Domain walls in early universe

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Introduction

Observations indicate that the universe is 100% baryo-asymmetric – it contains only matter, not antimatter

Observed baryon asymmetry cannot be explained by Standard Model $\Rightarrow$ physics beyond SM

There are models where the universe is globally:
1) asymmetric (matter $>$ antimatter initially)
2) symmetric $\Rightarrow$ domains of matter and antimatter with domain walls (DW) between them

Introduction

In symmetric models domain wall problem can arise – unacceptably high energy density of domain walls if they exist today

Solution: domain walls should exist only in the universe past and should disappear by now

Introduction

Another problem – if matter and antimatter domains exist and they are close enough, the gamma rays produced in annihilation along the boundary would be detectable.
Such gamma rays have not been observed

Solution: matter and antimatter domains should be separated by cosmologically large distances, i.e.
domain wall thickness should be cosmologically large

so let us consider
Under what conditions domain wall thickness can become cosmologically large

I. Evolution of thick domain walls in de Sitter universe. Stationary solutions

R. Basu and A. Vilenkin
Phys. Rev. D 50 (1994) 7150

In their paper Basu and Vilenkin (BV) address the question
what happens to the domain wall during inflation

The inflationary Universe is approximated by de Sitter spacetime, which has a constant expansion rate H,
and the scale factor evolves as

$$a(t) = e^{Ht} \tag{1}$$

The metric with spatially flat sections is given by

$$ds^2 = dt^2 - e^{2Ht}\left(dx^2 + dy^2 + dz^2\right) \tag{2}$$

BV consider a one component scalar field theory with a simple double-well potential

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - \frac{\lambda}{2} \left(\varphi^2 - \eta^2 \right)^2 \tag{3}$$

The corresponding equation of motion is

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \right) = -2\lambda\varphi \left(\varphi^2 - \eta^2 \right) \tag{4}$$

In flat spacetime, $H = 0$, and in one-dimensional static case, $\varphi = \varphi(z)$, the equation takes the form

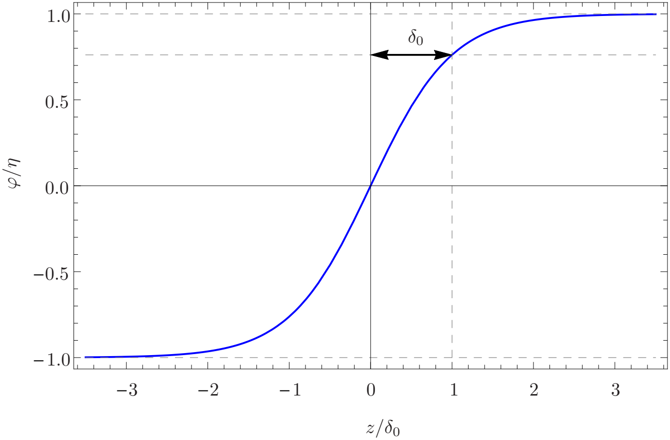
$$\frac{d^2\varphi}{dz^2} = 2\lambda\varphi \left(\varphi^2 - \eta^2 \right) \tag{5}$$

It has a kink-type solution, which describes a static infinite DW

One can assume that the wall is situated at $z = 0$ in xy-plane:

$$\varphi(z) = \eta \tanh \frac{z}{\delta_0}, \tag{6}$$

where $\boldsymbol{\delta_0 = 1/(\sqrt{\lambda}\eta)}$ is the flat-spacetime wall thickness



What happens in an expanding universe with constant $H > 0$?

In this case, if one looks for stationary solution, it is reasonable to suggest that the field φ depends only on $za(t) = z \exp Ht$, which is the proper distance from the wall
So, one can choose the following ansatz for φ:

$$\varphi = \eta \cdot f(u), \quad \text{where} \quad u = Hz \cdot e^{Ht}, \tag{7}$$

here u and f are dimensionless.

