Nonlocal infrared modifications of gravity and dark energy

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based on

MM,	PRD 2014, 1307.3898
Foffa, MM, Mitsou,	PLB 2014, 1311.3421; IJMPA 2014, 1311.3435
Kehagias and MM,	JHEP 2014, 1401.8289
MM and Mancarella,	PRD 2014, 1402.0448
Dirian, Foffa, Khosravi, Kunz, MM,	JCAP 2014, 1403.6068
Dirian, Foffa, Kunz, MM, Pettorino,	JCAP 2015, 1411.7692; JCAP 2016, 1602.03558
MM	PRD 2016, 1603.01515
Cusin, Foffa, MM, Mancarella,	PRD 2016, 1602.01078
MM (review)	Springer, 1606.08784
Dirian,	PRD 2017, 1704.04075
Belgacem, Dirian, Foffa, MM,	JCAP 2018, 1712.07066
	PRD 2018, 1712.08198
	PRD 2018, 1805.08731
Belgacem, Finke, Frassino, MM	JCAP 2019, 1812.11181

- do we really understand the IR limit of (quantum) GR?

- can a mass scale appear in the IR?

- can IR effects be described by non-local effective terms associated to a mass scale?

- can such nonlocal terms explain dark energy and provide a viable cosmological model?

- Example: DGP model (Dvali,Gabadadze, Porrati 2000)

$$S=\frac{1}{2}M_{(5)}^3\int d^5X\sqrt{-G}R(G)+\frac{1}{2}M_{(4)}^2\int d^4x\sqrt{-g}R(g)$$

- linearizing over flat space

$$\Box h_{\mu\nu}+\partial_y^2 h_{\mu\nu}=0$$

$$h_{\mu\nu}(x,y)=e^{-y\sqrt{-\Box}}h_{\mu\nu}(x)$$

$$\left(\mathcal{E}^{\mu\nu\rho\sigma}+m\sqrt{-\Box}\right)h_{\rho\sigma}=8\pi G\,T^{\mu\nu}\qquad m=2M_{(5)}^3/M_{(4)}^2$$

- a possible covariantization is (Dvali, Gabadadze, Shifman 2002)

$$\left(1+\frac{m}{\sqrt{-\Box}}\right)G_{\mu\nu}=8\pi G\,T_{\mu\nu}$$

(valid only to linear order)

example of how can emerge a nonlocal term, relevant in the IR, and associated to a mass scale

Is it conceivable that a mass scale and associated nonlocal terms emerge dynamically already in GR?

At the fundamental level, the action in QFT is local

However, the quantum effective action is nonlocal

$$e^{iW[J]} \equiv \int D\varphi \; e^{iS[\varphi]+i\int J\varphi} \qquad \frac{\delta W[J]}{\delta J(x)} = \langle 0|\varphi(x)|0\rangle_J \equiv \phi[J]$$

quantum effective action: $\Gamma[\phi] \equiv W[J] - \int \phi J$

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = -J(x)$$

the quantum EA gives the exact eqs of motion for the vev, that include the quantum corrections

- light particles $\leftrightarrow$ nonlocalities in the quantum effective action

these nonolocalities are well understood in the UV.

E.g. in QED

$$\Gamma_{\text{QED}}[A_\mu] = -\frac{1}{4}\int d^4x \left[F_{\mu\nu}\frac{1}{e^2(\Box)}F^{\mu\nu} + \mathcal{O}(F^4)\right] + \text{fermionic terms}$$

$$\frac{1}{e^2(\Box)} \simeq \frac{1}{e^2(\mu)} - \beta_0 \log\left(\frac{-\Box}{\mu^2}\right)$$

$$\log\left(\frac{-\Box}{\mu^2}\right) \equiv \int_0^\infty dm^2 \left[\frac{1}{m^2+\mu^2} - \frac{1}{m^2-\Box}\right]$$

it is just the running of the coupling constant in coordinate space

Note: we are not integrating out light particles from the theory!

The quantum effective action is especially useful in GR

$$e^{i\Gamma_m[g_{\mu\nu}]} = \int D\varphi \; e^{iS_m[g_{\mu\nu};\varphi]}$$

(vacumm quantum EA. We can also retain the vev's of the matter fields
ith the Legendre transform)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} \qquad \langle 0|T_{\mu\nu}|0\rangle = \frac{2}{\sqrt{-g}} \frac{\delta \Gamma_m}{\delta g^{\mu\nu}}$$

It gives the exact Einstein eqs including quantum matter loops

$$G_{\mu\nu} = 8\pi G \, \langle 0_{\rm in}|T_{\mu\nu}|0_{\rm in}\rangle$$

$\Gamma = S_{EH} + \Gamma_m$ is an action that, used at tree level, give the eqs of motion that include the quantum effects

In the UV the quantum effective action in GR can be computed perturbatively in an expansion in the curvature using heat-kernel techniques

Barvinsky-Vilkovisky 1985,1987

$$\Gamma = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\left[R\,k_R(\Box)R + \frac{1}{2}C_{\mu\nu\rho\sigma}k_W(\Box)C^{\mu\nu\rho\sigma}\right]$$

$m_s << E$ $$k(\Box) \simeq k(\mu) - \beta_k \log\left(\frac{-\Box}{\mu^2}\right) + c_1\frac{m_s^2}{\Box} + c_2\frac{m_s^4}{\Box^2} + \ldots$$

Gorbar-Shapiro 2003

$m_s >> E$ $$k(\Box) \simeq O(\Box/m_s^2)$$

In the IR, however, we have much less control, because of possible strong IR effects

A typical IR effect is dynamical mass generation

- infrared divergences of massless fields in dS lead to dynamical mass generation,

$$m^2_{\rm dyn} \propto H^2\sqrt{\lambda}$$

Starobinsky-Yokoyama 1994, Riotto-Sloth 2008, Burgess et al 2010, Rajaraman 2010,....

the graviton propagator has exactly the same IR divergences

Antoniadis and Mottola 1986,....

mass generation forbidden in GR forbidden by diff invariance ?

