Cosmology with nonlocal gravity

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Hot topics in Modern Cosmology
Spontaneous Workshop XIII
5-11 May 2019 - IESC, Cargese

1209.0836 & 1310.4329 with Dodelson
1608.02541 with Shafieloo
1711.08759
1809.06841 with Woodard
1811.04647 with Chu
1901.07832 with Amendola, Dirian & Nersisyan

The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007–2013)/ERC Grant Agreement No. 617656 "Theories and Models of the Dark Sector: Dark Matter, Dark Energy and Gravity."

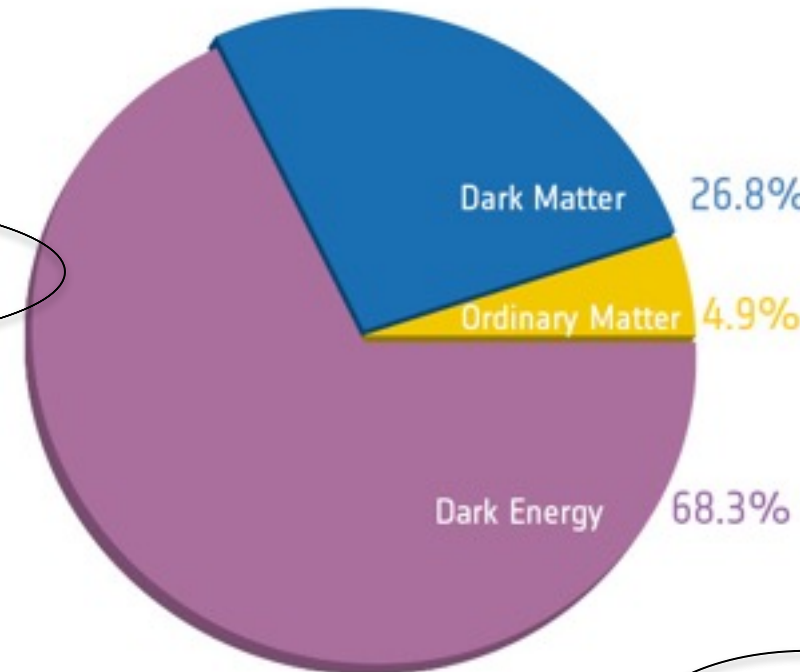
European Research Council
Established by the European Commission

Outline

- Motivation: Why changing GR? Why nonlocal gravity?
- Nonlocal gravity models for dark energy
- Nonlocal gravity models for dark matter
- What are the main issues regarding nonlocal gravity (or MG in general)?
    - Structure formation
    - Localization and Stability
    - Solar System tests and GW signatures
    - Derivation from first principles

Why changing GR? Why nonlocal gravity?

• GR works well up to the solar system scales, but needs too much "dark" stuff to describe cosmic motions on larger scales:



• Dark Energy for cosmic acceleration

• Dark Matter for galactic dynamics

None of them detected!

• Can we describe the same effects of the dark substances w/o invoking them but by modifying GR?

"The only local, metric-based, generally coordinate invariant, potentially stable models are the so-called f(R) models." Woodard astro-ph/0601672

Ostrogradsky theorem

• f(R) can give the early-time acceleration or inflation, e.g., Starobinsky inflation, but neither DE nor DM phenomena. Then, the remaining options are:

- Add fields other than the metric to carry part of the gravitational force
- Break general covariance
- Abandon locality

Two more requirements

- Solar system constraints: Don't ruin the successes of GR in the solar system
- Stability: Universe lasted 13.8 billion years

So, a lot of discussions about these issues in the MG literature…

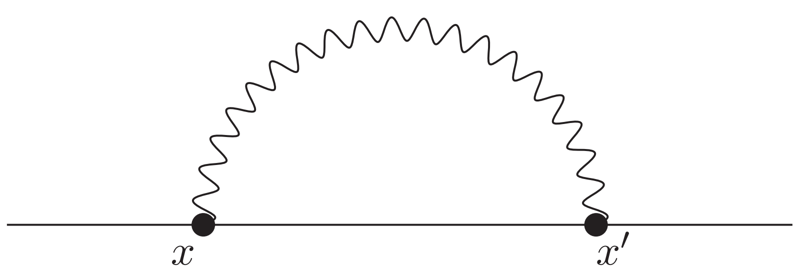
Origin of nonlocality

The quantum effective action is nonlocal: Maggiore's talk

Feynman taught us how to compute objects like correlation functions from the quantum effective action order by order in perturbation theory.

- Example: A scalar field interacting with a graviton field

A propagator is the inverse of a kinetic operator, i.e., a nonlocal object



If the field is massive, the propagator can be expanded and truncated, so it becomes local:

$$\frac{1}{\Box - m^2} = -\frac{1}{m}\left[1 + \frac{\Box}{m^2} + \cdots\right]$$

But, if the field is massless: $\frac{1}{\Box}$

"Nonlocality inevitably arises as quantum loop corrections of massless particles."

A nonlocal quantum effective action might derive from fundamental theory, however, no such derivation is currently available, so take a phenomenological approach.

What form of nonlocal actions would do the job?

Before moving on to introduce nonlocal models replacing DE let me remind you:

Dark Energy: What is making the universe accelerate?

$$H^2(t) \neq \frac{8\pi G}{3}(\rho_m + \rho_r)$$

Approaching a constant: observed!

Falling off with time

- Two options: Add more energy or modify gravity!

$$H^2(t) = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_{\rm new}) \quad \text{or} \quad H^2(t) = \frac{8\pi G_{\rm eff}(t)}{3}(\rho_m + \rho_r)$$

- ΛCDM

Works with $\Omega_\Lambda \approx 0.7\,, \quad \Omega_m \approx 0.3\,, \quad \Omega_r \approx 8.5 \times 10^{-5}$

But, why $\rho_\Lambda = \Omega_\Lambda \rho_c \sim (10^{-3}\text{eV})^4$ so small and why dominant now?

The so-called old & new problems of the cosmological constant

Hint on how to modify: Mimic the behavior of Λ w/o Λ!

A nonlocal model replacing DE: Deser-Woodard model

$$S_{\rm DW} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + Rf\left(\frac{1}{\Box}R\right)\right]$$ Deser & Woodard 0706.2151

- Nonlocality via $\frac{1}{\Box}$ and act it on R

→ $\frac{1}{\Box}R$ is dimensionless → no new mass parameter

- Btw, Idea of adding $R\frac{1}{\Box}R$ is not new

Polyakov 1981, Wetterich 1998, ... in different contexts

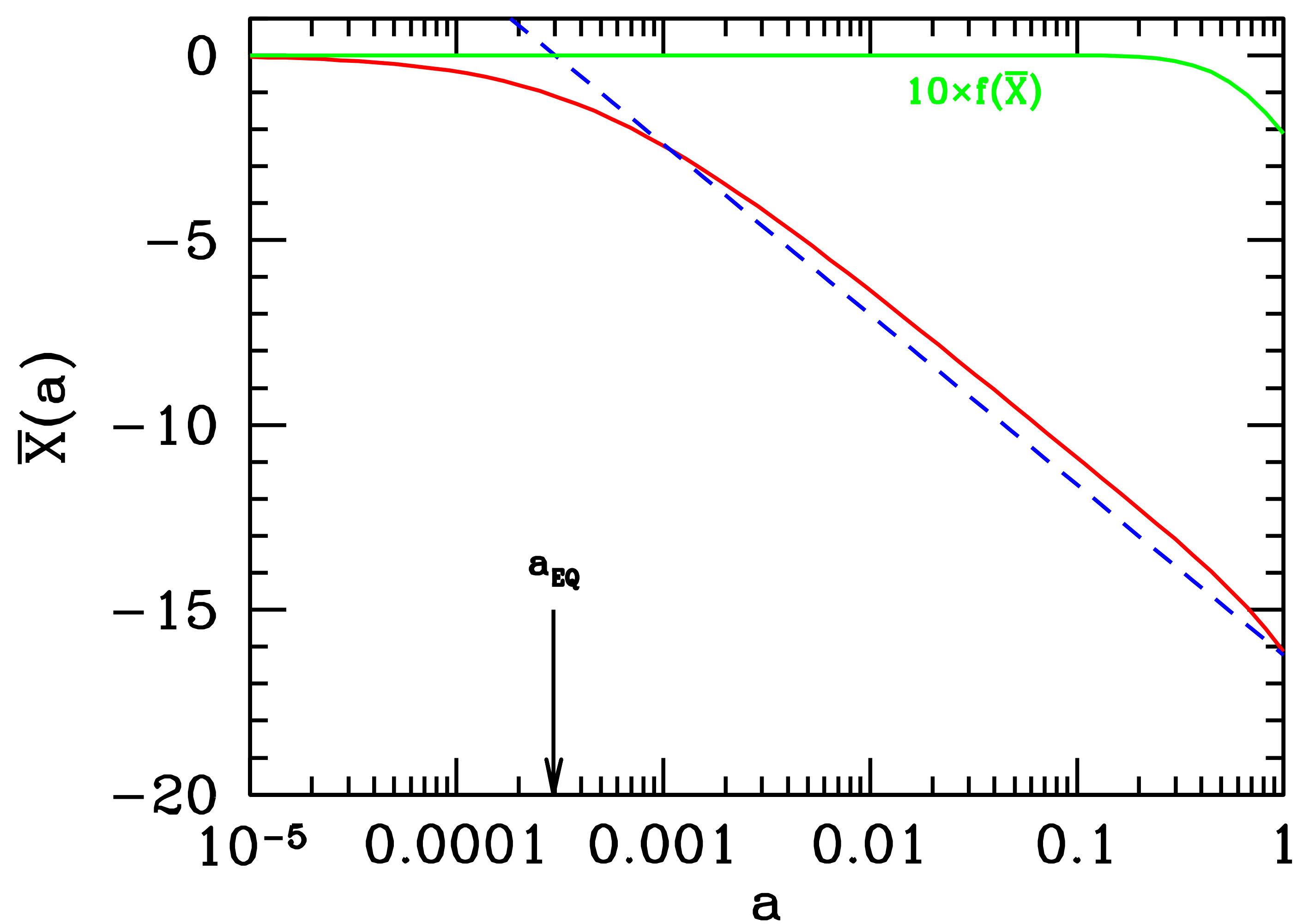
- Two built-in delays: desired property for mimicking Λ

$R = 0$ during radiation domination → no modification until $t_{\rm eq} \sim 10^5$ years

$\frac{1}{\Box}R \sim -\frac{4}{3}\ln\left(\frac{t}{t_{eq}}\right)$ during matter domination → $\frac{1}{\Box}R \sim -15$ at 10^{10} years

Finally the function f can be tuned to reproduce ΛCDM, but no huge fine-tuning is required, thanks to these delays!

