

# Non-Local Star as A Blackhole Mimicker

Review on non-local gravity

Phenomenology in astrophysics  
&  
A laboratory test via spin entanglement

**Anupam Mazumdar**

Hot Topics in Modern Cosmology, 5-11 May, 2019, IESC, Cargese

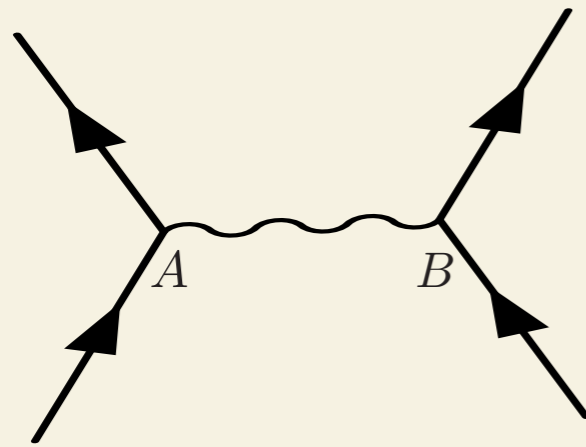
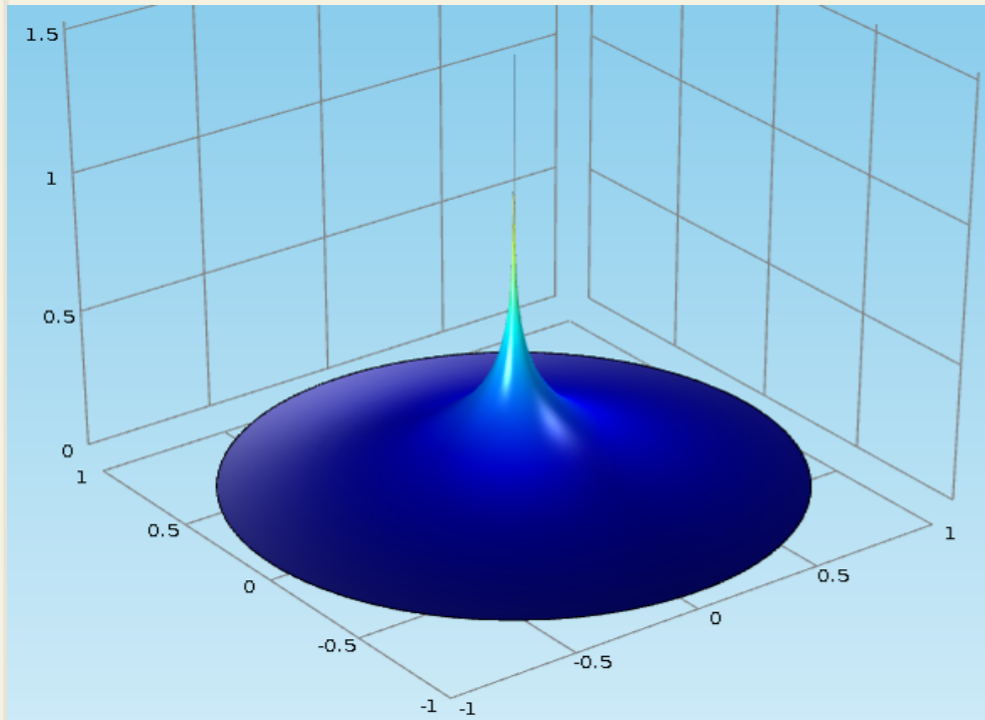


university of  
 groningen

faculty of science and engineering

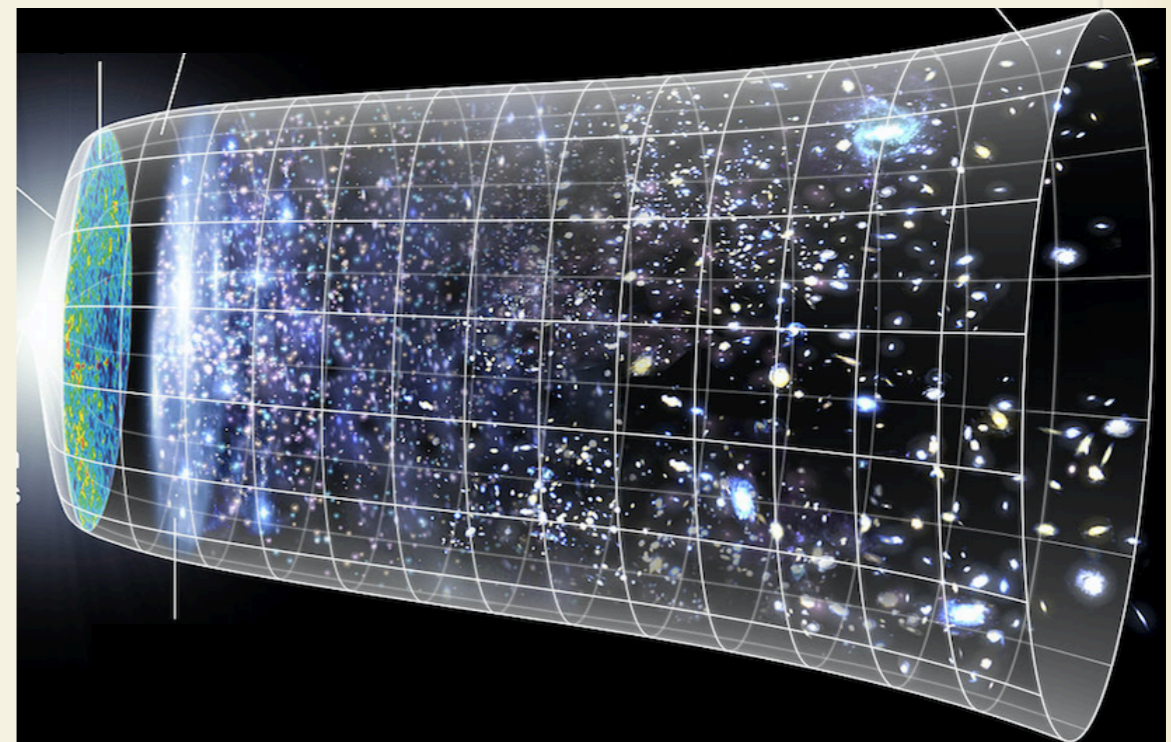
van swinderen institute for  
particle physics and gravity

# Locality in Spacetime & Singularities



$$V \sim \frac{1}{r}$$

**Graviton or Photon  
(mediator is massless)**



# Note on Locality

Finite derivative theory always has a point support

$$x^n \delta^n(x) = (-1)^n n! \delta(x)$$

Infinite derivatives acting on a delta source does not have any point support

$$e^{\alpha \nabla_x^2} \delta(x) = \frac{1}{\sqrt{2\pi}} \int dk e^{-\alpha k^2} e^{ik \cdot x} = \frac{1}{\sqrt{2\alpha}} e^{-x^2/4\alpha}$$



**Non-locality is the key perhaps for addressing Singularities!**

# Finite Derivative Gravity

$$S = \int \sqrt{-g} d^4x \left( \frac{R}{16\pi G} \right)$$

One loop pure gravitational action is renormalizable. The theory is not scale invariant, suffers from cosmological and Schwarzschild/Ring singularities

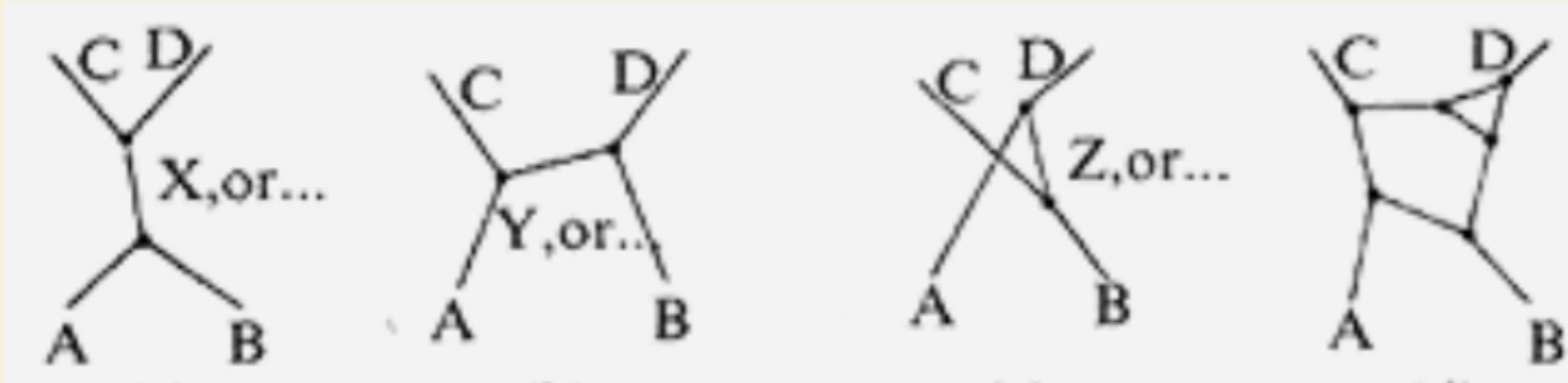
$$S = \int \sqrt{-g} d^4x [M_p^2 R + \alpha C^2] \leftarrow \text{Weyl term alone does not introduce singularities}$$

$$S = \int \sqrt{-g} d^4x [M_p^2 R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}]$$

Quadratic Curvature Gravity is renormalizable, but contains **“Ghosts”**:  
**Vacuum is Unstable + Cosmological/B-H Singularities remain**

$$S = \int \sqrt{-g} d^4x [M_p^2 R + \alpha(R\Box R + R\Box^2 R + \text{finite order}) + \beta(R^{\mu\nu}\Box R_{\mu\nu} + R^{\mu\nu}\Box^2 R_{\mu\nu} + \text{finite order}) + \dots]$$

# Out of fashion: Perturbative Quantum Gravity

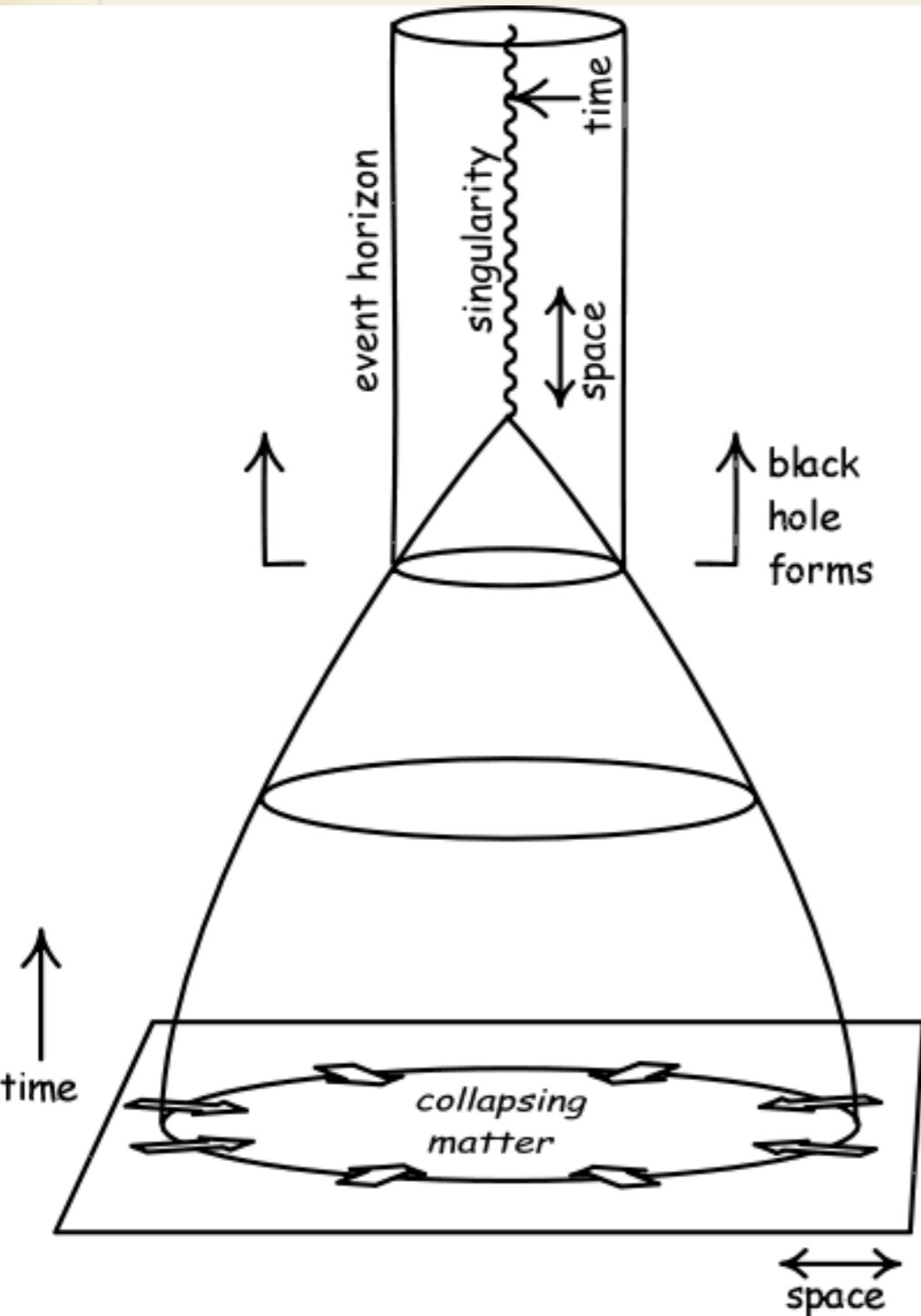


## Rise of String Theory



Return of Perturbative Quantum Gravity.  
Inspired by String Field Theory

# String Theory & Time Dependence



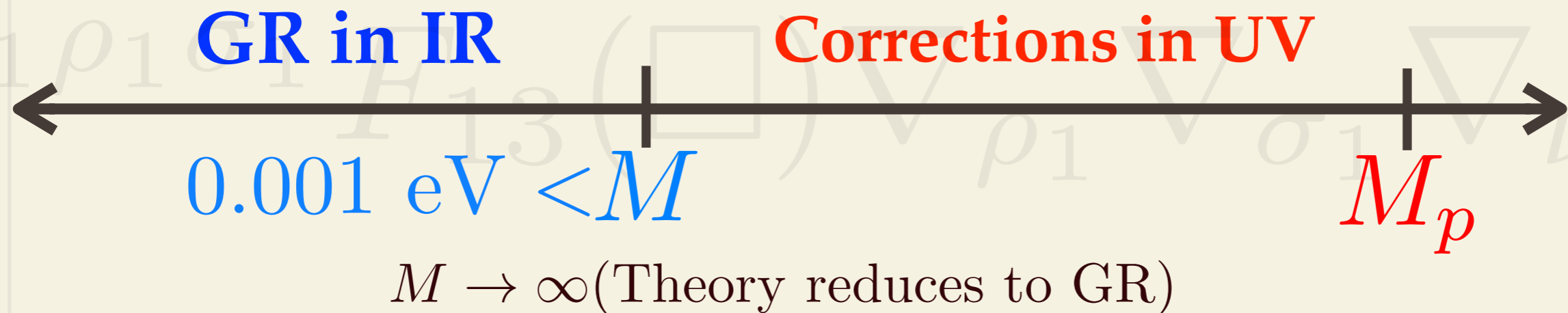
**Near The Singularity adiabatic collapse approximation breaks down !**

**Higher curvature corrections become important, such as alpha' corrections**

**Supersymmetry is not helpful**

Singularity problems in String Theory remains extremely hard !

# Phenomenology with UV Gravity



- (1) **Allow higher derivative contributions in metric by maintaining diffeomorphism invariance and Bianchi identities. This will lead to non-local interactions** ( Biswas,AM, Siegel, hep-th/0508194, Biswas, Gerwick, Koivisto, AM, 1110.5249 )
- (2) **Study the degrees of freedom around Maximally symmetric spacetimes and in general, e.g. Einstein spacetime.** ( Biswas, Koshelev, AM, 1606.01250, 1602.08475, 1905.0XXX )
- (3) **Construct higher derivative theory of gravity with spacetime torsion** ( Dombriz, Torralba, AM, 1812.04037 )

# Higher Curvature Gravity

$$S_q = \int d^4x \sqrt{-g} R_{\mu_1 \nu_1 \lambda_1 \sigma_1} \mathcal{O}_{\mu_2 \nu_2 \lambda_2 \sigma_2}^{\mu_1 \nu_1 \lambda_1 \sigma_1} R^{\mu_2 \nu_2 \lambda_2 \sigma_2}$$

All possible terms allowed by symmetry, parity invariant and torsion free

**Motivation:**

**String Field Theory**

**Unknown Infinite Functions  
of Covariant Derivatives**

$$\begin{aligned}
 S_q = \int d^4x \sqrt{-g} [ & R F_1(\square) R + R F_2(\square) \nabla_\mu \nabla_\nu R^{\mu\nu} + R_{\mu\nu} F_3(\square) R^{\mu\nu} + R_\mu^\nu F_4(\square) \nabla_\nu \nabla_\lambda R^{\mu\lambda} \\
 & + R^{\lambda\sigma} F_5(\square) \nabla_\mu \nabla_\sigma \nabla_\nu \nabla_\lambda R^{\mu\nu} + R F_6(\square) \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda} F_7(\square) \nabla_\nu \nabla_\sigma R^{\mu\nu\lambda\sigma} \\
 & + R_\lambda^\rho F_8(\square) \nabla_\mu \nabla_\sigma \nabla_\nu \nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1 \nu_1} F_9(\square) \nabla_{\mu_1} \nabla_{\nu_1} \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma R^{\mu\nu\lambda\sigma} \\
 & + R_{\mu\nu\lambda\sigma} F_{10}(\square) R^{\mu\nu\lambda\sigma} + R_{\mu\nu\lambda}^\rho F_{11}(\square) \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1} F_{12}(\square) \nabla^{\rho_1} \nabla^{\sigma_1} \nabla_\rho \nabla_\sigma R^{\mu\rho\nu\sigma} \\
 & + R_{\mu}^{\nu_1\rho_1\sigma_1} F_{13}(\square) \nabla_{\rho_1} \nabla_{\sigma_1} \nabla_{\nu_1} \nabla_\nu \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1} F_{14}(\square) \nabla_{\rho_1} \nabla_{\sigma_1} \nabla_{\nu_1} \nabla_{\mu_1} \nabla_\mu \nabla_\nu \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} ]
 \end{aligned}$$



# Gravitational Form Factors

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + R \mathcal{F}_1 \left( \frac{\square}{M^2} \right) R + R_{\mu\nu} \mathcal{F}_2 \left( \frac{\square}{M^2} \right) R^{\mu\nu} + R_{\mu\nu\lambda\sigma} \mathcal{F}_3 \left( \frac{\square}{M^2} \right) R^{\mu\nu\lambda\sigma} \right]$$

**Einstein-Hilbert  
Recovers IR**

**Ultra-violet modifications**

$$\frac{\square}{M^2}$$

$M \rightarrow \infty$  (Theory reduces to GR)

## Infinite Derivative Gravity (IDG)

*Biswas, AM, Siegel, [hep-th/0508194](#)*

*Biswas, Gerwick, Koivisto, AM, [gr-qc/1110.5249](#)*

*Biswas, Koshelev, AM, (extension for de Sitter & Anti-deSitter): [arXiv:1602.08475](#), [arXiv:1606.01250](#)*