Gauge-invariant (or diff-invariant) mass terms can be obtained with nonlocal operators

eg massive electrodynamics

Dvali 2006

$$\Gamma = -\frac{1}{4}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} - m_\gamma^2 F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu} \right)$$

in the gauge $\partial_\mu A^\mu = 0$ we have

$$\frac{1}{4}m_\gamma^2 F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu} = \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

it is a nonlocal but gauge-inv photon mass term!

equivalently, $\left(1 - \frac{m_\gamma^2}{\Box}\right)\partial_\mu F^{\mu\nu} = 0 \quad \rightarrow \quad \begin{cases} \partial_\mu A^\mu = 0 \\ (\Box - m_\gamma^2)A^\mu = 0 \end{cases}$

- Numerical results on the gluon propagator from lattice QCD and OPE are reproduced by adding to the quantum effective action a term

$$\frac{m_g^2}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu}$$

$$F_{\mu\nu} = F^a_{\mu\nu}T^a\,, \quad \Box^{ab} = D^{ac}_\mu D^{\mu,cb}\,, \quad D^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$$

(Boucaud et al 2001,Capri et al 2005,Dudal et al 2008)

it is a nonlocal but gauge invariant mass term for the gluons, generated dynamically by strong IR effects

Is it possible that a mass is dynamically generated in GR in the IR?

difficult non-perturbative question. Some hints:

- Euclidean lattice gravity suggests dynamical generation of a mass m, and a running of G_N

$$G(k^2) = G_N \left[1 + \left(\frac{m^2}{k^2} \right)^{\frac{1}{2\nu}} + \mathcal{O}\left(\frac{m^2}{k^2} \right)^{\frac{1}{\nu}} \right]$$

$\nu \approx 1/3$

Hamber 1999, .., 2017

- recent results based on causal dynamical triangulation find in the quantum effective action a mass for the conformal mode, just as in the model that we had previously postulated

(Knorr and Saueressig PRL 2018)

- non-perturbative functional RG techniques find interesting fixed-point structure in the IR, and strong-gravity effects

Wetterich 2017

- functional RG again indicates the dynamical emergence of an IR scale, related to the conformal mode instability in Euclidean space

Morris 2018

the dynamical emergences of a mass scale in the IR in gravity is a meaningful working hypothesis

Our approach: we will postulate some nonlocal effective action, which depends on a mass scale, and is supposed to catch IR effects in GR

- phenomenological approach. Identify a non-local modification of GR that works well

- attempt at a more fundamental understanding

- massive photon: can be described replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \to \quad \left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

(Dvali 2006)

- for gravity, a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \to \quad \left(1 - \frac{m^2}{\Box_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

however this is not correct since $\nabla^\mu(\Box_g^{-1} G_{\mu\nu}) \neq 0$

we lose energy-momentum conservation

- to preserve energy-momentum conservation:

$$G_{\mu\nu} - m^2 (\Box^{-1} G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

(Jaccard,MM, Mitsou, 2013)

however, instabilities in the cosmological evolution (Foffa,MM, Mitsou, 2013)

- $$G_{\mu\nu} - m^2 (g_{\mu\nu} \Box^{-1} R)^T = 8\pi G T_{\mu\nu}$$

(MM 2013)

stable cosmological evolution! ``RT model"

- a related model: (MM and M.Mancarella, 2014)

$$S_{\rm NL} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - m^2 R \frac{1}{\Box^2} R \right]$$

``RR model"

rewrite it as

$$\Gamma_{\rm RR} = \int d^4x \sqrt{-g}\, \left[\frac{m_{\rm Pl}^2}{2} R - R \frac{\Lambda_{\rm RR}^4}{\Box^2} R \right]$$

``RR model''

- $\Lambda_{\rm RR}$ generated dynamically, analog to $\Lambda_{\rm QCD}$

- $g_{\mu\nu}(x) = e^{2\sigma(x)}\eta_{\mu\nu} \quad \to \quad R = -6\Box\sigma + \mathcal{O}(\sigma^2)$

  is a mass term for the conformal mode!

- $\Lambda_{\rm RR}^4 \sim m^2 m_{\rm Pl}^2 \qquad m \sim H_0 \to \Lambda_{\rm RR} \sim {\rm meV}$

  $\Lambda_{\rm RR}$ is the fundamental scale. No ultralight particle provides a solution to the naturalness problem

- causality ok once intepreted as derived from a quantum effective action (eqs of motion for the in-in expectation values)

- issues of degrees of freedom, ghosts, must be adressed at the level of the fundamental action

can be written as a local scalar-tensor theory in terms of auxiliary fields that, however, are not dynamical

(their initial conditions are fixed in terms of those of the metric) we have explored several other models but it turns out to be very difficult to construct viable models

- $m^2 R_{\mu\nu} \Box^{-2} R^{\mu\nu}$ unstable background evolution
- $m^2 C_{\mu\nu\rho\sigma} \Box^{-2} C^{\mu\nu\rho\sigma}$ unstable tensor perturbations