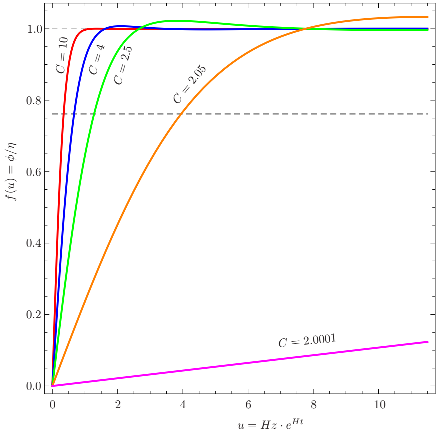
Equation of motion takes the form:

$$\left(1 - u^2\right) f'' - 4uf' = -2Cf\left(1 - f^2\right) \tag{8}$$

It is noteworthy that all parameters of the problem are combined into a single positive constant $\boldsymbol{C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 0}$

Since one is interested in the kink-type solutions, boundary conditions should be

$$f(0) = 0, \qquad f(\pm\infty) = \pm 1 \tag{9}$$



Stationary field configurations $f(u)$ for different values of C

BV noticed that stationary solutions can be found only for $C > 2$, but no explanation of this observation was given.

Our naive explanation is the following:

eq. of motion (8) and boundary condition $f(0) = 0$ lead to $f''(0) = 0$,

then expanding $f(u)$ into Taylor series near $u = 0$, one obtains that for sufficiently small positive ϵ and for $f'(0) > 0$:

for $C > 2$, $f''(\epsilon) < 0 \Rightarrow f(u)$ is concave downward like ordinary kink solution, $f(u) = \tanh u/\delta_0$

for $C \leq 2$, $f''(\epsilon) > 0 \Rightarrow f(u)$ is convex downward, it is not the kink-type solution

Conclusions for section I

- In the case of very thin DW, when thickness is much smaller than the de Sitter horizon, $\delta << H^{-1}$, i.e. $C >> 1$, the solution is well approximated by the flat-spacetime solution
- Beyond the critical value, $\delta_0 \geq H^{-1}/\sqrt{2}$, i.e. $C \leq 2$, there are no stationary solutions at all

II. Evolution of thick domain walls in de Sitter universe. Beyond the stationary solutions

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko

JCAP 1610 (2016) 10, 026

Beyond the stationary approximation we can find solution not only for $C > 2$, but also for $C \leq 2$.
To this end one should solve the original equation of motion (4) in the case when the field φ is a function of at least two independent variables, z and t:

$$\frac{\partial^2 \varphi}{\partial t^2} + 3H\frac{\partial \varphi}{\partial t} - e^{-2Ht}\frac{\partial^2 \varphi}{\partial z^2} = -2\lambda\varphi\left(\varphi^2 - \eta^2\right) \tag{10}$$

It is convenient to introduce dimensionless variables $\tau = Ht$, $\zeta = Hz$ and dimensionless function $f(\zeta, \tau) = \varphi(z, t)/\eta$
As a result one obtains the equation

$$\frac{\partial^2 f}{\partial \tau^2} + 3\frac{\partial f}{\partial \tau} - e^{-2\tau}\frac{\partial^2 f}{\partial \zeta^2} = 2Cf\left(1 - f^2\right), \tag{11}$$

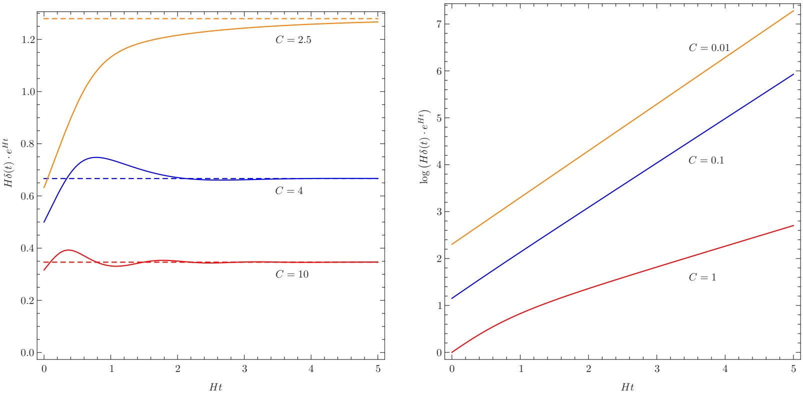
where constant $\boldsymbol{C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 0}$ as it was above

