Non-dynamical torsion from fermions and CMBR phenomenology

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based on

P. Dona & A. Marciano, arXiv:1605.09337 (PRD 2016)

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848 (EPJC 2019)

works in progress…

Cargese, 6th-11th of May 2019

Separate Universe assumption

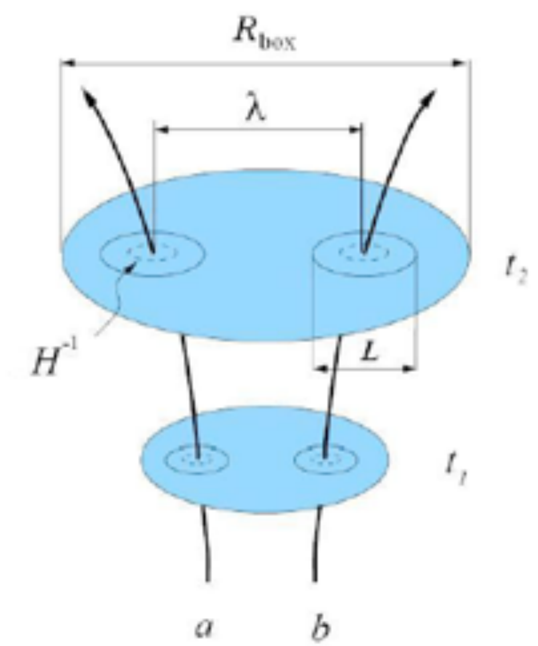
$$ds^2 = N^2 dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

ADM decomposition

$$K_{ij} = -\nabla_{(j} n_{i)} = \frac{1}{2N}\left(-\partial_t \gamma_{ij} + {}^{(3)}\nabla_{(i}N_{j)} + {}^{(3)}\nabla_{(j}N_{i)}\right)$$

$$K_{ij} = -\frac{\theta}{3}\gamma_{ij} + A_{ij} \qquad \theta = \frac{3}{N}\left(\frac{\dot a(t)}{a(t)} + \dot\psi\right)$$

$$N(t_1, t_2; x_j) = \frac{1}{3}\int_{\gamma(\tau)} \theta d\tau, \qquad n_\mu = (N, 0)$$



Perturbations

$$\epsilon = k/(a\,H)$$

$$\gamma_{ij} = a(t, x_i)\,\tilde\gamma_{ij}, \quad \tilde\gamma_{ij} = (e^h)_{ij}, \quad a(t, x_i) = a(t)\,e^{\psi(t, x_i)}$$

Curvature perturbation variable

Lyth, Malik & Sasaki, JCAP 2015

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\frac{d\rho(t,x_i)}{dt} + 3\tilde{H}(t,x_i)[\rho(t,x_i) + p(t,x_i)] = 0$$

$$\frac{d\rho(t)}{dt} + 3\frac{\dot{a}(t)}{a(t)}[\rho(t) + p(t)] = \dot{\psi}(t)$$

Uniform density slicing & adiabatic pressure

$$\psi(t_2, x^i) - \psi(t_1, x^i) = -\ln\left[\frac{a(t_2)}{a(t_1)}\right] - \frac{1}{3}\int_{\rho(t_1,x^i)}^{\rho(t_2,x^i)} \frac{d\rho}{\rho + p}$$

$$-\zeta(x^i) = \psi(t, x^i) + \frac{\delta\rho}{3(\rho + p)}$$

Macroscopic quantum states of matter I

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

I Classical background fields correspond to expectation values on macroscopic (condensed) states

$$\phi(x) := \langle\alpha|\hat{\phi}|\alpha\rangle$$

II Matter perturbations are evaluated as the the first order expansion of the expectation values on perturbed macroscopic states

$$\delta\phi(x) := \langle\alpha+\delta\alpha|\hat{\phi}|\alpha+\delta\alpha\rangle|_{O(\delta\alpha)}$$

Macroscopic quantum states of matter II

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

III **Density matrix and infrared mode of the macroscopic state**

$$\rho_{1-\mathrm{p}}(x-x') = \int_{k,k'} e^{-\imath(kx-k'x')}\langle a_k^\dagger a_{k'}\rangle$$

$$\rho_{1-\mathrm{p}}(t-t';\vec{x}-\vec{x}') = \frac{N_0}{V} + \int_k e^{-\imath\vec{k}\cdot(\vec{x}-\vec{x}')}\, n(k)$$

II **Off-diagonal long ranged order (ODLRO) and vanishing of correlations at large space-time distances**

$$\lim_{||x-x'||\to\infty} \rho_{1-\mathrm{p}}(t-t';\vec{x}-\vec{x}') = \langle\phi(x)\phi^\dagger(x')\rangle_0 \equiv n_0$$

Bosonic statistics and coherent states

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

$$\phi\left(x\right)=\int\frac{d^3k}{\left(2\pi\right)^3}\frac{1}{\sqrt{2E_k}}\left(a_k e^{-ikx}+a_k^\dagger e^{+ikx}\right)$$

Scalar field

Bosonic Hilbert space and infinite occupation numbers

$$\left|\alpha\right\rangle\equiv\prod_k\left|\alpha\left(k\right)\right\rangle=\prod_k e^{\alpha(k)a_k^\dagger-\alpha^*(k)a_k}\left|0\right\rangle=D\left(\alpha\right)\left|0\right\rangle$$

Displacement operator

$$D\left(\alpha\right)^\dagger\phi\left(x\right)D\left(\alpha\right)=\phi\left(x\right)+\phi_\alpha\left(x\right)$$

$$\left\langle\alpha\right|\mathcal{O}\left(\phi\left(x\right)\right)\left|\alpha\right\rangle=\left\langle0\right|\mathcal{O}\left(\phi\left(x\right)+\phi_\alpha\left(x\right)\right)\left|0\right\rangle$$

Matter perturbations at linear order

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \alpha + \delta\alpha | \widehat{T_{\mu\nu}(\phi)} | \alpha + \delta\alpha \rangle \Big|_{O(\delta\alpha)}$$

Expanding perturbations in the conservation equation

$$3(\zeta + \psi)\langle \alpha | \widehat{\rho} + \widehat{p} \,|\alpha\rangle = -\langle \alpha + \delta\alpha | \widehat{\rho} | \alpha + \delta\alpha \rangle \Big|_{O(\delta\alpha)}$$