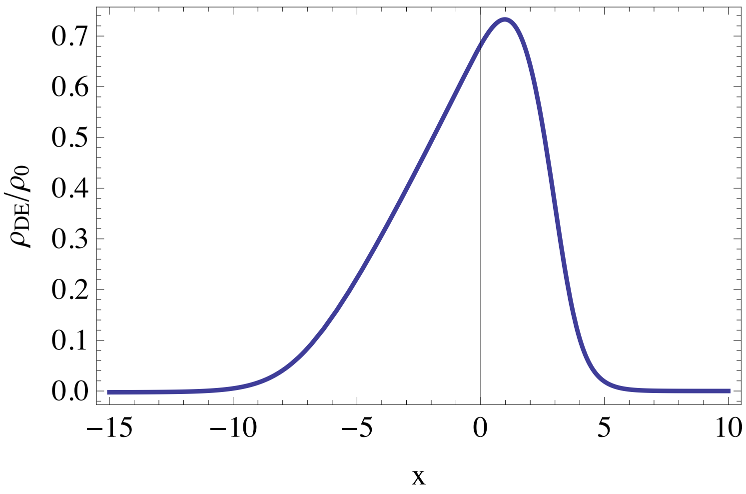
the models that work are those that correspond to a mass for the conformal mode!

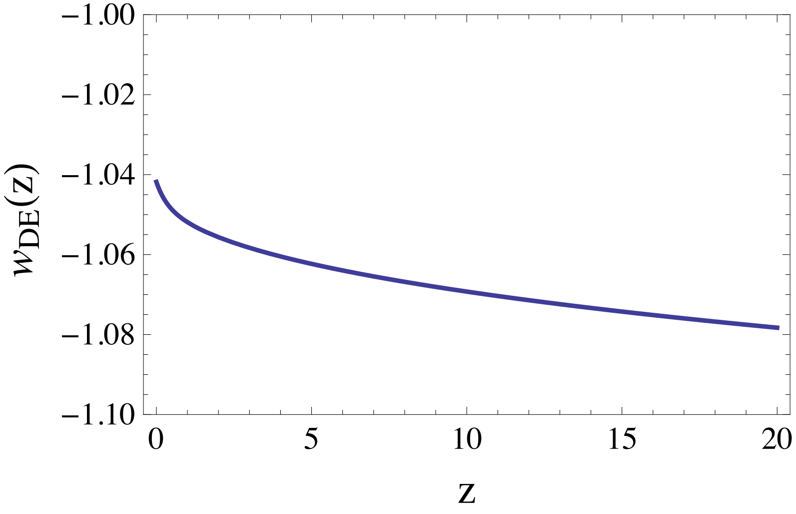
- $m^2 R \Delta_4^{-1} R$ $\quad \Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}g^{\mu\nu}\nabla_\mu R \nabla_\nu$ $\quad c_{GW} \neq c$!
- finally, even RR model recently ruled out by comaprison with Lunar Laser Ranging

Belgacem, Finke, Frassino, MM 2019
(Deser-Woodard model also ruled out)

Features of the RT model

- predicts a dynamical DE (with a phantom EoS) MM 2013





$$x \equiv \log a \qquad (x_{eq} \simeq -8.1)$$

$$\dot{\rho}_{DE} + 3(1 + w_{DE}) H \rho_{DE} = 0$$

Cosmological perturbations

- well-behaved? YES

Dirian, Foffa, Khosravi, Kunz, MM
JCAP 2014

this step is already non-trivial, see e.g. DGP or bigravity

- consistent with data? YES

- comparison with ΛCDM

Dirian, Foffa, Kunz, MM, Pettorino,
JCAP 2015, 2016
Dirian, 2017
Belgacem, Dirian, Foffa, MM 2018

implement the perturbations in a Boltzmann code
compute likelihood, χ^2, perform parameter estimation

- We test the non-local models against
  - Planck 2015 TT, TE, EE and lensing data,
  - isotropic and anisotropic BAO data,
  - JLA supernovae,
  - local H_0 measurements,
  - growth rate data

  and we perform Bayesian parameter estimation and model comparison
- we vary $\omega_b = \Omega_b h_0^2,\ \omega_c = \Omega_c h_0^2,\ H_0,\ A_s,\ n_s,\ z_{\rm re}$

  we have the same free parameters as in ΛCDM

the model turns out to fit the data at a level statistically equivalent to ΛCDM (actually slightly better)

Potentially distinguishable from ΛCDM with future cosmological observations (EUCLID, SKA, DESI ...)

predicts higher value of H_0

(RT: H_0= 68.9; ΛCDM: H_0= 67.9 with our CMB+BAO+SNe datasets)

- the model passes solar system tests (including LLR)
  - the limit m ➔ 0 is smooth (no vDVZ discontinuity)
  - G_{eff} reduces to G at short scales

- predicts GW propagation with c_{GW}=c (ok with GW170817)

- and predicts an interesting and novel effect in GW propagation leading to a GW luminosity distance, testable with ET and LISA

Belgacem, Dirian, Foffa, MM, PRD 2018a,2018b
Belgacem et al (LISA CosmoWG), to appear

Modified GW propagation

Belgacem, Dirian, Foffa, MM
PRD 2018, 1712.08108
and PRD 2018, 1805.08731

in GR : $$\tilde{h}''_A + 2\mathcal{H}\tilde{h}'_A + k^2\tilde{h}_A = 0$$

writing $$\tilde{h}_A(\eta, \mathbf{k}) = \frac{1}{a(\eta)}\tilde{\chi}_A(\eta, \mathbf{k})$$

we get $$\tilde{\chi}''_A + \left(k^2 - a''/a\right)\tilde{\chi}_A = 0$$

inside the horizon $a''/a \ll k^2$, so $\tilde{\chi}''_A + k^2\tilde{\chi}_A = 0$

1. GWs propagate at the speed of light
2. $h_A \propto 1/a$

   For coalescing binaries this gives $h_A \propto 1/d_L(z)$

In several modified gravity models:

$$\tilde{h}''_A + 2\mathcal{H}[1 - \delta(\eta)]\tilde{h}'_A + k^2\tilde{h}_A = 0$$

This appear to be completely generic in modified gravity:

(Belgacem et al., LISA CosmoWG, to appear)

- RR, RT non-local modifications of gravity
- DGP
- scalar-tensor theories (Brans-Dicke, Horndeski, DHOST,..)
- bigravity

the ``GW luminosity distance" is different from the standard (electromagnetic) luminosity distance !