Boundary conditions for the kink-type solution are

$$f(0,\tau) = 0, \qquad f(\pm\infty,\tau) = \pm 1 \tag{12}$$

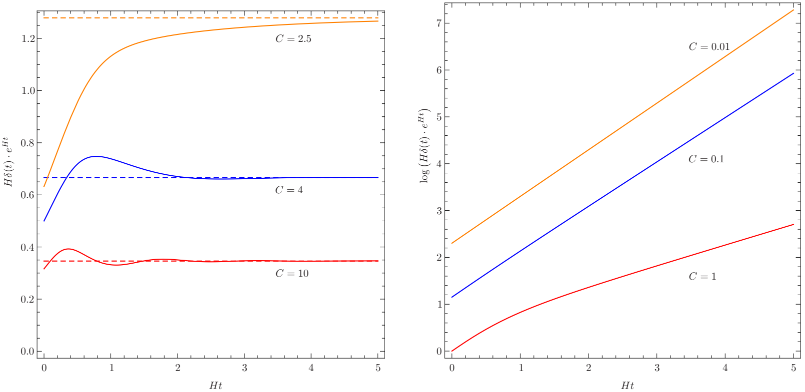
and we choose the initial configuration as domain wall with thickness δ_0 (with respect to coordinate z) and zero time derivative:

$$f(\zeta,0) = \tanh\frac{z}{\delta_0} = \tanh\sqrt{C}\zeta, \qquad \left.\frac{\partial f(\zeta,\tau)}{\partial\tau}\right|_{\tau=0} = 0 \tag{13}$$

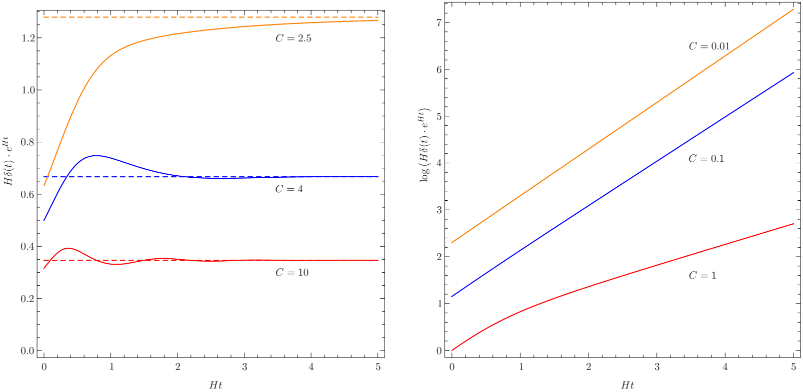


Time dependence of DW physical thickness, $a(t)\delta(t)$

We define coordinate thickness $\delta(t)$ as the value of z at the position where $\varphi/\eta = \tanh 1 \approx 0.76$



In the left plot ($C > 2$) – all curves tend to constant values corresponding to stationary solutions (dashed lines), $a(t)\delta(t) \to \delta_0$, i.e. $\delta(t) \to \delta_0 \cdot \exp\left(-Ht\right)$



Right plot contains curves for $C < 2$. Along the vertical axis the logarithm of the DW thickness is shown in order to compare the rate of the DW expansion with the exponential cosmological one, $a(t) = \exp Ht$

For $C \lesssim 0.1$ the rate of the DW expansion is the exponential one with a good accuracy, $\delta(t) \approx \delta_0$

Conclusions for section II

- We have shown that for large values of parameter $C > 2$ the initial kink configuration in a de Sitter background tends to the stationary solution obtained by Basu and Vilenkin.
- For $C \leq 2$ the stationary solution does not exist and the thickness of the wall infinitely grows with time.
- For $C \lesssim 0.1$ the rise is close to the exponential one $\Rightarrow$ transition regions between domains might be cosmologically large