Example: Chaotic Inflation

$$\langle \alpha + \delta\alpha |\, \widehat{\rho}\, | \alpha + \delta\alpha \rangle = \lim_{x \to y} \frac{1}{2} m^2 \, \langle \alpha + \delta\alpha |\, \widehat{\phi}(x)\, \widehat{\phi}(y)\, | \alpha + \delta\alpha \rangle = \frac{1}{2} m^2 \left[\phi_{\alpha+\delta\alpha}(x) \right]^2$$

Power spectrum of scalar perturbations

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

$$-\widehat{\Xi} = \widehat{\mathbb{1}}\,\psi(t, x^i) + \frac{\widehat{\rho}}{3\langle\alpha|\widehat{\rho} + \widehat{p}\,|\alpha\rangle}$$

Slow-roll condition

$$\langle\alpha|\ddot{\widehat{\phi}} + 3H\dot{\widehat{\phi}} + \widehat{V'(\phi)}|\alpha\rangle = 0 \quad \longrightarrow \quad 3H\phi_\alpha \simeq -V(\phi_\alpha)$$

Power spectrum

$$\mathcal{P}_\zeta = \lim_{x\to y} \langle\alpha + \delta\alpha|\,\widehat{\Xi}(x)\,\widehat{\Xi}(y)\,|\alpha + \delta\alpha\rangle\Big|_{O(\delta\alpha^2)}$$

Fermion fields and linear perturbations

Alexander, Brandenberger, Calcagni, Hui, Nicolis, Piazza, Prokopec, Sasaki, etc…

I **Pressure perturbations (non adiabatic) and conservation of curvature perturbations**

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{\rm na}$$

II **A no-go argument:**

$$\delta\phi \rightarrow \delta(\bar{\psi}\psi) = \delta\bar{\psi}\,\psi + \bar{\psi}\,\delta\psi$$

$$\psi(t) = \langle\alpha|\hat{\psi}|\alpha\rangle = \langle\alpha|R^\dagger(\varphi)\,R(\varphi)\hat{\psi}R^\dagger(\varphi)\,R(\varphi)|\alpha\rangle|_{\varphi=2\pi} = -\psi(t)$$

Fermion fields & macroscopic coherent states I

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

Pauli exclusion principle and quasi particles

$$a_k^\dagger \qquad \rightarrow \qquad c_k^\dagger = a_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger$$

BCS states as macroscopic coherent states

$$|\alpha\rangle \equiv e^{\int d^3k\, \alpha(k) c_k^\dagger - \alpha^*(k) c_k}\, |0\rangle \; = D(\alpha)\, |0\rangle$$

$$J_1 = \frac{1}{2}\left(a^\dagger b^\dagger + h.c.\right), \quad J_2 = -\frac{i}{2}\left(a^\dagger b^\dagger - h.c.\right), \quad J_3 = \frac{1}{2}\left(a^\dagger a + b^\dagger b - 1\right), \quad [J_i,\ J_j] = i\epsilon_{ij}{}^k J_k$$

BCS states are SU(2) coherent states

$$|\hat{n}\rangle = D(\hat{n})\, |j, -j\rangle = |\xi\rangle = \exp\left(\xi J^+ - \bar{\xi} J^-\right) |j, -j\rangle$$

Fermion fields & macroscopic coherent states II

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

Linear perturbations and SU(2) rotations

$$\langle \hat{n} | \bar{\psi} \psi | \hat{n} \rangle = \int_k \vec{\zeta}_k \cdot \langle \hat{n} | \vec{J}_k | \hat{n} \rangle = \int_k \vec{\zeta}_k \cdot \hat{n}_k$$

$$|\hat{n} + \delta\hat{n}\rangle \equiv D(\delta\hat{n}) |\hat{n}\rangle = |R(\hat{z}, \delta\hat{n})\, \hat{n}\rangle$$

$$\langle \hat{n} + \delta\hat{n} | \vec{J} | \hat{n} + \delta\hat{n} \rangle \approx \hat{n} + \delta\hat{n} \times \hat{n} = \hat{n} - \hat{n} \times \langle \delta\hat{n} | \vec{J} | \delta\hat{n} \rangle$$

Cargese, 6th-11th of May 2019

Bogolubov transformations & non-BD states

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

Adjoint action of displacement operators

$$D(\alpha)^\dagger a_k D(\alpha) = a_k + \alpha(k)$$

$$D(\alpha)^\dagger a_k^\dagger D(\alpha) = a_k^\dagger + \alpha^*(k)$$

U(1) bosonic case

$$\tilde{a} = \cos(|\xi|)\, a + \frac{\xi}{|\xi|} \sin(|\xi|)\, b^\dagger$$

$$\tilde{b}^\dagger = \cos(|\xi|)\, a - \frac{\bar{\xi}}{|\xi|} \sin(|\xi|)\, b^\dagger$$

SU(2) fermionic case

The macroscopic state obtained is the Bogolubov transform of the vacuum

Gravity with non-dynamical torsion I

Rovelli & Perez, CQG 2005; Freidel & Minic, PRD 2005

$$\mathcal{S}_{\mathrm{Holst}} = \frac{1}{2\kappa}\int_M d^4x\;|e|\;e^\mu_I e^\nu_J P^{IJ}{}_{KL} F_{\mu\nu}{}^{KL}(\omega) \qquad P^{IJ}{}_{KL} = \delta^{[I}_K\delta^{J]}_L - \epsilon^{IJ}{}_{KL}/(2\gamma)$$

$$\mathcal{S}_{\mathrm{Dirac}} = \int_M d^4x\;|e|\left\{\frac{1}{2}\left[\overline{\psi}\gamma^I e^\mu_I\left(1-\frac{\imath}{\alpha}\gamma_5\right)\imath\nabla_\mu\psi - m\overline{\psi}\psi\right] + \mathrm{h.c.}\right\}$$

Theory with torsion!