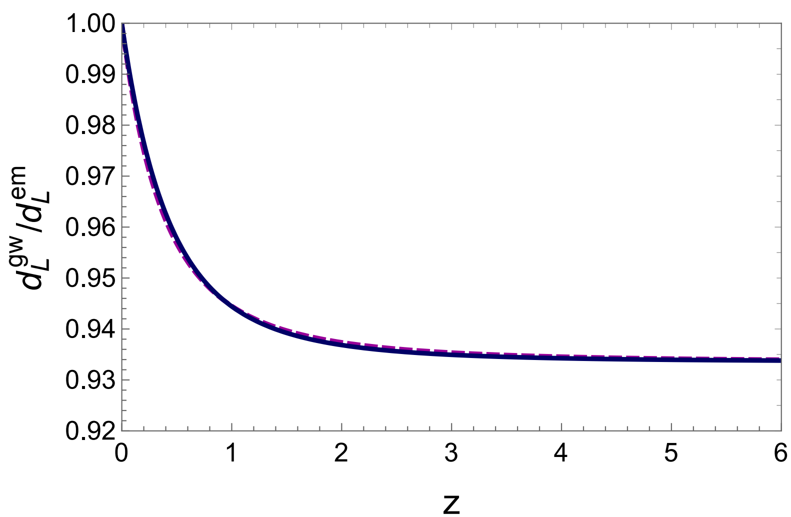
Deffayet and Menou 2007
Saltas et al 2014,
Lombriser and Taylor 2016,
Nishizawa 2017,
Belgacem et al 2017, 2018

in terms of δ(z) :

$$d_L^{\rm gw}(z) = d_L^{\rm em}(z) \exp\left\{ - \int_0^z \frac{dz'}{1+z'}\,\delta(z') \right\}$$

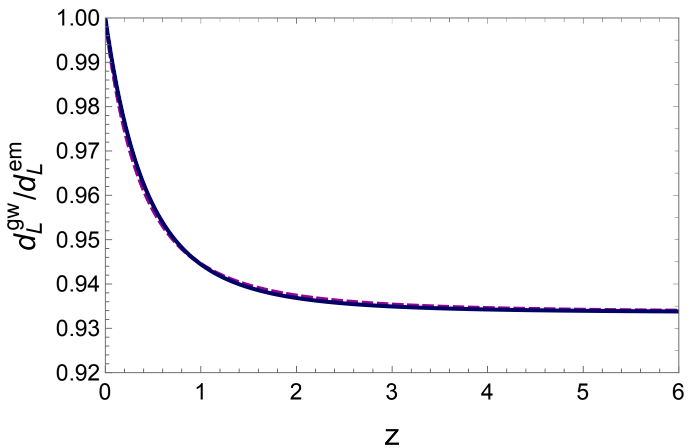
eg, prediction of the RT nonlocal model:

6% effect at z>1



a general parametrization of modified GW propagation

Belgacem, Dirian, Foffa, MM
PRD 2018, 1805.08731



$$\frac{d_L^{\rm gw}(z)}{d_L^{\rm em}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$

e.g. for the RT model:

$$\Xi_0 \simeq 0.934\,, \qquad n \simeq 2.6$$

However, the parametrization is very natural, and indeed we finds (LISA CosmoWG) that it fits the result of (almost) all modified gravity models

parametrizing extension of the DE sector:

background: (w_0, w_a); scalar pert: (Σ, μ) ; tensor pert: (Ξ_0, n)

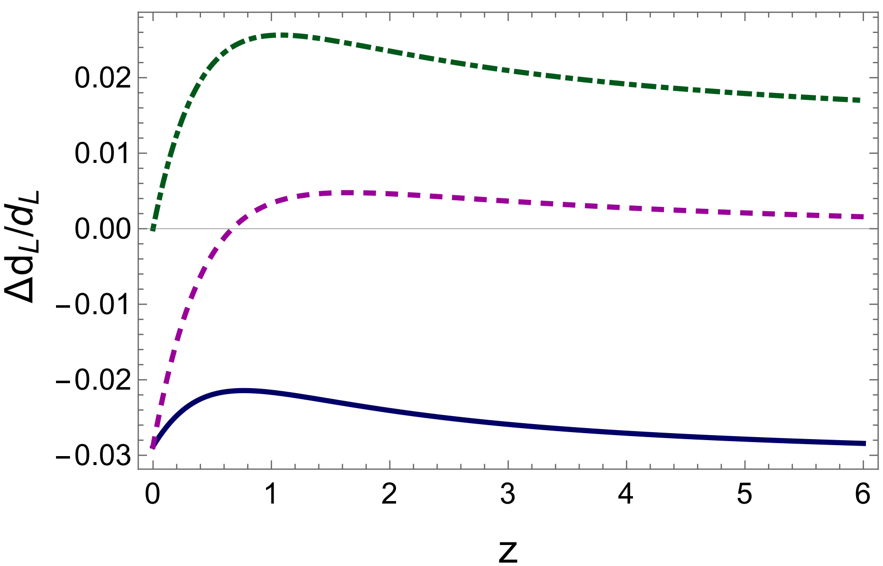
for standard sirens, the most important parameters are w_0, Ξ_0

at ET and LISA this propagation effect dominates over that from the dark energy EoS !