III. Evolution of thick domain walls during inflation

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko
Eur. Phys. J. C78 (2018) 10, 855

One of the inflation models that is in agreement with observational data is R^2-inflation

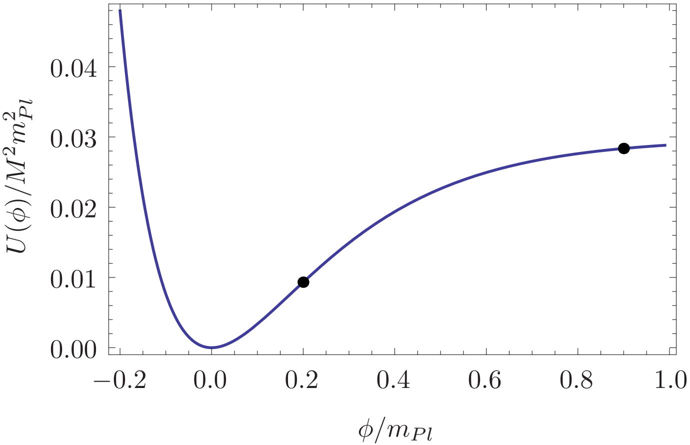
Modified theory of gravity with the action

$$S = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{R^2}{6M^2} \right) \tag{14}$$

is equivalent to the general relativity with a scalar field ϕ with the potential

$$U(\phi) = \frac{3M^2 m_{Pl}^2}{32\pi} \left(1 - e^{-4\sqrt{\pi/3}\,\phi/m_{Pl}} \right)^2 \tag{15}$$

We take $M = 2.6 \cdot 10^{-6}\, m_{Pl}$ in consistency with observational data, and choose $\phi_i = \phi(0) = 0.9\, m_{Pl}$, $\phi_e = 0.2\, m_{Pl}$

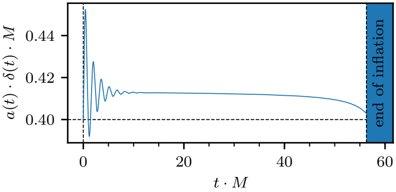


The evolution of the domain wall is basically defined by the parameter C.
Now it is not a constant, but a function of time and is defined as

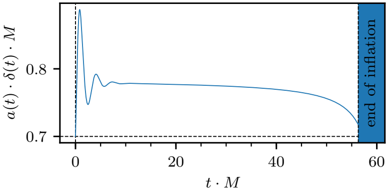
$$C(t) = \frac{1}{(H(t)\delta_0)^2} \tag{16}$$

Since during inflation $H(t)$ decreases, $C(t)$ increases with time.

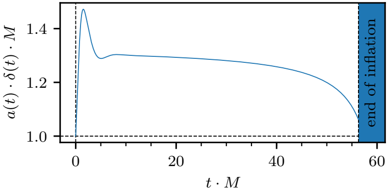
Let us also introduce time t_C such that $C(t_C) = 2$



