Chaotic Inflation and No-Scale Gravity

Alberto Salvio

Università di Roma Tor Vergata

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Hot topics in Modern Cosmology,
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INSTITUT D'ÉTUDES SCIENTIFIQUES DE CARGÈSE

Main questions

Can a *viable* field theory of fundamental interactions hold up to infinite energy?

If so, what are its *experimental signatures*?

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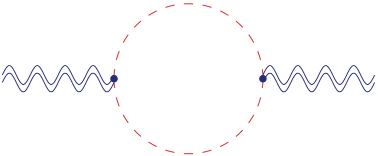
Quadratic-in-curvature terms in the action

$$S_{\rm quad} = \int d^4x \sqrt{-g}\, \mathscr{L}_{\rm quad}, \qquad \mathscr{L}_{\rm quad} = \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}$$

are unavoidably generated by matter loops, such as



$$(4\pi)^2 \frac{d\alpha}{d\ln\bar\mu} = \frac{N_V}{15} + \frac{N_f}{60} + \frac{N_s}{180} - \frac{(\delta_{ab} + 6\xi_{ab})(\delta_{ab} + 6\xi_{ab})}{72}$$

$$(4\pi)^2 \frac{d\beta}{d\ln\bar\mu} = -\frac{N_V}{5} - \frac{N_f}{20} - \frac{N_s}{60}$$

N_V, N_f, N_s are the numbers of vectors, Weyl fermions and real scalars ϕ_a with non-minimal couplings ξ_{ab} (that is $\mathscr{L} \supset -\xi_{ab}\phi_a\phi_b R/2$)

Quadratic gravity scenario

Adding the quadratic terms makes gravity (and all other interactions) renormalizable *[Stelle (1977)]*

Intuitive reason: in the UV the theory is the most general dimensionless one

The general quadratic gravity (QG) Lagrangian:

$$\mathscr{L} = \frac{R^2}{6f_0^2} - \frac{W^2}{2f_2^2} + \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm BSM}$$

where

- $W^2 \equiv W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$
- $\mathscr{L}_{\rm EH}$ is the Einstein-Hilbert piece plus a cosmological constant
- $\mathscr{L}_{\rm SM}$ is the SM $\mathscr{L}$ (plus $-\xi_H|H|^2R$):
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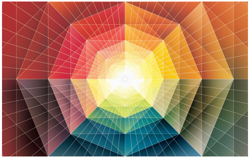
It is also possible to generate scales dynamically

The dimensionful terms (the Planck mass, the electroweak scale and the cosmological constant) can all be dynamically generated through dimensional transmutation (**Agravity**) *[Salvio, Strumia (2014)]*



Quanta Magazine (Simons Foundation)

The Ostrogradsky theorem

Classical Lagrangians that depend non-degenerately on the second derivatives have Hamiltonians unbounded from below *[Ostrogradsky (1848)]*

Indeed, looking at the spectrum (around the flat spacetime) :

(i) massless graviton

(ii) scalar ω with squared mass $M_0^2 \sim \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2$

(iii) massive spin-2 ghost with squared mass $M_2^2 = \frac{1}{2} f_2^2 \bar{M}_{\rm Pl}^2$
(a manifestation of the Ostrogradsky theorem)
It is associated with $\frac{W^2}{2 f_2^2}$

By linearizing the theory one finds the spin-2 Hamiltonians *[Salvio (2017)]*

$$H_{\rm graviton} = \sum_{\lambda=\pm 2} \int d^3q \left[\left(P_\lambda^{(1)}\right)^2 + q^2 \left(Q_\lambda^{(1)}\right)^2 \right]$$

$$H_{\rm ghost} = - \sum_{\lambda=\pm 2, \pm 1, 0} \int d^3q \left[\left(\tilde{P}_\lambda^{(1)}\right)^2 + (q^2 + M_2^2) \left(\tilde{Q}_\lambda^{(1)}\right)^2 \right]$$

Proceeding perturbatively

Let us split the metric $g_{\mu\nu}$ as follows:

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{cl}} + \hat{h}_{\mu\nu}$$

- $g_{\mu\nu}^{\mathrm{cl}}$ is a classical background that solves the classical EOMs
- $\hat{h}_{\mu\nu}$ is a *quantum* deviation

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Can we hope that something similar happens for gravitons?