recall that

$$d_L(z) = \frac{1+z}{H_0} \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_M (1+\tilde{z})^3 + \rho_{\rm DE}(\tilde{z})/\rho_0}}$$

(neglect radiation for standard sirens)

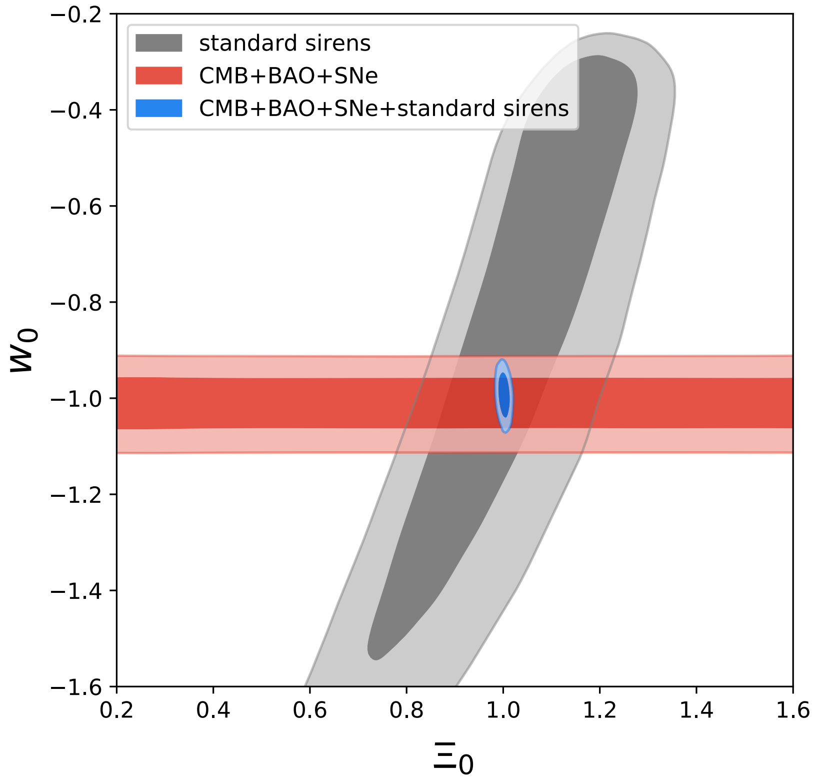


relative difference of e.m. luminosity distance RR-LCDM for the same values of Ω_M and H_0

relative difference with the respective best-fit parameters

relative difference of gw luminosity distance

Forecast for the Einstein Telescope



	Δw_0	$\Delta \Xi_0$
CMB+BAO+SNe+ET	0.032	0.008

with 10^3 standard sirens at ET, Ξ_0 can be measured to better than 1%

Some viable model predict up to 6% deviation

Conclusion: at the phenomenological level, this non-local model is quite satisfying

- solar system tests OK
- generates dynamically a dark energy
- cosmological perturbations work well
- comparison with CMB,SNe,BAO with modified Boltzmann code ok
- passes tests of structure formation
- higher value of H0
- speed of gravity ok
- same number of parameters as ΛCDM, and competitive with ΛCDM from the point of view of fitting the data
- testable predictions for near future cosmology experiments (Euclid/SKA/DESI) and GWs (LISA,ET)

Next step:

understanding from first principles where such non-local term comes from

Thank you!

Is the theory causal?

in a fundamental QFT nonlocality ➔ acausality. E.g.

$$\frac{\delta}{\delta\phi(x)}\int dx'\phi(x')(\Box^{-1}\phi)(x') = \frac{\delta}{\delta\phi(x)}\int dx'dx''\phi(x')G(x';x'')\phi(x'')$$

$$= \int dx'[G(x;x') + G(x';x)]\phi(x')$$

the quantum EA generates the eqs of motion of $\langle 0|g_{\mu\nu}|0\rangle$

$\langle 0_{\rm out}|g_{\mu\nu}|0_{\rm in}\rangle$ Feynman path integral, acausal eqs

$\langle 0_{\rm in}|g_{\mu\nu}|0_{\rm in}\rangle$ Schwinger-Keldish path integral, nonlocal but causal eqs

What are the dof of the theory?
Is there a ghost?

The RR model can be put into local form introducing two auxiliary fields

$$U = -\Box^{-1} R \,, \qquad S = -\Box^{-1} U$$

then the eqs of motion read

$$\begin{aligned} G_{\mu\nu} &= \frac{m^2}{6} K_{\mu\nu}(U, S) + 8\pi G T_{\mu\nu} \\ \Box U &= -R \\ \Box S &= -U \end{aligned}$$

$$\Lambda_{\rm RR}^4 = \frac{1}{12} m^2 m_{\rm Pl}^2$$

are U and S associated to real quanta? Then U would be a ghost

One cannot read the dof from the vacuum quantum effective action ! You need the fundamental theory

Example: Polyakov quantum EA in D=2

- is an example of the fact that we can get non-perturbative informations on the quantum EA

- since it can be computed exacly, it is useful to clarify conceptual aspects that create confusion in the literature

(dof, ghosts,propagating vs non-propagating fields, etc)

- consider gravity + N conformal matter fields in D=2

conformal anomaly: $$\langle 0|T^a_a|0\rangle = \frac{N}{24\pi}R$$

this result is exact. Only the 1-loop term contributes.

