On hydrodynamic approach to scalar condensate

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Hydrodynamics is simple after GR:

$$\partial_\mu T^{\mu\nu} = 0,$$

$$T = \mathrm{diag}(\rho, p, p, p).$$

Particles are conserved:

$$\partial_\mu J^\mu = 0.$$

And equation of state

$$p = p(\rho).$$

NR physics with attraction→ Jeans instability.

For relativistic problem: number of particles $\rightarrow$ global charge Q.
Condensate solution for complex field

$$\Phi = \mathrm{e}^{-\mathrm{i}\omega t} f$$

exists in large periodic box and instability can be obtained without Jeans swingle. Attractive gravitational force $\rightarrow$ Hubble flow (M. Khlopov, B. A. Malomed and I. B. Zeldovich, 1985)
$P(X)$-theories with shift symmetry, space-time symmetry breaking...
Recent progress of mode analysis (S. Grozdanov, P. K. Kovtun, A. O. Starinets and P. Tadić arXiv:1904.12862)

What if $f = f(x)$, localized configuration? KdV– great wave from one equation

Bose gas at $T = 0$

Ideal Bose gas can be localized in the harmonic trap:

$$\Psi \sim \exp(-\omega^2 x^2)$$

This system can be studied both experimentally and theoretically, as classical theory of the complex field Ψ with Lagrangian

$$\mathrm{i}\Psi^* \frac{d}{dt}\Psi - \frac{1}{2m}|\nabla\Psi|^2 - U(x)|\Psi|^2 + \lambda_0|\Psi|^4$$

Dimensionless combination $\lambda = m^2\lambda_0$ is not small! Nonlinear term is just a correction to potential — dilute gas approximation.

Although Rb is a metall at usual temperature.

Feild theory in semiclassical regime

In relativistic field theory one can obtain bag for the same field...
For the validity of semiclassical approximation one can use potential of the form

$$V = \frac{m^4}{g^2} U(g|\phi|/m),$$

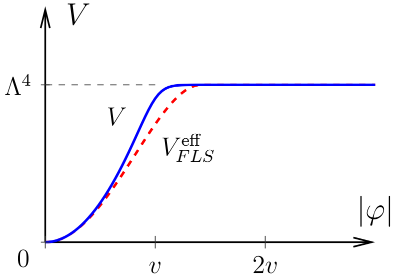
then, after redifinition $\phi = g\phi$ we obtain Lagrangian without small parameters and overall factor $1/g^2$ before action.
In this case semiclassiclal method is a saddle point approximation for path integral.
Soliton in classical theory is an analog of the bag — but nonlinear interaction is crucial.

Choice of potential

Potential which admits analytical solution

$$V(|\phi|) = m^2|\phi|^2\theta\left(1 - \frac{|\phi|^2}{v^2}\right) + m^2v^2\theta\left(\frac{|\phi|^2}{v^2} - 1\right)$$

is precisely what we need if we use smooth regularization and $g = m/v$, $\Lambda = m^2v^2$.



For general potential $V(Z), Z = \Phi^*\Phi$ clumping occurs for unstable condensate solution,

$$\partial^2 V/\partial Z^2 < 0,$$

thus for $V(z) = M^2 Z - \lambda Z^2$ there is instability for $\lambda > 0$.
For periodic box it can be presented in the form:

$$\partial Q/\partial \omega < 0.$$

One more interesting feature:

$$\frac{\partial E}{\partial Q} = \omega$$

and angular velocity ω can be considered as chemical potential.

One can find nontopological solitons in:

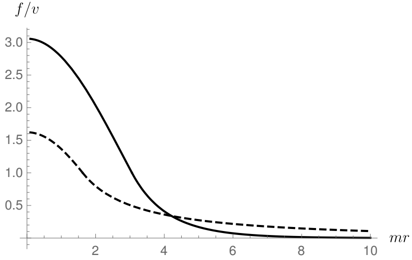
- KdV Equation, $(1+1)$ model;
- Single complex scalar field theories, Q-balls in any dimensions;
- Ultralight bosonic DM — central core of galaxies (W. Hu, R. Barkana and A. Gruzinov, PRL 200);
- Axion miniclusters, $(1-\cos\phi) \sim (m^2\phi^2 - \lambda\phi^4)$.
  Similar objects in NR limit — oscillons (not stationary solutions!).