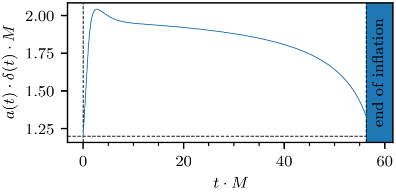
$\delta_0 = 0.4 \cdot M^{-1}$



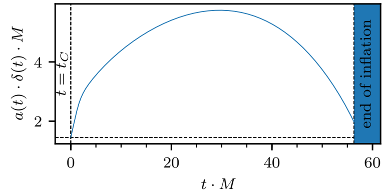
$\delta_0 = 0.7 \cdot M^{-1}$



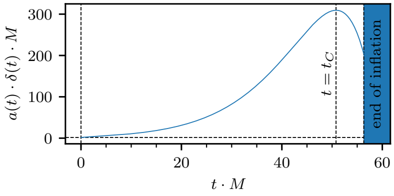
$\delta_0 = 1.0 \cdot M^{-1}$



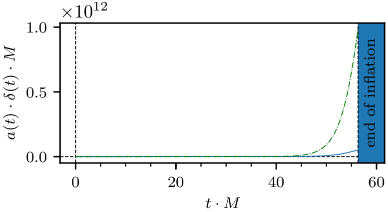
$\delta_0 = 1.2 \cdot M^{-1}$



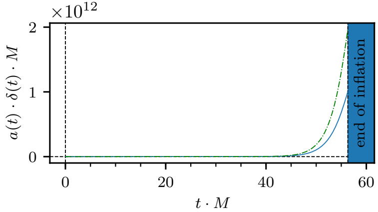
$\delta_0 \approx 1.4507 \cdot M^{-1}$ $(t_C = 0)$



$\delta_0 = 1.7 \cdot M^{-1}$



$\delta_0 = 5.0 \cdot M^{-1}$



$\delta_0 = 10 \cdot M^{-1}$

Green dash-dotted line corresponds to $a(t) \cdot \delta_0 \cdot M$,
$\delta(t) \approx \delta_0$ for large δ_0

Conclusions for section III

In the inflationary universe

- If δ_0 is quite small ($\boldsymbol{C(t) > 2}$ during all the time of inflation), then DW thickness, $a(t)\delta(t)$, tends to constant value, δ_0, therefore is not cosmologically large.
- If $\delta_0 \sim H(t)^{-1}$, then DW thickness can grow up to quite large values during inflation, but at some moment it starts to diminish and tends to δ_0 at the end of inflation.
- Domain walls, which were thick initially ($\boldsymbol{C(t) < 2}$ during all the time of inflation), expand until the end of inflation. In this case coordinate thickness almost does not change, $\boldsymbol{\delta(t) \approx \delta_0}$, and domain wall expansion is entirely due to growth of scale factor, $a(t)$.
  Thickness of such DW can be cosmologically large at the end of inflation.

IV. Separated matter and antimatter domains with vanishing domain walls

based on

A.D. Dolgov, S.I. Godunov, A.S. Rudenko, and I.I. Tkachev,
JCAP 1510 (2015) 10, 027

A few years ago we considered a model which may lead to baryo-symmetric universe with cosmologically large domains of matter and antimatter separated by cosmologically large distances, without the domain wall problem [*A.D. Dolgov, S.I. Godunov, A.S. Rudenko, and I.I. Tkachev, JCAP 1510 (2015) 10, 027; arXiv:1506.08671*].

We considered therein ϕ^2-inflation with the inflaton potential $\boldsymbol{U = m^2\phi^2/2}$

A coupling of the pseudoscalar field χ (which made the domain wall) to the inflaton field ϕ was introduced on purpose to generate a non-zero value of χ and to keep it during baryogenesis as a source of CP violation.

Model

Now we recalculate the model in R^2-inflation with the inflaton potential

$$U(\phi) = \frac{3M^2 m_{Pl}^2}{32\pi}\left(1 - e^{-4\sqrt{\pi/3}\,\phi/m_{Pl}}\right)^2 \tag{17}$$

Action:

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\,\left(\mathcal{L}_\phi + \mathcal{L}_\chi + \mathcal{L}_{\rm int}\right), \tag{18}$$

where

$$\mathcal{L}_\phi = \frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - U(\phi), \tag{19}$$

$$\mathcal{L}_\chi = \frac{1}{2}\,g^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi - \frac{1}{2}m^2\chi^2 - \frac{\lambda}{4}\chi^4, \tag{20}$$

$$\mathcal{L}_{\rm int} = \mu^2\chi^2 V(\phi), \tag{21}$$

we choose $M = 2.6\cdot 10^{-6}\,m_{Pl}$, $m = 10^{-10}\,m_{Pl}$, $\lambda = 10^{-3}$, $\mu = 2\cdot 10^{-5}\,m_{Pl}$