The evolution of $\bar{X} = \bar{\Box}^{-1}\bar{R}$



- Dashed line: $\overline{X} = -2\ln(a/a_{\rm EQ})$ for the purely matter dominated era, $a \propto t^{2/3}$
- Red curve: The evolution of $\overline{X} \equiv \overline{\Box}^{-1}\overline{R}$ when $\overline{R}$ is for ΛCDM
- Green curve: $f(\bar{X})$ chosen to fit the ΛCDM expansion history

Comparison with other nonlocal models replacing DE

Both DW & RR are ruled out by LLR
1812.11181 Belgacom, Finke, Fassino, Maggiore

- ΛCDM: $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R - 2\Lambda]$
- Deser-Woodard 0706.2151: $S_{\rm DW} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + Rf\left(\frac{1}{\Box}R\right)\right]$

The nonlocal function nontrivially modulates the scalar curvature.

RR model

- Maggiore-Mancarella 1402.0448: $S_{\rm MM} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - m^2 R\frac{1}{\Box^2}R\right]$
- Vardanyan-Akrami-Amendola-Silvestri 1702.08908:

$$S_{\rm VAAS} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + m^2\frac{1}{\Box}R\right]$$

- Amendola-Burzilla-Nersisyan 1707.04628:

$$S_{\rm ABN} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - m^4\frac{1}{\Box^2}R\right]$$

Replace Λ by a mass parameter and give a modulation with a nonlocal scalar.

Digression: Simple Exercise with $\frac{1}{\Box}$

Specialization to FLRW $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx_j$

Acting on a function of time $f(t)$

$$\Box = \frac{1}{\sqrt{-g}}\partial_\rho(\sqrt{-g}g^{\rho\sigma}\partial_\sigma) \rightarrow -\frac{1}{a^3}\partial_t(a^3\partial_t)$$

$$\left[\frac{1}{\Box}f\right](t) = -\int_{t_t}^{t} dt' \frac{1}{a^3(t')}\int_{t'_t}^{t'} dt'' a^3(t'')f(t'')$$

typos: the lower limits are t_i

What is $\Box^{-1}R$ for $a(t) \sim t^{2/3}$ matter domination?

$$R = 6(\dot{H} + 2H^2) = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$

$$\frac{1}{\Box}R = -\int_{t_t}^{t} dt' \frac{1}{a^3(t')}\int_{t'_t}^{t'} dt'' a^3(t'')R(t'')$$

$$\rightarrow -\frac{4}{3}\left[\ln\left(\frac{t}{t_{eq}}\right) - 1 + \frac{t_{eq}}{t}\right] \quad \text{for} \quad a(t) = t^{2/3}$$

At leading order: $\frac{1}{\Box}R \sim -\ln(t),\quad \frac{1}{\Box^2}R \sim t^2\ln(t) \quad \left(\rightarrow\ R\frac{1}{\Box^2}R \sim \ln(t)\right)$

Derivation of conserved & causal field equations

- The invariant action guarantees the conservation of field equations obtained from its variation, whether it is local or nonlocal.

- Variation produces both adv & ret propagators: Equations not causal!

$$S[\phi] = \int dx' \phi(x') \frac{1}{\Box} \phi(x') = \int dx' \phi(x') \int dx'' G(x'; x'') \phi(x'')$$

$$\frac{\delta S}{\delta \phi(x)} = \int dx' \Big[G(x; x') + G(x'; x) \Big] \phi(x')$$

Example taken from
Belgacem, Dirian, Foffa, Maggiore 1712.07066

- Ideally one should derive eqns from Schwinger-Keldysh formalism, which guarantees the equations causal and conserved, but such a derivation is currently not possible…

- DW used a trick: Replace all advanced Green's functions $\frac{1}{\Box}_{\mathrm{adv}}$ by retarded ones $\frac{1}{\Box}_{ret}$

- The resulting equations are causal and conserved!
- Other nonlocal models all use the same trick of DW to get their eqns of motion.

Modified field equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\Delta G_{\mu\nu} = \Big[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\Big]\Big\{f\Big(\frac{1}{\Box}R\Big) + \frac{1}{\Box}\Big[Rf'\Big(\frac{1}{\Box}R\Big)\Big]\Big\} + \Big[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\Big]\partial_\rho\Big(\frac{1}{\Box}R\Big)\partial_\sigma\Big(\frac{1}{\Box}\Big[Rf'\Big(\frac{1}{\Box}R\Big)\Big]\Big)$$

- Localization: Introduce two auxiliary fields *X* and *U* defined as

$$\Box X \equiv R\ ,\ \ \Box U \equiv Rf'(X)$$

Nojiri and Odintsov 0708.0924

$$\Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu](f + U) + \Big[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\Big]\partial_\rho X \partial_\sigma U$$

- Specialize to FLRW: Modified field eqns and auxiliary eqns at the 0th order

$$\begin{aligned} 3H^2 + [3H^2 + 3H\partial_t](\overline{f} + \overline{U}) + \frac{1}{2}\partial_t\overline{X}\partial_t\overline{U} &= 8\pi G\rho \\ -(2\dot{H} + 3H^2) - [2\dot{H} + 3H^2 + 2H\partial_t + \partial_t^2](\overline{f} + \overline{U}) + \frac{1}{2}\partial_t\overline{X}\partial_t\overline{U} &= 8\pi G p \end{aligned}$$

$$\begin{aligned} -(\partial_t^2 + 3H\partial_t)\overline{X} &= 6(\dot{H} + 2H^2) \\ -(\partial_t^2 + 3H\partial_t)\overline{U} &= 6(\dot{H} + 2H^2)\, f' \end{aligned}$$

Reconstruction of f

The function f can be reconstructed for any given expansion history $H(t)$

Generic technique for the reconstruction: Deffayet & Woodard 0904.0961

- Step 1: Compute f as a function of time $\zeta \equiv 1+z$ once $h(\zeta) \equiv H(\zeta)/H_0$ is given

$$f(\zeta) = -2\int_\zeta^\infty d\zeta_1\,\zeta_1\phi(\zeta_1) - 6\Omega_\Lambda \int_\zeta^\infty d\zeta_1 \frac{\zeta_1^2}{h(\zeta_1)I(\zeta_1)} \int_{\zeta_1}^\infty d\zeta_2 \frac{I(\zeta_2)}{\zeta_2^4 h(\zeta_2)} + 2\int_\zeta^\infty d\zeta_1 \frac{\zeta_1^2}{h(\zeta_1)I(\zeta_1)} \int_{\zeta_1}^\infty d\zeta_2 \frac{r(\zeta_2)\phi(\zeta_2)}{\zeta_2^5}$$

- Step 2: Compute $\overline{X}$ as a function ζ

$$\overline{X}(\zeta) = -\int_\zeta^\infty \frac{d\zeta_1\zeta_1^2}{h(\zeta_1)} \int_{\zeta_1}^\infty d\zeta_2 \frac{r(\zeta_2)}{\zeta_2^4 h(\zeta_2)} = -\int_\zeta^\infty \frac{d\zeta_1\zeta_1^2}{h(\zeta_1)} I(\zeta_1)$$

- Step 3: Convert $\zeta = \zeta(\overline{X})$ then plug into $f(\zeta)$ in Step 1, which gives $f = f(\overline{X})$