# Perturbative Unitarity

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + R\mathcal{F}_1 \left( \frac{\square}{M^2} \right) R + R_{\mu\nu} \mathcal{F}_2 \left( \frac{\square}{M^2} \right) R^{\mu\nu} + R_{\mu\nu\lambda\sigma} \mathcal{F}_3 \left( \frac{\square}{M^2} \right) R^{\mu\nu\lambda\sigma} \right]$$

$$2\mathcal{F}_1 + \mathcal{F}_2 + 2\mathcal{F}_3 = 0 \quad a(\square) = 1 - \frac{1}{2} \mathcal{F}_2(\square) \frac{\square}{M_s^2} - 2\mathcal{F}_3(\square) \frac{\square}{M_s^2}$$

$$\Pi(k^2) = \frac{1}{a(k^2)} \left[ \frac{P^{(2)}}{k^2} - \frac{P^0}{2k^2} \right]$$

$$a(k^2) = e^{\gamma(k^2)}$$

**Demand no extra poles other than massless graviton in GR**

**Entire Function**

**Simplest choice:**  $a(k^2) = e^{k^2/M_s^2}$

# Complete Equations of Motion

$$P^{\alpha\beta} \approx \frac{\alpha_c}{8\pi G} \left( 4G^{\alpha\beta} \mathcal{F}_1(\square_s) \mathcal{R} + g^{\alpha\beta} \mathcal{R} \mathcal{F}_1(\square_s) \mathcal{R} - 4 \left( \nabla^\alpha \nabla^\beta - g^{\alpha\beta} \square \right) \mathcal{F}_1(\square_s) \mathcal{R} \right. \\ \left. - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta} (\Omega_{1\sigma}^\sigma + \bar{\Omega}_1) + 4\mathcal{R}_\mu^\alpha \mathcal{F}_2(\square_s) \mathcal{R}^{\mu\beta} \right. \\ \left. - g^{\alpha\beta} \mathcal{R}_\nu^\mu \mathcal{F}_2(\square_s) \mathcal{R}_\mu^\nu - 4\nabla_\mu \nabla^\beta (\mathcal{F}_2(\square_s) \mathcal{R}^{\mu\alpha}) + 2\square (\mathcal{F}_2(\square_s) \mathcal{R}^{\alpha\beta}) \right. \\ \left. + 2g^{\alpha\beta} \nabla_\mu \nabla_\nu (\mathcal{F}_2(\square_s) \mathcal{R}^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta} (\Omega_{2\sigma}^\sigma + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \right)$$

Conroy, Biswas, Koshelev, AM  
1308.2319, JCAP 2014

$$= T^{\alpha\beta} = 0,$$

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1n} \sum_{l=0}^{n-1} \nabla^\alpha \mathcal{R}^{(l)} \nabla^\beta \mathcal{R}^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1n} \sum_{l=0}^{n-1} \mathcal{R}^{(l)} R^{(n-l)},$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} \mathcal{R}_\nu^{\mu;\alpha(l)} \mathcal{R}_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} \mathcal{R}_\nu^{\mu(l)} \mathcal{R}_\mu^{\nu(n-l)},$$

$$\Delta_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} [\mathcal{R}_\sigma^{\nu(l)} \mathcal{R}^{(\beta\sigma;\alpha)(n-l-1)} - \mathcal{R}_\sigma^{\nu;\alpha(l)} \mathcal{R}^{\beta\sigma(n-l-1)}]_{;\nu}.$$

$$P^{\alpha\beta} = -\frac{1}{2} [1 + 2\mathcal{F}_2(\square)\square + 4\mathcal{F}_3(\square)\square] \square h^{\alpha\beta} \\ -\frac{1}{2} [-1 - 2\mathcal{F}_2(\square)\square - 4\mathcal{F}_3(\square)\square] \partial_\sigma (\partial^\alpha h^{\sigma\beta} + \partial^\beta h^{\alpha\sigma}) \\ -\frac{1}{2} \left[ 1 - 8\mathcal{F}_1(\square)\square - 2\mathcal{F}_2(\square)\square + \frac{4}{3}\mathcal{F}_3(\square)\square \right] (\partial^\beta \partial^\alpha h + \eta^{\alpha\beta} \partial_\mu \partial_\nu h^{\mu\nu}) \\ -\frac{1}{2} \left[ -1 + 8\mathcal{F}_1(\square)\square + 2\mathcal{F}_2(\square)\square - \frac{4}{3}\mathcal{F}_3(\square)\square \right] \eta^{\alpha\beta} \square h \\ -\frac{1}{2} \left[ 8\mathcal{F}_1(\square)\square + 4\mathcal{F}_2(\square)\square + \frac{8}{3}\mathcal{F}_3(\square)\square \right] \square^{-1} \nabla^\alpha \nabla^\beta \partial_\mu \partial_\nu h^{\mu\nu},$$

$$R^{(m)} \equiv \square^m R$$

Equations of motion  
in  
weak field limit

# 2nd order perturbations of the full action

## Perturbations in higher derivative gravity beyond maximally symmetric spacetimes

K. Sravan Kumar,<sup>1</sup> Shubham Maheshwari,<sup>1</sup> and Anupam Mazumdar<sup>1</sup>

<sup>1</sup>Van Swinderen Institute, University of Groningen, 9747 AG, Groningen, The Netherlands

We study (covariant) scalar-vector-tensor (SVT) perturbations of infinite derivative gravity (IDG), at the quadratic level of the action, around conformally-flat, covariantly constant curvature backgrounds which are *not* maximally symmetric spacetimes (MSS). This extends a previous analysis of perturbations done around MSS, which were shown to be ghost-free. We motivate our choice of backgrounds which arise as solutions of IDG in the UV, avoiding big bang and black hole singularities. Contrary to MSS, in this paper we show that, generically, all SVT modes are coupled to each other at the quadratic level of the action. For simple cases of the full IDG action, we illustrate this mixing and a case where the action can be diagonalized and ghost-free solutions can be constructed. Our study is widely applicable for both non-singular cosmology and black hole physics where backgrounds depart from MSS. In appendices, we provide SVT perturbations around *arbitrary* and conformally-flat (FLRW) backgrounds which can serve as a compendium of useful results when studying SVT perturbations of various higher derivative gravity models.

$$1. \delta^2 \mathcal{L}_{EH+\Lambda} = \frac{M_P^2}{2} \delta^2 \left[ \sqrt{-g} (R - 2\Lambda) \right] = \frac{M_P^2}{2} \delta_0$$

$$2. \delta^2 \mathcal{L}_{R^2} = \delta^2 \left[ \frac{1}{2} \sqrt{-g} R \mathcal{F}_1(\square_s) R \right]$$

$$3. \delta^2 \mathcal{L}_{S^2} = \delta^2 \left[ \frac{1}{2} \sqrt{-g} S^\nu{}_\mu \mathcal{F}_2(\square_s) S^\mu{}_\nu \right]$$

$$4. \delta^2 \mathcal{L}_{C^2} = \delta^2 \left[ \frac{1}{2} \sqrt{-g} C^{\rho\sigma}{}_{\mu\nu} \mathcal{F}_3(\square_s) C^{\mu\nu}{}_{\rho\sigma} \right]$$

Any generic spacetime background

Scalar-Vector-Tensor decomposition

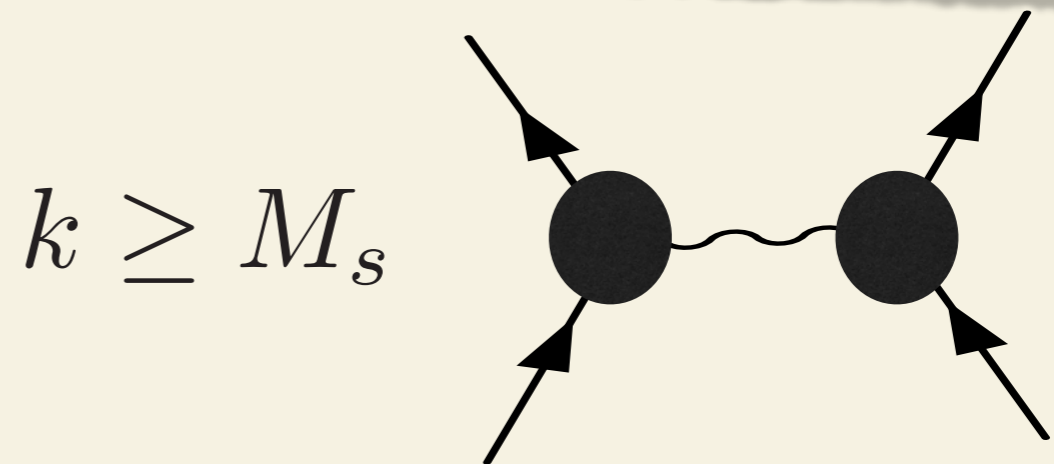
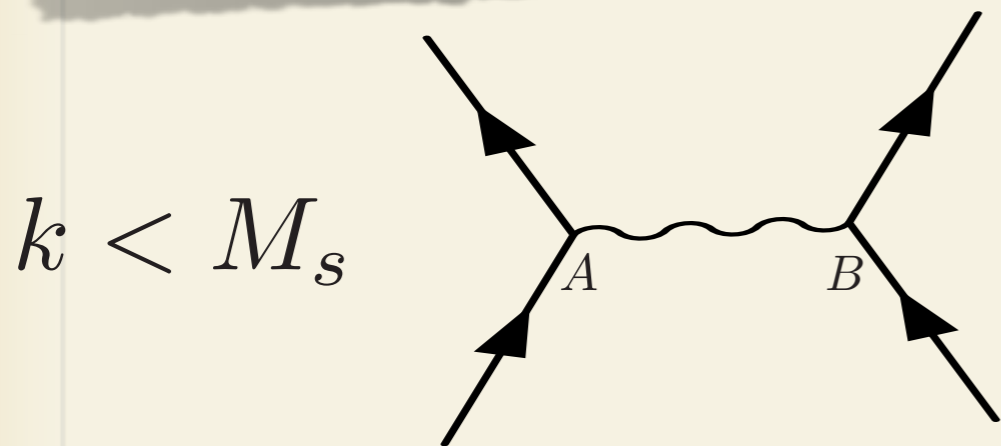
$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu + \left( \bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4} \bar{g}_{\mu\nu} \bar{\square} \right) B + \frac{1}{4} \bar{g}_{\mu\nu} h.$$

$$\bar{\nabla}^\mu \hat{h}_{\mu\nu} = 0, \quad \bar{g}^{\mu\nu} \hat{h}_{\mu\nu} = 0, \quad \bar{\nabla}^\mu A_\mu = 0$$

# A Simple Action

By perturbative unitarity constraints:

$$S = \int d^4x \sqrt{-g} \left[ M_p^2 \frac{R}{2} + R \left[ \frac{e^{-\square/M_s^2} - 1}{\square} \right] R - 2R_{\mu\nu} \left[ \frac{e^{-\square/M_s^2} - 1}{\square} \right] R^{\mu\nu} \right]$$



$$\Pi(k^2) = \frac{1}{a(k^2)} \left[ \frac{P^{(2)}}{k^2} - \frac{P^0}{2k^2} \right] \quad a(k^2) = e^{k^2/M_s^2}$$

**Massless Graviton: massless spin-2 and spin-0 components propagate**

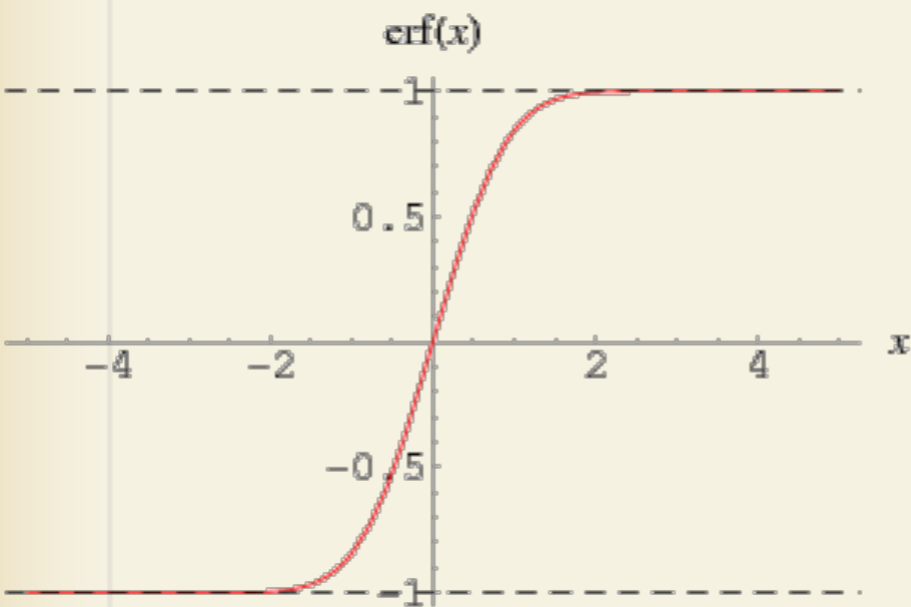
# Gravitational Potential

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

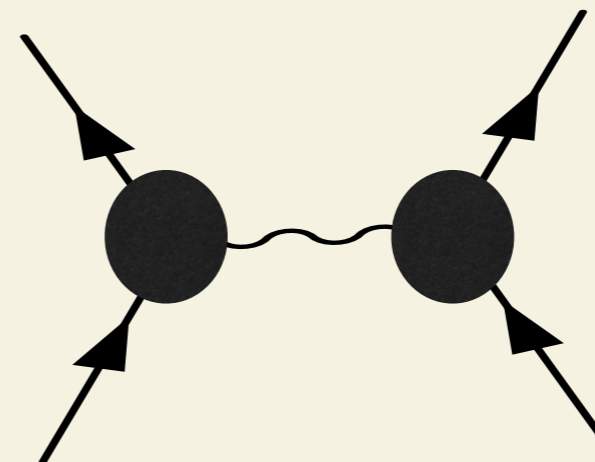
$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r} \operatorname{erf} \left( \frac{rM}{2} \right)$$

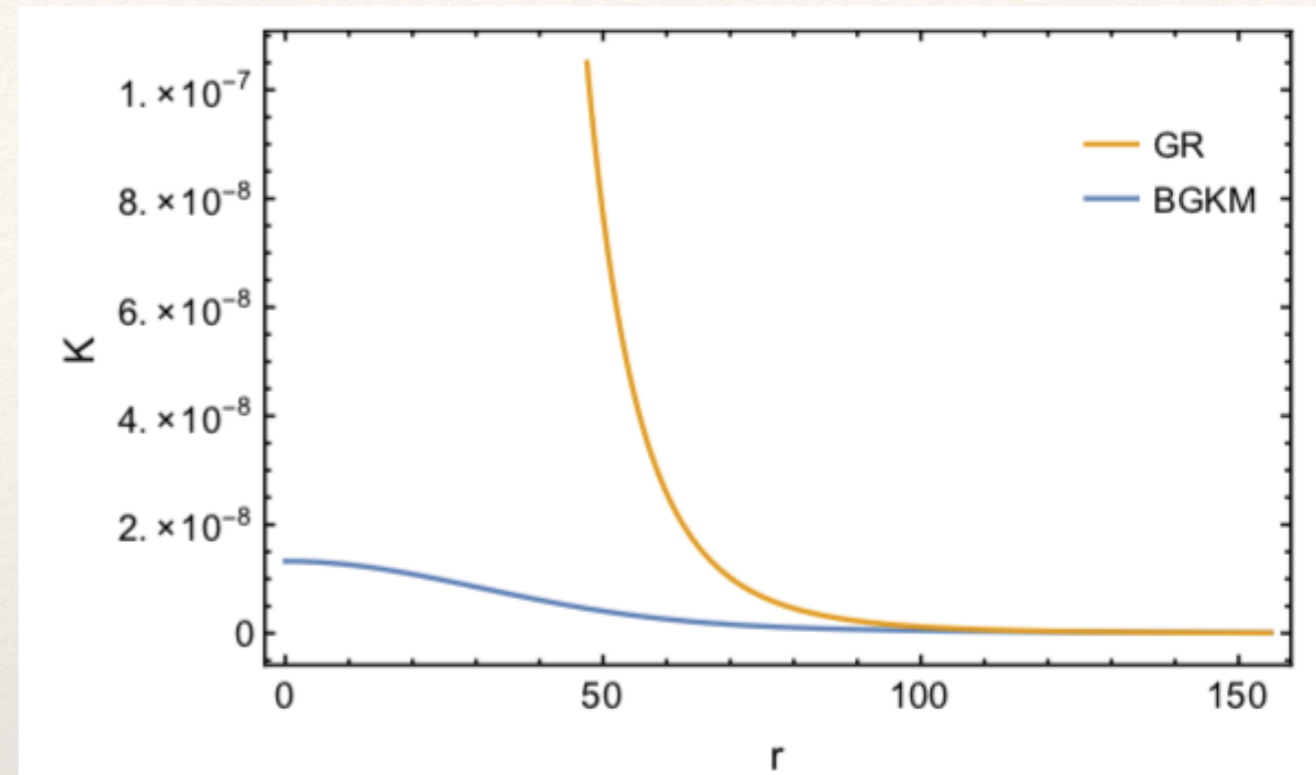
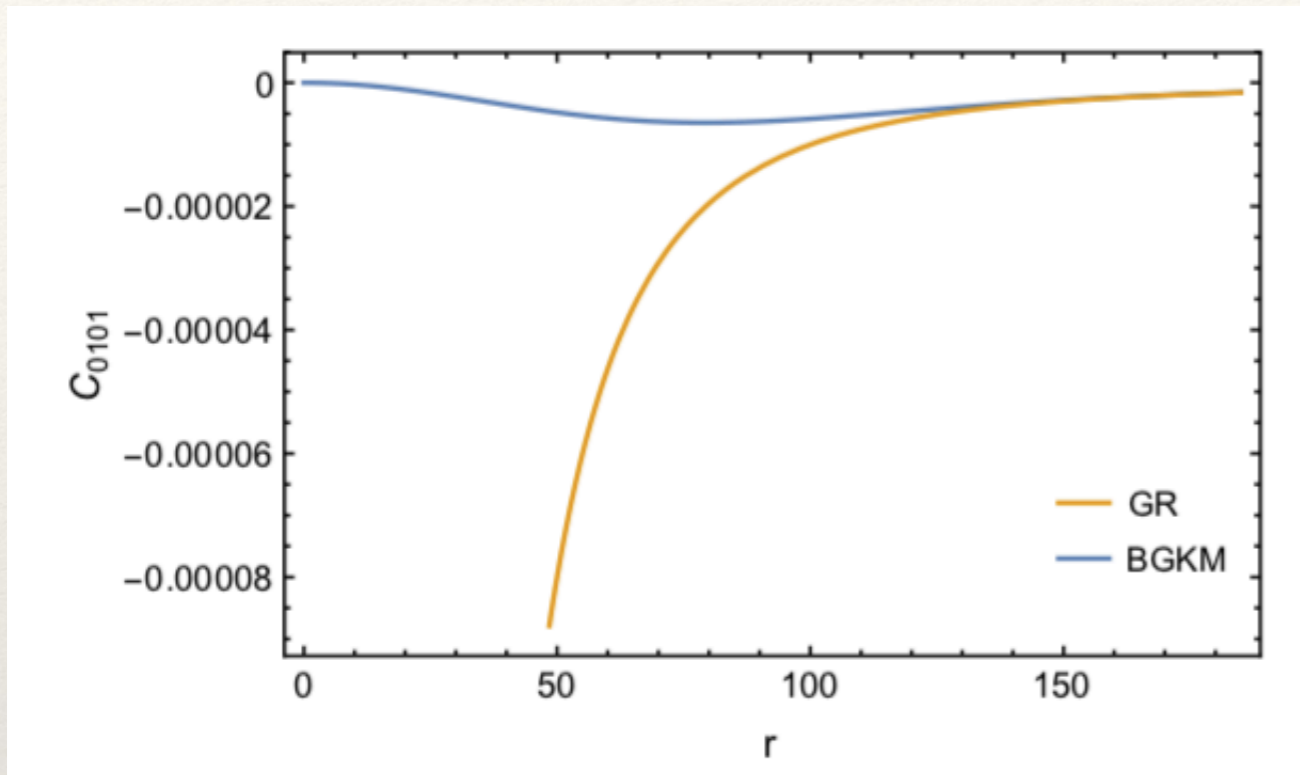
$$\Phi \sim \frac{mM_s}{M_p^2} < 1 \Rightarrow mM_s < M_p^2$$