In D=2: $$g_{\mu\nu} = e^{2\sigma}\eta_{\mu\nu} \qquad R = -2\Box\sigma$$

$$\frac{\delta\Gamma}{\delta\sigma} = -\frac{N}{12\pi}\Box\sigma$$

$$\Gamma[\sigma] - \Gamma[\sigma=0] = -\frac{N}{24\pi}\int d^2x\, e^{2\sigma}\sigma\Box\sigma$$

in D=2, $\Gamma[0]=0$. This is the exact quantum EA we can covariantize $\Gamma[\sigma] = -\frac{N}{24\pi}\int d^2x\, e^{2\sigma}\sigma\Box\sigma$

using $\sqrt{-g} = e^{2\sigma}$

$$R = -2\Box\sigma$$

we get

$$\begin{aligned}\Gamma &= \frac{N}{48\pi}\int d^2x\,\sqrt{-g}\,\sigma R \\ &= -\frac{N}{96\pi}\int d^2x\,\sqrt{-g}\,R\Box^{-1}R\end{aligned}$$

Polyakov quantum effective action

$$\Gamma = \frac{N}{48\pi}\int d^2x\,\sqrt{-g}\,\sigma R$$
$$= -\frac{N}{96\pi}\int d^2x\,\sqrt{-g}\,R\Box^{-1}R$$

we could localize it introducing again $U = -\Box^{-1}R$

By definition $\Box U = -R$ ie $\Box U = 2\Box\sigma$

The most general solution is $U = 2\sigma + U_{\rm hom}$

however, only U=2σ gives back the original action

U is fixed in terms of the metric. There are no creation/annihilation operators associated to U

Similarly, consider

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$
$$= -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m_\gamma^2}{\Box}\right)F^{\mu\nu}$$

we could localize it, preserving gauge-invariance, introducing $U^{\mu\nu} = -\Box^{-1}F^{\mu\nu}$

however, there is no new dof associated to $U^{\mu\nu}$

The initial conditions on the original fields (metric,A_μ) fix in principle the initial conditions on the auxiliary fields

If we had a derivation of the RR model from a fundamental action, the inital conditions on the auxiliary fields would be fixed in terms of the initial conditions on the metric. However, we do not have a derivation. So, how do we choose in practice the initial conditions on U,S ?

in cosmology, at the background level initial conditions at early times are taken to be homogeneous: a priori the space of theories is parametrized by

$$\{U(t_0), \dot{U}(t_0), S(t_0), \dot{S}(t_0)\}$$

however, for the RR (and the RT) model the cosmological solutions are an attractor in this space : 3 stable directions, one marginal parameter $U=u_0$

at the level of cosmological perturbations, what are the initial conditions on the auxiliary fields?

recall that for the Polyakov action $U = -\Box^{-1} R = 2\sigma$

in this case, writing $U(t, \mathbf{x}) = \bar{U}(t) + \delta U(t, \mathbf{x})$

$$\bar{U}(t_{\rm in}) = \bar{\sigma}(t_{\rm in})$$

$$\delta U(t_{\rm in}, \mathbf{x}) = \delta\sigma(t_{\rm in}, \mathbf{x}) \,, \quad \delta\dot{U}(t_{\rm in}, \mathbf{x}) = \delta\dot{\sigma}(t_{\rm in}, \mathbf{x})$$

in general, δU, δV must vanish if the metric perturbation vanish

(they are not independent dof!)

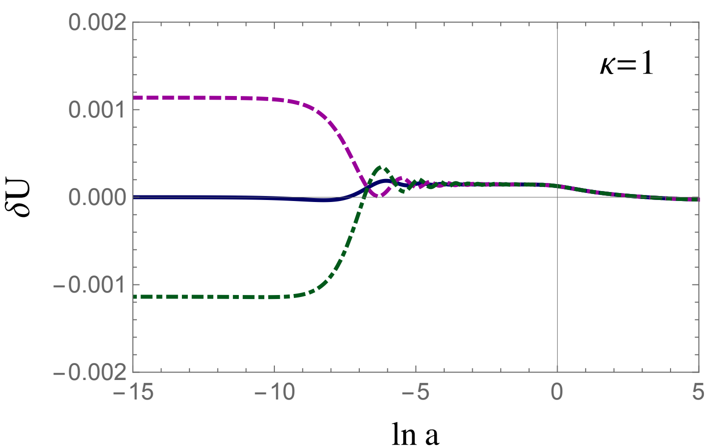
for the RR model we set

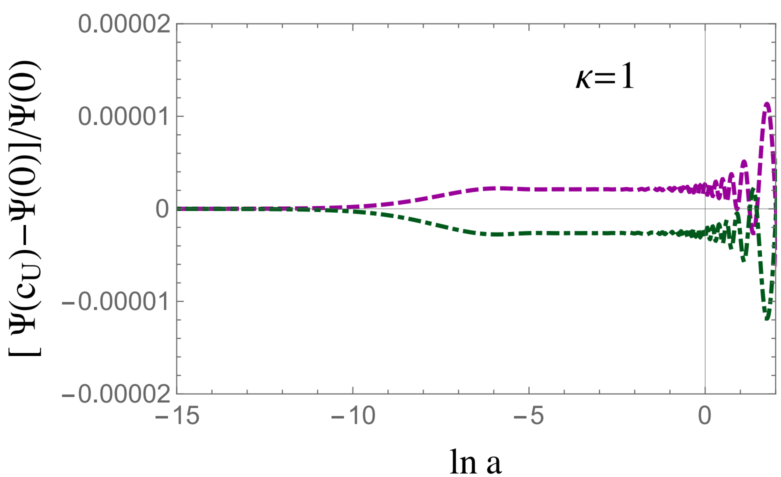
$$U(t_{\rm in},\mathbf{x}) = c_U\Phi(t_{\rm in},\mathbf{x})\,, \qquad U'(t_{\rm in},\mathbf{x}) = c_U\Phi'(t_{\rm in},\mathbf{x})\,,$$
$$V(t_{\rm in},\mathbf{x}) = c_V\Phi(t_{\rm in},\mathbf{x})\,, \qquad V'(t_{\rm in},\mathbf{x}) = c_V\Phi'(t_{\rm in},\mathbf{x})\,,$$