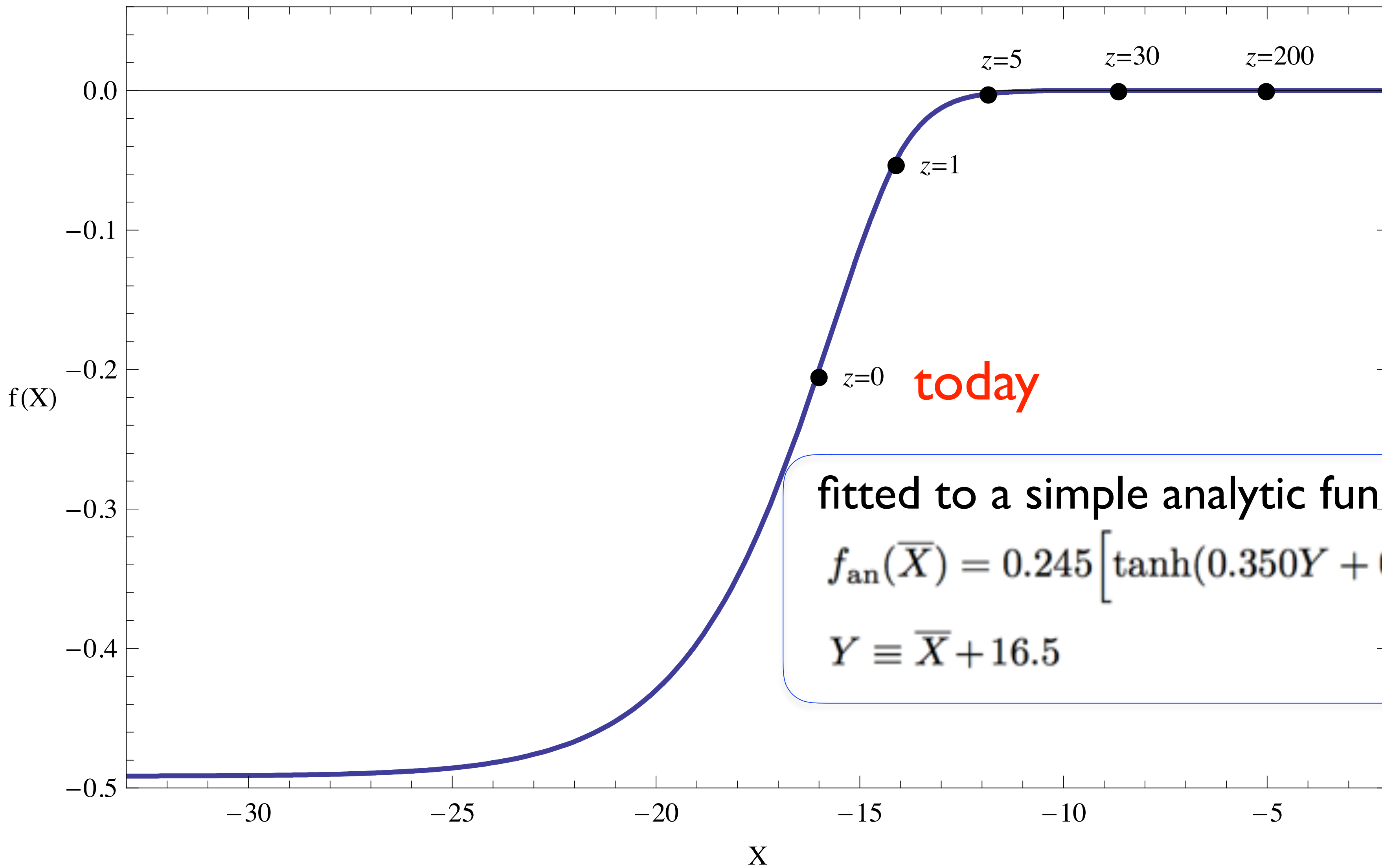
Note: f is a solution of the ODE obtained from the 0th order equations

$$\ddot{F} + 5H\dot{F} + \left(6H^2 + 2\dot{H}\right)F = -6H_0^2\Omega_\Lambda$$

where $F = f + \frac{1}{\Box}\left(R\frac{df}{dX}\right), \; X \equiv \frac{1}{\Box}R\,, \; \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$

For the case of ΛCDM, $h(\zeta) = \sqrt{\Omega_\Lambda + \Omega_m \zeta^3 + \Omega_r \zeta^4}$

with $\{\Omega_\Lambda, \Omega_m, \Omega_r\} = \{0.72, 0.28, 8.5 \times 10^{-5}\}$



fitted to a simple analytic function

$$f_{\rm an}(\overline{X}) = 0.245\left[\tanh(0.350Y + 0.032Y^2 + 0.003Y^3) - 1\right]$$

$$Y \equiv \overline{X} + 16.5$$

For the case of $f(R)$, the only solution which gives ΛCDM is $f(R) = R - 2\Lambda$

Dunsby et al. 1005..2205

Scalar Perturbations

Now, the background expansion is fixed (the same as the one in ΛCDM).
The next question is how it would be the structure formation in this model.

- To see the growth of matter fluctuations we take scalar perturbations.
- Note: Growth of perturbations is a unique prediction of the model

Take scalar perturbations around the FLRW metric in the Newtonian gauge

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$$

Also for auxiliary fields

$$X = \overline{X} + \delta X \ , \ \ U = \overline{U} + \delta U$$

Matter overdensity

$$\delta = \frac{\delta\rho}{\rho} \quad \text{from} \quad \delta T_{00} = \rho\delta$$

→ Five perturbation variables Φ, Ψ, δ, δX, δU

Perturbation equations

Two sets of perturbation equations

Set A: For five variables Φ, Ψ, δ, δX, δU in the localized version

$$
\begin{aligned}
k^2\Phi + k^2\left[\Phi(\bar{f}+\bar{U}) + \frac{1}{2}(\bar{f}'\delta X + \delta U)\right] &= 4\pi G a^2 \rho \\
(\Phi+\Psi) + (\bar{f}'\delta X + \delta U) + (\Phi+\Psi)(\bar{f}+\bar{U}) &= 0 \\
\ddot{\delta} + 2H\dot{\delta} &= -\frac{k^2}{a^2}\Psi \\
\left(-\partial_t^2 - 3H\partial_t - \frac{k^2}{a^2}\right)\delta X &= 2\frac{k^2}{a^2}(\Psi + 2\Phi) \\
\left(-\partial_t^2 - 3H\partial_t - \frac{k^2}{a^2}\right)\delta U &= 2\frac{k^2}{a^2}(\Psi + 2\Phi)\bar{f}'
\end{aligned}
$$

Set B: For three variables Φ, Ψ, δ in the nonlocal version

$$
\begin{aligned}
k^2\Phi + k^2\left\{\Phi(\bar{f}+\bar{U}) + k^2\int_{t_i}^{t}\frac{dt'}{a^2(t')}G(t,t';k)\left[\Psi(\vec{k},t') + 2\Phi(\vec{k},t')\right]\left[f'(\bar{X}(t)) + f'(\bar{X}(t'))\right]\right\} &= 4\pi G a^2\rho\delta \\
(\Phi+\Psi) + (\Phi+\Psi)(\bar{f}+\bar{U}) + 2k^2\int_{t_i}^{t}\frac{dt'}{a^2(t')}G(t,t';k)\left[\Psi(\vec{k},t') + 2\Phi(\vec{k},t')\right]\left[f'(\bar{X}(t)) + f'(\bar{X}(t'))\right] &= 0 \\
\ddot{\delta} + 2H\dot{\delta} &= -\frac{k^2}{a^2}\Psi
\end{aligned}
$$

Integral solutions for δX and δU

$$
\begin{aligned}
\delta X(\vec{k},t) &= \int_{t_i}^{t} dt' G(t,t';k)\frac{2k^2}{a(t')^2}\left[\Psi(\vec{k},t') + 2\Phi(\vec{k},t')\right] \\
\delta U(\vec{k},t) &= \int_{t_i}^{t} dt' G(t,t';k)\frac{2k^2}{a(t')^2}\left[\Psi(\vec{k},t') + 2\Phi(\vec{k},t')\right] f'(\bar{X}(t'))
\end{aligned}
$$

Construction of Green's function:
1209.0836 Park & Dodelson

Two different implementations for the sub-horizon limit

Sub-horizon limit $k \gg Ha$ $\approx$ Quasi-static limit: Drop time derivatives

$$
\begin{aligned}
k^2\Phi + k^2\left[\Phi(\bar{f}+\bar{U}) + \frac{1}{2}(\bar{f}'\delta X + \delta U)\right] &= 4\pi G a^2 \rho \\
(\Phi+\Psi) + (\bar{f}'\delta X + \delta U) + (\Phi+\Psi)(\bar{f}+\bar{U}) &= 0 \\
\ddot{\delta} + 2H\dot{\delta} &= -\frac{k^2}{a^2}\Psi \\
\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right)\delta X &= -2\frac{k^2}{a^2}(\Psi + 2\Phi) \\
\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right)\delta U &= -2\frac{k^2}{a^2}(\Psi + 2\Phi) f'(\bar{X})
\end{aligned}
$$

Set A: Dodelson, Park 1310.4329 and Park 1711.08759

Found an error

Forced Harmonic Oscillators

Found discrepancy

$$
\begin{aligned}
k^2\Phi + k^2\left[\Phi(\bar{f}+\bar{U}) + \frac{1}{2}(\bar{f}'\delta X + \delta U)\right] &= 4\pi G a^2 \rho \\
(\Phi+\Psi) + (\bar{f}'\delta X + \delta U) + (\Phi+\Psi)(\bar{f}+\bar{U}) &= 0 \\
\ddot{\delta} + 2H\dot{\delta} &= -\frac{k^2}{a^2}\Psi \\
\delta X &= -2(\Psi + 2\Phi) \\
\delta U &= -2(\Psi + 2\Phi) f'(\bar{X})
\end{aligned}
$$

Set C: Nersisyan, Fernandez Cid, Amendola 1701.00434

An excuse why I made an error in my numerical code

Set B: For three variables Φ, Ψ, δ in the nonlocal version

1310.4329 Dodelson & Park

$$k^2\Phi + k^2\left\{\Phi(\bar{f}+\bar{U}) + k^2\int_{t_i}^{t}\frac{dt'}{a^2(t')}G(t,t';k)\left[\Psi(\vec{k},t')+2\Phi(\vec{k},t')\right]\left[f'(\bar{X}(t))+f'(\bar{X}(t'))\right]\right\} = 4\pi G a^2\rho\delta$$

$$(\Phi+\Psi) + (\Phi+\Psi)(\bar{f}+\bar{U}) + 2k^2\int_{t_i}^{t}\frac{dt'}{a^2(t')}G(t,t';k)\left[\Psi(\vec{k},t')+2\Phi(\vec{k},t')\right]\left[f'(\bar{X}(t))+f'(\bar{X}(t'))\right] = 0$$

$$\ddot{\delta} + 2H\dot{\delta} = -\frac{k^2}{a^2}\Psi$$

Discretized to solve this:

$$\int \rightarrow \sum$$

$$\frac{df}{dt} \rightarrow f(t+\Delta t) - f(t)$$

- Set B (integro-differential eqns)
11 hours of running time with Mathematica in a laptop with the i7 processor in 2017, and 3 days with an old computer in 2013