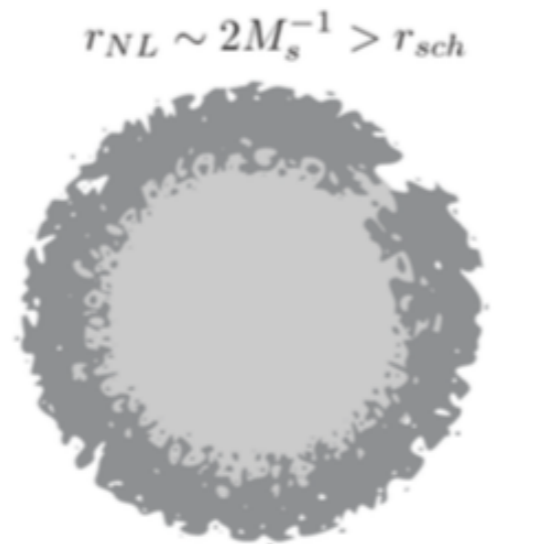
**Interaction becomes Non-Local**



# Towards Conformal Solution



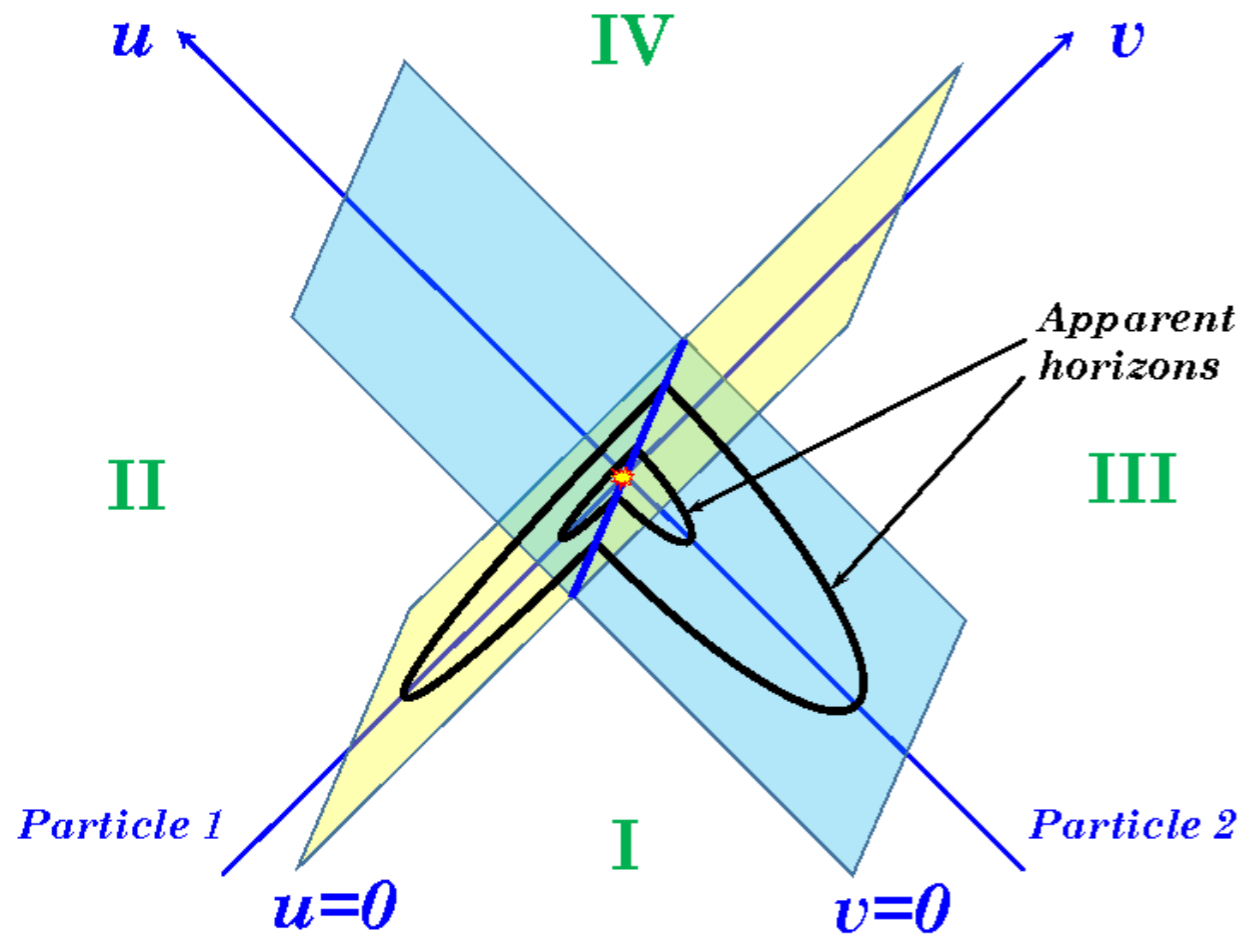
Schwarzschild's blackhole



Non-local, compact object  
in infinite derivative gravity

*Such non-local objects could be BHs provided linear solution is promoted all the way to non-linear level.*

# Head-on Collisions



$$ds^2 = -(1 + 2\varphi_d) dt^2 + (1 - 2\psi_d + 2\varphi_d)(dy^2 + d\zeta_\perp^2)$$
$$x = (y, \zeta_\perp), \quad \zeta_\perp = (\zeta^2, \dots, \zeta^{d+1}).$$

Aichelburg-Sexl metric

**Take Penrose limit  
(Ultra-relativistic limit)**

**No formation of  
apparent horizon**

**No Singularity**

**Very consistent with the quantum picture**



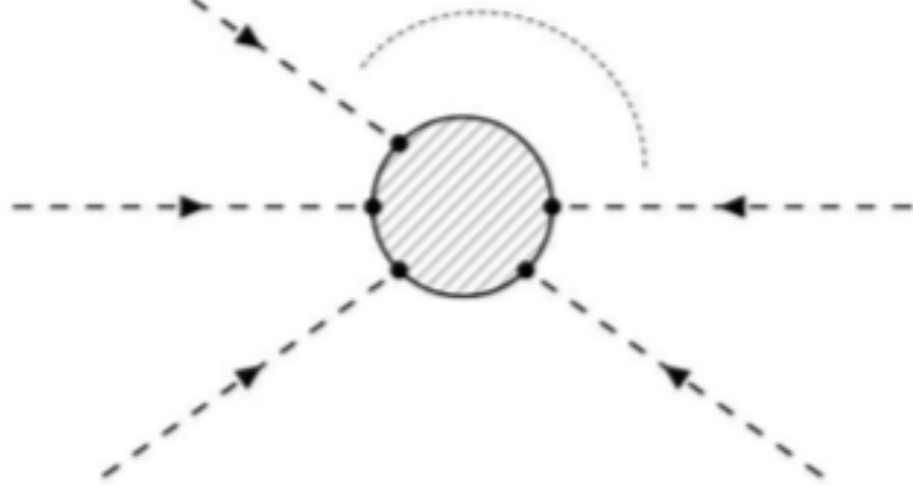
# Collective Behavior & Emergent Scale

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

$$S_{\text{free}} = \frac{1}{2} \int d^4x (\phi \square a(\square) \phi) \quad a(\square) = e^{-\square/M^2}$$

$$S_{\text{int}} = \frac{1}{M_p} \int d^4x \left( \frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \square \phi a(\square) \phi - \frac{1}{4} \phi \partial_\mu \phi a(\square) \partial^\mu \phi \right)$$

$$\mathcal{M}_N \xrightarrow{N \gg 1} \lambda^{3(N-2)} e^{-Np^2/M_s^2} = \lambda^{3(N-2)} e^{-p^2/M_{\text{eff}}^2}$$



$$M_{\text{eff}} \sim \frac{M_s}{\sqrt{N}}$$

*Persists with zero external momenta*

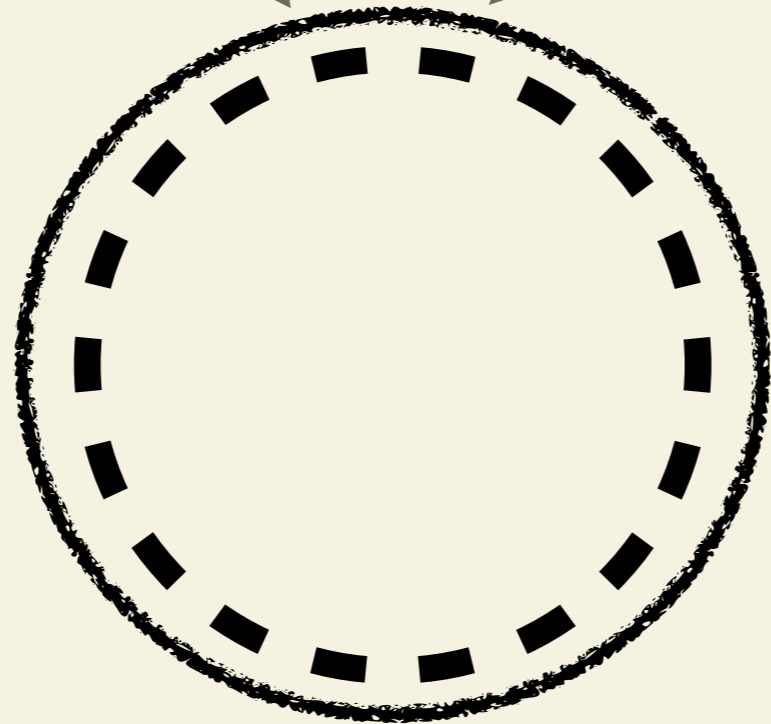
*N-gravitons behave like a condensate*

PRD (2019) 1812.01441 [hep-th]

# Gravitationally Bound System

No Singularity

No Event Horizon



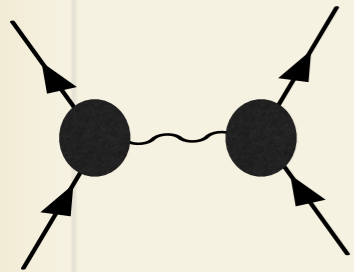
$$R_{Sch} = 2M$$

$$R_{Buch} = 2.25M$$

**Buchdahl Star: for constant energy density: Most Compact after Black hole**

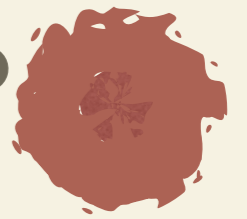
**Q: Could we realize Buchdahl Star in Quantum Gravity?**

# Complementarity Principle



$$\Phi \sim \frac{mM_s}{M_p^2} < 1 \Rightarrow mM_s < M_p^2$$

$$r_{NL} \sim 1/M_s$$



$$\Phi \sim \frac{m_o M_{\text{eff}}}{M_p^2} < 1 \Rightarrow m_o M_{\text{eff}} < M_p^2$$

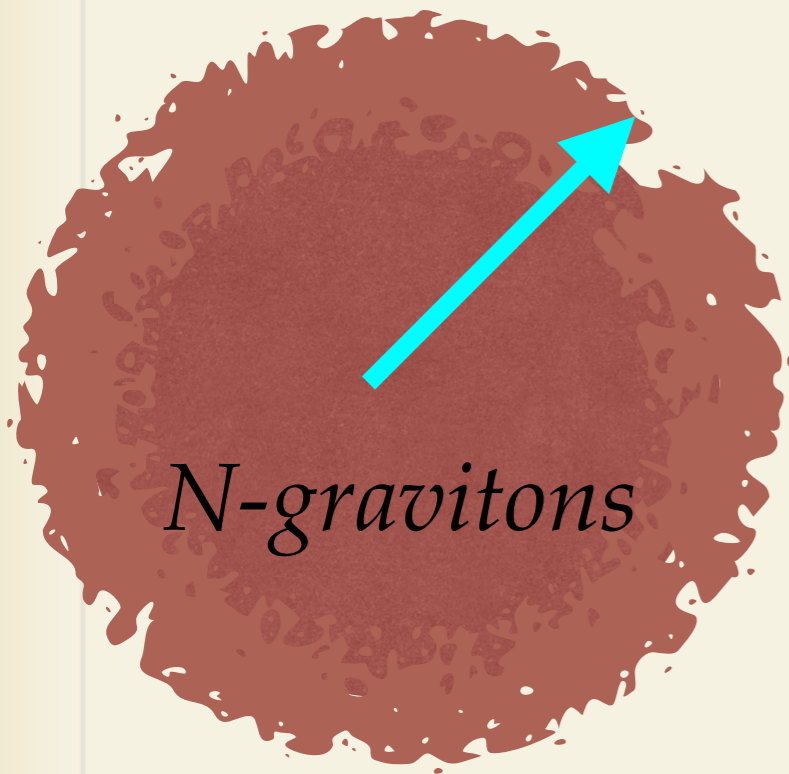
$$M_{\text{eff}} = \frac{M_s}{\sqrt{N}}$$

$$r_{NL} \sim 1/M_{\text{eff}}$$



**Gravitational potential remains weak throughout the regime :  
We can promote linear solution in the entire manifold**

# Compact Non-local Star



$$\lambda \sim M_{\text{eff}}^{-1} = \sqrt{N} M_s^{-1}$$

Radius

$$E_g \sim M_{\text{eff}} = M_s / \sqrt{N}$$

Energy of Gravitons

**Forms a gravitationally  
bound system**

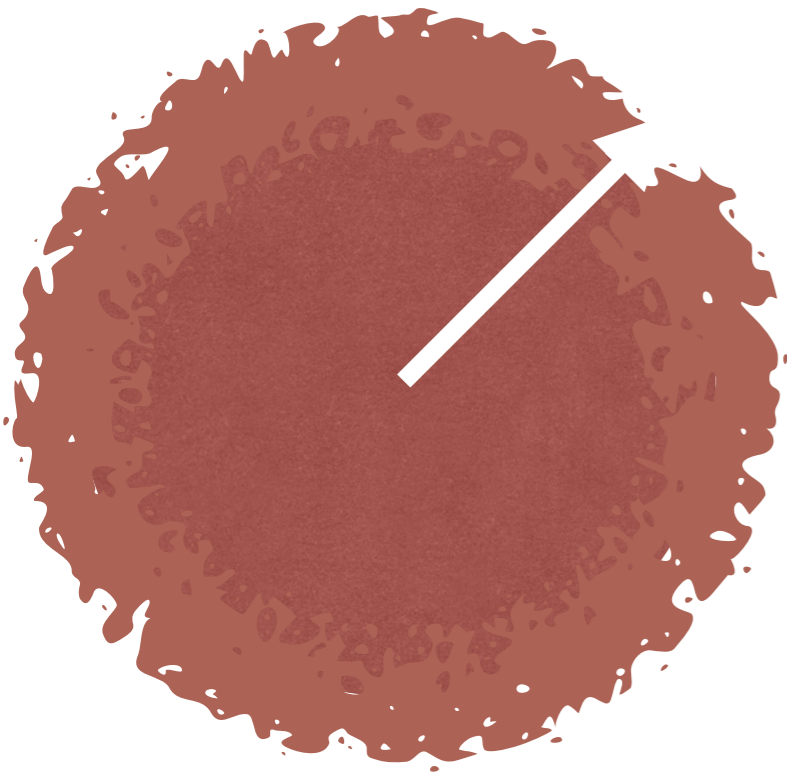
Mass of  $N$  gravitons interacting non – locally

$$E_{\text{tot}} = m_{\circ} = N M_{\text{eff}} = N \frac{M_s}{\sqrt{N}} = \sqrt{N} M_s$$

For a solar mass object :  $N \sim 10^{82}$

**Ensemble of large  $N$  weakly coupled-Gravitons**

# Bekenstein States



$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

$$S \sim \hbar \left( \frac{4G^2 m_\odot^2}{L_p^2} + \frac{L_{\text{eff}}^2}{L_p^2} \right) \equiv \hbar s,$$

$$s \sim \frac{L_{\text{eff}}^2}{L_p^2} = N \frac{L_s^2}{L_p^2} = N \frac{M_p^2}{M_s^2}$$

*Bekenstein States*  $\mathcal{N} \sim e^{N(L_s/L_p)^2} = e^{N(M_p/M_s)^2}$

For a solar mass object :  $\mathcal{N} = e^{10^{82}(M_p/M_s)^2}$

What happens when I throw a chalk, neutrino, ....., anything... inside?

$$\tau = \left( \frac{L_s}{L_p} \right)^9 \tau_{bh} = \left( \frac{M_p}{M_s} \right)^9 \tau_{bh}$$

**Longer life time than a Blackhole**

**The Non-local star absorbs everything, even better than a Blackhole!!!**

# Metric of a Non-local Star

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi(r))dr^2 + r^2d\Omega^2,$$

where

$$\Phi(r) = \begin{cases} -\frac{Gm}{r}\text{Erf}\left(\frac{r}{2L_{\text{eff}}}\right), & r \lesssim r_{\text{NL}}, \\ -\frac{Gm}{r}, & r > r_{\text{NL}}, \end{cases}$$

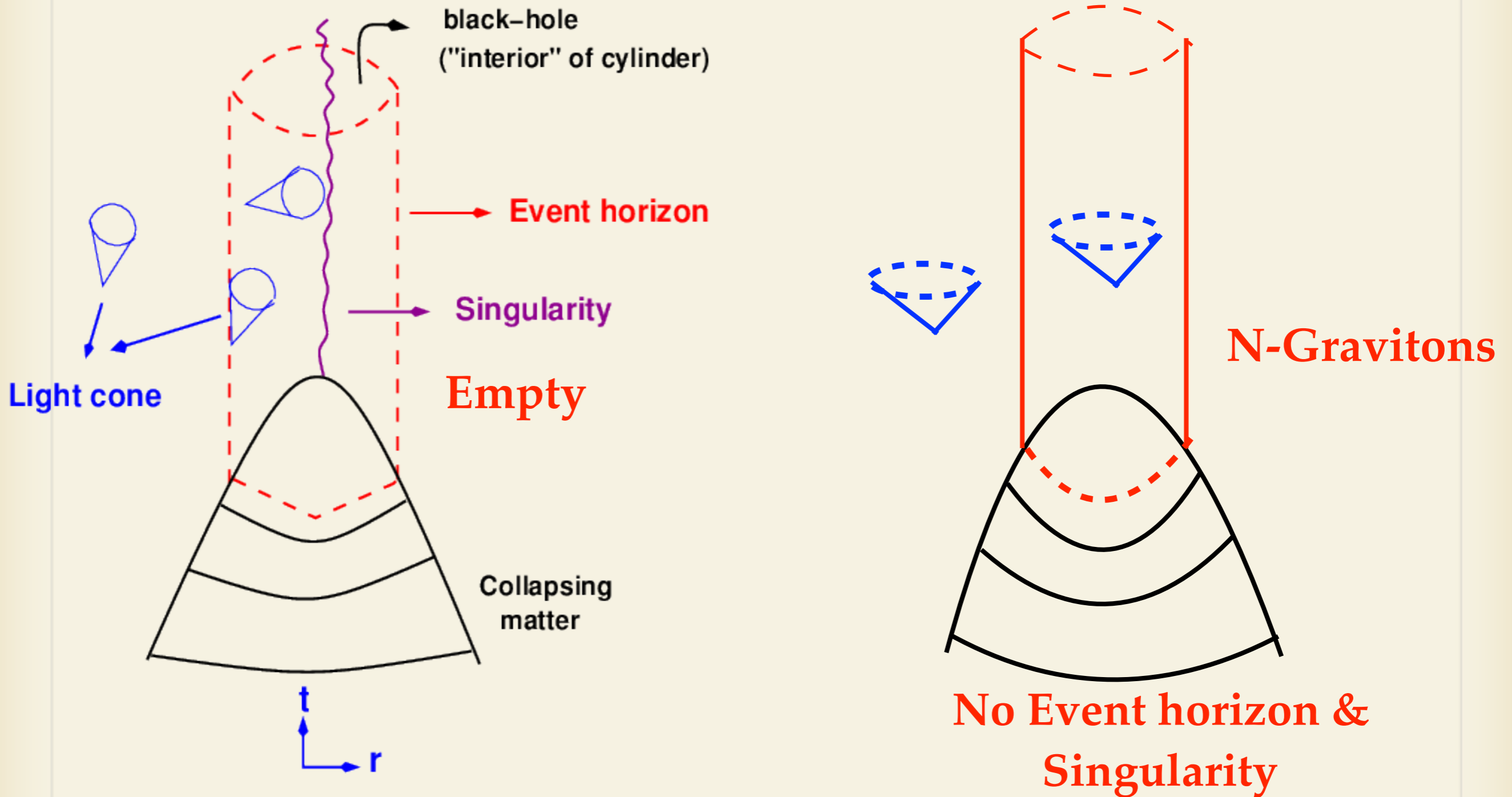
and

$$\Psi(r) = \begin{cases} -\frac{Gm}{r}\text{Erf}\left(\frac{r}{2L_{\text{eff}}}\right) + \frac{Gm e^{-r^2/4L_{\text{eff}}^2}}{\sqrt{\pi}L_{\text{eff}}}, & r \lesssim r_{\text{NL}} \\ -\frac{Gm}{r}, & r > r_{\text{NL}} \end{cases}$$

$$r_{\text{NL}} \sim 2L_{\text{eff}} = r_{\text{sch}}(1 + \epsilon) > 2Gm \frac{2}{\sqrt{\pi}}. \quad \epsilon \gtrsim 0.128$$