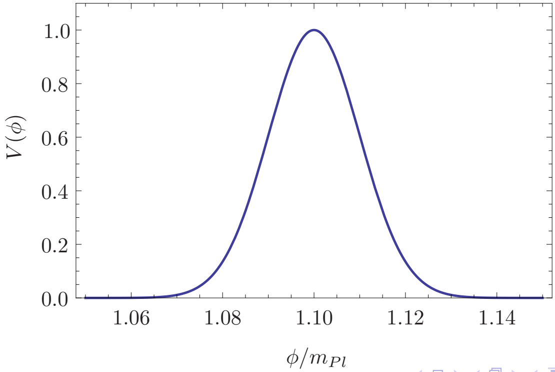
Model

$V(\phi)$ is non-zero only in a narrow range of ϕ

(interaction between ϕ and χ is non-negligible only during short period of time),

e.g. in a toy model $\boldsymbol{V(\phi) = \exp\left[-\frac{(\phi-\phi_0)^2}{2\phi_1^2}\right]}$

we choose $\phi_0 = 1.1\, m_{Pl}$, $\phi_1 = 0.01\, m_{Pl}$



FLRW metric: $ds^2 = dt^2 - a^2(t)\, d\mathbf{x}^2$

Hubble parameter: $H(t) = \frac{\dot a(t)}{a(t)} = \sqrt{\frac{8\pi\rho(t)}{3m_{Pl}^2}}$

Energy density:

$$\rho(t) = \frac{\dot\phi^2}{2} + U(\phi) + \frac{\dot\chi^2}{2} + \frac{m^2}{2}\chi^2 + \frac{\lambda}{4}\chi^4 - \mu^2\chi^2 V(\phi) \qquad (22)$$

Equations of motion:

$$\ddot\phi + 3H\dot\phi + U'(\phi) - \mu^2\chi^2 V'(\phi) = 0, \qquad (23)$$

$$\ddot\chi + 3H\dot\chi + m^2\chi + \lambda\chi^3 - 2\mu^2\chi V(\phi) = 0, \qquad (24)$$

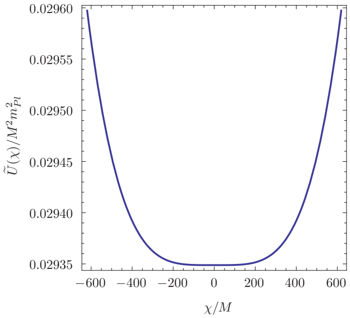
here it is assumed that $\phi = \phi(t)$ and $\chi = \chi(t)$

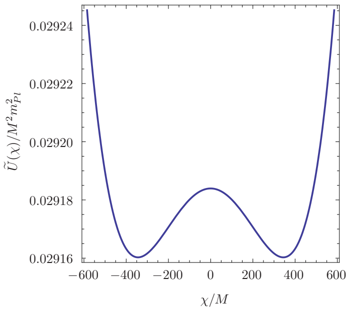
Model

During inflation (usual slow-roll regime) ϕ decreases and when it reaches vicinity of ϕ_0, two minima appear in the potential

$$\widetilde{U}(\chi)\Big|_{\phi=const} = \frac{\lambda}{4}\chi^4 + \left(\frac{m^2}{2} - \mu^2 V(\phi)\right)\chi^2 + U(\phi),$$

so the point $\chi = 0$ becomes local maximum

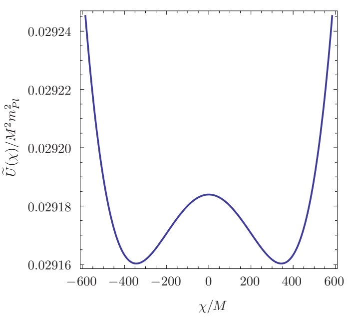


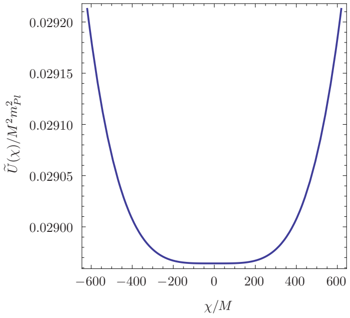


Model

In spatial regions where χ turns out to be positive/negative (due to fluctuations) it rolls down to the positive/negative minimum $\Rightarrow$ domains with opposite signs of CP violation appear [*Lee, 1974*]