- Set A (differential eqns)
a few seconds with NDSolve in Mathematica

Note: the Green function is highly oscillating, so needs the number of iterations very big…

- So, simplified the code by somehow localizing it in my head and made an error…

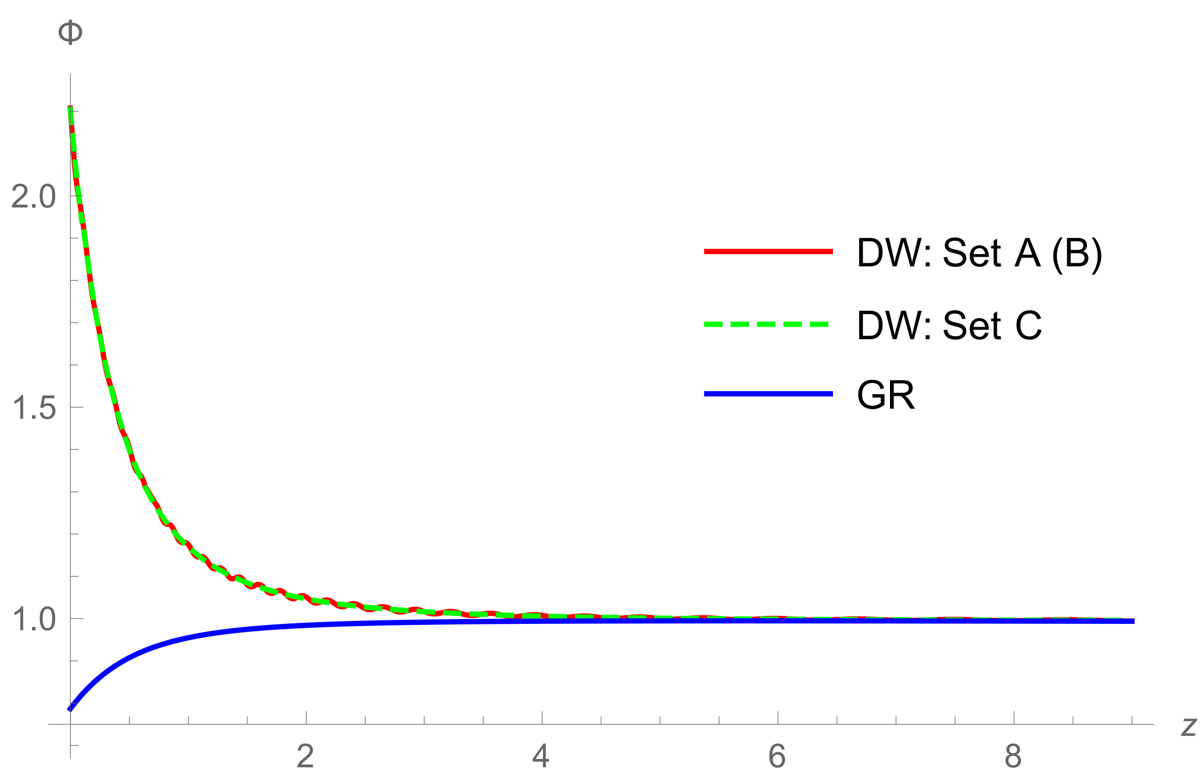
- Actually got it right (which was weaker growth) but back then almost sure we would get stronger growth: So couldn't believe the correct result…

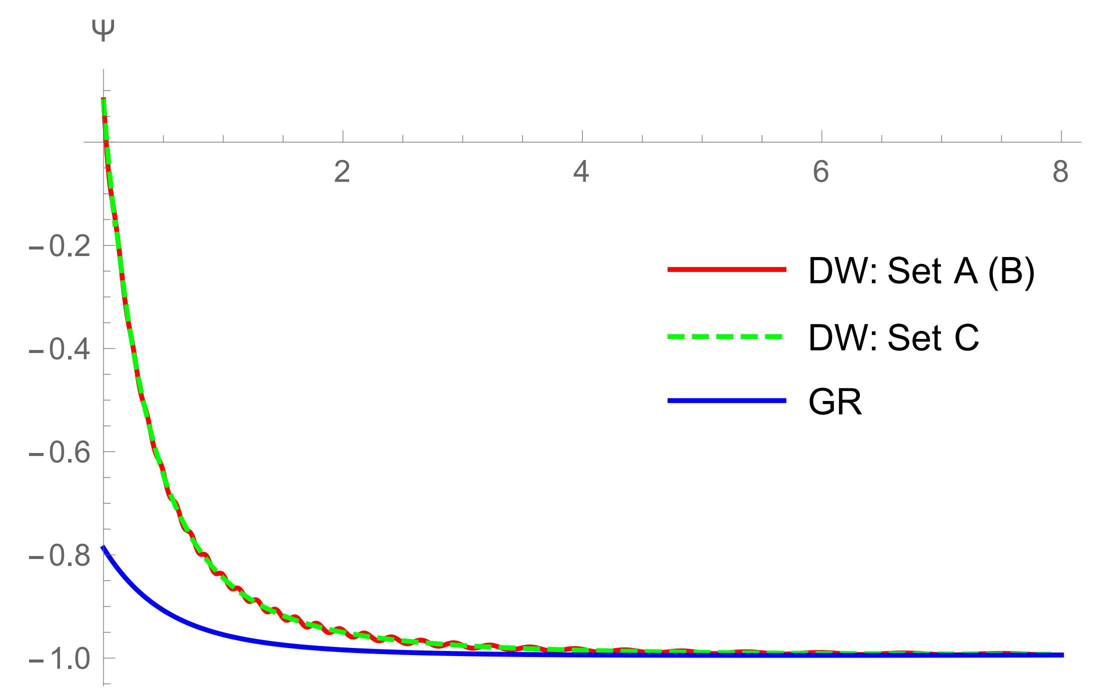
Solutions of perturbation equations

Initial conditions at $z_i = 9$

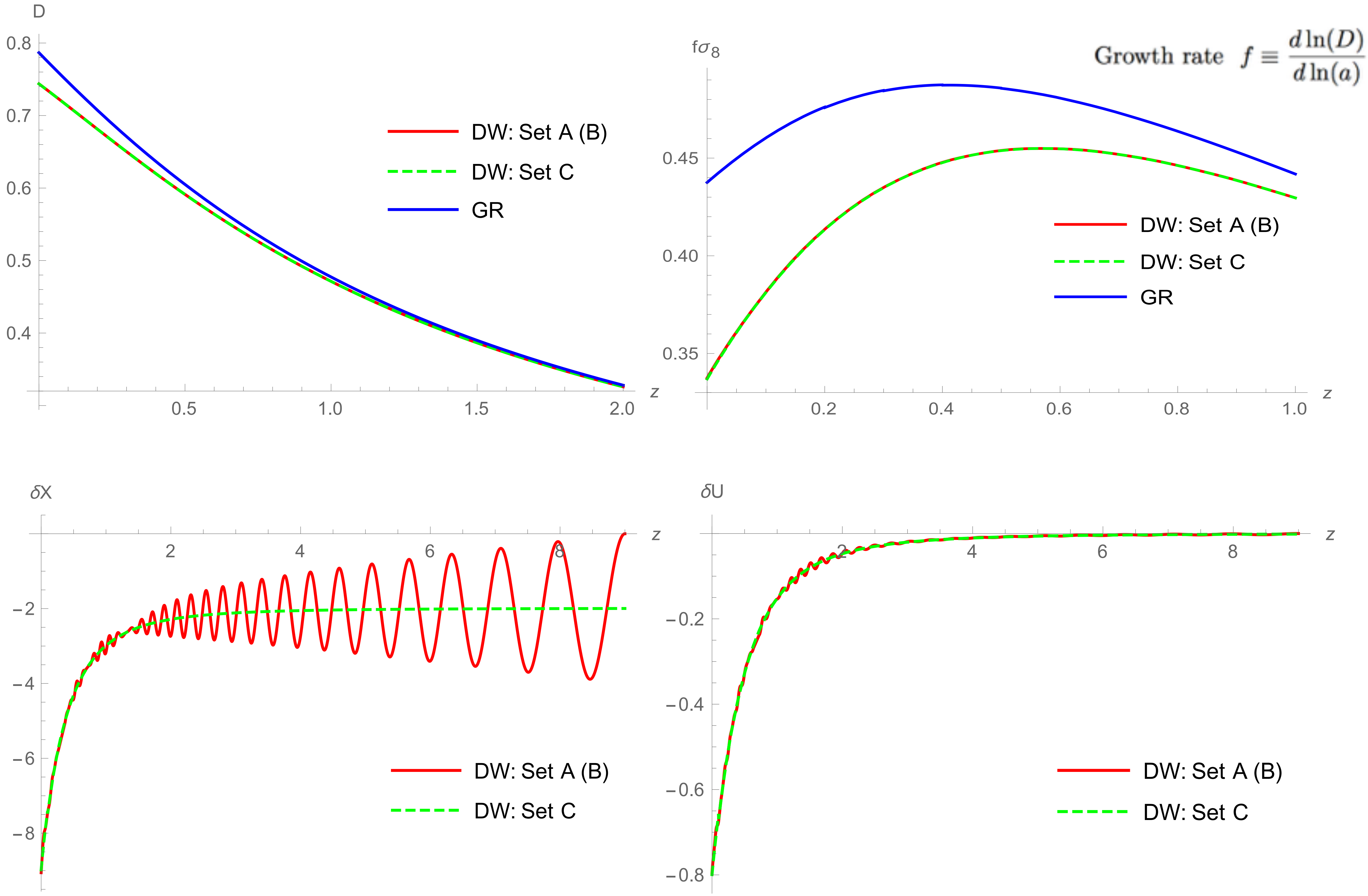
$$\Phi(z_i) = \Phi_{\rm GR}(z_i)\ ,\ \Psi(z_i) = \Psi_{\rm GR}(z_i) = -\Phi_{\rm GR}(z_i)$$
$$\delta(z_i) = \delta_{\rm GR}(z_i) = \frac{2k^2 a(z_i)}{3H_0^2\Omega_m}\Phi_{\rm GR}(z_i)\ ,\ \delta'(z_i) = \delta'_{\rm GR}(z_i)$$
$$\delta X(z_i) = 0\ ,\ \delta U(z_i) = 0\ ,\ \delta X'(z_i) = 0\ ,\ \delta U'(z_i) = 0$$