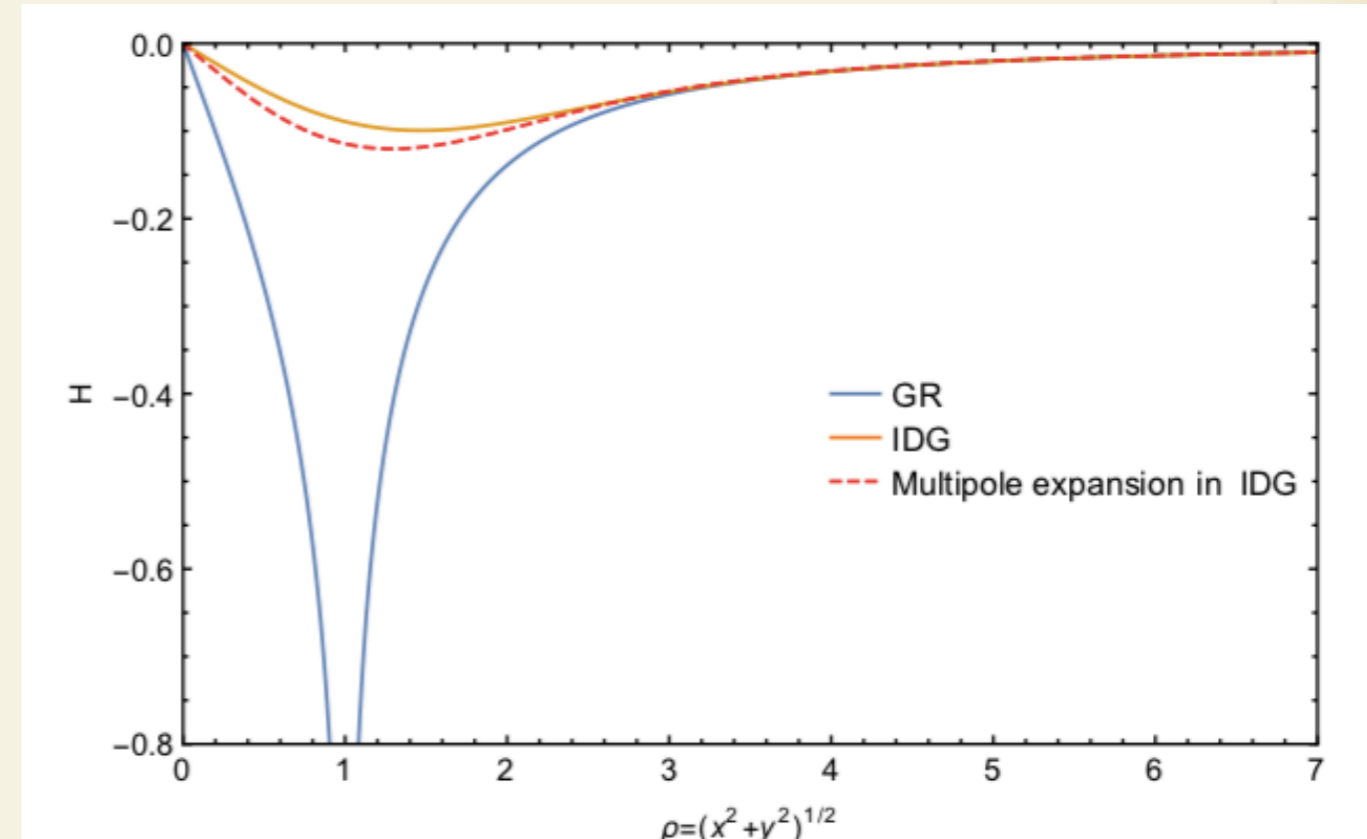
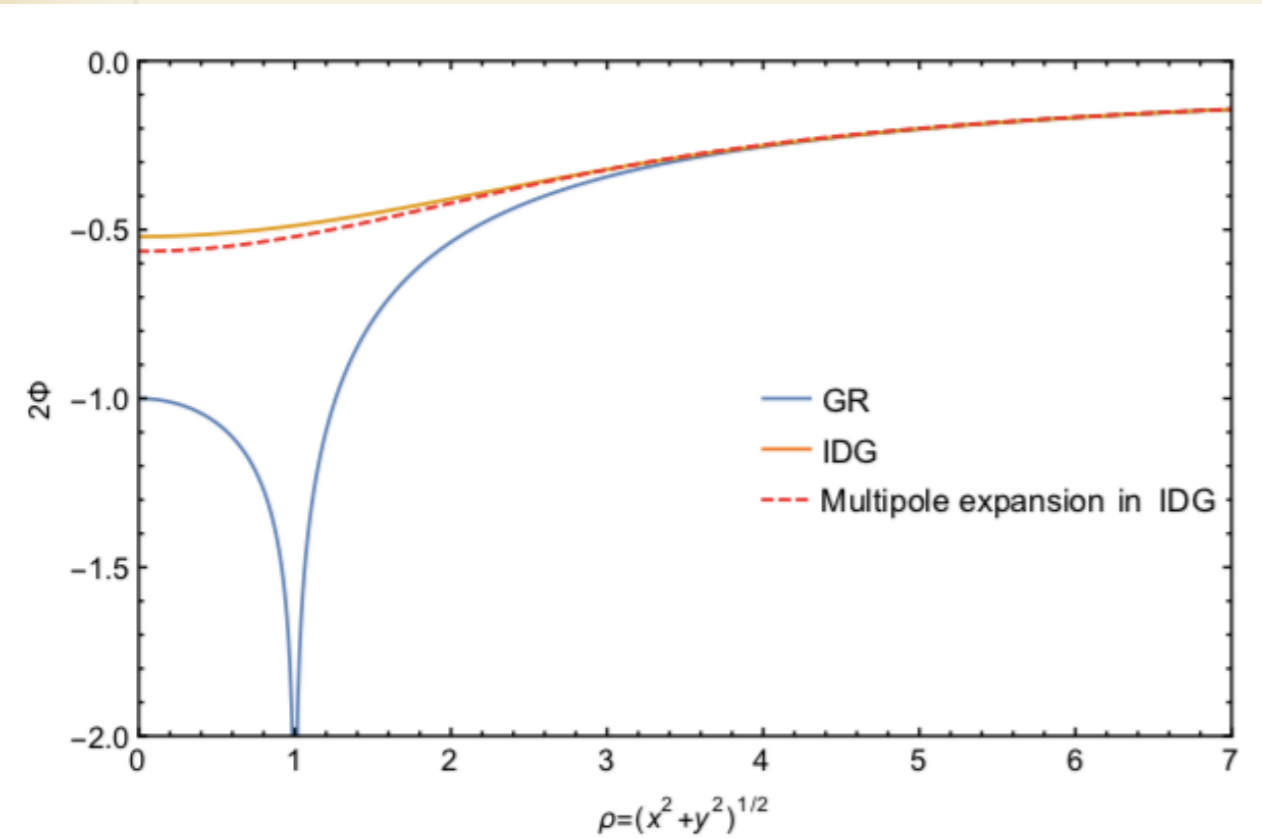
$$r > 2.256M$$

# Schwarzschild vs. Non-local Stars



**How Compact is the Non-local Star?**

# Conformally flat: Rotating solution with No Ring Singularity



$$ds^2 = -(1 + 2\Phi)dt^2 + 2\vec{h} \cdot d\vec{x}dt + (1 - 2\Psi)d\vec{x}^2,$$

$$\Phi(0) = -\frac{Gm}{a} \text{Erf} \left( \frac{M_s a}{2} \right)$$

$$ds^2 = -\left(1 - \frac{2Gm}{r} \text{Erf} \left( \frac{M_s r}{2} \right)\right) dt^2 + \left(1 + \frac{2Gm}{r} \text{Erf} \left( \frac{M_s r}{2} \right)\right) (dr^2 + r^2 d\Omega^2) - 4GJ \left[ \frac{1}{r} \text{Erf} \left( \frac{M_s r}{2} \right) - \frac{M_s}{\sqrt{\pi}} e^{-\frac{M_s^2 r^2}{4}} \right] \sin^2 \theta d\varphi dt.$$

$$a < \frac{2}{M_s} \quad (\text{radius of the ring} < \text{scale of non-locality})$$

*At non-linear level only solution survives is a conformally flat metric*

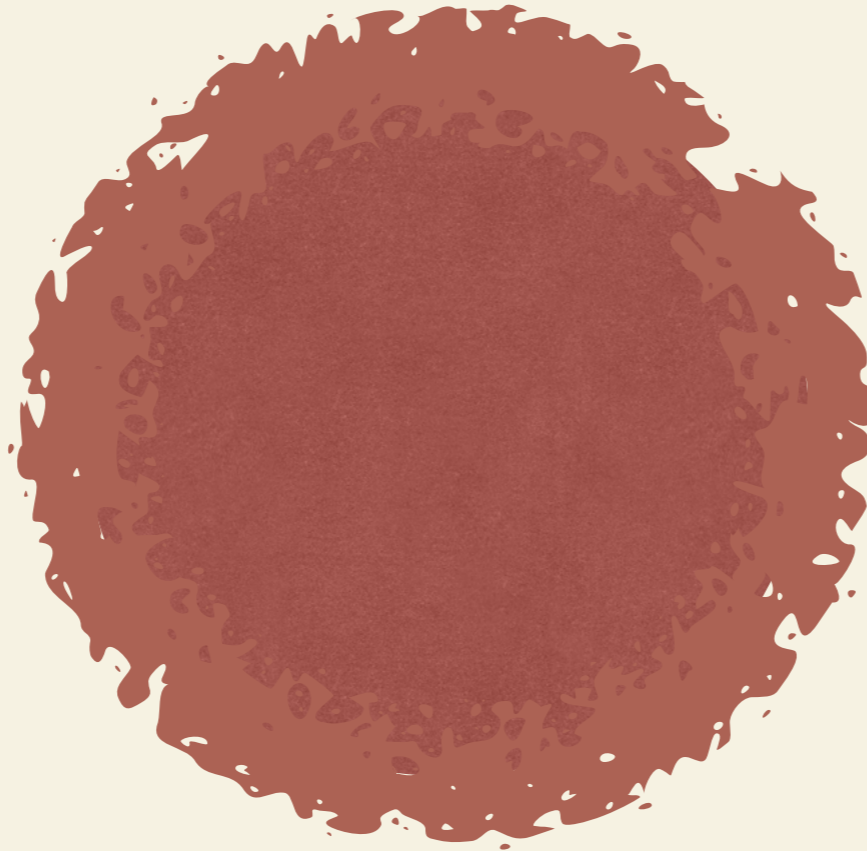


# Non-Local Star

No Horizon

No Singularity

Ultra Stable



Ultra-Compact

$$r > 2.256M$$

Satisfies Buchdahl bound

$$r > 2.5M$$

Shadow:  $5.84 M$

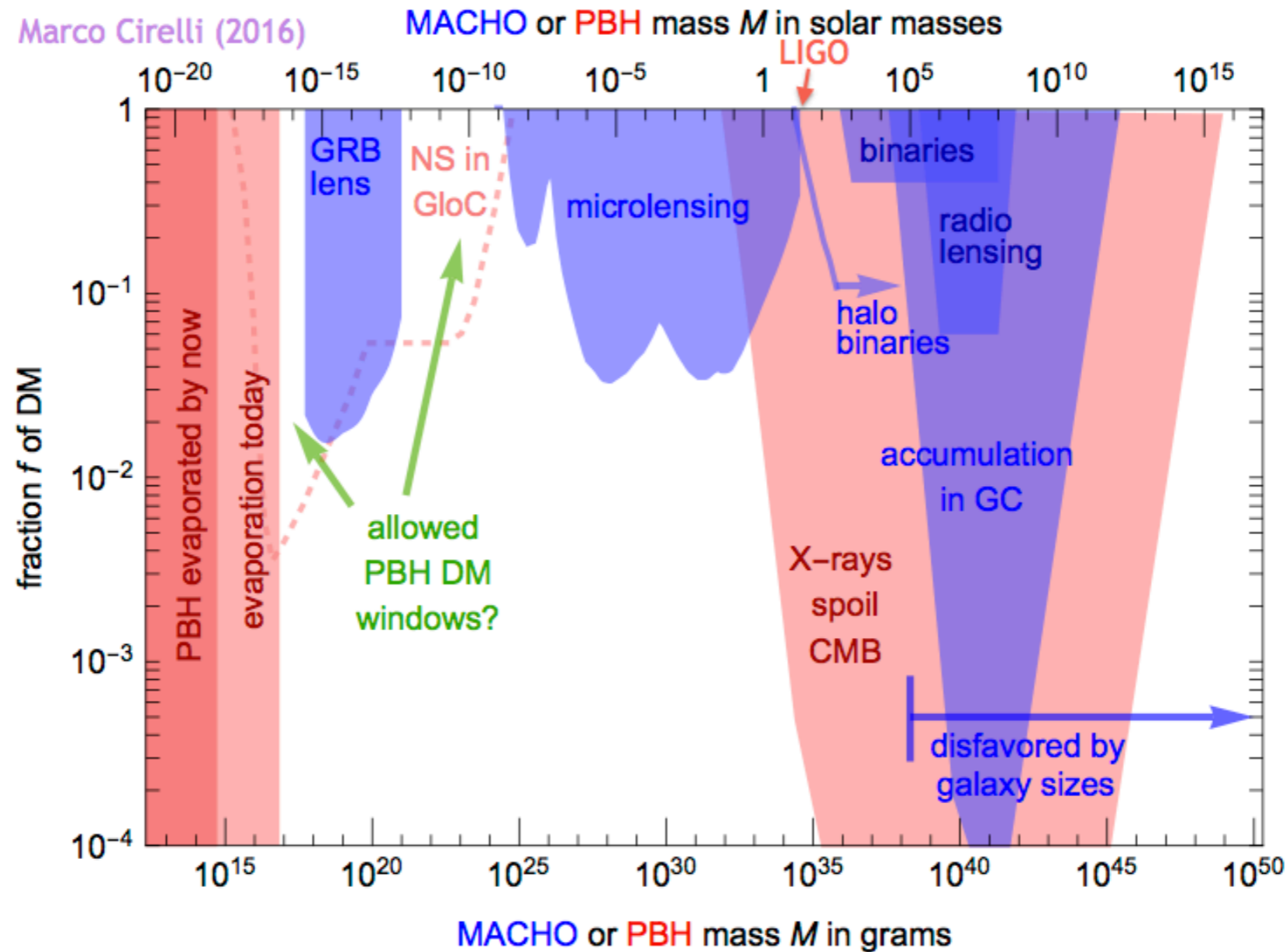
**As a Blackhole Mimicker !**

	radius	horizon	photosphere	$\mu$	absorption	life time
blackhole	$2Gm$	YES	YES	0	1	$L_p^4 \frac{m^3}{\hbar^3}$
nonlocal star	$2Gm(1 + \epsilon)$	NO	YES	0.11	$0.977 \lesssim \kappa \lesssim 1$	$\left(\frac{L_s}{L_p}\right)^8 L_s^4 \frac{m^3}{\hbar^3}$

*A self consisted solution within Ghost free, singularity free IDG*

*Buoninfante, AM, 1903.01542*

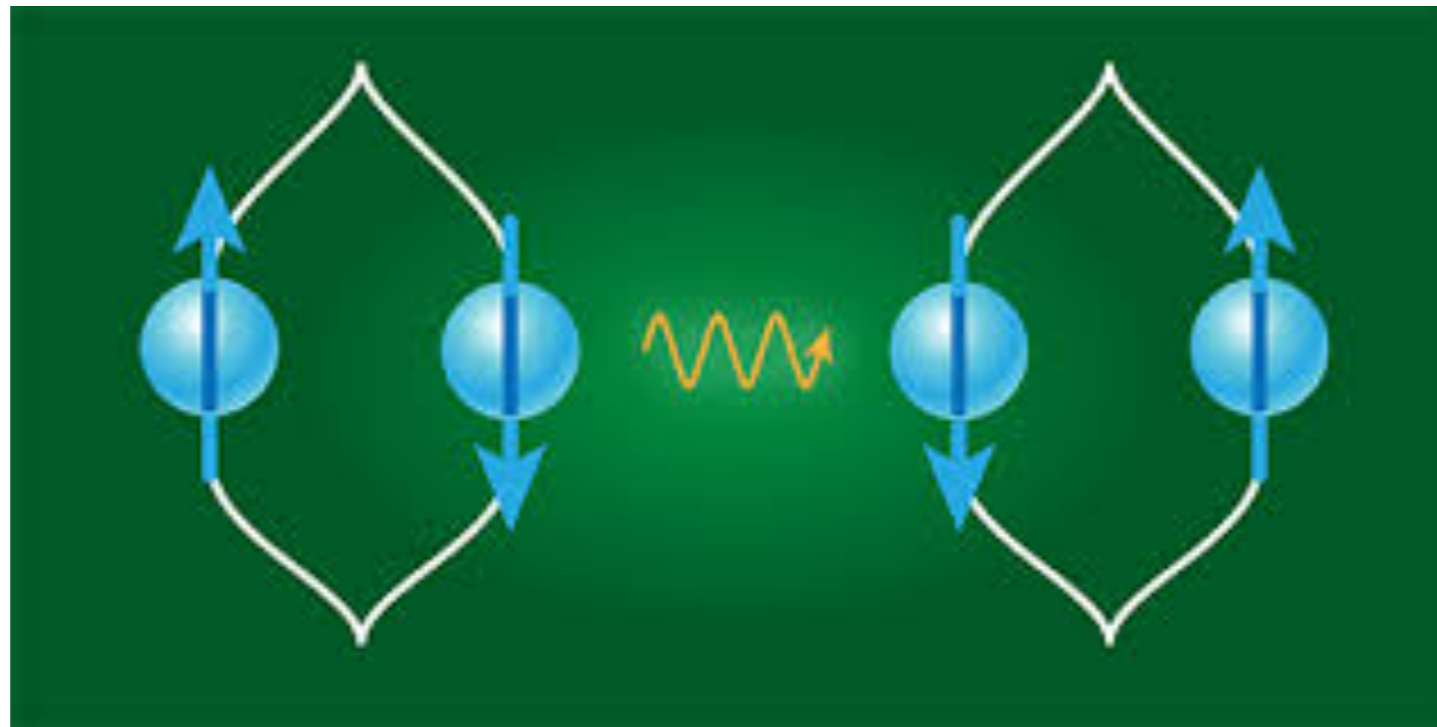
# Non-local Star & Dark Matter Candidate



$$m_{\circ} > 10^{14} \left( \frac{M_s}{M_p} \right)^3 \text{ g} \quad M_s \sim 10^{16} \text{ GeV}, \quad M_p \sim 10^{19} \text{ GeV}; \quad m_{\circ} > 10^9 \text{ g}$$

Perfect candidate for a dark Matter, which opens up a new parameter space as compared to primordial blackholes