Yes, renormalizability implies that the *quantum* Hamiltonian governing $\hat{h}_{\mu\nu}$ is bounded from below *[Stelle (1977)]*

However, the space of states must be endowed with an indefinite metric (with respect to which the "position" q and momentum p operators are self-adjoint)

Probability

The presence of an indefinite metric leads to the question:

How can we define probabilities consistently?

A derivation of probability

- Define observable any operator A with complete eigenstates $\{|a\rangle\}$ *[Salvio (2018)]*: for any state $|\psi\rangle$ there is a decomposition

$$|\psi\rangle = \sum_a c_a |a\rangle$$

  One can show that the basic operators q, p and H have complete eigenstates at *any* order in perturbation theory

- Interpret $|a\rangle$ as the state where A assumes certainly the value a
  (call it the deterministic Born rule)

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Experimentalists prepare a large number N of times the same state, so consider

$$|\Psi_N\rangle \equiv \underbrace{|\psi\rangle ... |\psi\rangle}_{N \text{ times}} = \sum_{a_1 ... a_N} c_{a_1} ... c_{a_N} |a_1\rangle ... |a_N\rangle$$

Define a frequency operator F_a which counts the number N_a of times there is the value a in the state $|a_1\rangle ... |a_N\rangle$:

$$F_a |a_1\rangle ... |a_N\rangle = N_a |a_1\rangle ... |a_N\rangle$$

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Strumia (2017) showed that

$$\lim_{N\to\infty} F_a |\Psi_N\rangle = B_a |\Psi_N\rangle, \qquad B_a \equiv \frac{|c_a^2|}{\sum_b |c_b^2|}$$

(all coefficients in the basis $|a_1\rangle ... |a_N\rangle$ converge to the same quantities)

The emergent norms to compute probabilities

$\{|a\rangle\}$ is complete so we can ***define*** a "norm" operator P_A:

$$\langle a'|P_A|a\rangle \equiv \delta_{aa'}$$

where for any pair of states $|\psi_1\rangle$, $|\psi_2\rangle$, we denote the indefinite metric with $\langle\psi_2|\psi_1\rangle$.
The definition above provides a positive metric (a norm):

$$\langle\psi_2|\psi_1\rangle_A \equiv \langle\psi_2|P_A|\psi_1\rangle_A = \sum_a c^*_{a2} c_{a1}$$

(which is positive for $|\psi_1\rangle = |\psi_2\rangle$)

$$B_a \equiv \frac{|c_a^2|}{\sum_b |c_b^2|} = \frac{|\langle a|\psi\rangle_A|^2}{\langle\psi|\psi\rangle_A}$$

We recover the full probabilistic Born rule, but expressed in terms of the ***positive norm*** not in terms of the indefinite one

- All probabilities are positive
- The probabilities sum up to one at any time (the theory is unitary)

Higgs hierarchy problem

Condition to solve the Higgs hierarchy problem

The theory is renormalizable

$\Longrightarrow$ one can absorb the loop divergences and compute δM_h:

$$\delta M_h^2 \sim \frac{\bar{M}_{\rm Pl}^2 f^4}{(4\pi)^2}, \quad \text{naturalness} \to f_2 \sim \sqrt{\frac{4\pi M_h}{\bar{M}_{\rm Pl}}} \sim 10^{-8}$$

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$$M_2 = \frac{1}{\sqrt{2}} f_2 \bar{M}_{\rm Pl} \sim 10^{10} \text{GeV}$$

[Salvio, Strumia (2014)]

The quantization proposed in *Anselmi's talk* leads to the same result

Let us go back to the the following metric splitting

$$g_{\mu\nu} = g^{\rm cl}_{\mu\nu} + \hat{h}_{\mu\nu}$$

- $\hat{h}_{\mu\nu}$ is a *quantum* deviation
- $g^{\rm cl}_{\mu\nu}$ is a classical background that solves the classical EOMs.
  Do the Ostrogradsky theorem lead to runaway solutions?