Set A and B give the same solutions as they should do, so let's compare Set A and C

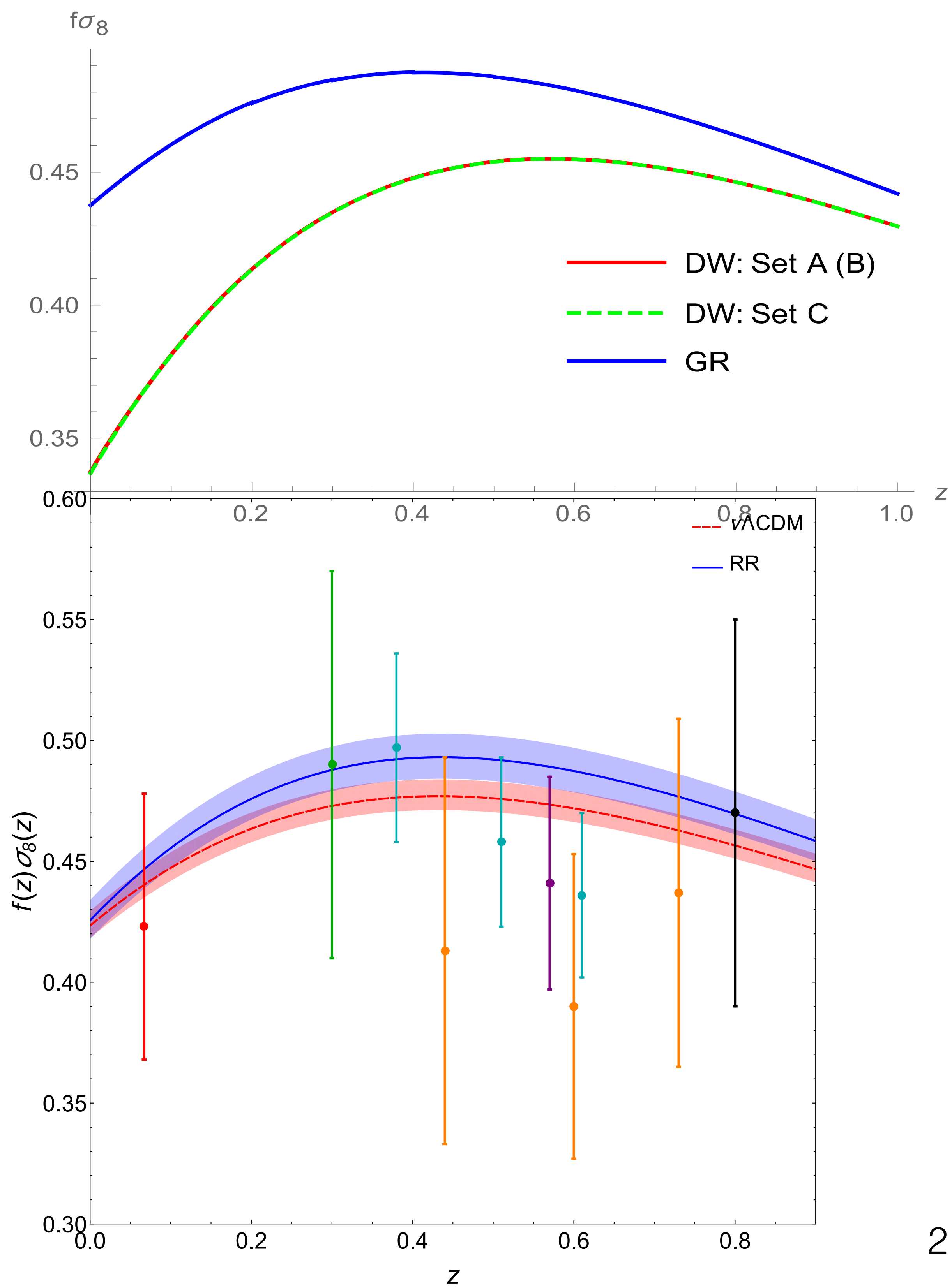




Solutions of perturbation equations



Comparison with RR model by MM



$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + Rf\left(\frac{1}{\Box}R\right)\right]$$

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

Figure 9 from Belgacem, Dirian, Foffa, Maggiore 1712.07066

Alternative definitions of effective gravitational constant

$$\frac{k^2}{a^2}\Phi \equiv G_{\text{eff}\Phi} \times 4\pi G\rho\delta \quad \text{vs.} \quad -\frac{k^2}{a^2}\Psi \equiv G_{\text{eff}\Psi} \times 4\pi G\rho\delta$$

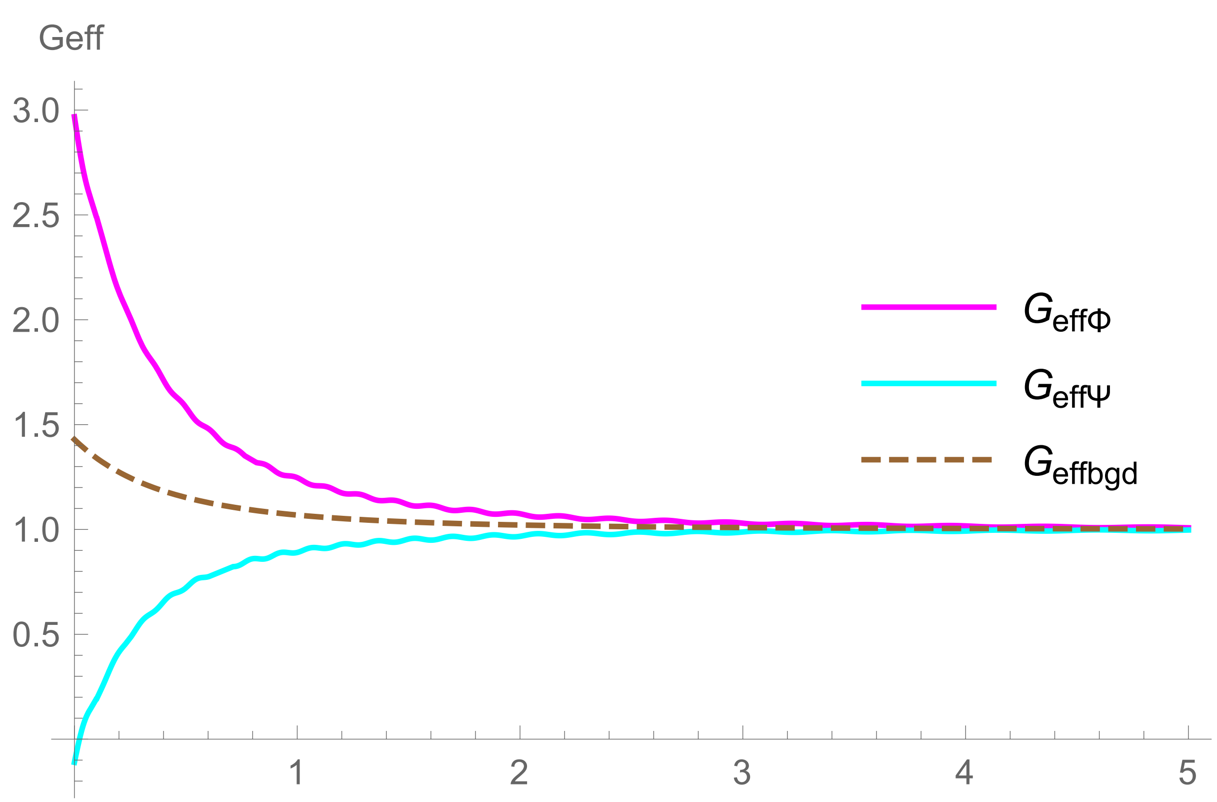
Dodelson, Park 1310.4329 — Nersisyan, Fernandez Cid, Amendola 1701.00434

The same if the gravitational slip $\eta = \frac{\Phi+\Psi}{\Phi}$ vanished like in GR, but not in DW

$$G_{\text{eff}\Phi} = \frac{1}{1+\overline{f}+\overline{U}+\frac{1}{2\Phi}(\overline{f}'\delta X+\delta U)} \longrightarrow G_{\text{effbgd}} = \frac{1}{1+\overline{f}+\overline{U}}$$