($V=H_0^2S$) and vary c_U,c_V at a level $-10<c_U,c_V<10$

result: very little dependence of the final results

Belgacem, Dirian, Foffa, MM 2018





Absence of vDVZ discontinuity

- the propagator is continuous for m=0

$$D^{\mu\nu\rho\sigma}(k) = \frac{i}{2k^2}\left(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}-\eta^{\mu\nu}\eta^{\rho\sigma}\right) + \frac{1}{6}\left(\frac{i}{k^2}-\frac{i}{k^2-m^2}\right)\eta^{\mu\nu}\eta^{\rho\sigma},$$

- write the eqs of motion of the non-local theory in spherical symmetry:

A. Kehagias and MM 2014

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

for mr <<1: low-mass expansion

for $r >> r_S$: Newtonian limit (perturbation over Minowski)

match the solutions for $r_S << r << m^{-1}$ (this fixes all coefficients)

- result: for $r \gg r_s$

$$A(r) = 1 - \frac{r_S}{r}\left[1 + \frac{1}{3}(1 - \cos mr)\right]$$

$$B(r) = 1 + \frac{r_S}{r}\left[1 - \frac{1}{3}(1 - \cos mr - mr \sin mr)\right]$$

for $r_s \ll r \ll m^{-1}$: $A(r) \simeq 1 - \frac{r_S}{r}\left(1 + \frac{m^2 r^2}{6}\right)$

the limit $m \to 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3}\frac{r_S}{r}\left(1 - \frac{r_S}{12 m^4 r^5}\right)$$

vDVZ discontinuity

breakdown of linearity below $r_V = (r_s/m^\wedge 4)^{1/5}$

Background evolution

- consider $$\Gamma_{\rm RR} = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{6} R \frac{1}{\Box^2} R \right]$$

localize using $$U = -\Box^{-1} R \,, \quad S = -\Box^{-1} U$$

in FRW we have 3 variables: H(t), U(t), W(t)=H^2(t)S(t).

define $x = \ln a(t)$, $h(x) = H(x)/H_0$,

$\gamma = (m/3H_0)^2$ $\zeta(x) = h'(x)/h(x)$

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$
$$W'' + 3(1-\zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U$$

- there is an effective DE term, with

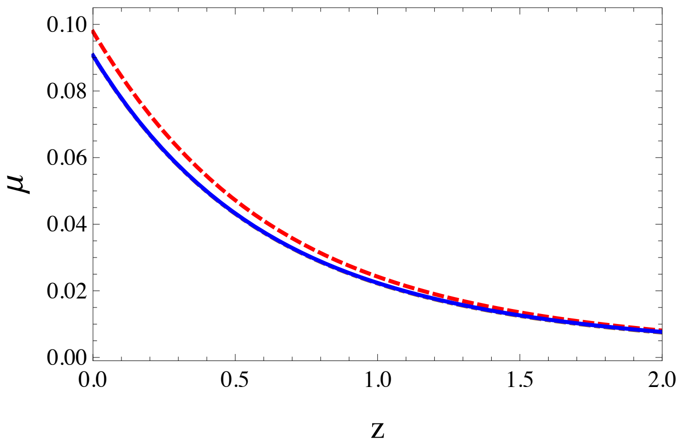
$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad \qquad \rho_0 = 3H_0^2/(8\pi G)$$

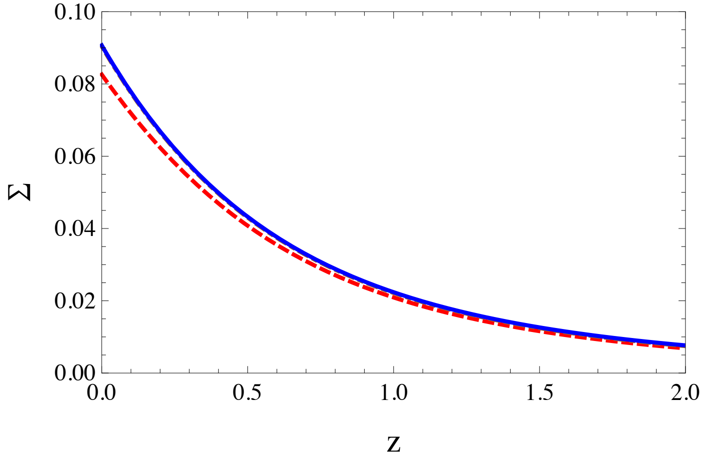
- define w_{DE} from $\dot{\rho}_{\rm DE} + 3(1 + w_{\rm DE}) H \rho_{\rm DE} = 0$

- the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$ (plus the inital conditions, parametrized by u_0, c_U, c_W)

- the perturbations are well-behaved and differ from ΛCDM at a few percent level

$$\Psi = [1 + \mu(a;k)]\Psi_{\rm GR}$$
$$\Psi - \Phi = [1 + \Sigma(a;k)](\Psi - \Phi)_{\rm GR}$$