Classical theory

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- Recall that in the *free-field limit*

$$H_{\rm ghost} = -\sum_{\lambda=\pm 2,\pm 1,0} \int d^3q \left[\left(P^{(1)}_\lambda\right)^2 + (q^2+M_2^2)\left(Q^{(1)}_\lambda\right)^2 \right]$$

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- The intermediate case $0 < f_2 < 1$ must have intermediate energy thresholds (above which the runaways are activated)

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This argument can be made precise in quadratic gravity. The whole cosmology can only involve energies below this threshold and avoid runaways

→ "ghost metastability"

[Salvio (2019)]

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First perform the field redefinition $\quad g_{\mu\nu} \to \frac{\bar{M}_{\rm Pl}^2}{f} g_{\mu\nu}, \qquad f \equiv \bar{M}_{\rm Pl}^2 - \frac{2R}{3f_0^2} > 0,$

(where the Ricci scalar above is computed in the Jordan frame metric) that gives

$$S = \int d^4x\sqrt{-g}\left(-\frac{W^2}{2f_2^2} - \frac{\bar{M}_{\rm Pl}^2}{2}R + \mathscr{L}_m^E\right) \qquad \text{“Einstein frame action”}$$

The Einstein-frame matter Lagrangian, $\mathscr{L}_m^E$, also contains an effective scalar ω, which corresponds to the R^2 term in the Jordan frame: the part of the Lagrangian that depends only on ω is given by

$$\mathscr{L}_m^\omega = \frac{(\partial\omega)^2}{2} - U(\omega), \qquad U(\omega) = \frac{3f_0^2\bar{M}_{\rm Pl}^4}{8}\left(1 - e^{-2\omega/\sqrt{6}\bar{M}_{\rm Pl}}\right)^2$$

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To make the ghost explicit consider an auxiliary field $\gamma_{\mu\nu}$ *[Hindawi, Ovrut, Waldram (1996)]* :

$$S = \int d^4x \sqrt{-g} \left[\frac{M_2^2 \bar{M}_{\rm Pl}^2}{8} \left(\gamma_{\mu\nu}\gamma^{\mu\nu} - \gamma^2 \right) - \frac{\bar{M}_{\rm Pl}^2}{2} G_{\mu\nu}\gamma^{\mu\nu} - \frac{\bar{M}_{\rm Pl}^2}{2} R + \mathscr{L}_m^E \right]$$

where $G_{\mu\nu}$ is the Einstein tensor and $\gamma \equiv \gamma_{\mu\nu} g^{\mu\nu}$. Expanding around $\eta_{\mu\nu}$ gives a mixing between $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ and $\gamma_{\mu\nu}$ that can be removed by expressing $h_{\mu\nu} = \bar{h}_{\mu\nu} - \gamma_{\mu\nu}$. The tensors $\bar{h}_{\mu\nu}$ and $\gamma_{\mu\nu}$ represent the graviton and the ghost

Interactions of the ghost and energy thresholds

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Next, $\frac{M_2^2}{8}\left(\gamma_{\mu\nu}\gamma^{\mu\nu} - \gamma^2\right)$ leads to mass and interaction terms of the schematic form

$$\frac{M_2^2}{2}\left(\phi_2^2 + \frac{\phi_2^3}{\bar{M}_{\rm Pl}} + \frac{\phi_2^4}{\bar{M}_{\rm Pl}^2} + \ldots\right),$$

(ϕ_2 represents the canonically normalized spin-2 fields: graviton and ghost)

The mass term has the same order of magnitude of the interactions for $\phi_2 \sim \bar{M}_{\rm Pl}$, which gives $M_2^2\phi_2^2/2 = M_2^4/f_2^2 \equiv E_2^4$, where

$$E_2 \equiv \frac{M_2}{\sqrt{f_2}} = \sqrt{\frac{f_2}{2}}\bar{M}_{\rm Pl}$$

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Analogously, one can show that the energy E in the matter sector must satisfy

$$\boxed{E \ll E_m} \qquad E_m \equiv \sqrt[4]{f_2}\,\bar{M}_{\rm Pl} \qquad \text{(matter sector)}$$