G_{effbgd} from $G_{\text{eff}\Phi}$ by taking the background modifications only

$$H^2 = \frac{8\pi G \times G_{\text{effbgd}}}{3}\rho$$



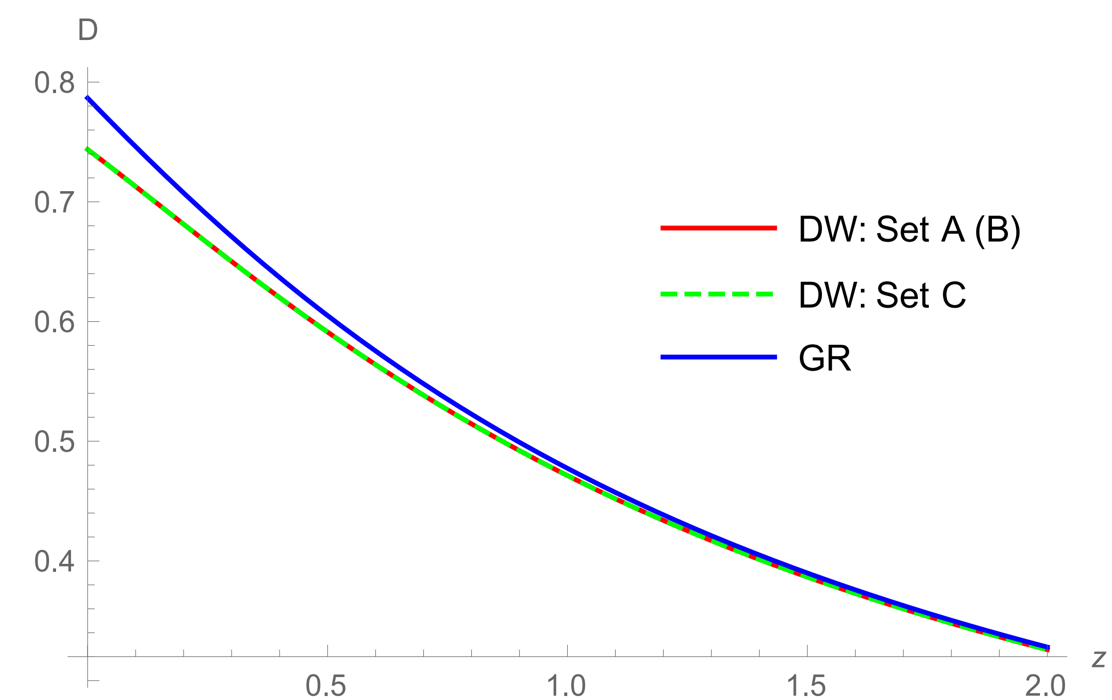
Growth equation

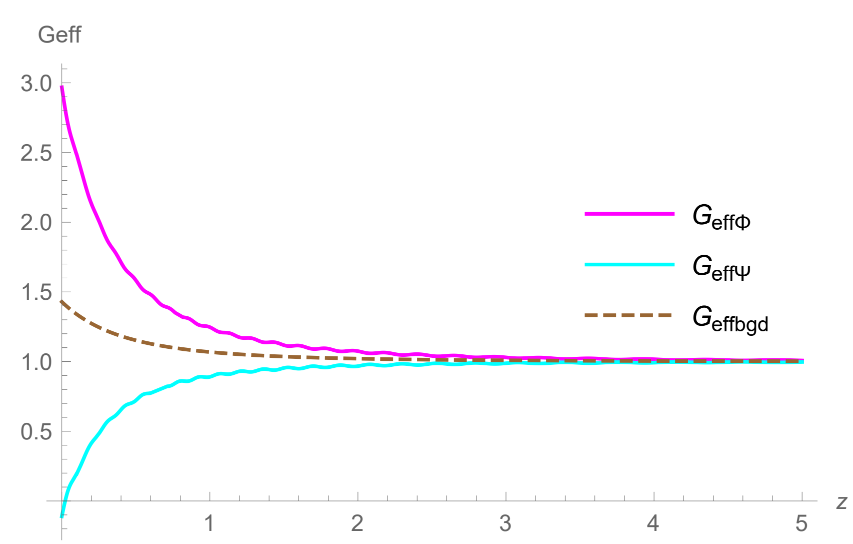
$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H_0^2\Omega_m a^{-3}\delta$$

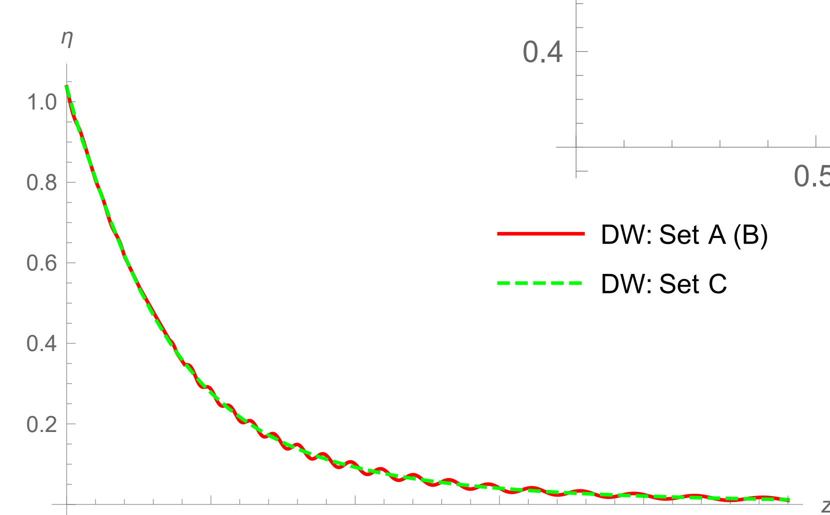
$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H_0^2\Omega_m a^{-3}(1-\eta)G_{\text{eff}\Phi}\delta \quad \text{vs.} \quad \ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H_0^2\Omega_m a^{-3}G_{\text{eff}\Psi}\delta$$

Dodelson, Park 1310.4329 — Nersisyan, Fernandez Cid, Amendola 1701.00434

$$\begin{aligned}(1-\eta)G_{\text{eff}\Phi} &= G_{\text{eff}\Psi} > 1 \quad \text{enhanced growth} \\ (1-\eta)G_{\text{eff}\Phi} &= G_{\text{eff}\Psi} < 1 \quad \text{suppressed growth}\end{aligned}$$







Structure formation in nonlocal cosmology (nonlocal models replacing DE)

- DW nonlocal gravity model can reproduce ΛCDM w/o Λ.
- Growth of structure predicted by the DW model (when the background is set the same as ΛCDM) is lower than in ΛCDM.

- MM nonlocal gravity model sets the mass parameter: $m \sim H_0$ then give a slow variation using the nonlocal scalar, so approximately reproduce ΛCDM.
- Growth of structure predicted by the MM model (when the background is set close to ΛCDM) is higher than in ΛCDM.

- What range of k values would hold these conclusion?
- Full linear equations rather than taking the quasi static limit?
- Growth with some non-ΛCDM backgrounds? Preliminary analysis for the DW model: Park & Shafieloo 1608.02541

Gravitational wave signatures

- Metric perturbations around the flat spacetime background, where $X = 0$
- At linear order: the gravitational wave polarizations in the DW model are the same as in GR, i.e., the speed of GWs is the same as the speed of light, except the effective Newton's constant is rescaled as $G_{\rm GW} = \dfrac{G}{1+f(0)}$

0807.3778 Koivisto
1811.04647 Chu & Park

- At quadratic order: the gravitational energy-momentum flux due to an isolated system turns out to scale as $1/r$, which would lead to a divergent total GW energy-momentum at infinity. This divergent flux can be avoided if we set

$$f'(0) = 0 = f''(0)$$

1811.04647 Chu & Park

Localization and Stability

• The DW nonlocal action can be re-cast in a localized form by introducing two auxiliary scalar fields: 0708.0924 Nojiri & Odintsov

$$S_{\rm DW} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R + Rf(X) + g^{\mu\nu}\partial_\mu X\partial_\nu U + UR\Big]$$

• One of the two scalar degrees of freedom turns out to be a ghost field, hence the localized version suffers from a kinetic energy instability.

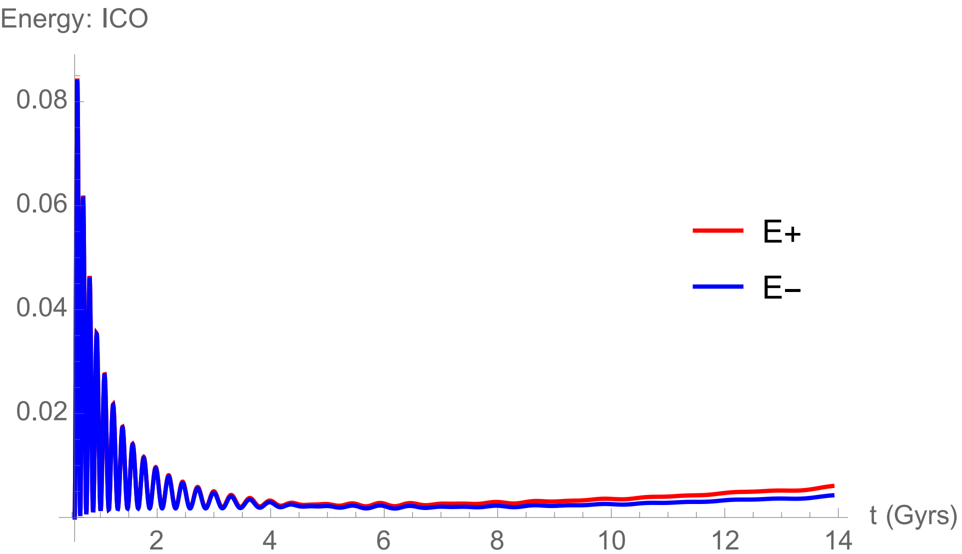
• However, the original nonlocal model is a constrained version of its localized cousin in which the auxiliary scalars and their first derivatives vanish on the initial value surface, so it can avoid the kinetic instability. 1307.6639 Deser & Woodard