[Alexander, Biswas, Cai, Magueijo, Prokopec, Kibble, Poplawski…]

Gravity with non-dynamical torsion II

S. Alexander, Y. Cai & A. Marciano PLB 2015

Theory with torsion

$$e_I^\mu \, C_{\mu JK} = \frac{\kappa}{4} \, \frac{\gamma}{\gamma^2+1} \, \left(\beta \, \epsilon_{IJKL} \, J^L - 2\theta \, \eta_{I[J} \, J_{K]}\right)$$

$$J^L = \overline{\psi}\gamma^L\gamma_5\psi$$

$$\mathcal{S}_{GR} = \frac{1}{2\kappa}\int_M d^4x|e|e_I^\mu e_J^\nu R_{\mu\nu}^{IJ}$$

$$\mathcal{S}_{\rm Dirac} = \frac{1}{2}\int_M d^4x|e|\left(\,\overline{\psi}\gamma^I e_I^\mu \imath\widetilde{\nabla}_\mu\psi - m\overline{\psi}\psi\right) + {\rm h.c.}$$

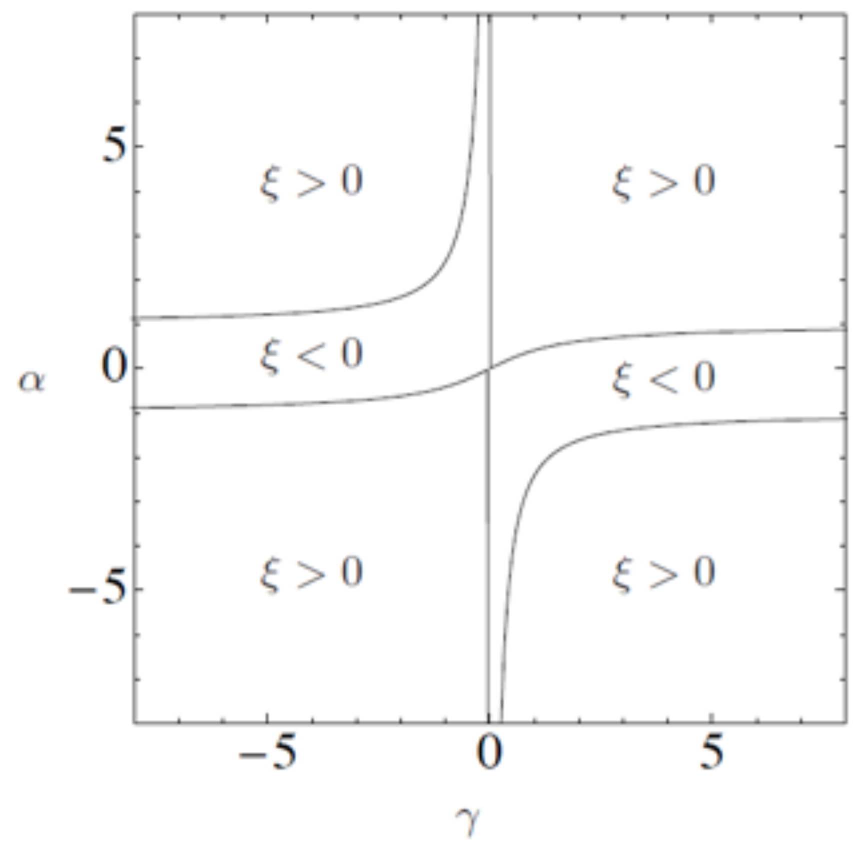
$$\mathcal{S}_{\rm Int} = -\xi\kappa\int_M d^4x|e|\, J^L \, J^M \, \eta_{LM}$$

Gravity with non-dynamical torsion III

S. Alexander, C. Bambi, A. Marciano & L. Modesto, [arXiv:1402.5880] PRD 90 (2014) 123510

A. Addazi, S. Alexander, Y. Cai & A. Marciano, arXiv: 1712.04848 (CPC 2018)

A. Addazi & A. Marciano, arXiv:1810.05513



$$\xi = \frac{3}{16}\frac{\gamma^2}{\gamma^2+1}\left(1+\frac{2}{\alpha\gamma}-\frac{1}{\alpha^2}\right)$$

NJL mechanism applied to SM fermions

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

$$\int \sqrt{-g} d^4x \bar{\Psi}(\imath\gamma^\mu(x)\nabla_\mu - M)\Psi + \frac{\lambda}{2N_f}[(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + (\bar{\Psi}\imath\gamma_5\Psi)(\bar{\Psi}\imath\gamma_5\Psi)]$$

N_f number of fermions of the SM

$\lambda = \xi\kappa$

$$\downarrow$$

$$\int \sqrt{-g}\, d^4x\, [\bar{\Psi}\imath\gamma^\mu(x)\nabla_\mu\Psi - \frac{N_f}{2\lambda}(|\Pi|^2 + |\Sigma|^2) - \bar{\Psi}(\Sigma + \imath\gamma_5\Pi)\Psi]$$

Effective potential I

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

Integrating out fermionic DOF

$$S_{\rm eff} = \int d^4x\sqrt{-g}\{-\frac{1}{\lambda}(|\Pi|^2+|\Sigma|^2)$$
$$-\imath \ln \mathrm{Det}\{\imath I \gamma^\mu(x)\nabla_\mu - (\Sigma + \imath\gamma_5\Pi)\}\}$$

with negligible corrections controlled by the number of fermions

$$S_{\rm eff}[\Pi,\Sigma] + O(1/N_f)$$

$$\downarrow$$

$$V(\Pi,\Sigma) = \frac{1}{2\lambda}(|\Pi|^2+|\Sigma|^2) + \imath \mathrm{Tr}\ln\langle x|\imath\gamma^\mu(x) I \nabla_\mu - (\Sigma+\imath\gamma_5\Pi)|z\rangle$$

Effective matrix potential

Effective potential II

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

Π, Σ classical slowly varying fields

Introduce $A = \Sigma + \imath\gamma_5\Pi$ and estimate the trace within the proper time method

$$V = \frac{1}{2\lambda}(|\Pi|^2 + |\Sigma|^2) - \imath \mathrm{Tr} \ln S(x, x, A)$$

$$S(x, y; A) = \langle x|(\imath I\gamma^\mu \nabla_\mu - A)^{-1}|y\rangle$$

where the propagator is associate the classical matrix equation

$$(\imath I\gamma^\mu(x)\nabla_\mu - A)S(x, y; A) = I\frac{1}{\sqrt{-g(x)}}\delta^4(x - y)$$