Possible to illustrate the argument in simple models

Relations with chaotic inflation *[Linde (1983)]*

For a natural Higgs mass ($f_2 \sim 10^{-8}$, $M_2 \sim 10^{10}$ GeV)

$$E_2 \sim 10^{-4}\bar{M}_{\rm Pl}, \qquad E_m \sim 10^{-2}\bar{M}_{\rm Pl}$$

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these conditions justify the use of homogeneous and isotropic solutions to describe the classical part of inflation (Linde's idea)

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The chaotic theory automatically ensures that the conditions to avoid runaway solutions are satisfied (verified for Starobinsky inflation, hilltop inflation and other models). The runaways above the energy thresholds give a rational for a homogeneous and isotropic universe

Explicit nonlinear calculations (assuming the built in Starobinsky's inflation)

$$ds^2 = dt^2 - a(t)^2 \sum_{i=1}^{3} e^{2\alpha_i(t)} dx^i dx^i$$

$$\alpha_1 \equiv \beta_+ + \sqrt{3}\beta_-, \quad \alpha_2 \equiv \beta_+ - \sqrt{3}\beta_-, \quad \alpha_3 = -2\beta_+.$$

One can reduce the system to first-order equations through the definitions

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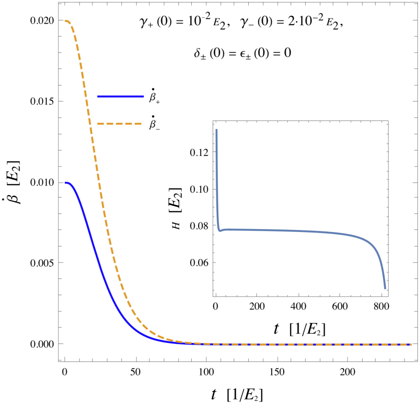
Small initial values for the anisotropy

$(|\gamma_\pm(0)| \ll E_2, \sqrt{|\delta_\pm(0)|} \ll E_2,$
$\sqrt[3]{|\epsilon_\pm(0)|} \ll E_2$ and $\sqrt{\bar{M}_{\rm Pl} H} \ll E_m)$

do not create problems: the anisotropy quickly goes to zero and one recovers the GR behavior

Example in the figure: $f_2 = 10^{-8}$, $f_0 \approx 1.6 \cdot 10^{-5}$, $\phi(0) \approx 5.5\bar{M}_{\rm Pl}$ and $\sqrt{\pi_\phi(0)} \approx 7.1 \cdot 10^{-6}\bar{M}_{\rm Pl}$



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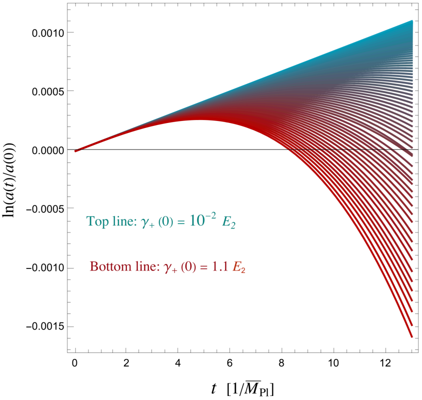
The patches where those conditions are *not* satisfied quickly collapse:

The scale factor in the Jordan frame shrinks as shown in the figure →

Example in the figure: $\gamma_-(0) = 10^{-1}E_2$, $\delta_\pm(0) = 0$, $\epsilon_\pm(0) = 0$, $f_2 = 10^{-8}$, $f_0 \approx 1.6 \cdot 10^{-5}$, $R(0) \approx 1.3 \cdot 10^2 f_0^2 \bar{M}_{\rm Pl}^2$ and $H(0) = 1.2E_2$.