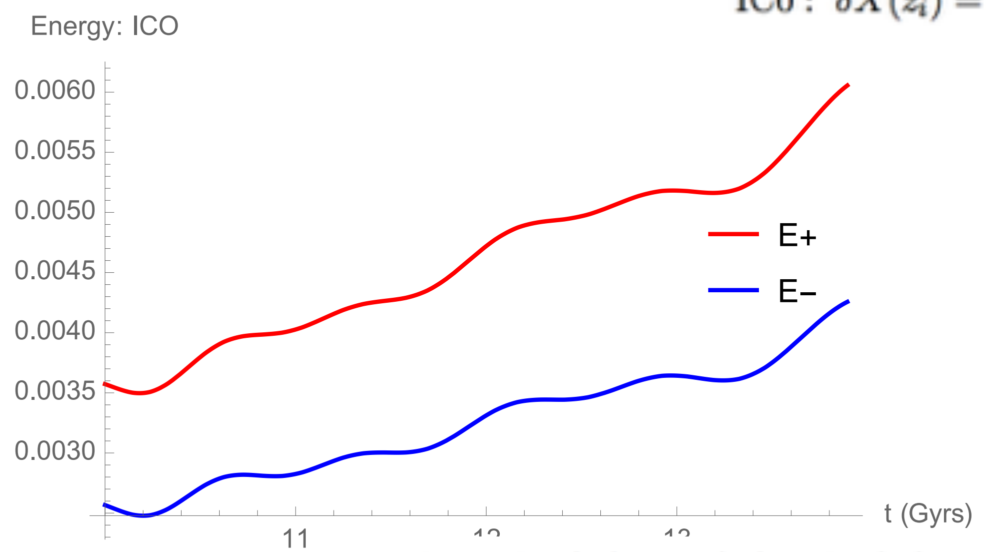
• It has been explicitly checked that the evolution of permitted perturbations does not lead to explosive excitation of the ghost mode in the original nonlocal model but it does so in the localized version. 1809.06841 Park & Woodard

• A similar analysis can be applied to other classes of nonlocal models.

Evolution of kinetic energies of normal (-) and ghost (+) scalars

$$\mathrm{IC0}: \ \delta X(z_i) = 0 \,, \ \delta U(z_i) = 0 \,, \ \delta X'(z_i) = 0 \,, \ \delta U'(z_i) = 0$$

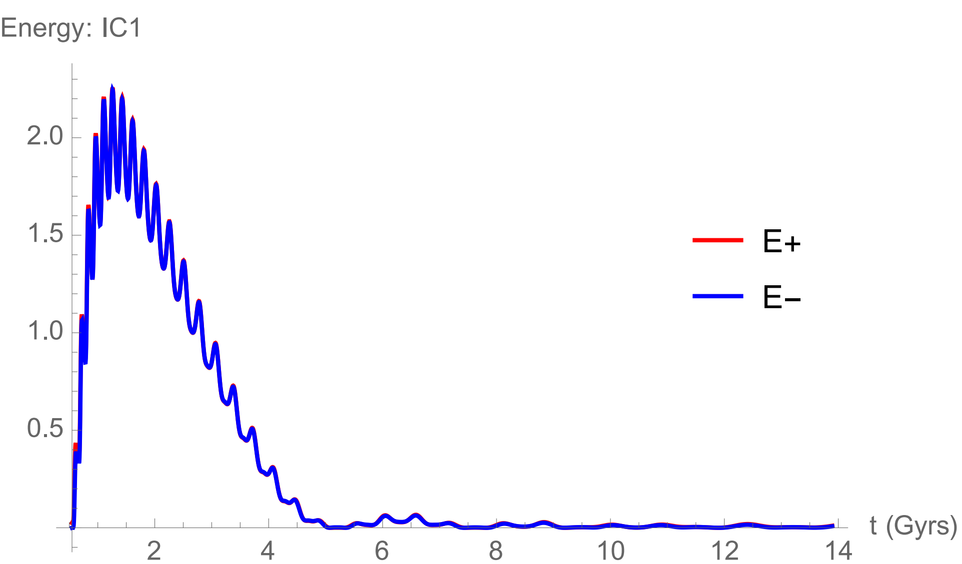


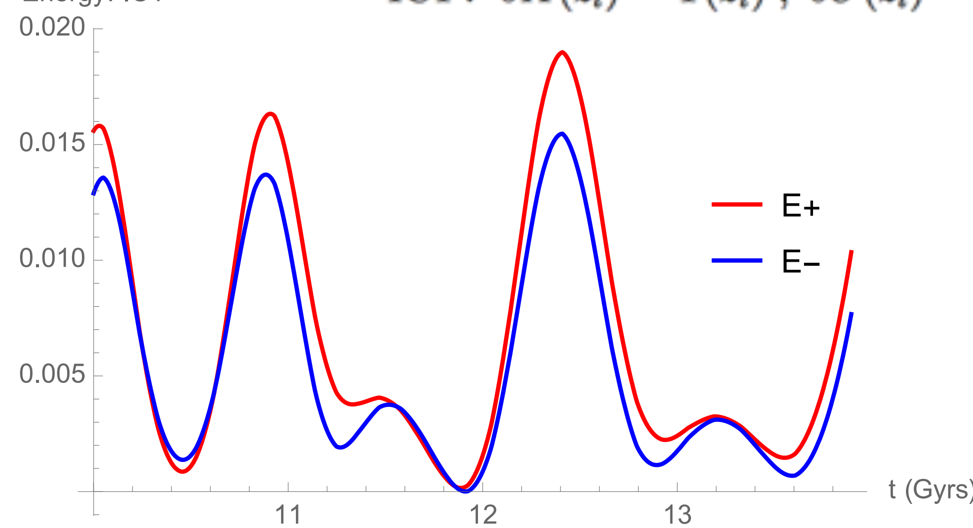


$$E_{\pm} \equiv \frac{1}{2}\delta \dot{A}_{\pm}^2 + \frac{1}{2}\frac{k^2}{a^2}\delta A_{\pm}^2$$

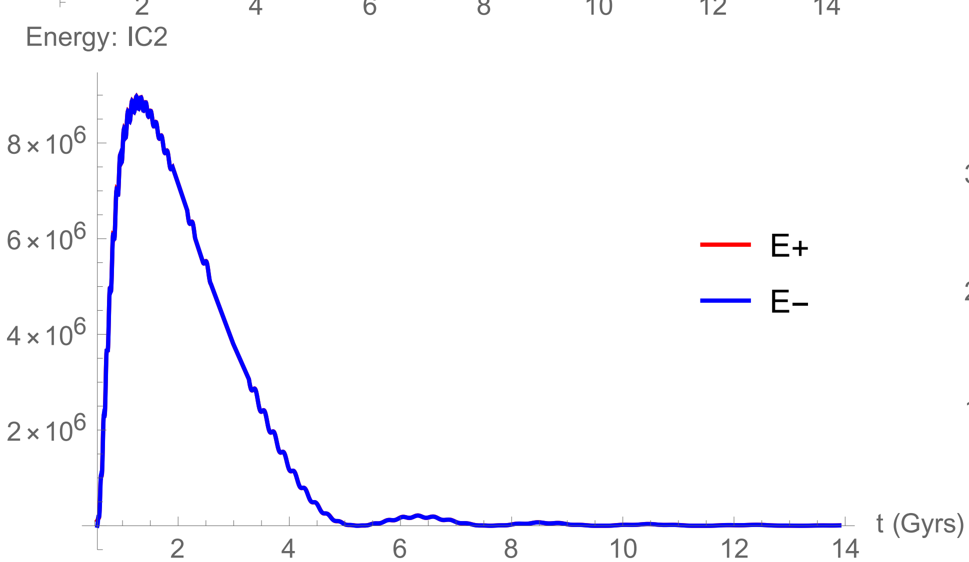
$$\delta A_{\pm} = \tfrac{1}{2}(\delta X \pm \delta U)$$

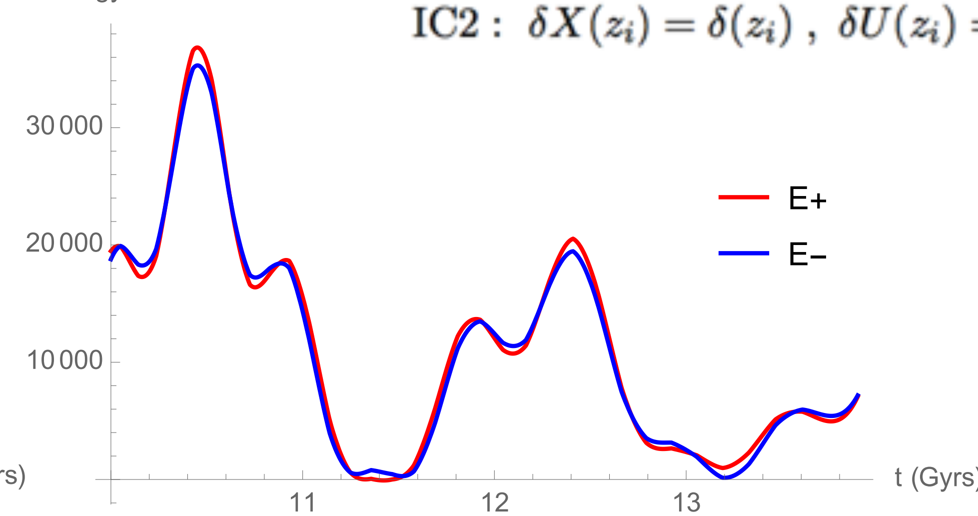
$$\mathrm{IC1}: \ \delta X(z_i) = \Phi(z_i) \,, \ \delta U(z_i) = \Phi(z_i) \,, \ \delta X'(z_i) = \Phi'(z_i) \,, \ \delta U'(z_i) = \Phi'(z_i)$$





$$\mathrm{IC2}: \ \delta X(z_i) = \delta(z_i) \,, \ \delta U(z_i) = \delta(z_i) \,, \ \delta X'(z_i) = \delta'(z_i) \,, \ \delta U'(z_i) = \delta'(z_i)$$





Solar System tests

- A perfect screening inside the solar system

$$\Box \sim -\partial_t^2 + \nabla^2 \quad \longrightarrow \quad \frac{1}{\Box} \text{ provides } \pm \text{ sign}$$

- $X = \frac{1}{\Box} R < 0$ for cosmology
- $X = \frac{1}{\Box} R > 0$ for gravitationally bound systems

Setting $f(X) = 0$ for $X \geq 0$ makes no change in the solar system.