Effective potential III

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

The calculate the propagator, one performed the background expansion $A = \bar{A} + \delta A$

$$\ln \mathrm{Det}\,\{\imath I \gamma^\mu(x)\nabla_\mu - A\} = \mathrm{Tr}\ln\{\imath I\gamma^\mu\nabla_\mu - A\}$$

$$= \mathrm{Tr}\ln\{\imath I\gamma^\mu(x)\nabla_\mu - A\} - \int d^4\mathrm{Tr}\{\delta A(x)\, S_F(x,x)\}$$

$$-\frac{1}{2}\int d^4x \int d^4y\, \delta A(x)\, S_F(x,y)\, \delta A(y)\, S_F(y,x) + \ldots$$

with fermion propagator

$$\sqrt{-g}(\imath I\gamma^\mu(x)\nabla_\mu - M)S_F(x,y) = \imath\delta^4(x-y)I$$

Effective potential IV

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

In the large N one can calculate the bubble diagram x into x

$$S(x,x;A) = \int \frac{d^4q}{(2\pi)^4}\Big[(I\gamma^a q_a + A)\frac{1}{q^2-|A|^2} - \frac{1}{12}R(I\gamma^a q_a + A)\frac{1}{(q^2-|A|^2)^2} + \frac{2}{3}R_{\mu\nu}q^\mu q^\nu(I\gamma^a q_a + A)\frac{1}{(q^2-|A|^2)^3} - \frac{1}{8}\gamma^a[\gamma^c,\gamma^d]R_{cda\mu}q^\mu\frac{1}{(q^2-|A|^2)^2}\Big]$$

within the weakly varying curvature approximation $\dot{R} \simeq 0$

$$V(A) = \tilde{V}(A) - \frac{1}{(4\pi)^2}\frac{R}{6}\left[-|A|^2\ln\left(1+\frac{\Lambda^2}{|A|^2}\right) + \frac{\Lambda^2|A|^2}{\Lambda^2+|A|^2}\right]$$

$$\tilde{V} = V_0 + \frac{1}{2\lambda}|A|^2 - \frac{1}{4\pi^2}\left[|A|^2\Lambda^2 + \Lambda^4\ln\left(1+\frac{|A|^2}{\Lambda^2}\right) - |A|^4\ln\left(1+\frac{\Lambda^2}{|A|^2}\right)\right]$$

Inflaton from SM fermions

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

In FLRW, impose either a custodial global symmetry or a gauge flavor symmetry

↓

$$V(a) = V_0 + \frac{1}{2\lambda}|a|^2$$
$$-\frac{1}{4\pi^2}\left[|a|^2\Lambda^2 + \Lambda^4 \ln\left(1+\frac{|a|^2}{\Lambda^2}\right) - |a|^4 \ln\left(1+\frac{\Lambda^2}{|a|^2}\right)\right]$$
$$-\frac{1}{(4\pi)^2}(\dot{H}+2H^2)\left[-|a|^2 \ln\left(1+\frac{\Lambda^2}{|a|^2}\right) + \frac{\Lambda^2|a|^2}{\Lambda^2+|a|^2}\right],$$

$$\Lambda^2 = c(\xi\kappa)^{-1}$$

↓

$$\frac{\epsilon[a]}{M_{Pl}^2} = \frac{1}{2}\left(\frac{V'[a]}{V[a]}\right)^2, \quad \frac{\eta[a]}{M_{Pl}^2} = \frac{V''[a]}{V[a]}$$

Consistency with data on CMBR

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

$$\Delta_R^2 \simeq \frac{V_0}{24\pi^2 M_{Pl}^4 \epsilon} \qquad \Delta^2_{R,\,\mathrm{exp}} = 2,215 \times 10^{-9}$$

$$\epsilon = 1/(2N/3+1)^{3/2} \qquad \eta = -1/(2N/3+1)$$

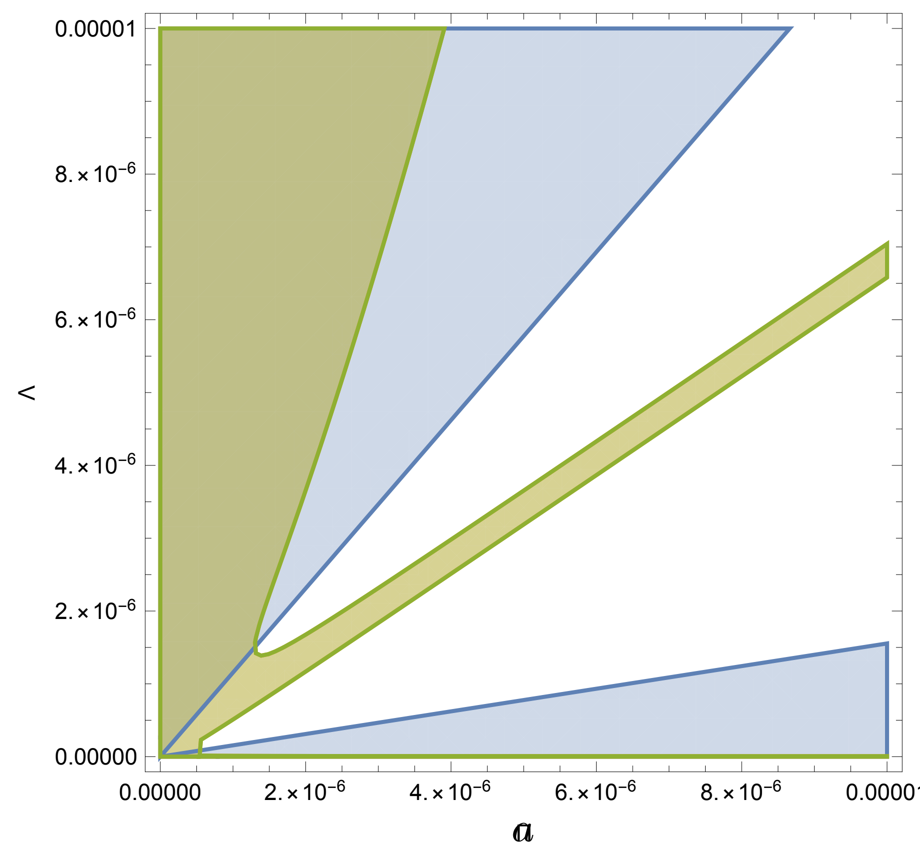
$$n_s - 1 = -2\,\epsilon - \eta \qquad n_{s,\mathrm{exp}} = 0.968 \pm 0.006$$