# Testing Quantum Aspects of Linearized Gravity in a Lab



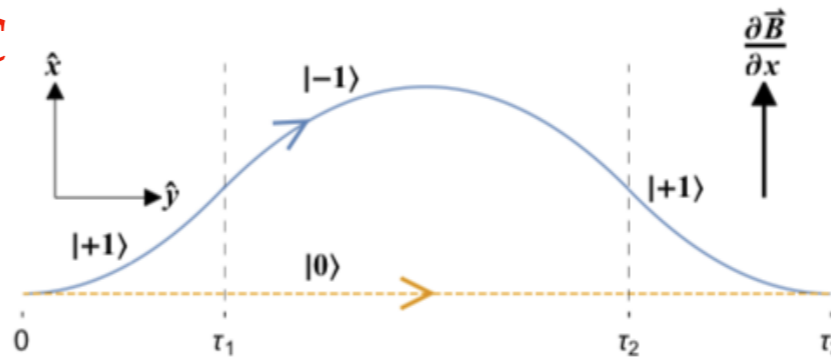
**Bose + AM + Morley + Ulbricht + Toros + Paternostro + Geraci + Barker + Kim + Milburn,**  
**Phy. Rev. Lett. [ArXiv: 1707.06050]**

See Also: Marletto and Vedral appeared on the same day [1707.06036], Phys. Rev. Lett.  
Belenchia, Wald, Giacomini, Castro-Ruiz, Brukner, Aspelmeyer [1807.0715], Phys. Rev. D  
M. Christodoulou and C. Rovelli, 1808.05842 [gr-qc]

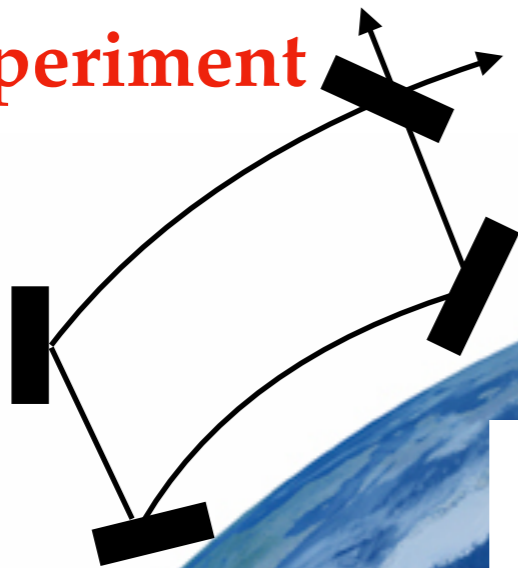
# Gravitational Induced Phase is Detectable !

$$\Delta\phi \sim i \frac{S(G, \dots)}{\hbar}$$

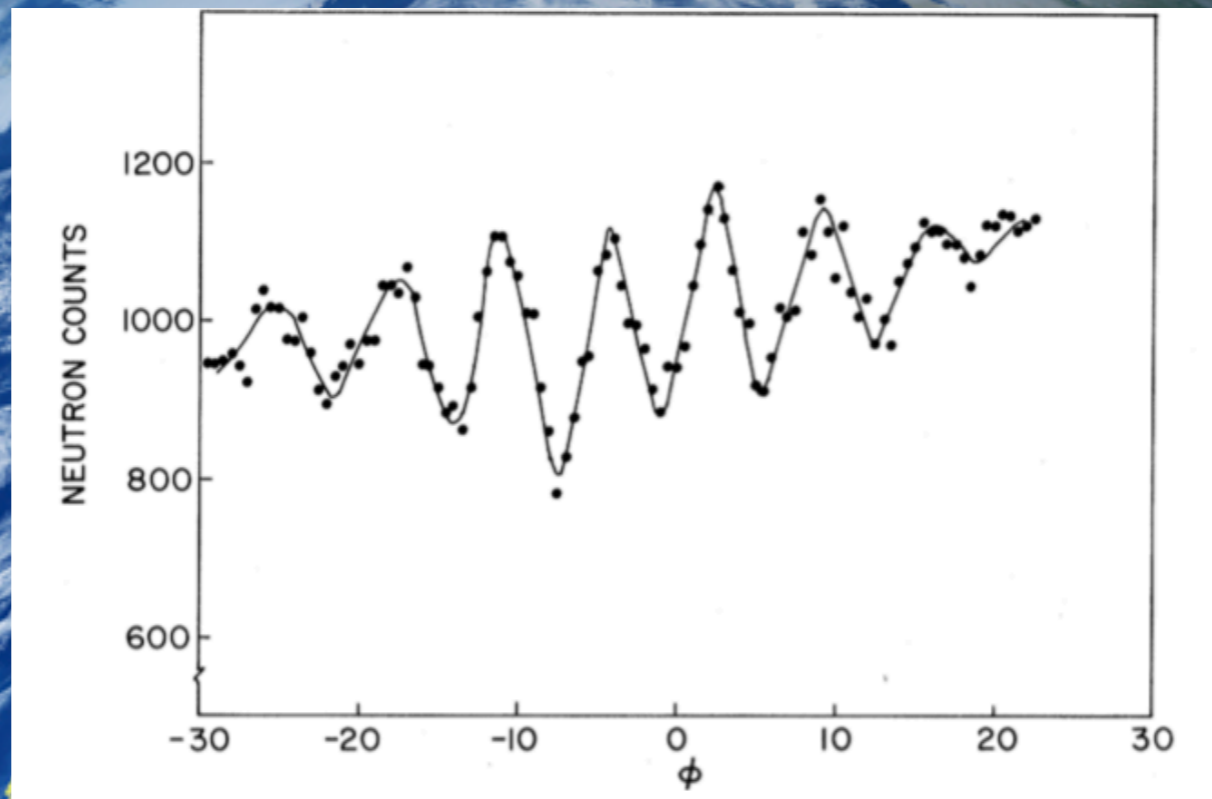
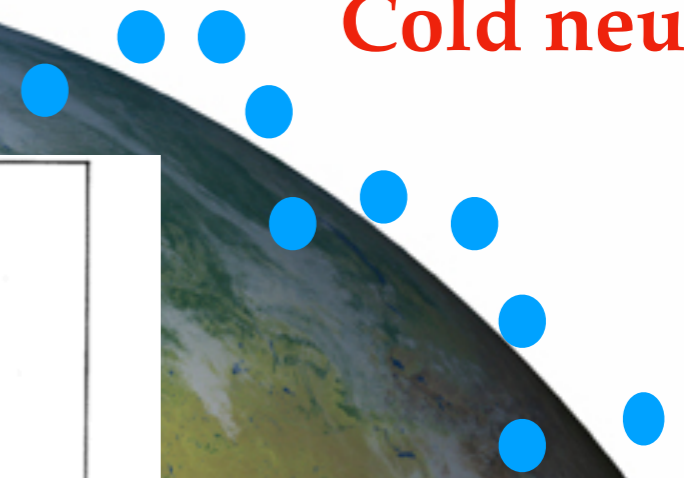
MIMAC



COW Experiment



Bouncing  
Cold neutron



# Levels of Excitements ...



**Can we put a graviton in a quantum superposition?**

**Can we study coalescing atoms, and see the loss of gravitons ( quantized ) in a laboratory?**

**Can we witness quantum entanglement due to gravitons ?**

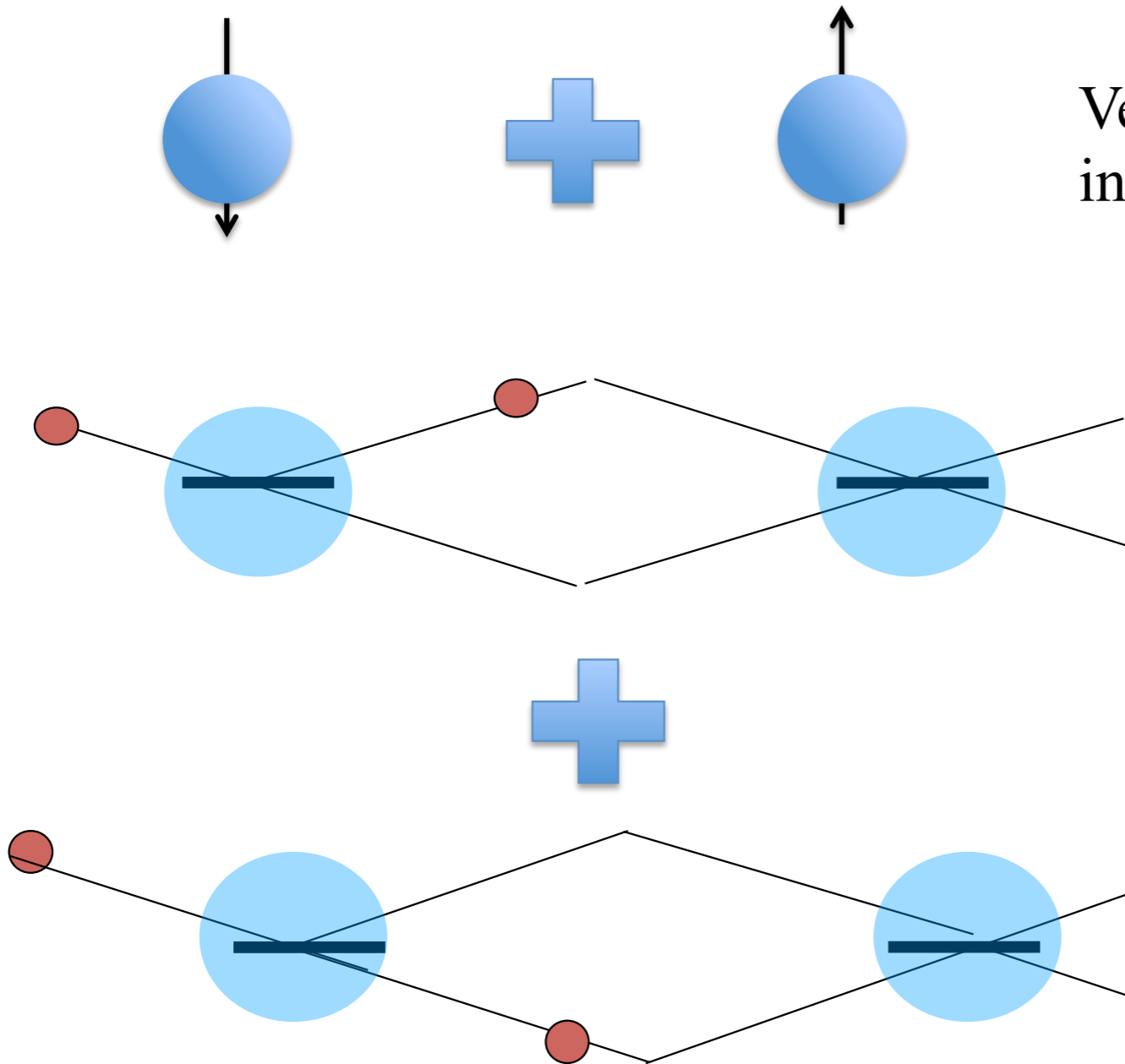


# Quantum World: “Off-Shell/Virtual”

The Superposition Principle **Underpins** Quantum Mechanics



Very familiar  
in experiments

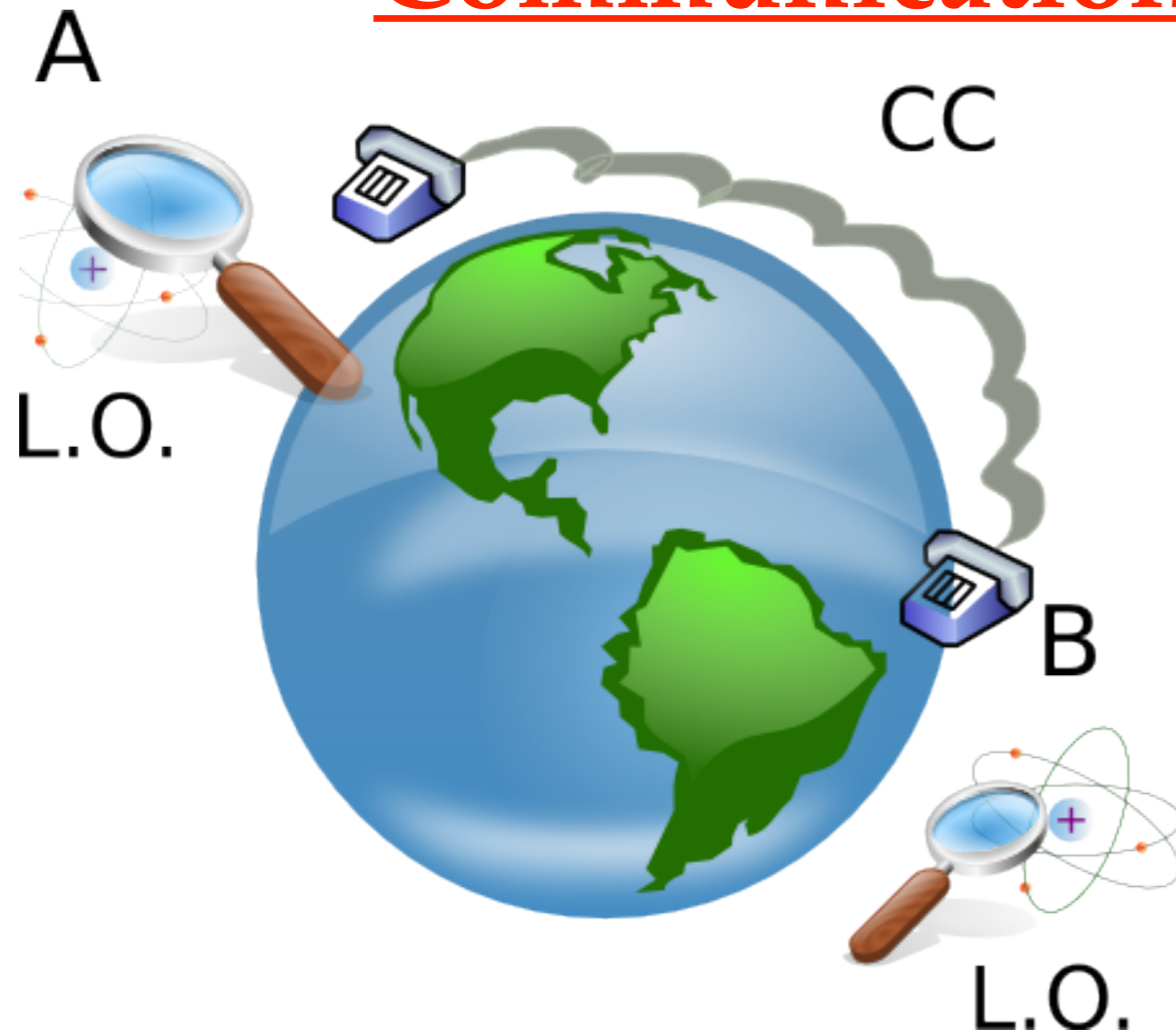


*Off-shellness is critical  
even before creating  
superposition of  
macroscopic objects  
in a lab!!*

*Localization of 2 distinct  
classical paths*

If you *decohere* (kill superpositions) nonclassical features of quantum mechanics go away. Even old quantum mechanics: the right difference between energy levels obtained only through a superposition of localized states.

# Local Operations & Classical Communication (LOCC)



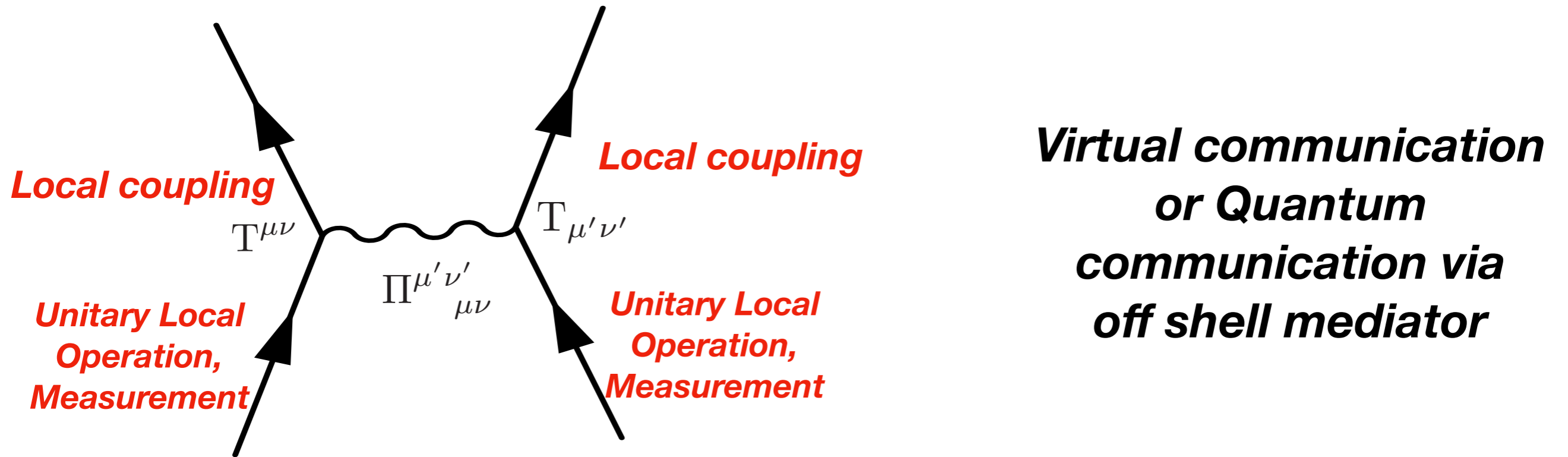
***Cannot create  
Entanglement***

Bennett, et.al, (1996)

**LOCC keeps Separable state remains Separable ( Cannot  
create entanglement )**

# Quantumness of the Mediator

## Graviton as an Off-shell/Virtual mediator

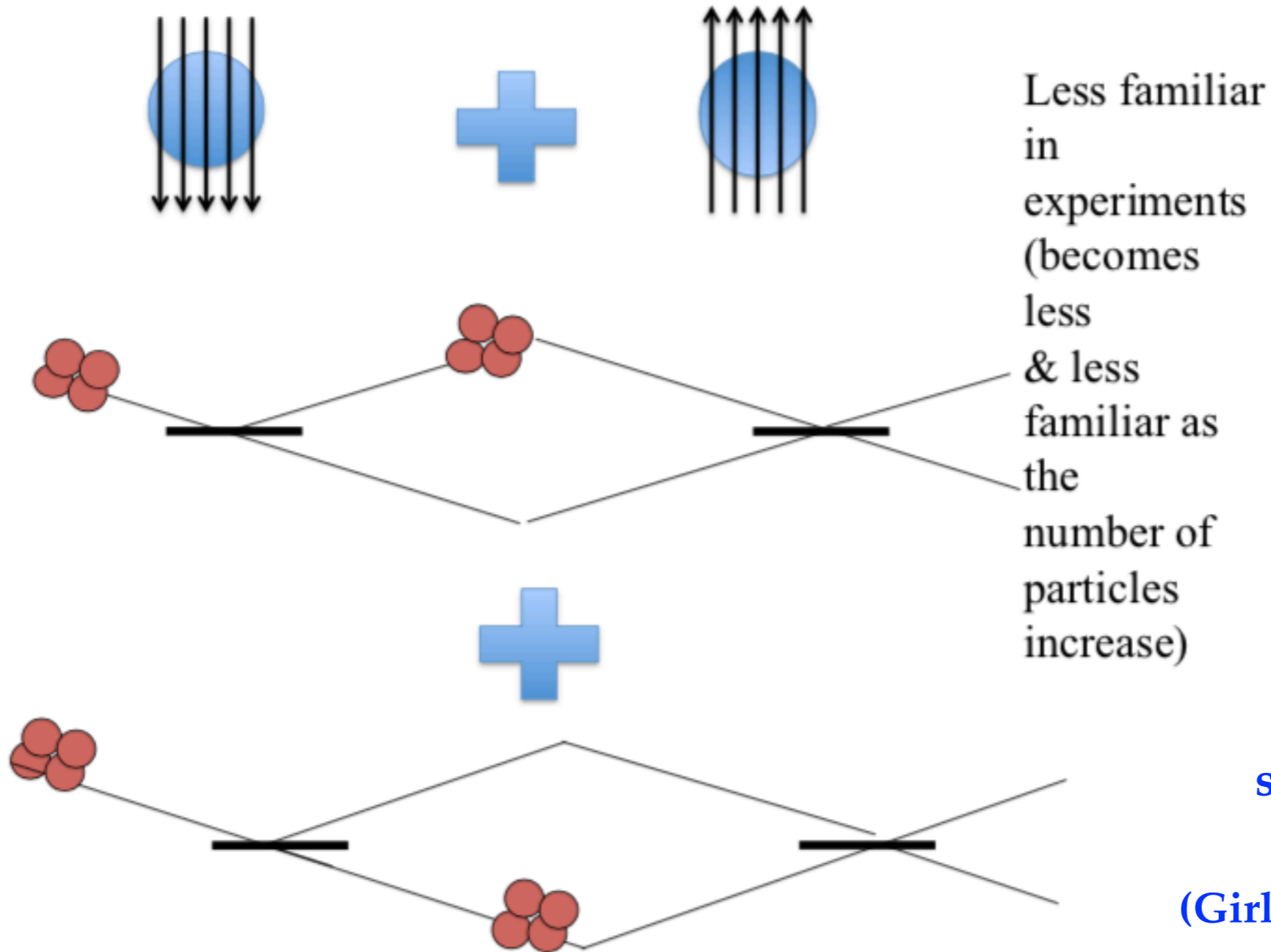


$$\Pi(k^2) \sim \frac{P^{(2)}}{k^2} - \frac{P^{(0)}}{2k^2} \quad \dots \rightarrow \quad V \sim \frac{1}{r}$$

Graviton propagator in terms of spin projection operators in 4d, Minkowski space time



# Challenges: Macroscopic & Maintaining the Superposition



Any mechanism from cooling to creating a potential is a source for Decoherence!