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Check that ***all*** linear modes around dS are bounded (for a fixed initial condition) for any wave number q

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Scalar modes: they are like in GR plus an gravity-isocurvature mode:

$$g_B(\eta, q) \equiv \frac{H}{\sqrt{2q}} \left(\frac{3}{q^2} + \frac{3i\eta}{q} - \eta^2 \right) e^{-iq\eta} + \mathcal{R} - \text{term}$$

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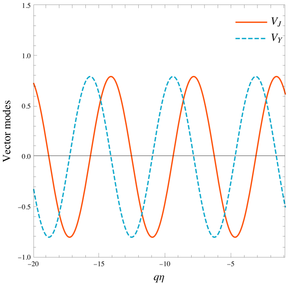
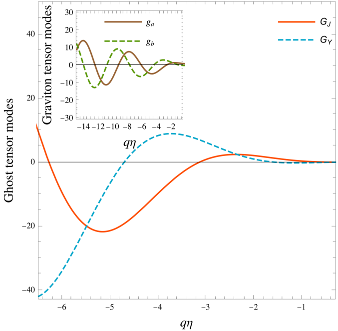
where η is the conformal time ($a^2 d\eta^2 = dt^2$, $\eta < 0$)

Vector and tensor modes:





Observational consequences

Observational consequences: $M_2 > H$

No differences compared to GR

Observational consequences: $M_2 < H$

The modifications:

- r gets suppressed

$$r \to \frac{r}{1 + \frac{2H^2}{M_2^2}}$$

  models that are excluded for a large r (e.g. quadratic inflation) become viable
- There is an isocurvature mode (which fullfils the observational bounds) corresponding to the scalar component of the spin-2 ghost (the vector components and the other tensor component decay with time)

Indeed,

- $P_{\mathcal{R}}$ is not changed by the ghost (so n_s is not changed either)
- while the tensor power spectrum is modified:

$$P_t \to \frac{P_t}{1 + \frac{2H^2}{M_2^2}}$$

- The isocurvature power spectrum P_B is the same as the tensor power spectrum in Einstein's gravity, except that it is smaller by a factor of $3/16 \approx 1/5$:

$$P_B = \frac{3}{2\bar{M}_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2$$

  and the correlation $P_{\mathcal{R}B}$ is highly suppressed

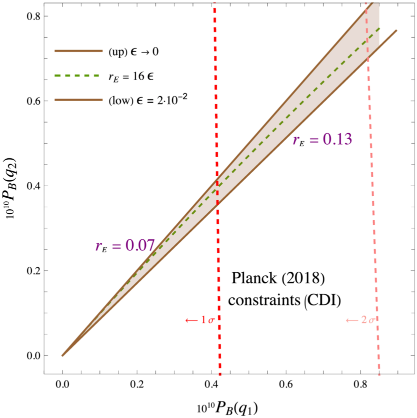
[Ivanov, Tokareva (2016)], [Salvio (2017)]

Ghost-isocurvature power spectrum ($M_2 < H$)

$q_1 = 0.002\ \mathrm{Mpc}^{-1}$ and $q_2 = 0.1\ \mathrm{Mpc}^{-1}$.

The strongest constraints from *Planck (2018)* have been taken

[Salvio (2017)], *[Salvio (2019)]*



Conclusions

- QG is renormalizable and solves the hierachy problem
- The price to pay: a ghost.
- We have provided a possible way of quantizing the theory
- The runaway solutions can be avoided even at energies exceeding the ghost mass
- Quadratic gravity (combined with Higgs mass naturalness) leads to testable predictions for the inflationary observables

Thank you very much for your attention!

Weinberg (2018): *"try crazy ideas ... something will come up"*

Extra slides

Trading negative energies with negative norm

Diagonalization of the Hamiltonian

For $V = 0$ the Hamiltonian is

$$H = \omega_1 \left(-\tilde{a}_1^\dagger \tilde{a}_1 + \frac{1}{2} \right) + \omega_2 \left(\tilde{a}_2^\dagger \tilde{a}_2 + \frac{1}{2} \right)$$

We have $\quad [\tilde{a}_1, \tilde{a}_1^\dagger] = -1, \qquad [\tilde{a}_2, \tilde{a}_2^\dagger] = 1, \qquad$ (all other commutators vanish)

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Onset of “negative norms”