$$N = \frac{1}{M_{Pl}^2} \int_{\phi_{end}}^{\phi} d\phi \frac{V'}{V} \simeq 60 \qquad \Lambda \simeq 2 \times 10^{15} \div 10^{16}\ \mathrm{GeV}$$

$$\epsilon \simeq 0.003 \qquad |\eta| \simeq 0.02$$

Predictions on primordial tensor spectrum

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848



$$\lambda^{-1} = \frac{\Lambda^2}{2\pi^2}$$

$$\Lambda \simeq 2 \times 10^{15} \div 10^{16} \text{ GeV}$$

↓

$$\mathcal{P}_T = \frac{2H_{inf}^2}{\pi^2 M_{Pl}^2} \simeq \frac{2V_0}{3\pi M_{Pl}^4} \sim 10^{-13} \div 10^{-11} \longrightarrow r = 10^{-4} \div 10^{-2}$$

Critical branches

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

There exist two critical branches for compatibility with data

I) $$\lambda^{-1} = \frac{\Lambda^2}{2\pi^2}$$

$$\Lambda \simeq 2 \times 10^{15} \div 10^{16}\,\text{GeV} \qquad V_0 = \Lambda^4$$

II) $$\lambda^{-1} = \frac{\Lambda^2}{\pi^2}$$

$$|a| \ll \Lambda$$

Reheating mechanism and graceful exit

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

For the second critical branch, perturbative reheating can be easily achieved

$|a| \ll \Lambda$

$$V(a) = V_0 + \frac{1}{2\lambda}|a|^2 - \frac{1}{4\pi^2}\left[|a|^2\Lambda^2 + \Lambda^4 \ln\left(1+\frac{|a|^2}{\Lambda^2}\right) - |a|^4 \ln\left(1+\frac{\Lambda^2}{|a|^2}\right)\right] - \frac{1}{(4\pi)^2}(\dot{H}+2H^2)\left[-|a|^2 \ln\left(1+\frac{\Lambda^2}{|a|^2}\right) + \frac{\Lambda^2|a|^2}{\Lambda^2+|a|^2}\right],$$

$\Lambda^2 = c(\xi\kappa)^{-1}$

Quartic term

Quadratic term

The model converge to the form of the Coleman-Weinberg potential for $|a| \ll \Lambda$

Graceful exit mechanism from inflation, with a reliable re-heating mechanism

- A. Cerioni, F. Finelli, A. Tronconi and G. Venturi, Phys. Lett. B 2009
- G. Barenboim, E. J. Chun and H. M. Lee, Phys. Lett. B 2014

Conclusions

i) Fermionic matter cosmological perturbations

ii) Spinorial perturbations entail non-isotropic d.o.f. not present in the scalar field perturbations

iii) Spinorial contributions can be recast in terms of a multi-field approach in inflationary scenarios

iv) Inflation can be sourced by an infrared collective mode generated by condensation of SM fermions

v) Constraints are satisfied, but phenomenology is reacher

vi) A falsifiable prediction for r can be provided

Merci

Thank you!

Grazie!

谢谢

Cargese, 6th-11th of May 2019

More on QCD relaxation phenomena on Friday!

Andrea Addazi

Cargese, 6th-11th of May 2019

The effective four-fermion action casts in the large N-approximation as

$$S = \frac{1}{2\kappa}\int d^4x|e|\overline{\Psi}\left(2\gamma^I e_I^\mu \nabla_\mu - M\right)\Psi + \frac{1}{4}\frac{\xi\kappa}{N_f}\int d^4x|e|\left(\overline{\Psi}\gamma^5\gamma^L\Psi\right)\left(\overline{\Psi}\gamma^5\gamma_L\Psi\right)$$

$$= \frac{1}{2\kappa}\int d^4x|e|\overline{\Psi}\left(2\gamma^I e_I^\mu \nabla_\mu - M\right)\Psi + \frac{1}{4}\frac{\xi\kappa}{N_f}\int d^4x|e|\left[(\overline{\Psi}\Psi)^2 + \left(\overline{\Psi}\iota\gamma_5\Psi\right)\left(\overline{\Psi}\iota\gamma_5\Psi\right) + \left(\overline{\Psi}\gamma^L\Psi\right)\left(\overline{\Psi}\gamma_L\Psi\right)\right]$$

$$= \int \sqrt{-g}d^4x\overline{\Psi}\left(2\gamma^\mu(x)\nabla_\mu - M\right)\Psi + \frac{\lambda}{2N_f}\left[(\overline{\Psi}\Psi)^2 + \left(\overline{\Psi}\iota\gamma_5\Psi\right)\left(\overline{\Psi}\iota\gamma_5\Psi\right)\right]$$

By introducing auxiliary fields Π. The total action can be as

$$S = S_{EH} + S_\Pi$$

$$S_\Pi = \int \sqrt{-g}d^4x\left[\overline{\Psi}\gamma^\mu(x)\nabla_\mu\Psi - \frac{N_f}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) - \overline{\Psi}\left(\Sigma + \iota\gamma_5\Pi\right)\Psi\right]$$

The generating functional of correlation functions

$$Z[\eta,\bar{\eta}] \equiv \int \mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\Pi\mathcal{D}\Sigma \exp\left[\iota S + \iota\int dx^4\left[\bar{\eta}(x)\Psi(x) + \eta(x)\overline{\Psi}(x)\right]\right]$$
$$= \int \mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\Pi\mathcal{D}\Sigma e^{\iota S + \iota\bar{\eta}\Psi + \iota\overline{\Psi}\eta}$$

where η, $\bar{\eta}$ are grassmannian source functions, setting the sources to zero $\eta = \bar{\eta} = 0$, we get effective partition function

$$Z[0,0] = \int \mathcal{D}\Pi\mathcal{D}\Sigma \exp iN_f\left\{\int\int\sqrt{-g}d^4x\left[-\frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) + (\Sigma + \iota\gamma_5\Pi)(\eta + \bar{\eta})\right] - i\ln Det\left[\iota\gamma^\mu\nabla_\mu - (\Sigma + \iota\gamma_5\Pi)\right]\right\}$$
$$= \int\left[D(\Sigma + \iota\gamma_5\Pi)\right]\exp iN_f\left\{\int\int\sqrt{-g}d^4x\left[-\frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right)\right] - \iota\ln Det\left[\iota\gamma^\mu\nabla_\mu - (\Sigma + \iota\gamma_5\Pi)\right]\right\}$$
$$= \int \mathcal{D}\Pi\mathcal{D}\Sigma e^{\iota N_f S_{\text{eff}}}$$

where

$$S_{\text{eff}} = \int d^4x\sqrt{-g}\left\{-\frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) - \iota\ln Det\left\{\iota I\gamma^\mu\nabla_\mu - (\Sigma + \iota\gamma_5\Pi)\right\}\right\}$$