Arndt, Zeilinger and collaborators have demonstrated that superposition is attainable for  $10^5$  amu  
(Girlich, et.al., Nature, Arndt group)

Such superpositions are also called GHZ states or NOON states or Schroedinger Cat States

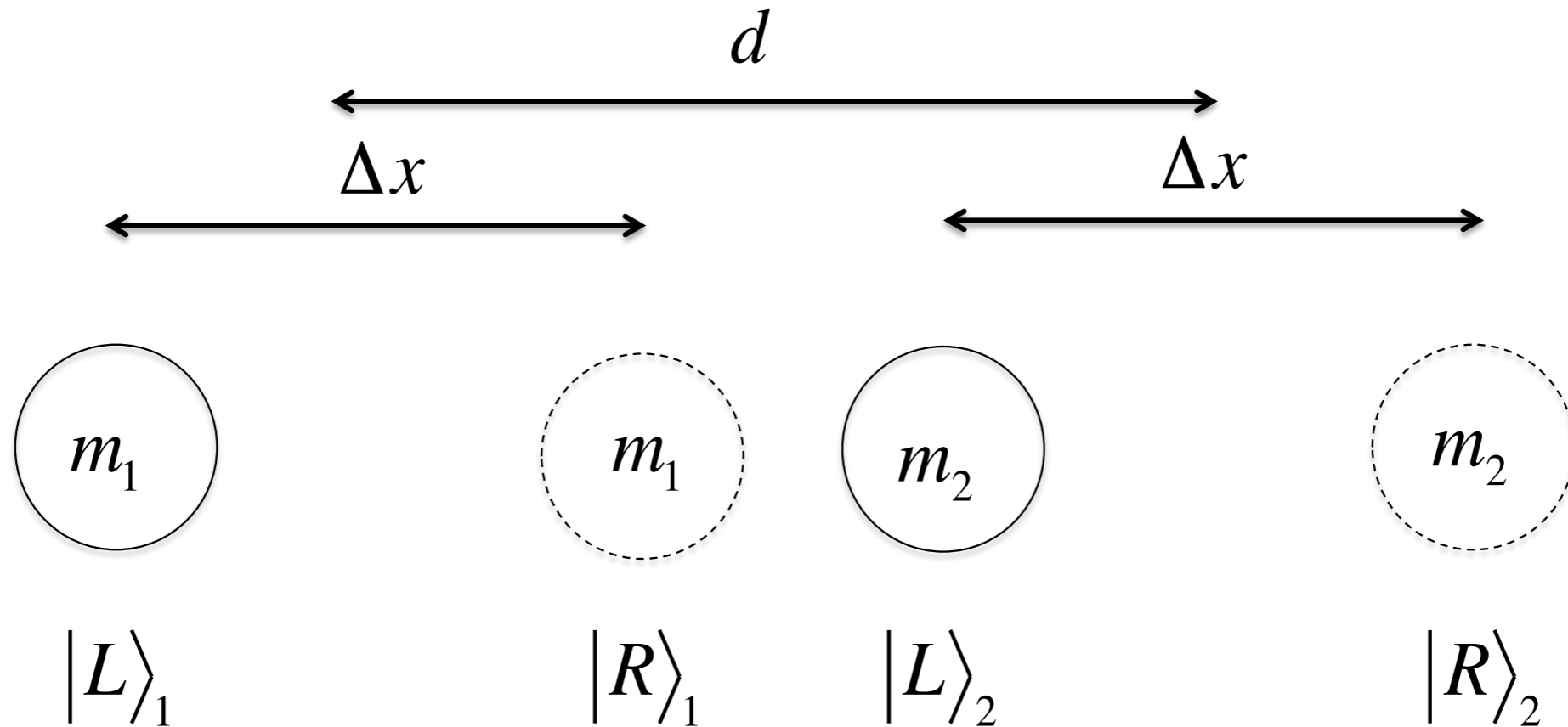
*S. Bose, K. Jacobs, P. L. Knight, Phys. Rev. A 59 (5), 3204 (1999). [arXiv: 1997], S. Bose, PRL (2006)*

*Armour, Blencowe, Schwab, PRL 2002, Marshall, Simon, Penrose, Bouwmeester, PRL 2003.*

*M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, PRL 111, 180403 (2013),*

# 2 Masses & Virtual Graviton

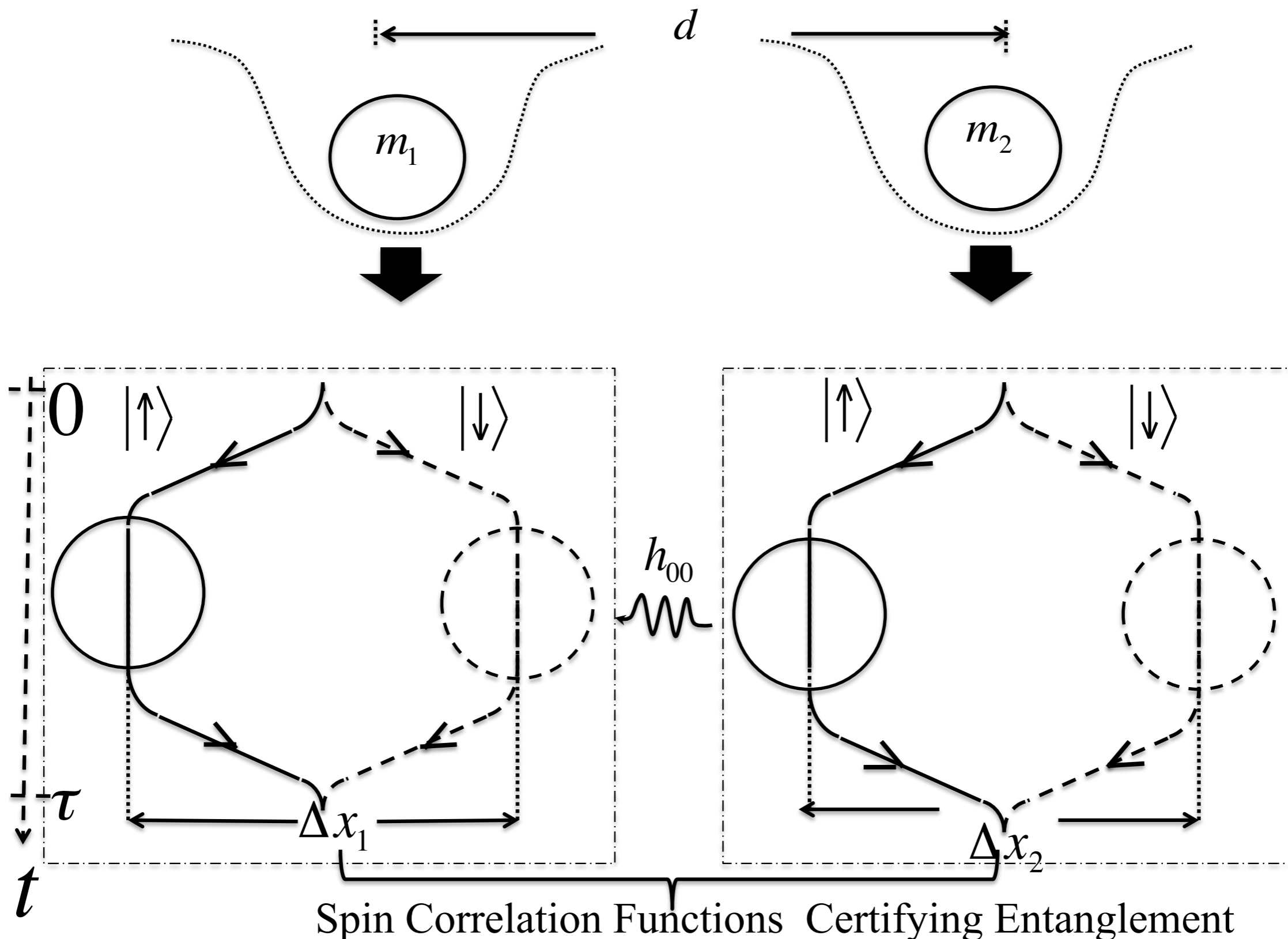
A Schematic of two matter-wave interferometers near each other

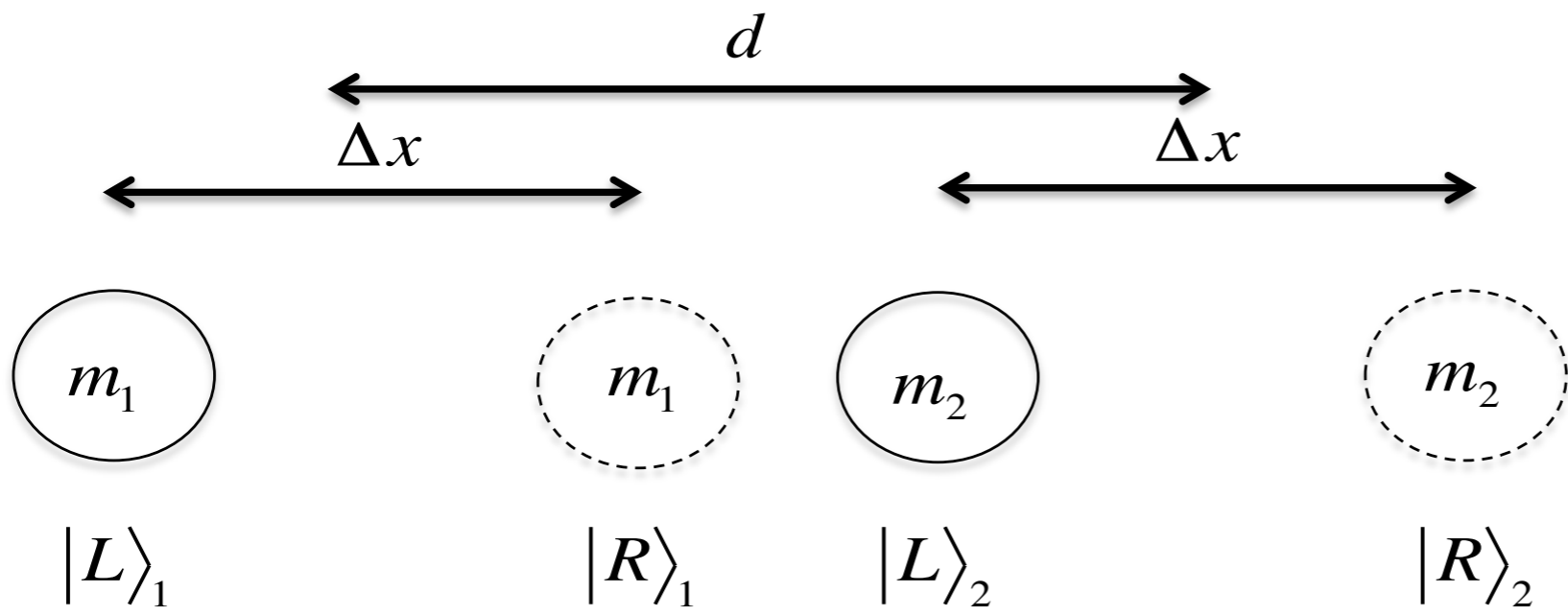


Consider two neutral test masses *held* in a superposition, each exactly as a path encoded qubit (states  $|L\rangle$  and  $|R\rangle$ ), near each other.

**Assume there are no other interactions for the time being other than pure gravitation**

# 2 Stern Gerlachs





$$\begin{aligned}
 |\Psi(t=0)\rangle_{12} &= \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1) \frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2) \\
 &= \frac{1}{2}(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2) \\
 \rightarrow |\Psi(t=\tau)\rangle_{12} &= \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2 \\
 &\quad + e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2),
 \end{aligned}$$

where

$$\begin{aligned}
 \phi_{RL} &\sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \quad \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)}, \\
 \phi_{LL} = \phi_{RR} &\sim \frac{Gm_1m_2\tau}{\hbar d}
 \end{aligned}$$

# Entanglement Phase

Step 4: Witness spin entangled state:

$$|\Psi(t = t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 \frac{1}{\sqrt{2}} (|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}} |\downarrow\rangle_2) \right. \\ \left. + |\downarrow\rangle_1 \frac{1}{\sqrt{2}} (e^{i\Delta\phi_{RL}} |\uparrow\rangle_2 + |\downarrow\rangle_2) \right\} |C\rangle_1 |C\rangle_2$$

through the correlations:

$$\mathcal{W} = |\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle - \langle \sigma_y^{(1)} \otimes \sigma_z^{(2)} \rangle|$$

we have

$$\Delta\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d - \Delta x)} \gg \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$
$$\Delta\phi_{LR} + \Delta\phi_{RL} \sim \mathcal{O}(1)$$

For mass  $\sim 10^{-14}$  kg (microspheres), separation at closest approach of the masses  $\sim 200$  microns (to prevent Casimir interaction), **time  $\sim 1$  seconds**, gives:

Scale of superposition  $\sim 100$  microns,  **$\Delta\phi_{RL} \sim 1$**

Planck's Constant fights Newton's Constant!

# Experimental Protocol



$10^{-14} Kg$

Radius : 100nm

Frequency of harmonic potential : 0.1MHz

Temperature : mK

## Neutralising e.m. charges

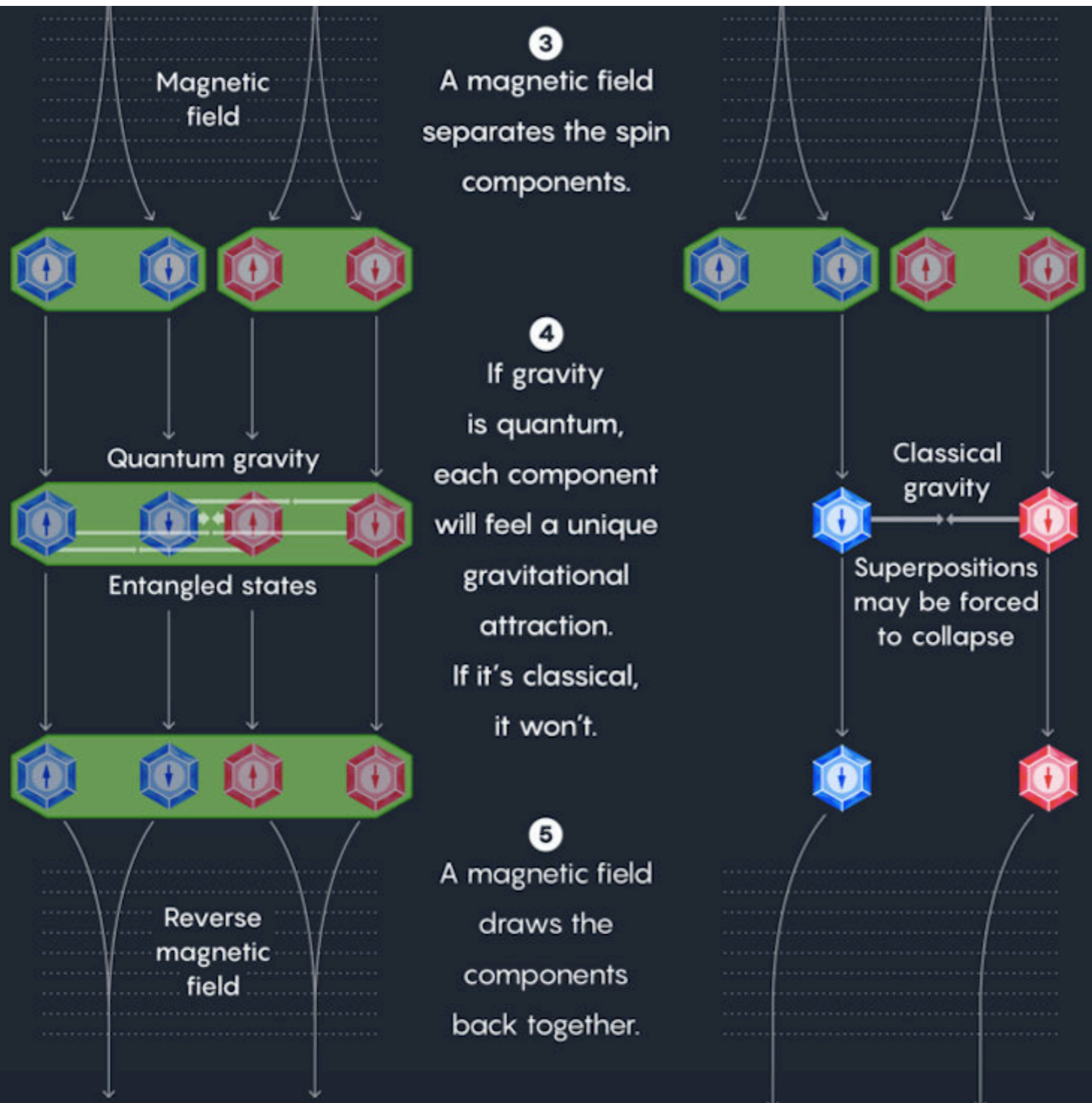
A magnetic field gradient of  $\sim 10^6$  T/m and a time  $\tau_{acc} \sim 500$  m/s<sup>2</sup>,  
 $\Delta x \sim 250\mu\text{m}$ ,  $d-\Delta x \sim 200\mu\text{m}$

T. Krisnanda, M. Zuppardo, M. Paternostro, T. Paterek, arXiv:1607.01140. Superconducting sphere with half a micrometer separation (magnetically levitating)

C. Wan, M. Scala, G. W. Morley, ATM. A. Rahman, H. Ulbricht, J. Bateman, P. F. Barker, S. Bose, and M. S. Kim, Phys. Rev. Lett. 117, 143003 (2016);  
M. Frimmer, K. Luszcz, S. Ferreiro, V. Jain, E. Hebestreit, and L. Novotny, Phys. Rev. A 95, 061801 (2017).