General feature

$$\frac{\partial E}{\partial Q} = \omega$$

preserved for inhomogeneous solutions.

Explicit solutions for parabolic-pieswise potential in $3+1$-dimensions



Very similar profiles for $(1+1)$-theories. Thin-wall approximation is not valid!

Difference between $(1+1)$ and $(3+1)$ for Energy-Momentum tensor: $T_{\mu\nu} = \partial_\mu \Phi^* \partial_\nu \Phi + \partial_\mu \Phi \partial_\nu \Phi^* - \eta_{\mu\nu} \mathcal{L}$.

In $(1+1)$ $T_{\mu\nu}$ is diagonal, T_{01} component is absent! for stationary configuration
Moreover, $T_{11} = 0$ on E.O.M. for scalar field with any potential.
Nothing similar to the pressure...
Only nontrivial energy and charge densities.

In $(3+1)$ there is pressure and even shear, T_{ij} is not diagonal. (M. Mai and P. Schweitzer, PRD2012)

$$T_{00} = \rho_E(r) \ , \quad T_{ij} = \left(\frac{x_i x_j}{r^2} - \frac{1}{d}\delta_{ij} \right) s(r) + \delta_{ij} p(r) \ , \tag{1}$$

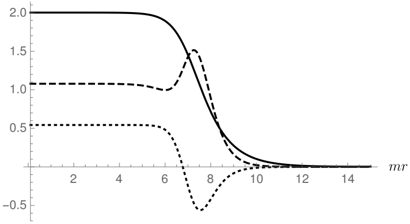
$s(r)$ determines the distribution of the shear force, and pressure is

$$p(r) = \omega^2 f^2 - \frac{1}{d} f'^2 - V(f^2) \ , \quad d = 3 \ , \tag{2}$$

which is negative outside the core, but

$$\int_0^\infty p(r) r^2 \mathrm{d}r = 0.$$

The last (von Laue) condition can be also obtained by scaling.



Profiles in polynomial model ($|\Phi|^6$) beyond thin-wall approximation: Q-ball (solid), energy density and pressure (dotted). Obviously, $\partial p(r)/\partial\rho(r) < 0$ outside the core... But the soliton is stable in linear approximation. There is a problem with equation of state which we did not find for homogeneous condensate.

Stability criterion in terms of chemical potential μ, the cost of new particle in the system:

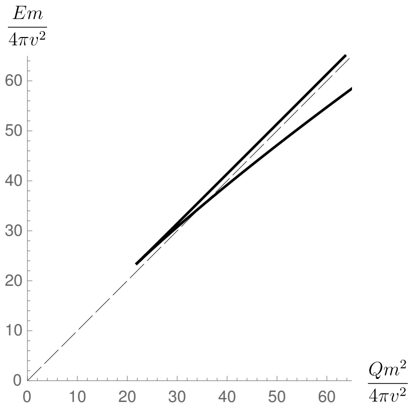
$$\partial\mu/\partial N < 0$$

– more easy to add new particle.

In relativistic theory one can find instability mode if

$$\partial Q/\partial\omega > 0.$$

For some potential one can find normal modes of soliton (A.Kovtun, E.N., S. Shkerin) or explicit instability for unstable configuration.



$\partial Q/\partial\omega > 0$ for the upper branch. There is also a region of metastability for classical Q-ball. (D. Levkov, E. Nugaev and A. Popescu, JHEP2017)

Soliton in field theory is nonperturbative semiclassical solution, interaction of all particles are crucial.

$$< \text{soliton}|Q \text{ particles} >= 0$$

in perturbation theory. All space-time translation symmetries are violated for linear perturbations by terms

$$\Phi^2\phi^{2*} + \phi^2\Phi^{2*}.$$

Thus, only $E - \omega Q$ is conserved for excitation. This is also a drawback for hydrodynamical approach, which is based on $\partial_\mu T^{\mu\nu} = 0$.

Conclusions

- Explicit profiles beyond thin-wall approximation provides illustration of difference between $(1+1)$ and $(3+1)$ models in terms of $T_{\mu\nu}$.
- Equations of motion contain crucial information for soliton and can not be replaced by single equation of state.
- All translation symmetries are violated by background. Complications for hydrodynamical approach.
- Angular velocity ω plays role of chemical potential.
- There is an example of semiclassical theory in which one can solve E.O.M. and find whole spectrum of linear perturbations.