Such CP violation is operative only when χ sits near the minimum, but in our model this minimum disappeared during inflation $\Rightarrow$ baryon asymmetry (if generated) exponentially inflated away





Successful baryogenesis should take place after the end of inflation

Model

Main features of the model:

1. Inflaton field ϕ always gives the main contribution to the energy density $\rho \Rightarrow$ standard slow-roll inflation

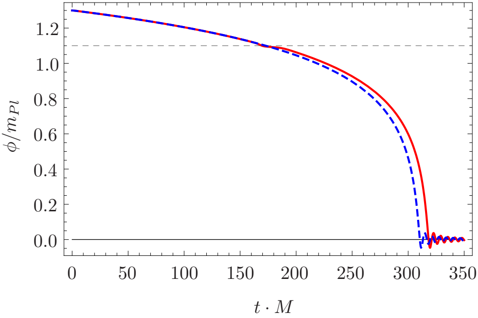
2. Inflation should last quite long after domains were formed (supposing that seed of the domain $\sim 1/H$ and present size of domain ~ 100 Mpc $\Rightarrow \phi_0 = 1.1\, m_{Pl}$)

3. The field χ remains non-vanishing for some time after the end of inflation and during baryogenesis
(non-zero χ can induce CP violation)

Evolution of the inflaton field $\phi(t)$

Slow-roll inflation

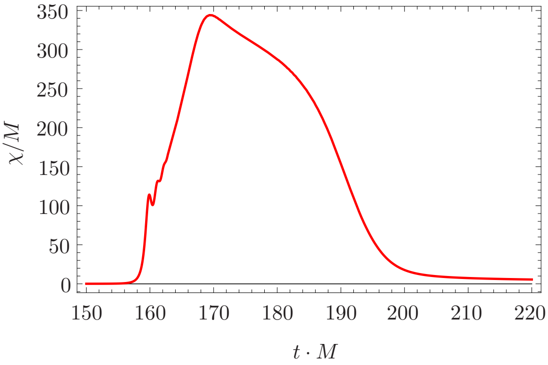
$\boldsymbol{\phi(t)}$ [red plot] only slightly deviates from standard one [blue plot].
Inflation lasts $\simeq$ 60 e-foldings ($e^{60} \sim 10^{26}$) after $\phi(t)$ passes ϕ_0



$\phi_0 = 1.1\, m_{Pl}$, $\phi_{in} = \phi(0) = 1.3\, m_{Pl}$

$\phi(t) = 0$ at $t \approx 318\, M^{-1}$ (beginning of reheating)

Evolution of the field $\chi(t)$



$\chi_{in} = \chi(0) = 0.1M$

When reheating starts (at $t \approx 318\,M^{-1}$) the field χ is quite large:
$\chi \approx 2.4M \sim 10^{14}$ GeV

Distances between domains

Since inflaton ϕ always gives the main contribution to $H \Rightarrow$ the Hubble parameter remains almost the same in the case of $\chi = 0$ as in the case of non-zero χ considered above $\Rightarrow$ size of region where $\chi = 0$ is only order of magnitude less than the domain size.

Therefore, model predicts the domains of cosmological size with the distances between them of cosmological size also

Reheating

The stage of inflation is followed by the stage of reheating, during which the heavy X-particles (e.g. vector bosons) can be produced through decay of inflaton field.

In the case of comparatively large coupling between X-particles and inflaton the decay of the inflaton occurs rapidly, during only one or few oscillations.

Assume that produced X-particles can decay into fermions (not SM in general). If the corresponding couplings are large enough, the X-particles decay very quickly.