1307.6639 Deser & Woodard

This choice amounts to setting to zero $f(0) = 0$ as well as all its derivatives $f^{(n \geq 1)}(0)$.

- It turns out $\frac{1}{\Box} R$ is negative for both cosmology & gravitationally bound systems

1812.11181 Belgacem, Finke, Frassino & Maggiore

- So, this way of screening does not work any more; the DW model will have significant deviations in the solar system scales (in which GR works very well), which means the DW model is ruled out.
- Belgacem et al explicitly showed both the DW & RR are ruled out by LLR.

Deser-Woodard Nonlocal Cosmology II

$$\mathcal{L}_{\rm nonlocal} \equiv \frac{1}{16\pi G} R\Big[1 + f\Big(Y[g]\Big)\Big]\sqrt{-g}$$

$$X[g] \equiv \frac{1}{\Box} R \; , \; Y[g] \equiv \frac{1}{\Box}\Big(g^{\mu\nu}\partial_\mu X[g] \partial_\nu X[g]\Big) \; ; \; \Box \equiv \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g} g^{\mu\nu} \partial_\nu\Big)$$

Deser & Woodard 1902.08075

Y changes sign from cosmology (Y>0) to gravitationally bound systems (Y<0);
So setting f(Y) = 0 for Y<0 prevents any modifications inside grav. bound systems.

Again, f can be constructed by requiring that the background expansion is exactly LCDM.

More tests - cosmological perturbations regarding the growth of structures and stability analysis, etc. should be done…

- This is not the end of the story; "DW-II may not work for the LLR test either because Geff in this model may still depend on time in smaller scales including the solar system scale" according to Maggiore… need an explicit check!

Nonlocal gravity replacing dark matter

- A nonlocal, metric based, generally coordinate invariant model replacing DM but assuming the existence of the cosmological constant of ΛCDM.

1106.4984 Deffayet, Esposito-Farese and Woodard

- It has been shown that:
    - Agrees with GR in the solar system region;
    - Reproduces the MOND force w/o DM resulting in the Tully-Fisher relation;
    - Enough lensing consistent with the data

1405.0393 Deffayet, Esposito-Farese and Woodard

- A nonlocal function (like in the DW nonlocal model) reproducing the ΛCDM expansion history without CDM has been constructed.

1608.07858 Kim, Rahat, Sayeb, Tan, Woodard and Yu

- Failed to produce sufficient structure formation, thus the immediate task is to improve the model in a way to drive more structure formation.

1804.01669 Tan and Woodard

A hint towards derivation from first principles

• Eventually if a (nonlocal) model passes all the phenomenological tests, it will be necessary to derive such a model from fundamental theory. Do you have at least a hint towards it?

"A nonlocal quantum effective action might derive from fundamental theory through the gravitational vacuum polarization of infrared gravitons vastly produced during primordial inflation."

• For the case of the de Sitter background, $a(t) = e^{Ht}$, the nonlocal scalar is:

$$\frac{1}{\Box} R\Big|_{\rm dS} = -4\ln(a) + \cdots$$

• One loop contributions to the graviton self-energy from a MMC scalar in de Sitter induce corrections to the Newtonian potential associated with a static point mass:

$$\Psi(t,r) = -\frac{GM}{ar}\left\{1 + \frac{G}{20\pi(ar)^2} - \frac{GH^2}{10\pi}\left[\frac{1}{3}\ln(a) + 3\ln(Har)\right] + \cdots\right\}$$

1510.03352 SP, Prokopec and Woodard

• c.f. In flat space, we reproduced the long-known result (Radkowski, 1970, Donoghue 1993) (done by the scattering amplitude technique) using the in-in formalism.

$$\Psi_{flat} = -\frac{GM}{r}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{r^2} + O\left(G^2\right)\right\}$$

1007.2662 SP and Woodard

Set-up

- Lagrangian of gravity plus a MMC scalar

$$\mathcal{L} = \frac{1}{16\pi G}\Big[R - 2\Lambda\Big]\sqrt{-g} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}$$

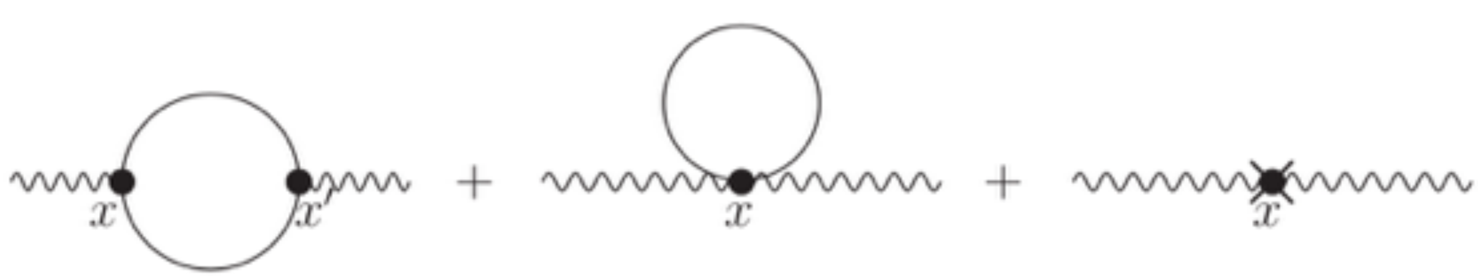
- Define the graviton field $h_{\mu\nu}$ as
  $g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa h_{\mu\nu},\ \overline{g}_{\mu\nu} = a^2\eta_{\mu\nu},\ \kappa^2 = 16\pi G,\ a = -\frac{1}{H\eta},\ H = \sqrt{\frac{1}{3}\Lambda}$

- Vary the one-particle irreducible (1PI) effective action w.r.t $h_{\mu\nu}$ to get
  the quantum corrected, linearized Einstein field equation

$$\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int d^4x' \big[{}^{\mu\nu}\Sigma^{\rho\sigma}\big](x;x')h_{\rho\sigma}(x') = \frac{\kappa}{2}\mathcal{T}^{\mu\nu}_{\rm lin}(x)$$

- $\mathcal{D}^{\mu\nu\rho\sigma}$: Lichnerowicz operator
  defined such that $\mathcal{D}^{\mu\nu\rho\sigma}h_{\mu\nu}$ is the linearized Einstein tensor, $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}(R - 2\Lambda)$

- $-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x')$: graviton self-energy = 1PI graviton 2-point function
  : quantum correction to the Lichnerowicz operator



A three-step procedure

1. Compute using dimensional regularization and renormalize the one-loop contribution to the graviton self-energy from a MMC scalar on de Sitter background by subtracting off the ultraviolet divergences using the R^2 and C^2 counterterms

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')$$

1101.5804 SP & Woodard
1403.0896 Leonard, SP, Prokopec & Woodard

2. Convert the in-out self-energy to the retarded one of the Schwinger-Keldysh formalism

$$\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') \to \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm Ret}\right](x;x')$$

3. Solve the quantum corrected, linearized Einstein field equation

$$\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')h_{\rho\sigma}(x') = \frac{\kappa}{2}\mathcal{T}^{\mu\nu}_{\rm lin}(x)$$

1510.03352 SP, Prokopec & Woodard

Discussion points

• For the case of the de Sitter background, $a(t) = e^{Ht}$, the nonlocal scalar is:

$$\frac{1}{\Box} R\Big|_{\rm dS} = -4\ln(a) + \cdots$$

• One loop contributions to the graviton self-energy from a MMC scalar in de Sitter induce corrections to the Newtonian potential associated with a static point mass:

$$\Psi(t,r) = -\frac{GM}{ar}\left\{1 + \frac{G}{20\pi(ar)^2} - \frac{GH^2}{10\pi}\left[\frac{1}{3}\ln(a) + 3\ln(Har)\right] + \cdots\right\}$$

1510.03352 SP, Prokopec and Woodard

• Another example of ln(a) correction: One loop corrected conformally coupled scalar mode functions in de Sitter

1708.01831 Boran, Kahya and SP

$$u_{\rm CC} \sim \frac{1}{\sqrt{2k}}\left\{\frac{1}{a} + GH^2\left[\frac{42323}{2^5\cdot 15\pi}\ \ln(a)\ -\ \frac{530953}{2^6\cdot 15\pi}\ - \left(\Delta c_4 - \frac{3}{4}\right)\ \frac{\ln(a)}{a}\right] + \mathcal{O}(G^2H^4)\right\}$$

• And many more examples

• Even though the loop counting parameter GH^2 is extremely small, the ln(a) will grow and eventually overcome it, then perturbation theory will break down; Would be necessary to use a nonperturbative resummation method such as Starobinsky's stochastic technique…

•The C^2 counterterm gives a ghost (Ostrogradsky thm)… how can we handle it? Talks by Mazumdar, Salvio, …