H. Pino, J. Prat-Camps, K. Sinha, B. P. Venkatesh, and O. Romero-Isart, arXiv:1603.01553v2

# Challenges & Sources of Decoherence



Electronic spins coherent for 1s (in steps 1 and 3), which should be possible for macro-diamond below 77 K

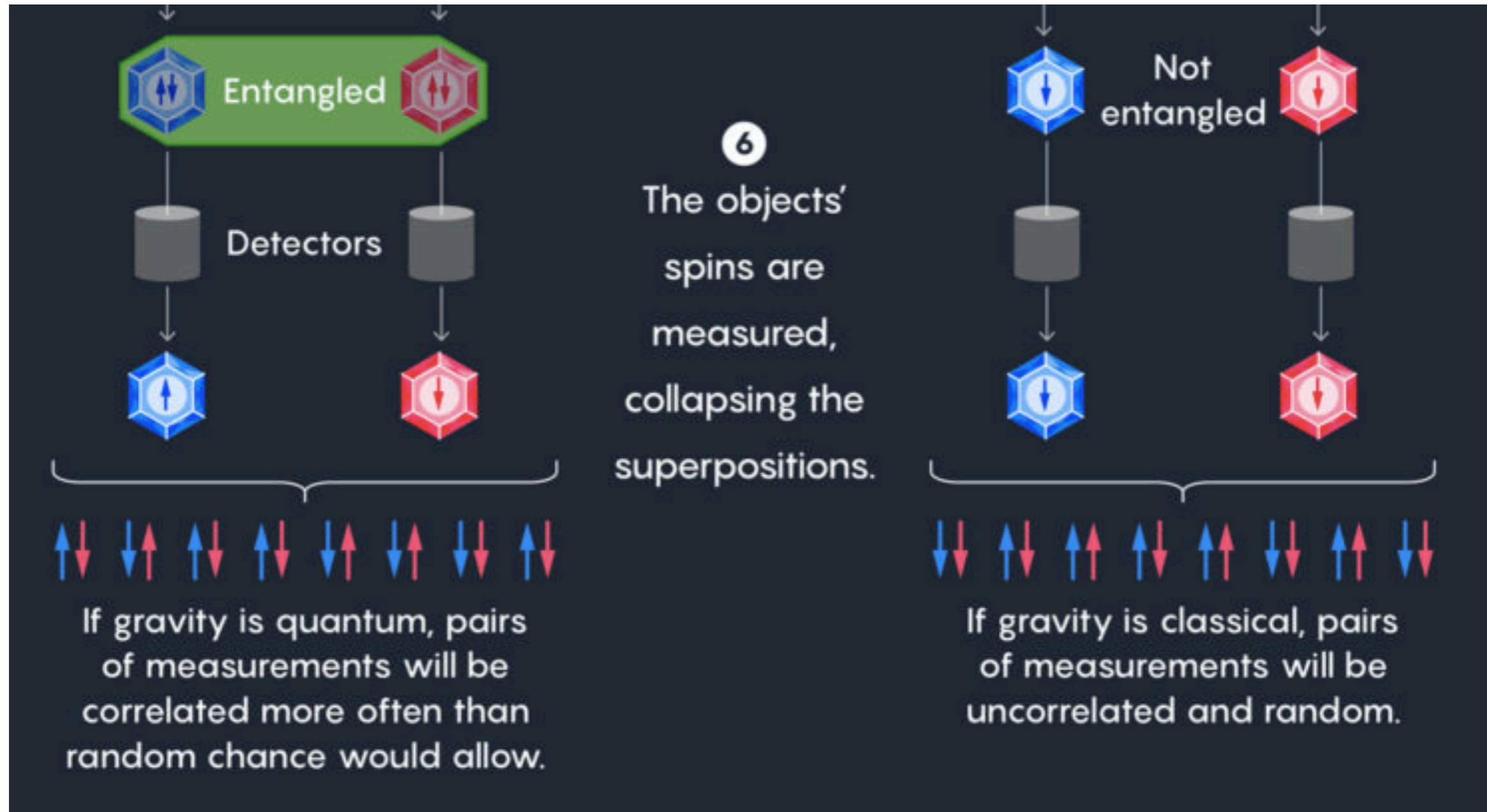
N. Bar-Gill, L.M. Pham, A. Jarmola, D. Budker, R. L. Walsworth, *Nature Comm*, 4, 1743 (2013),

S. Knowles, D. M. Kara and M. Atatüre, *Nature Materials* 13, 21 (2014),

Kaltenbaek, Aspelmeyer, (2015)

To estimate collisional and thermal decoherence times of the orbital degree of freedom we consider the pressure  $P = 10^{-15} Pa$  and the temperature 0.15 K. the collisional decoherence time for a superposition size of  $\Delta x \sim 250\mu m$  is the same order of magnitude as the total microsphere's fall time  $\tau + 2\tau_{acc} \sim 3.5 s$

# Measuring Spin Correlation & Establishing the Entanglement



$$\mathcal{W} = |\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle - \langle \sigma_y^{(1)} \otimes \sigma_z^{(2)} \rangle| \quad \text{If } \mathcal{W} > 1 \implies \text{Graviton is quantum}$$

**Basis Dependent Witness, similar to Bell's**

**Basis Independent Witness:**

$$S_A = -\text{Tr}_A \rho_A \log \rho_A = S_B$$



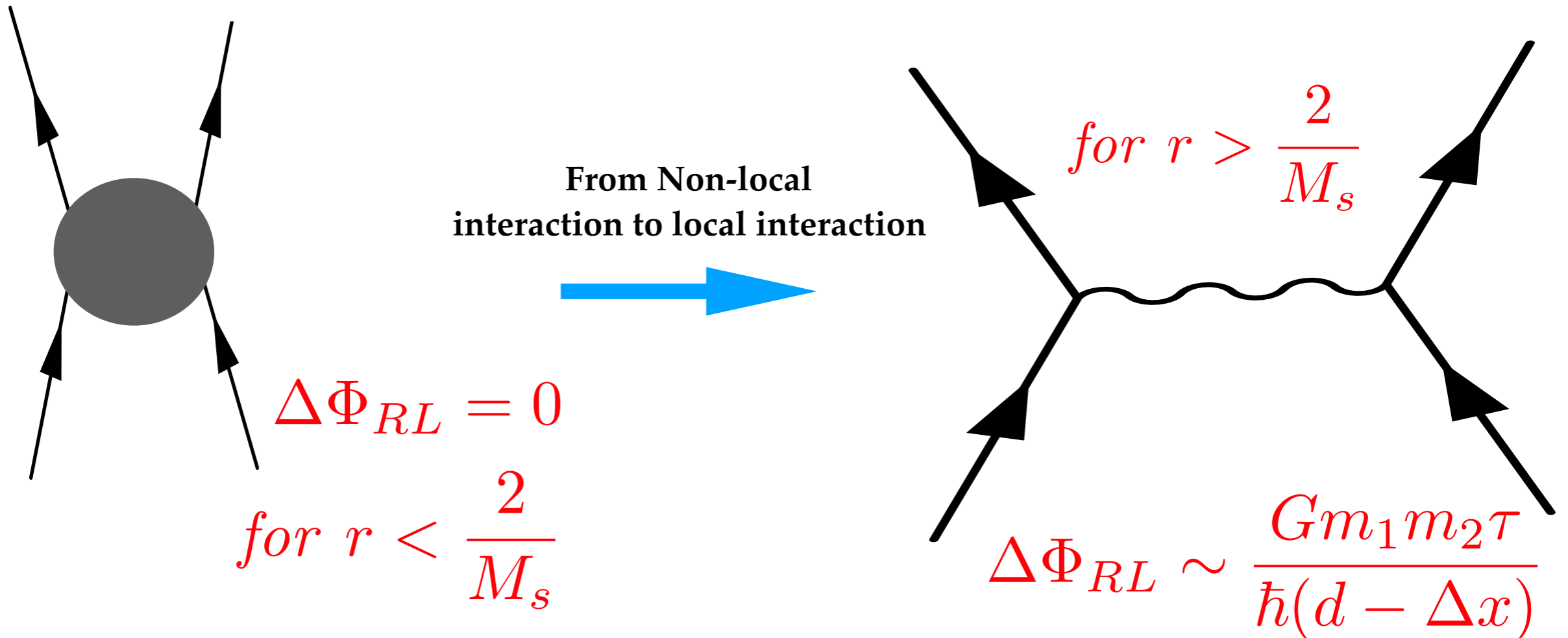
# Quantum Entanglement Vanishes as we resolve Gravitational Singularity

Step 4: Witness spin entangled state:

$$|\Psi(t = t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 \frac{1}{\sqrt{2}} (|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}} |\downarrow\rangle_2) \right. \\ \left. + |\downarrow\rangle_1 \frac{1}{\sqrt{2}} (e^{i\Delta\phi_{RL}} |\uparrow\rangle_2 + |\downarrow\rangle_2) \right\} |C\rangle_1 |C\rangle_2$$

through the correlations:

$$\mathcal{W} = \left| \langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle - \langle \sigma_y^{(1)} \otimes \sigma_z^{(2)} \rangle \right|$$



# Conclusion: We can potentially test linearized Quantum Gravity in a Lab !

Alice, Bob and Eve

**We are  
all  
entangled  
:  
Gravity is  
QUANTUM !**

**Now we can test it !**



# Relevant Publications

*T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar,*

*“Towards singularity and ghost free theories of gravity,” [arXiv:1110.5249 [gr-qc]].*

*T. Biswas, A. Mazumdar and W. Siegel,*

*“Bouncing universes in string-inspired gravity,” [hep-th/0508194].*

*L. Buoninfante, A. S. Koshelev, G. Lambiase and A. Mazumdar,*

*“Classical properties of non-local, ghost- and singularity-free gravity,” [arXiv:1802.00399 [gr-qc]].*

*L. Buoninfante, A. S. Koshelev, G. Lambiase, J. Marto and A. Mazumdar,*

*“Conformally-flat, non-singular static metric in infinite derivative gravity,” [arXiv:1804.08195 [gr-qc]].*

*L. Buoninfante, A. S. Cornell, G. Harmsen, A. S. Koshelev, G. Lambiase, J. Marto and A. Mazumdar,*

*“Towards nonsingular rotating compact object in ghost-free infinite derivative gravity,” [arXiv:1807.08896 [gr-qc]].*

*L. Buoninfante, A. Ghoshal, G. Lambiase and A. Mazumdar,*

*“Transmutation of nonlocal scale in infinite derivative field theories,” arXiv:1812.01441 [hep-th].*

*A. Ghoshal, A. Mazumdar, N. Okada and D. Villalba,*

*“Stability of infinite derivative Abelian Higgs models,” [arXiv:1709.09222 [hep-th]].*

*Bose+AM+Morley+Ulbricht+Toros+Paternostro+Geraci+Barker+Kim+Milburn, PRL (2017) [1707.06050]*

# Extra Slides

# Degrees of freedom & Ghosts

$$\Pi(k^2) = \frac{1}{k^2} \left[ P^{(2)} - \frac{P^{(0)}}{2} \right] - \frac{P^{(2)}}{k^2 - m_2^2} + \frac{P^{(0)}}{2(k^2 - m_0^2)}$$

$$m_2 = -(\beta/2)^{-1/2}, \quad m_0 = (\alpha + \beta)^{-1/2}$$

Massive Spin-0 & Massive Spin-2 ( Ghost ) Stelle (1977)

*Utiyama (1960), De Witt (1961), Stelle (1977)*

## Modification of Einstein's GR

*Modification of  
Graviton Propagator*

*Extra propagating  
degrees of freedom*

**Challenge: How to get rid of the extra dof ?**

# Avoiding Ghosts: Stability

Higher derivative theories generically carry Ghosts ( -ve Residue )

$$S = \int d^4x \phi \square (\square + m^2) \phi \Rightarrow \square (\square + m^2) \phi = 0$$
$$\Delta(p^2) \sim \frac{1}{p^2} - \frac{1}{p^2 - m^2}$$

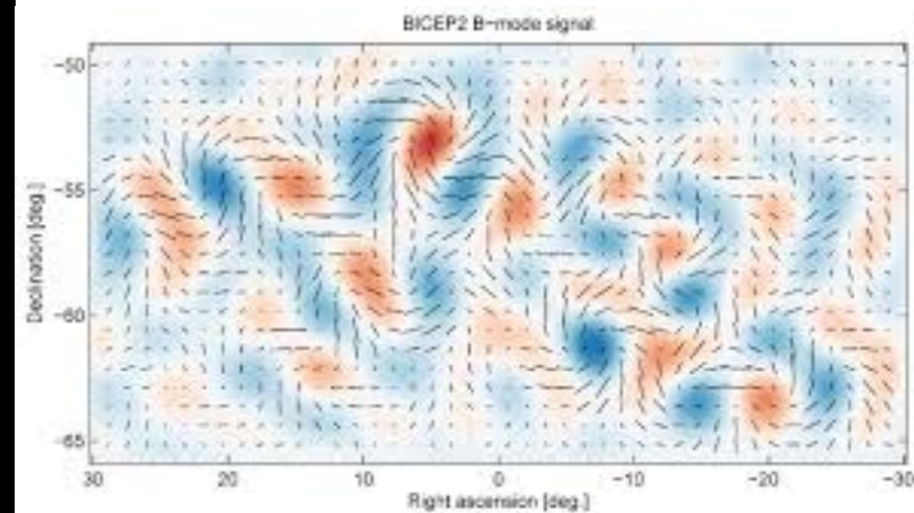
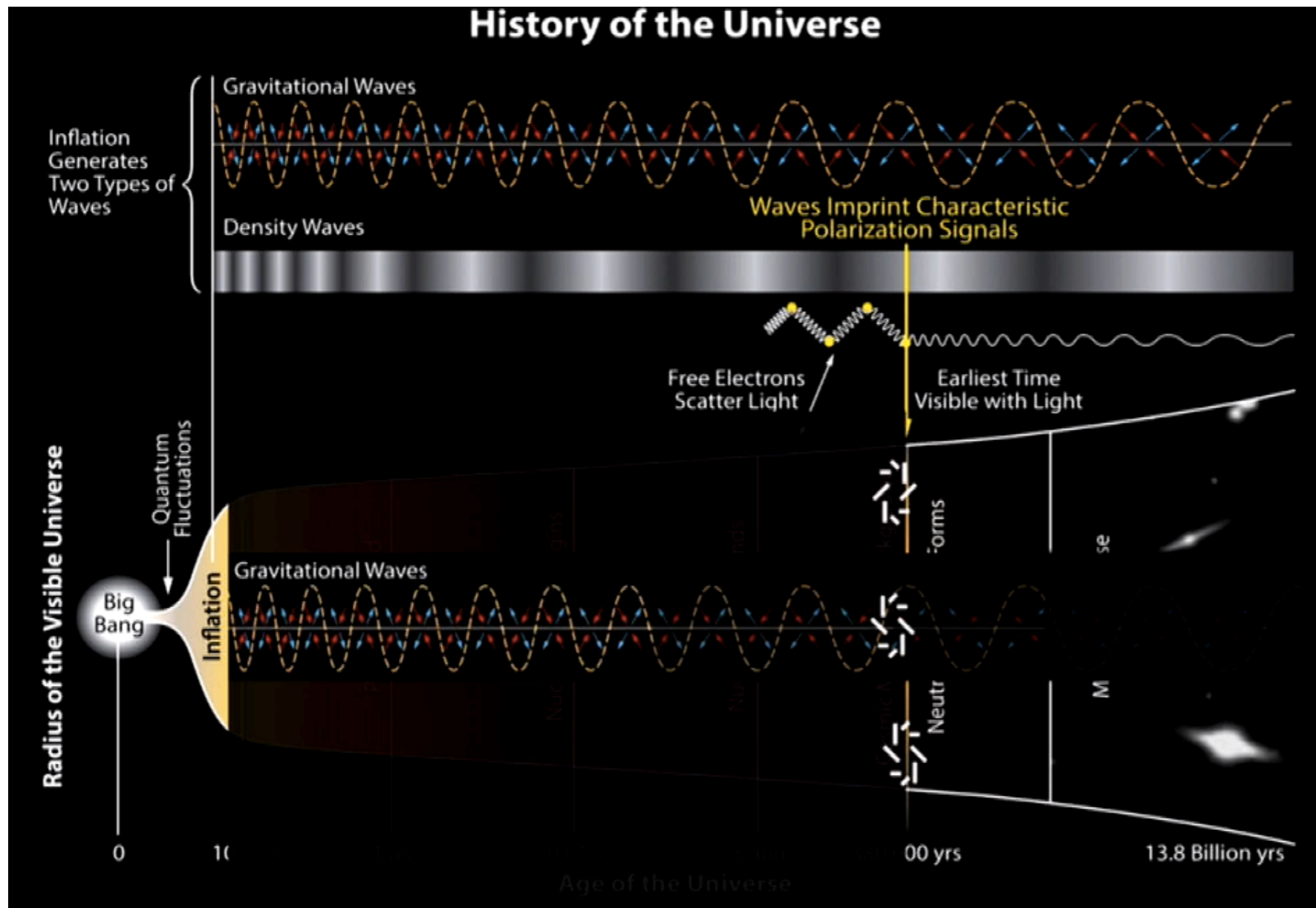
**Propagator with first order poles**

**Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!**

$$S = \int d^4x \phi e^{-\square/M^2} (\square + m^2) \phi \Rightarrow e^{-\square/M^2} (\square + m^2) \phi = 0$$
$$\Delta(p^2) = \frac{e^{-p^2/M^2}}{p^2 - m^2}$$

**No extra states other than the original dof.**

# Gravity being Classical or Quantum

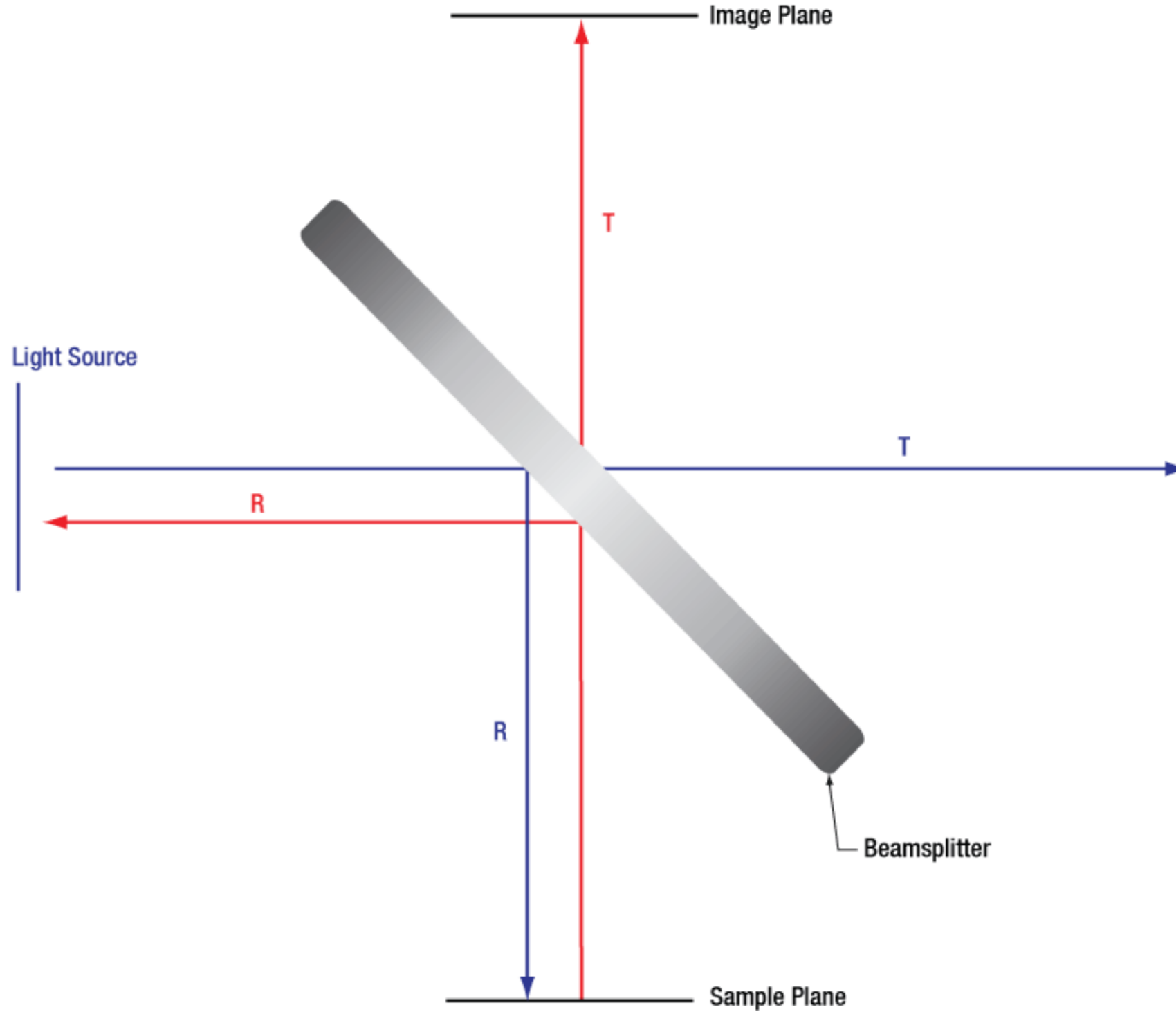
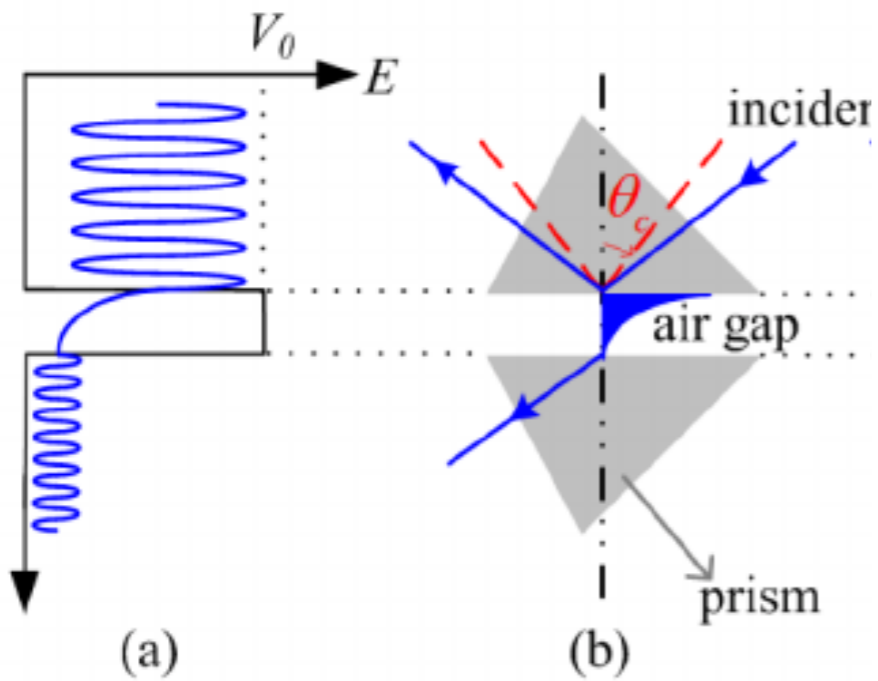
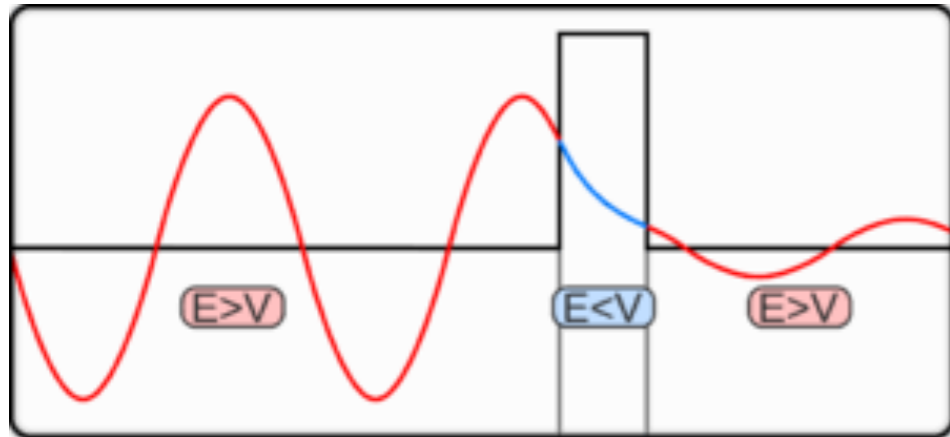