In the leading order of the $1/N$ expansion the generating functional $W[\eta, \overline{\eta}]$ (in the sources to zero $\eta = \bar{\eta} = 0$) for connected Green functions is given by

$$NW[0,0] = -i \ln Z[0,0]$$
$$= \int \sqrt{-g} d^4x \left\{ -\frac{1}{2\lambda} \left(|\Pi|^2 + |\Sigma|^2 \right) - \imath \ln Det \left\{ \imath I \gamma^\mu \nabla_\mu - (\Sigma + \imath \gamma_5 \Pi) \right\} \right\} + O\left(\frac{1}{N}\right)$$

The effective action $F[\Pi_c]$ is defined to be the Legendre transform of $W[\eta, \overline{\eta}]$

$$\Gamma[\Pi_c] = W[\eta, \overline{\eta}] - \int \sqrt{-g} d^4x \Pi_c(x) \left[\eta(x) + \overline{\eta}(x) \right]$$

For the sources to zero $\eta = \bar{\eta} = 0$, $\Gamma[\Pi_c] = W[0,0]$. Where.

$$\Pi_c = \frac{\delta W[\eta, \overline{\eta}]}{\delta \eta \delta \overline{\eta}} = \Pi + O\left(\frac{1}{N}\right)$$

$$\Gamma[0,0] = -\int \sqrt{-g} d^4x \frac{1}{2\lambda} \left(|\Pi|^2 + |\Sigma|^2 \right) - i \ln Det \left[i\gamma^\mu \nabla_\mu - \left(|\Pi|^2 + |\Sigma|^2 \right) \right] + O\left(\frac{1}{N}\right)$$

$$V(\Pi) = -\frac{\Gamma(\Pi)}{\int \sqrt{-g} d^4x}$$

$$\begin{aligned} V(\Pi) =& \frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) + \iota Tr \ln Det\left[i\gamma^\mu \nabla_\mu - \left(|\Pi|^2 + |\Sigma|^2\right)\right] \\ =& \frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) - Tr\int d^4x \ln\left[i\gamma^\mu \nabla_\mu - \left(|\Pi|^2 + |\Sigma|^2\right)\right] \\ =& \frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) - Tr\int d^4x \sqrt{-g}\int_0^\Pi dA S(x,x;A) + const. \end{aligned}$$

$$A = \Sigma + \iota\gamma_5 \Pi \qquad S(x,y;A) = \left\langle x \left| (uI\gamma^\mu \nabla_\mu - A)^{-1} \right| y \right\rangle$$

$$\left(iI\gamma^\mu(x)\nabla_\mu - A\right) S(x,y;A) = I\frac{1}{\sqrt{-g(x)}}\delta^4(x-y)$$

$$V(\Pi) = \frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) - \iota Tr \int_0^{\Pi} dA S(x,x;A) + O\left(\frac{1}{N}\right)$$
$$= \frac{1}{2\lambda}\left(|\Pi|^2 + |\Sigma|^2\right) - \iota Tr \ln S(x,x;A)$$

We expand around the background $A = \overline{A} + \delta A$

$$\ln Det\left\{\iota I \gamma^{\mu}(x)\nabla_{\mu} - A\right\} = Tr \ln\left\{\iota I \gamma^{\mu}\nabla_{\mu} - A\right\}$$
$$= Tr \ln\left\{\iota I \gamma^{\mu}(x)\nabla_{\mu} - A\right\} - \int d^4 Tr\left\{\delta A(x) S_F(x,x)\right\}$$
$$- \frac{1}{2}\int d^4x \int d^4y \delta A(x) S_F(x,y) \delta A(y) S_F(y,x) + \dots$$

where S_F is the fermion propagator and satisfied

$$\sqrt{-g}\left(iI\gamma^{\mu}(x)\nabla_{\mu} - M\right) S_F(x,y) = i\delta^4(x-y)I$$

$$\left[\delta^i_k \nabla^{\mu}\nabla_{\mu} + Q^i_k(x)\right] G^k_j(x,x') = \delta^i_j \delta(x,x')$$

$$\int dv_x \delta(x,x') f(x) = f(x')$$

$$\nabla_{\mu} G^i_j(x,x') = \partial_{\mu} G^i_j(x,x') + \Gamma^{\cdots i}_{\mu k}(x) G^k_j(x,x')$$