As usual $[a_1, N_1] = a_1$ and $[a_2, N_2] = a_2$ by defining $N_1 \equiv -\tilde{a}_1^\dagger \tilde{a}_1$ and $N_2 \equiv \tilde{a}_2^\dagger \tilde{a}_2$
The spectrum of N_1 is bounded from below if you introduce an indefinite metric:

$$-\nu_n n = \langle n | a_1^\dagger a_1 | n \rangle$$

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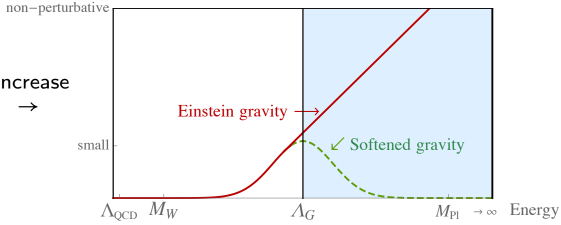
$$-\nu_n n = \langle n | a_1^\dagger a_1 | n \rangle = |c|^2 \langle n-1 | n-1 \rangle = |c|^2 \nu_{n-1}$$

Quadratic gravity is a realization of "softened gravity"

(Einstein) gravitational interactions increase with energy

Idea (*softened gravity*):

consider theories where the increase of the gravitational coupling $\rightarrow$ *stops at some* $\Lambda_G \ll M_{\rm Pl}$.



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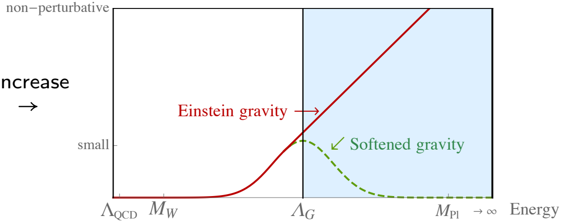

One can then have the gravitational contribution to the Higgs mass under control:

$$\delta M_h^2 \approx \frac{G_N \Lambda_G^4}{(4\pi)^2}$$

Requiring naturalness → $\Lambda_G \lesssim 10^{11}$ GeV *[Giudice, Isidori, Salvio, Strumia (2014)]*

In quadratic gravity $\Lambda_G \sim M_2$

Classical dynamics: a simple scalar field example

To simplify consider

$$\mathscr{L} = -\frac{1}{2}\phi\Box\phi - \frac{c_4}{2}\phi\Box^2\phi - V(\phi)$$

It is a toy version of out theory:

- $-\frac{1}{2}\phi\Box\phi$ represents the Einstein-Hilbert part
- $-\frac{c_4}{2}\phi\Box^2\phi$ represents the quadratic terms
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Two-derivative form

Add $\frac{c_4}{2}\left(\Box\phi - \frac{A-\phi/2}{c_4}\right)^2$ (vanishes when the EOM of the auxiliary field A are used)

$$\Longrightarrow \mathscr{L} = -\frac{1}{2}\phi_+\Box\phi_+ + \frac{1}{2}\phi_-\Box\phi_- + \frac{m^2}{2}\phi_-^2 - V(\phi_+ + \phi_-)$$

where $m^2 \equiv 1/c_4$ has to be taken positive to avoid tachyonics.

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The EOMs are

$$\Box\phi_+ = -V'(\phi_+ + \phi_-), \qquad \Box\phi_- = -m^2\phi_-^2 + V'(\phi_+ + \phi_-).$$

For definiteness take $V(\phi) = \lambda\phi^4/4$, where $\lambda > 0$, which stabilizes ϕ_+, while ϕ_- feels

$$v(\varphi) = \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4, \qquad \varphi = \text{typical order of magnitude of field values}$$

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Ghost metastability

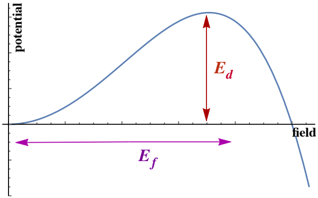
For

$$\varphi \ll E_f \equiv \frac{m}{\sqrt{\lambda/2}}$$

and

$$E \ll E_d \equiv \frac{m}{(4\lambda)^{1/4}}$$

(where E is the energy associated with the field derivatives)
the runaway solutions are avoided



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Example in the figure: $\lambda = 10^{-2}$, $\phi_+(0) = 10^{-2}E_f$, $\phi_-(0) = 10^{-2}E_f$, $\dot\phi_+(0) = (1.5 \cdot 10^{-1}E_d)^2$ and $\dot\phi_-(0) = -(10^{-2}E_d)^2$.