge renormalization.

$$a_{\text{bare}} \frac{m_{\text{bare}}}{m_P} \int dt\, \phi \qquad \rightarrow \qquad a(\Lambda) \frac{m(\Lambda)}{m_P} \int dt\, \phi$$

Renormalization of the charge

$$\begin{cases} a_{\text{bare}} \to a(\Lambda) \\ \tilde{G} = G_N(1 + 2a^2) \end{cases}$$

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$$\Rightarrow \tilde{G}_{AB} = \tilde{G}\left[1 + (4\tilde{\beta} - \tilde{\gamma} - 3)\left(\frac{E_A}{m_A} + \frac{E_B}{m_B}\right)\right]$$

Renormalization of the charge

$$\frac{\tilde{G} m_1 m_2}{r}$$

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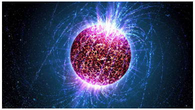

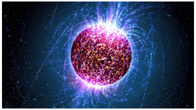

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Nordtvedt (1968)

Outline

Multipole expansion

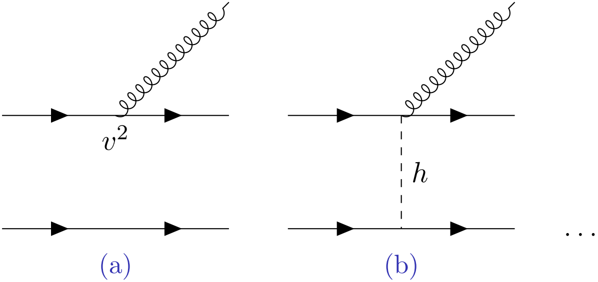

Figure: Some Feynman diagrams for the emission of one radiation scalar.

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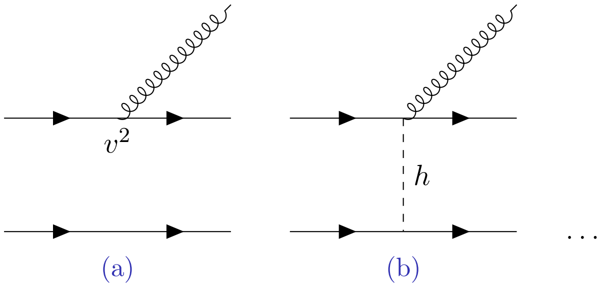

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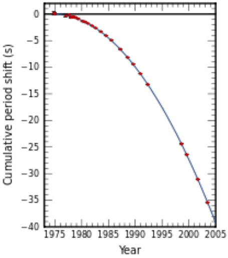
Net result :

$$S_{\text{int}} = \frac{1}{2}\int dt I_h^{ij} R_{0i0j} + \frac{1}{m_P}\int dt \left(I_\phi \bar{\phi} + I_\phi^i \partial_i \bar{\phi} + \frac{1}{2} I_\phi^{ij} \partial_i \partial_j \bar{\phi} \right) + \dots$$

Radiated power

Quadrupole formula :

$$P_h = \frac{G_N}{5} \left\langle \dddot{I}_h^{ij\,2} \right\rangle + \dots$$



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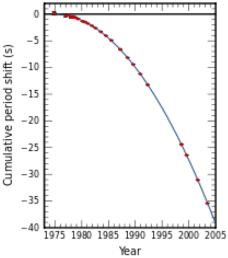

Monopole, dipole and quadrupole scalar radiation :

$$P_\phi = 2G_N \left(\left\langle \dot{I}_\phi^{\,2} \right\rangle + \frac{1}{3} \left\langle \ddot{I}_\phi^{i\,2} \right\rangle + \frac{1}{30} \left\langle \dddot{I}_\phi^{ij\,2} \right\rangle + \dots \right)$$

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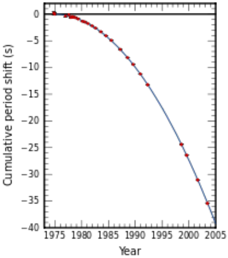

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$I_\phi = \mathrm{Const} + \mathrm{v}^2\mathrm{correction},\ I_\phi^i \propto a_1 - a_2$