**Correct observation, but wrong interpretation**

A. Ashoorioon, P. S. Bhupal Dev and A. Mazumdar,  
"Implications of purely classical gravity for inflationary tensor modes,"  
Mod. Phys. Lett. A 29, no. 30, 1450163 (2014)  
[arXiv:1211.4678 [hep-th]].

L. M. Krauss and F. Wilczek,  
"Using Cosmology to Establish the Quantization of Gravity,"  
Phys. Rev. D 89, no. 4, 047501 (2014)  
[arXiv:1309.5343 [hep-th]].

# Quantum "Off-shell"



***Quantum Tunnelling is an off-shell process***



# Linearized Quantum Gravity

***Graviton must obey the quantum superposition principle***



$h_{\mu\nu}$  are also localized

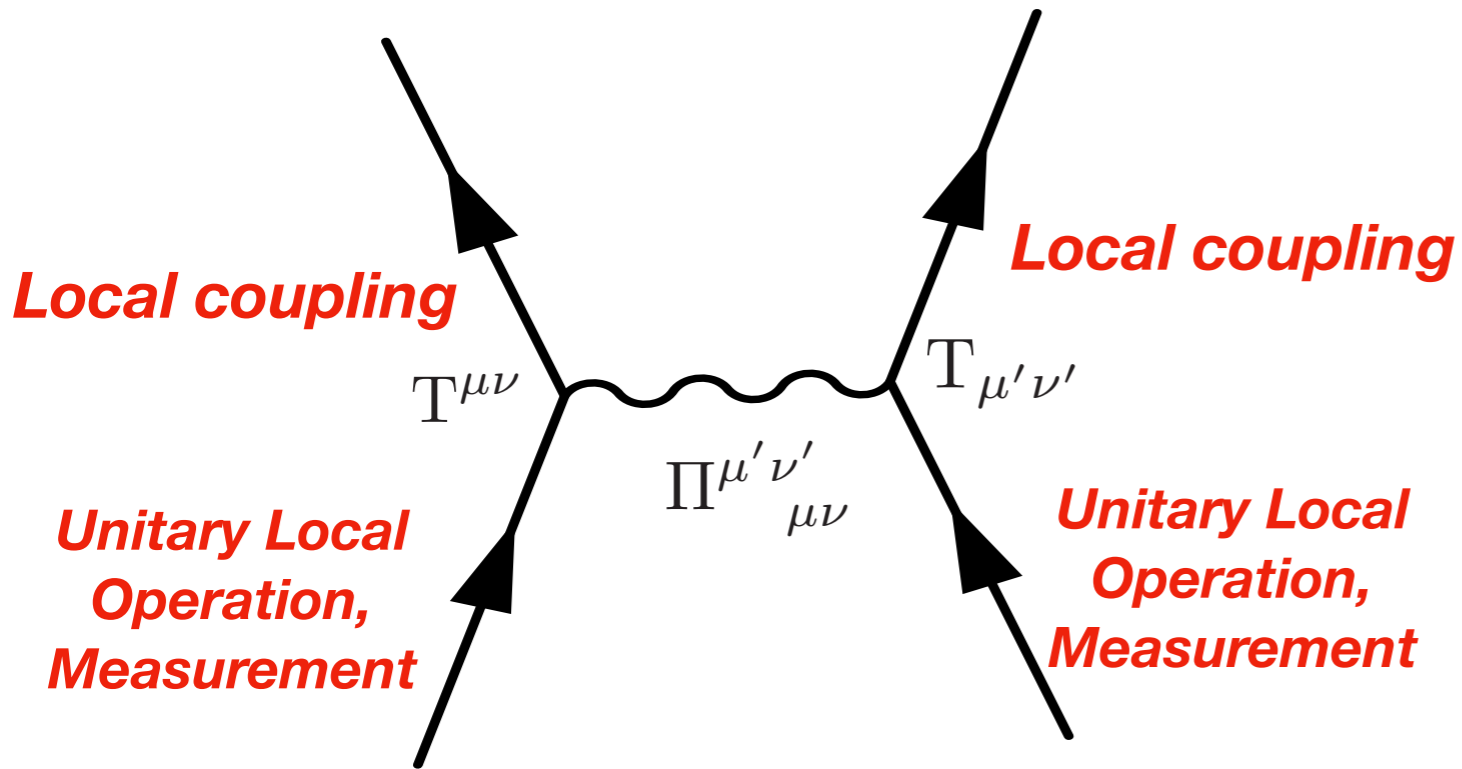
$$G_{\mu\nu} \propto T_{\mu\nu}$$

***Provided we can prepare masses in Fock state & control decoherence.***

***This will not be so for Bose-Einstein Condensate, or if matter is in a Coherent state!!***

# Linearized Quantum Gravity

## Graviton as an Off-shell/Virtual mediator



*Virtual communication or Quantum communication via off shell mediator*

$$\Pi(k^2) \sim \frac{P^{(2)}}{k^2} - \frac{P^{(0)}}{2k^2}$$

↓

$$V \sim \frac{1}{r}$$

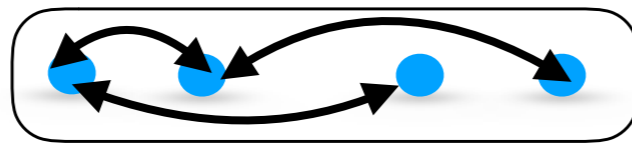
Product State



$$|\Psi\rangle = |\Psi\rangle_A \otimes |\Psi\rangle_B$$



Entangled State



$$|\Psi\rangle \neq |\Psi\rangle_A \otimes |\Psi\rangle_B$$



# Probing UV gravity

(1) GR:  $\lim_{k^2 \rightarrow 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$

(2) F(R) Gravity:

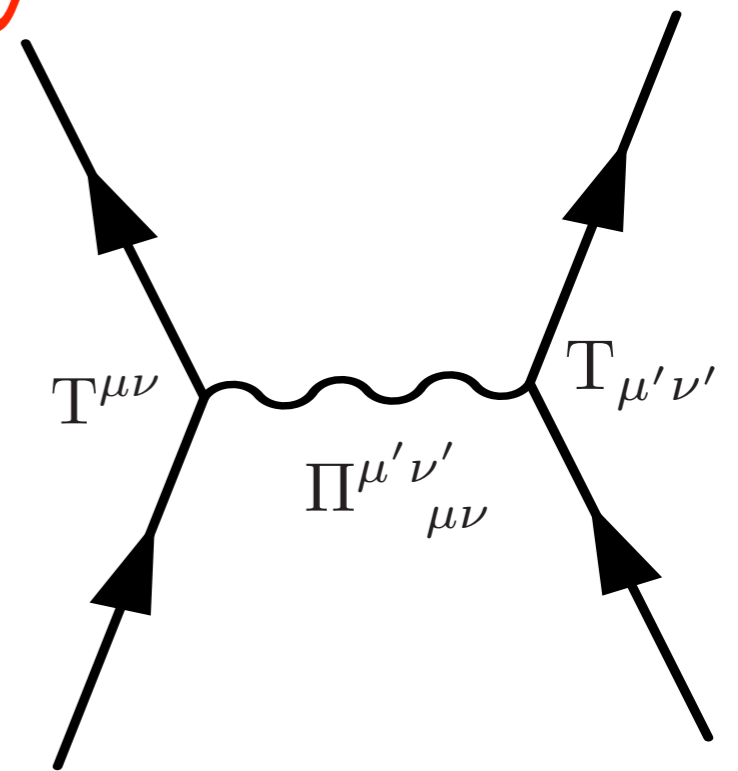
$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \dots$$

$$\Pi = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 + m^2}, \quad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(3) Weyl Gravity:

$$\mathcal{L} = R - \frac{1}{m^2} C^2 \quad C^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 2R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2$$

$$\Pi = \frac{\mathcal{P}^2}{k^2 (1 - (k/m)^2)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2 + m^2}$$



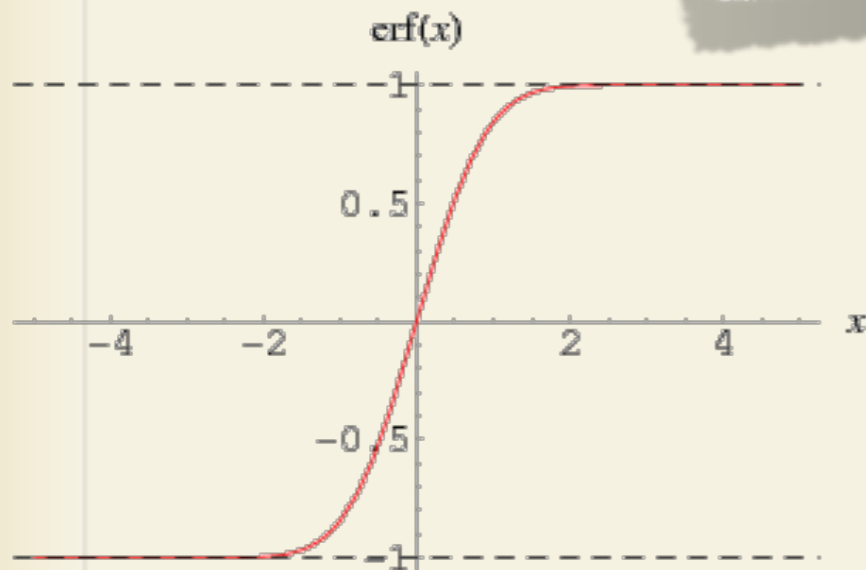
$$V \sim \frac{1}{r} e^{-mr} + \dots$$

# Infinite Derivative Gravity (IDG): Non-Singular gravity

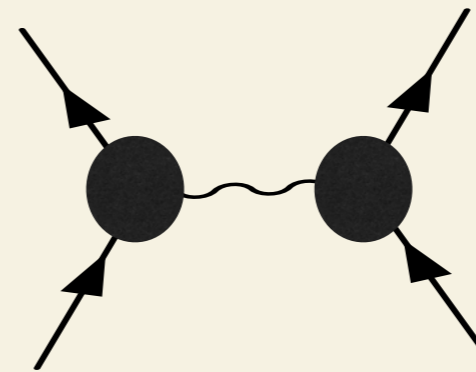
$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r} \operatorname{erf} \left( \frac{rM}{2} \right)$$



**Interaction becomes Non-Local**



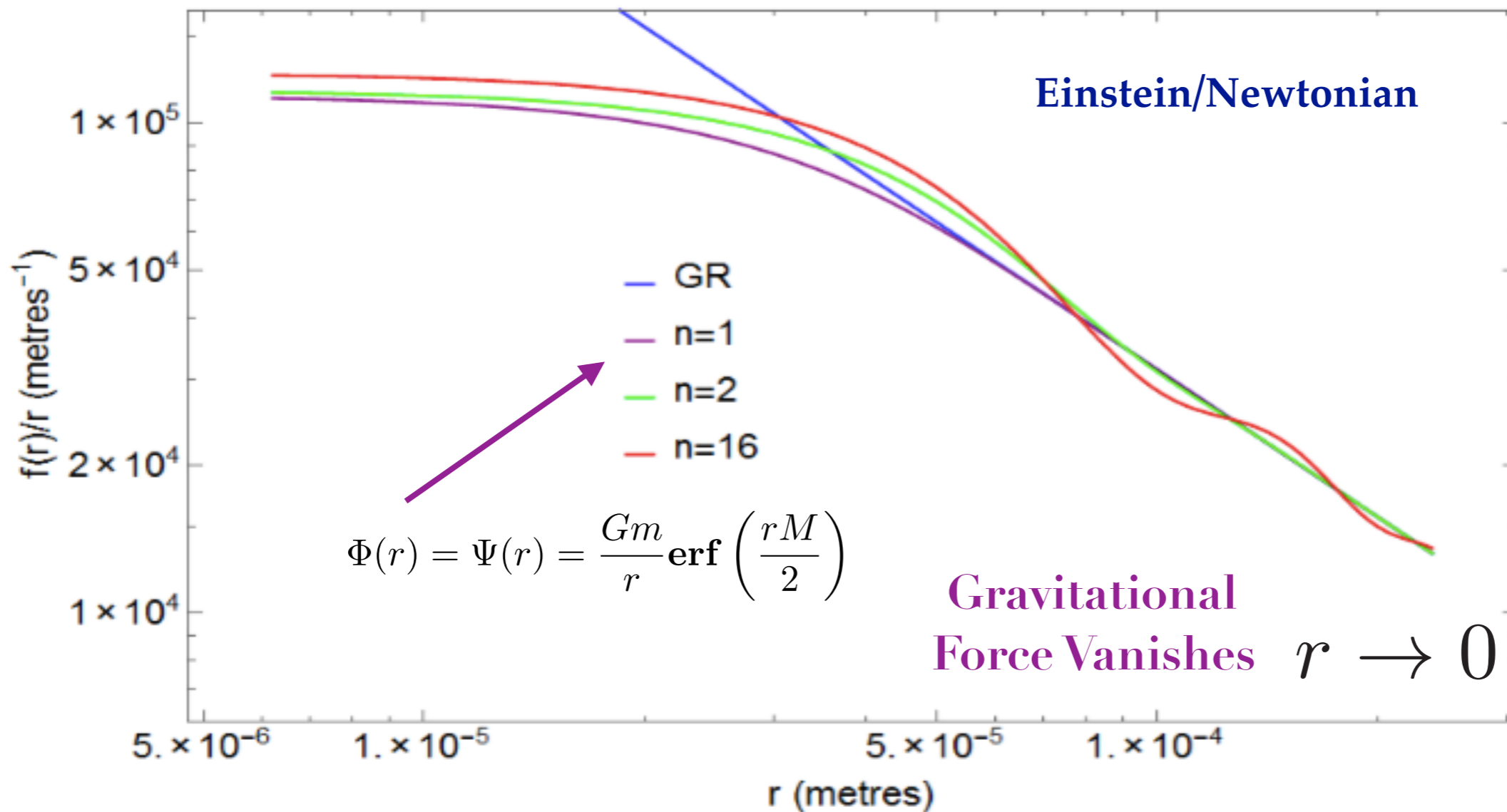
*Biswas, Mzumdar, Siegel, JCAP (2005),*

*Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (2012) (gr-qc/1110.5249)*

*Dombriz, Jose, Mazumdar, With Space-time Torsion: 1812.04037*

# Resolution of Singularity at short distances

$$a(\square) = e^{\gamma(\square)} \quad \text{Any Entire Function: } \gamma(\square) = -\frac{\square}{M^2} - \sum_N a_N \left(\frac{\square}{M^2}\right)^N$$

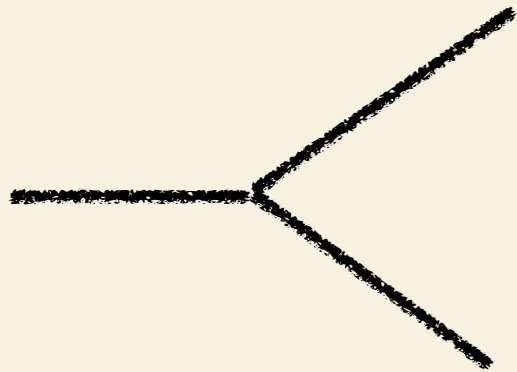


$$mM \ll M_p^2 \implies m \ll M_p$$

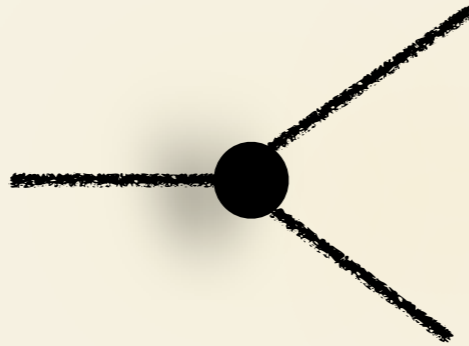
Current Bound :  $M > 0.01 \text{ eV}$        $m \leq 10^{25} \text{ grams}$

# Local vs Non-Local Field Theory

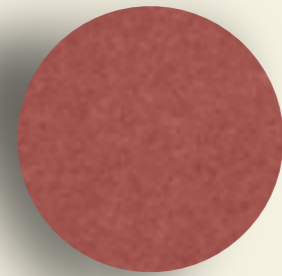
$$S = \int d^4x \left[ -\frac{1}{2} \phi e^{\frac{\square+m^2}{M^2}} (\square + m^2) \phi - \frac{\lambda}{4!} \phi^4 \right] \quad \Pi(p^2) = -\frac{ie^{-\frac{p^2+m^2}{M^2}}}{p^2 + m^2}$$



$$P^2 < M^2$$



$$P^2 \geq M^2$$



$$r \sim M^{-1}$$

**Scale of Non-Locality**

$$\delta m^2 \sim \lambda M^2 \quad \Gamma_4 \sim -\lambda^2 e^{-2m^2/M^2} [1 + \mathcal{O}(m^2/M^2)]$$

$$\sigma_{NL}(f\bar{f} \rightarrow f'\bar{f}') = e^{-s/M^2} \sigma_L(f\bar{f} \rightarrow f'\bar{f}')$$