$$
\begin{aligned}
\delta^i_k \nabla^\mu \nabla_\mu G^k_j =& \delta^i_k \nabla^\mu \left(\partial_\mu G^k_j + \Gamma^{\dots k}_{\mu m} G^m_j\right) \\
=& \delta^i_k \left(\eta^{\mu\nu} + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma\right) \nabla_\nu \left(\partial_\mu G^k_j + \Gamma^{\dots k}_{\mu m} G^m_j\right) \\
=& \delta^i_k \left(\eta^{\mu\nu} + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma\right) \left(\partial_\mu \nabla_\nu G^k_j + \nabla_\nu \Gamma^{\dots k}_{\mu m} G^m_j\right) \\
=& \delta^i_k \left(\eta^{\mu\nu} + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma\right) \left[\partial_\mu \left(\partial_\nu G^k_j + \Gamma^{\dots k}_{\nu m} G^m_j\right) + \Gamma^{\dots k}_{\mu m} \left(\partial_\nu G^m_j + \Gamma^{\dots m}_{\nu n} G^n_j\right)\right] \\
=& \delta^i_k \left(\eta^{\mu\nu} \partial_\mu \partial_\nu G^k_j + \eta^{\mu\nu} \partial_\mu \Gamma^{\dots k}_{\nu m} G^m_j\right) + \delta^i_k \left(\frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \partial_\nu G^k_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \Gamma^{\dots k}_{\nu m} G^m_j\right) \\
&+ \delta^i_k \left(\eta^{\mu\nu} \Gamma^{\dots k}_{\mu m} \partial_\nu G^m_j + \eta^{\mu\nu} \Gamma^{\dots k}_{\mu m} \Gamma^{\dots m}_{\nu n} G^n_j\right) + \delta^i_k \left(\frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \Gamma^{\dots k}_{\mu m} \partial_\nu G^m_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \Gamma^{\dots k}_{\mu m} \Gamma^{\dots m}_{\nu n} G^n_j\right) \\
=& \eta^{\mu\nu} \partial_\mu \partial_\nu G^i_j + \eta^{\mu\nu} \partial_\mu \Gamma^{\dots i}_{\nu m} G^m_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \partial_\nu G^i_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \Gamma^{\dots i}_{\nu m} G^m_j \\
&+ \eta^{\mu\nu} \Gamma^{\dots i}_{\mu m} \partial_\nu G^m_j + \eta^{\mu\nu} \Gamma^{\dots i}_{\mu m} \Gamma^{\dots m}_{\nu n} G^n_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \Gamma^{\dots i}_{\mu m} \partial_\nu G^m_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \Gamma^{\dots i}_{\mu m} \Gamma^{\dots m}_{\nu n} G^n_j \\
\simeq& \eta^{\mu\nu} \partial_\mu \partial_\nu G^i_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \partial_\nu G^i_j + 2\eta^{\mu\nu} \Gamma^{\dots i}_{\nu m} \partial_\mu G^m_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \partial_\nu G^i_j \\
=& \eta^{\mu\nu} \partial_\mu \partial_\nu G^i_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \partial_\nu G^i_j + \frac{2}{4} \eta^{\mu\nu} R_{\mu\rho\hat{a}\hat{b}} \left(J^{\hat{a}\hat{b}}\right)^i_j y^\rho \partial_\nu G^m_j + \frac{2}{3} R^{\mu}_{..\nu} y^\nu \partial_\mu G^i_j \\
=& \eta^{\mu\nu} \partial_\mu \partial_\nu G^i_j + \frac{1}{3} R^{\mu..\nu}_{.\rho\sigma} y^\rho y^\sigma \partial_\mu \partial_\nu G^i_j + \frac{1}{2} R^{\mu}_{\nu\hat{a}\hat{b}} \left(J^{\hat{a}\hat{b}}\right)^i_j y^\nu \partial_\mu G^i_j + \frac{2}{3} R^{\mu}_{..\nu} y^\nu \partial_\mu G^i_j
\end{aligned}
$$

$$
\mathcal{J}^{\hat{a}\hat{b}} = \frac{1}{4}\left[\gamma^{\hat{a}}, \gamma^{\hat{b}}\right]
$$

$$\left[\eta^{\mu\nu}\partial_\mu\partial_\nu G^i_j + \frac{1}{3}R^{\mu\ldots\nu}_{\ \ ..\rho\sigma}y^\rho y^\sigma\partial_\mu\partial_\nu G^i_j + \frac{1}{2}R^\mu_{\ \nu\hat{a}\hat{b}}\left(J^{\hat{a}\hat{b}}\right)^i_{\ l}y^\nu\partial_\mu G^l_j + \frac{2}{3}R^\mu_{\ ..\nu}y^\nu\partial_\mu G^i_j + Q^i_l G^l_j + \cdots\right] = \delta^i_j\delta(x,y)$$

$$\left[\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{3}R^{\mu\ldots\nu}_{\ \ ..\rho\sigma}y^\rho y^\sigma\partial_\mu\partial_\nu + \frac{1}{2}R^\mu_{\ \nu\hat{a}\hat{b}}\left(J^{\hat{a}\hat{b}}\right)^i_{\ l}y^\nu\partial_\mu + \frac{2}{3}R^\mu_{\ ..\nu}y^\nu\partial_\mu + Q^i_l + \cdots\right]G^i_j = \delta^i_j\delta(x,y)$$

$$G^i_j(x,x') = \int\frac{d^4q}{(2\pi)^4}e^{iq\cdot y}\left(G^i_{2j} + G^i_{3j} + G^i_{4j} + \cdots\right)$$

$$(i\gamma_\mu\nabla^\mu + A)\,G(x,x';A) = S(x,x';A) \qquad G(x,x';A) = \int\frac{d^4q}{(2\pi)^4}e^{ipx'}\tilde{G}(p,x';A)$$

$$\left[\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{3}R^{\mu\ldots\nu}_{\ \ ..\rho\sigma}y^\rho y^\sigma\partial_\mu\partial_\nu + \frac{1}{8}R^\mu_{\ ..\nu ab}\left[\gamma^a,\gamma^b\right]\partial_\mu + \frac{2}{3}R^\mu_{\ ..\nu}y^\nu\partial_\mu\right]\tilde{G}(q,x';A) = 1$$

$$\tilde{G}(q,x';A) = \frac{1}{q^2-A^2} - \frac{1}{12}\frac{R}{(q^2-A^2)^2} + \frac{2}{3}\frac{R^{\mu\nu}q_\mu q_\nu}{(q^2-A^2)^3} + O\left(R_{;\mu},(R)^2\right)$$

weak-curvature expansion

$$S\left(x,x';A\right)=\left(i\gamma_\mu\nabla^\mu+A\right)\int\frac{d^4q}{(2\pi)^4}e^{-ip(x-x')}\tilde{G}(p,y)$$

$$=\int\frac{d^4q}{(2\pi)^4}\exp^{-iq(x-x')}\left[\frac{\gamma^a q_a+A}{q^2-A^2}-\frac{1}{12}R\frac{\gamma^a q_a+A}{\left(q^2-A^2\right)^2}\right.$$

$$\left.+\frac{2}{3}R^{\mu\nu}q_\mu q_\nu\frac{\gamma^a q_a+A}{\left(q^2-A^2\right)^3}-\frac{1}{8}\gamma^a\left[\gamma^a,\gamma^b\right]R_{cda\mu}q^\mu\frac{1}{\left(q^2-A^2\right)^2}\right]$$