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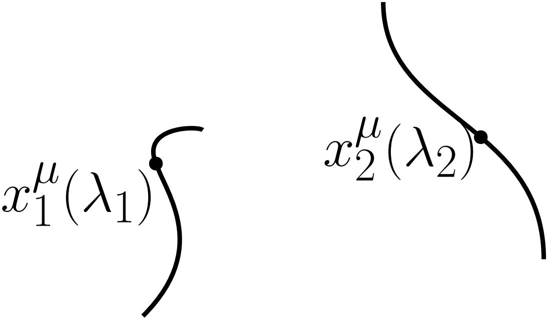

$$S_{pp} = -m \int d\tau$$

$$+a \frac{m}{m_P} \int d\tau \phi$$

$$+b \frac{m}{m_P^2} \int d\tau \phi^2 + \dots$$

$\lambda_a \to \lambda'_a(\lambda_a) \Rightarrow$ Use $d\tau = \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$

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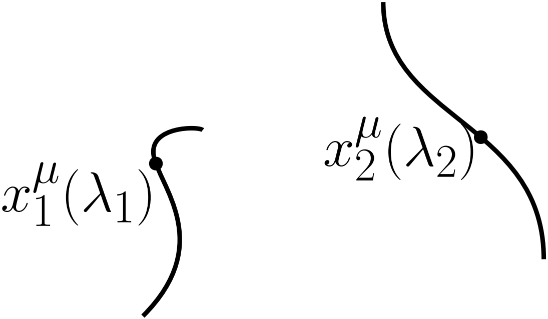
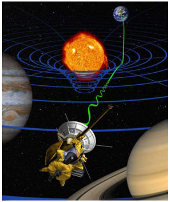


$$S_{pp} = -m \int d\tilde{\tau}$$

Conformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi) g_{\mu\nu}$$



$$\lambda_a \to \lambda'_a(\lambda_a) \Rightarrow \text{Use } \; d\tilde{\tau} = \sqrt{\tilde{g}_{\mu\nu} dx^\mu dx^\nu}$$

Bekenstein disformal coupling ('92)

Are there other ways to couple matter and still preserve causality and the equivalence principle ?

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Finsler geometry : disformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + \frac{B(\phi)}{M^2}\partial_\mu\phi\partial_\nu\phi$$

Bekenstein disformal coupling ('92)

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$$\int d\tilde{\tau} \supset \int d\tau \left(\frac{d\phi}{d\tau} \right)^2$$

The disformal energy

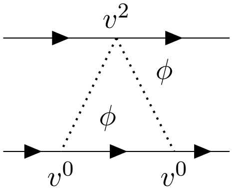

Figure: Disformal conservative diagram

The disformal energy

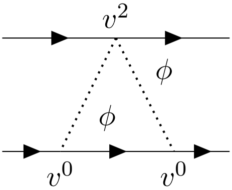

Figure: Disformal conservative diagram

$$L_{\text{dis}} = 4a^2 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{M^2} \left(\frac{d}{dt} \frac{1}{r} \right)^2, \quad r = |\mathbf{x}_1 - \mathbf{x}_2|$$

The disformal dissipated power

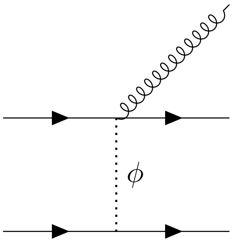

Figure: Disformal dissipative diagram

The disformal dissipated power

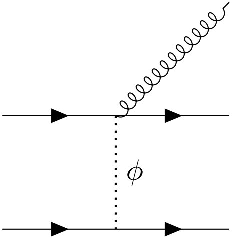

Figure: Disformal dissipative diagram

Disformal monopole :

$$I_{\text{dis}} = 8a\frac{G_N m_1 m_2}{M^2}\frac{d^2}{dt^2}\frac{1}{r}$$

Circular orbits

$$\dot{r} = 0 \Rightarrow L_{\text{dis}} = I_{\text{dis}} = 0 \; !$$

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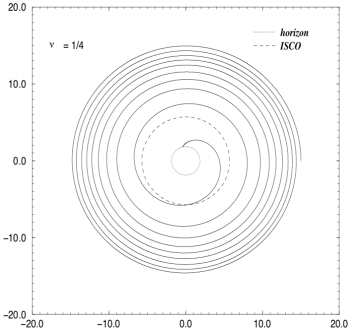

Radiation reaction

$$\Rightarrow L_{\rm dis} = \mathcal{O}(v^{14}), \quad I_{\rm dis} = \mathcal{O}(v^{12})$$

Outline

Conclusions

- Binary systems : ideal probe of strong-field GR
- Effective theory point of view : operators allowed by symmetries
- Disformal couplings : no effect on circular orbits