Therefore, field χ may remain non-zero yet to the moment when X-particles have been completely decayed:
e.g. $\chi \sim 10^{12}$ GeV at $t = 400\, M^{-1}$
(non-zero χ can induce CP violation in the X-boson decays)

Non-zero χ can induce CP violation in X-boson decays

The field χ is real and pseudoscalar,
so it interacts with the produced fermions as

$$\mathcal{L}_{\chi\psi\psi} = g_{kl}\chi\bar{\psi}^k i\gamma_5\psi^l = ig_{kl}\chi(\bar{\psi}^k_L\psi^l_R - \bar{\psi}^k_R\psi^l_L),$$

where k and l denote the fermion flavor.

Free fermion Lagrangian may contain mass terms

$$-m_{\psi kl}\bar{\psi}^k\psi^l = -m_{\psi kl}(\bar{\psi}^k_L\psi^l_R + \bar{\psi}^k_R\psi^l_L)$$

The sum of these terms and $\mathcal{L}_{\chi\psi\psi}$ can be presented in matrix form

$$-(\bar{\psi}_R M_\psi \psi_L + \bar{\psi}_L M^\dagger_\psi \psi_R),$$

here $M_\psi = m_\psi + ig\chi$ is a non-Hermitian matrix,
whereas m_ψ and g are Hermitian ones

Non-zero χ can induce CP violation in X-boson decays

Using simultaneously two unitary transformations
$\psi_R \to \psi'_R = U_R \psi_R$ and $\psi_L \to \psi'_L = U_L \psi_L$
one can always diagonalize the matrix M_ψ.

The elements of transformation matrices U_R and U_L depend on the magnitude of the field χ

The mass terms take the simple form

$$-m'_{\psi ab} \bar{\psi}^a \psi^b,$$

where ψ^a and ψ^b are the mass eigenstates and
m'_ψ is diagonal matrix with real diagonal elements

Non-zero χ can induce CP violation in X-boson decays

However, the interaction of fermions with vector boson X_μ remains the same under these transformations:

$$g_{Rkl} X_\mu \bar{\psi}^k_R \gamma^\mu \psi^l_R = g'_{Rab} X_\mu \bar{\psi}^a_R \gamma^\mu \psi^b_R,$$

here $g'_R = U_R g_R U_R^\dagger$ is matrix of coupling constants in mass eigenstate basis (analogously $g'_L = U_L g_L U_L^\dagger$).

The constants g'_{ab} are complex in general case, and if there are at least three species of fermions, one cannot rotate away simultaneously all phases in complex matrices $g'_{R,L}$ [*M. Kobayashi, T. Maskawa, 1973*].

The complexity of the coupling constants means that
CP is violated in the X-boson decays

Sakharov criteria

The model may satisfy Sakharov criteria for successful baryogenesis without fine tuning of parameters

1. Breaking of C and CP invariance

The magnitude of CP violation depends on the value of χ through the matrices $U_{R,L}$ and hence coupling constants $g'_{R,L}$. Since χ is essentially non-zero after the end of inflation and during baryogenesis, CP-odd effects can be large enough.

We assume that interactions with X-boson involve fermions with certain chirality, and thus these interactions break C-invariance

2. Deviation from thermal equilibrium

State of matter during reheating is out of thermal equilibrium

3. Non-conservation of baryon number B

Assume also that
the baryon number is not conserved in X-boson decays

Baryon asymmetry generated in the decay of one X-particle:

$$\delta_X = \frac{1}{\Gamma_{X,tot}} \sum_f \Gamma(X \to f) B_f$$

The ratio of the baryon number density to the entropy density can be estimated as

$$\Delta_B = \frac{n_B - n_{\bar{B}}}{s} \sim 10^{-3} \delta_X$$

Thus, to get observed value $\Delta_B \simeq 0.86 \cdot 10^{-10}$ it is sufficient to have only $\boldsymbol{\delta_X \sim 10^{-7}}$

Conclusions for section IV

- The considered model may lead to baryo-symmetric universe with cosmologically large domains of matter and antimatter separated by cosmologically large distances, avoiding the domain wall problem.
- Inflation is an essential ingredient of the scenario. A coupling of the pseudoscalar field χ to the inflaton field was introduced on purpose to generate a non-zero value of χ and to keep it during baryogenesis as a source of CP violation.

Thank you for your attention!