$$+O\left(R_{;\mu},(R)^2\right)$$

↓

$$S(x,x;A)=\int\frac{d^4q}{(2\pi)^4}\left[\left(I\gamma^a q_a+A\right)\frac{1}{q^2-|A|^2}-\frac{1}{12}R\left(I\gamma^a q_a+A\right)\frac{1}{\left(q^2-|A|^2\right)^2}\right.$$

$$\left.+\frac{2}{3}R_{\mu\nu}q^\mu q^\nu\left(I\gamma^a q_a+A\right)+\frac{1}{\left(q^2-|A|^2\right)^3}-\frac{1}{8}\gamma^a\left[\gamma^c,\gamma^d\right]R_{cda\mu}q^\mu\frac{1}{\left(q^2-|A|^2\right)^2}\right]$$

$$
\begin{aligned}
V(A) =& \frac{1}{2\lambda}A^2 - iTr\int_0^{\Pi} dA \int \frac{d^4q}{(2\pi)^4}\left[(\gamma^a q_a + A)\frac{1}{q^2 - A^2}\right. \\
& - \frac{1}{12}R\left(\gamma^{\dot{a}} q_{\overline{a}} + A\right)\frac{1}{\left(q^2 - |A|^2\right)^2} \\
& + \frac{2}{3}R_{\mu\nu}q^\mu q^\nu \left(\gamma^a q_a + A\right)\frac{1}{\left(q^2 - |A|^2\right)^3} \\
& \left. - \frac{1}{8}\gamma^a \left[\gamma^c, \gamma^d\right] R_{cda\mu} q^\mu \frac{1}{\left(q^2 - |A|^2\right)^2}\right] \\
=& V(0) + \frac{1}{2\lambda}A^2 - \frac{1}{(4\pi^2)}\left[A^2\Lambda^2 + \Lambda^4 \ln\left(1 + \frac{A^2}{\Lambda^2}\right) - \sigma^4 \ln\left(1 + \frac{\Lambda^2}{A^2}\right)\right] \\
& - \frac{1}{(4\pi)^2}\frac{R}{6}\left[-\sigma^2 \ln\left(1 + \frac{\Lambda^2}{A^2}\right) + \frac{\Lambda^2 A^2}{\Lambda^2 + A^2}\right]
\end{aligned}
$$

$$V(a) = V_0 + \frac{1}{2\lambda}a^2 - \frac{1}{4\pi^2}\left[a^2\Lambda^2 + \Lambda^4 \ln\left(1+\frac{a^2}{\Lambda^2}\right) - a^4 \ln\left(1+\frac{\Lambda^2}{a^2}\right)\right] - \frac{1}{(4\pi)^2}\left(\dot{H}+2H^2\right)\left[-a^2\ln\left(1+\frac{\Lambda^2}{a^2}\right) + \frac{\Lambda^2 a^2}{\Lambda^2+a^2}\right]$$

$$\frac{\epsilon[a]}{M_{Pl}^2} = \frac{1}{2}\left(\frac{V'[a]}{V[a]}\right)^2, \frac{\eta[a]}{M_{Pl}^2} = \frac{V''[a]}{V[a]}$$

$$\epsilon = -\frac{\dot{H}}{H^2}, \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

$$V' = \frac{1}{\lambda}a - \frac{1}{4\pi^2}\left[2a\Lambda^2 + \frac{2\Lambda^2 a}{1+a^2/\Lambda^2} - 4a^3\ln\left(1+\frac{\Lambda^2}{a^2}\right) + \frac{2\Lambda^2 a}{1+\Lambda^2/a^2}\right]$$
$$+ \frac{H^2}{(4\pi)^2}(\epsilon-2)\left[-2a\ln\left(1+\frac{\Lambda^2}{a^2}\right) + \frac{2\Lambda^2 a^{-1}}{1+\Lambda^2/a^2} + \frac{2\Lambda^2 a}{\Lambda^2+a^2} - \frac{2\Lambda^2 a^3}{(\Lambda^2+a^2)^2}\right]$$

$$V'' = \frac{1}{\lambda} - \frac{1}{4\pi^2}\left[2\Lambda^2 + \frac{2\Lambda^2}{1+a^2/\Lambda^2} + \frac{4a^2}{(1+a^2/\Lambda^2)^2} - 12a^2\ln\left(1+\frac{\Lambda^2}{a^2}\right) - \frac{8\Lambda^2}{1+\Lambda^2/a^2} + \frac{2\Lambda^2}{1+\Lambda^2/a^2} - \frac{4\Lambda^4 a^{-2}}{(1+\Lambda^2/a^2)^2}\right]$$
$$+ \frac{H^2}{(4\pi)^2}(\epsilon-2)\left[-2\ln\left(1+\frac{\Lambda^2}{a^2}\right) + \frac{4\Lambda^2 a^{-2}}{1+\Lambda^2/a^2} - \frac{2\Lambda^2 a^{-2}}{1+\Lambda^2/a^2} + \frac{2\Lambda^2}{\Lambda^2+a^2} + \frac{4\Lambda^2 a^2}{(\Lambda^2+a^2)^2} - \frac{6\Lambda^2 a^2}{(\Lambda^2+a^2)^2} + \frac{8\Lambda^2 a^4}{(\Lambda^2+a^2)^3}\right]$$
$$= \frac{1}{\lambda} - \frac{1}{4\pi^2}\left[2\Lambda^2 + \frac{2\Lambda^2}{1+a^2/\Lambda^2} + \frac{4a^2}{(1+a^2/\Lambda^2)^2} - 12a^2\ln\left(1+\frac{\Lambda^2}{a^2}\right) - \frac{6\Lambda^2}{1+\Lambda^2/a^2} - \frac{4\Lambda^4 a^{-2}}{(1+\Lambda^2/a^2)^2}\right]$$
$$+ \frac{H^2}{(4\pi)^2}(\epsilon-2)\left[-2\ln\left(1+\frac{\Lambda^2}{a^2}\right) + \frac{2\Lambda^2 a^{-2}}{1+\Lambda^2/a^2} + \frac{2\Lambda^2}{\Lambda^2+a^2} - \frac{2\Lambda^2 a^2}{(\Lambda^2+a^2)^2} + \frac{8\Lambda^2 a^4}{(\Lambda^2+a^2)^3}\right]$$