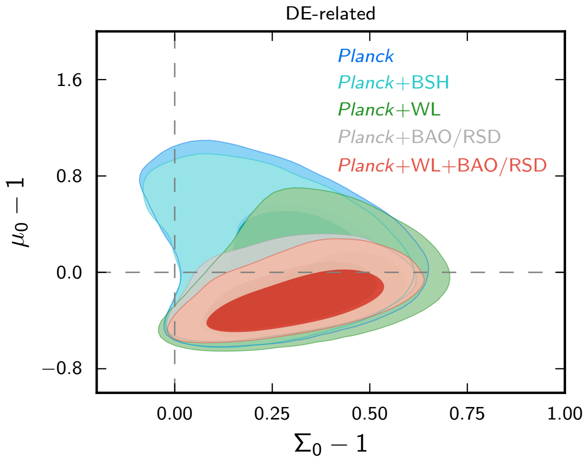




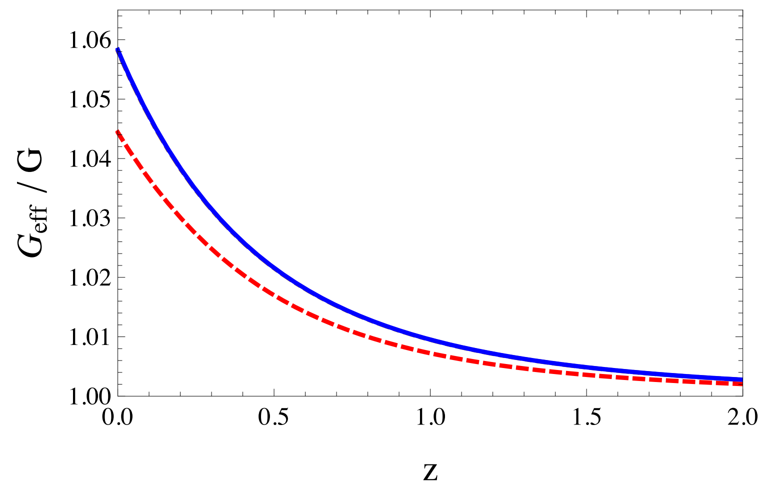
- deviations at z=0.5 of order 4%

- consistent with data:

(Ade et al., Planck XV, 2015)

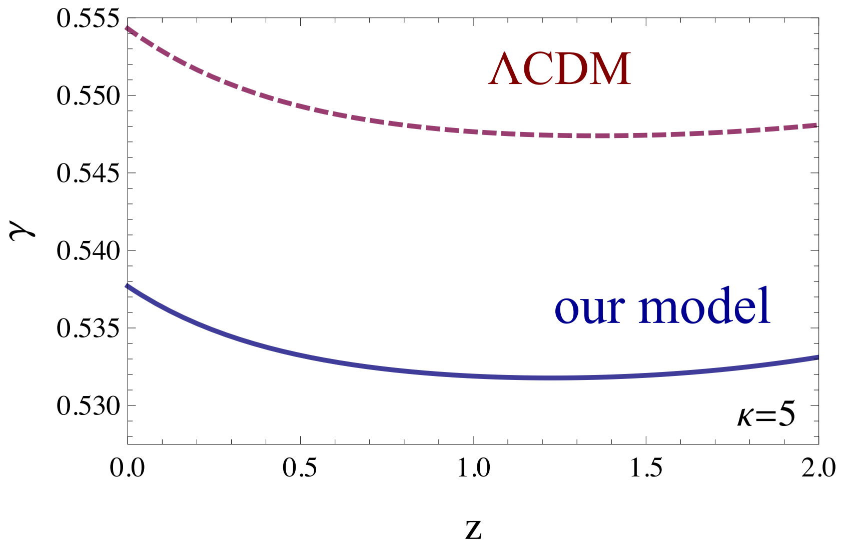


effective Newton constant (RR model)



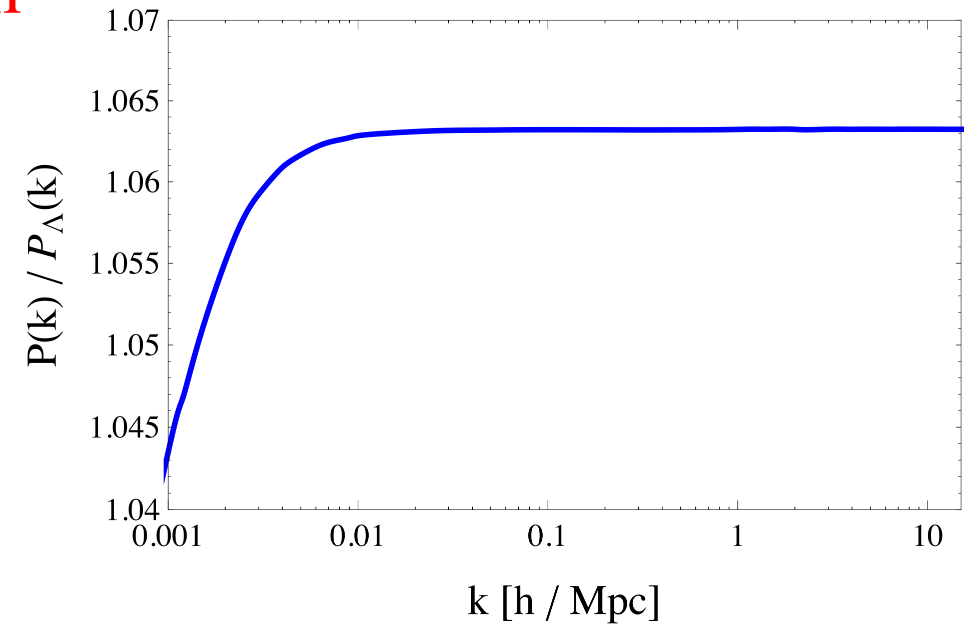
growth index:

$$\frac{d\log\delta_M(a;k)}{d\ln a} = [\Omega_M]^{\gamma(z;k)}$$



- linear power spectrum

matter power spectrum
compared to ΛCDM



DE clusters but its linear
power spectrum is small
compared to that of matter