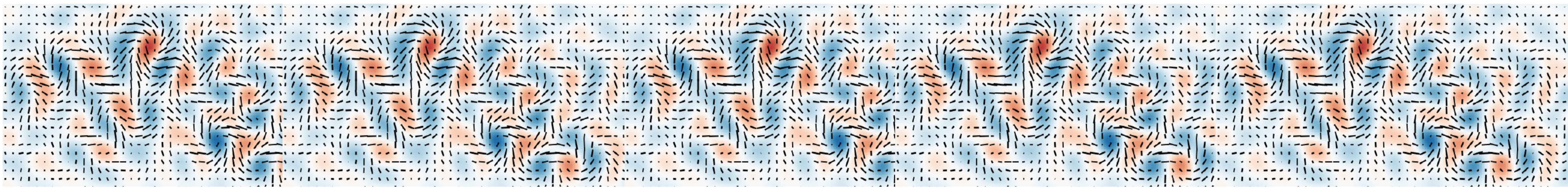

Reconstructing the B-mode power spectrum using simulated CMB maps

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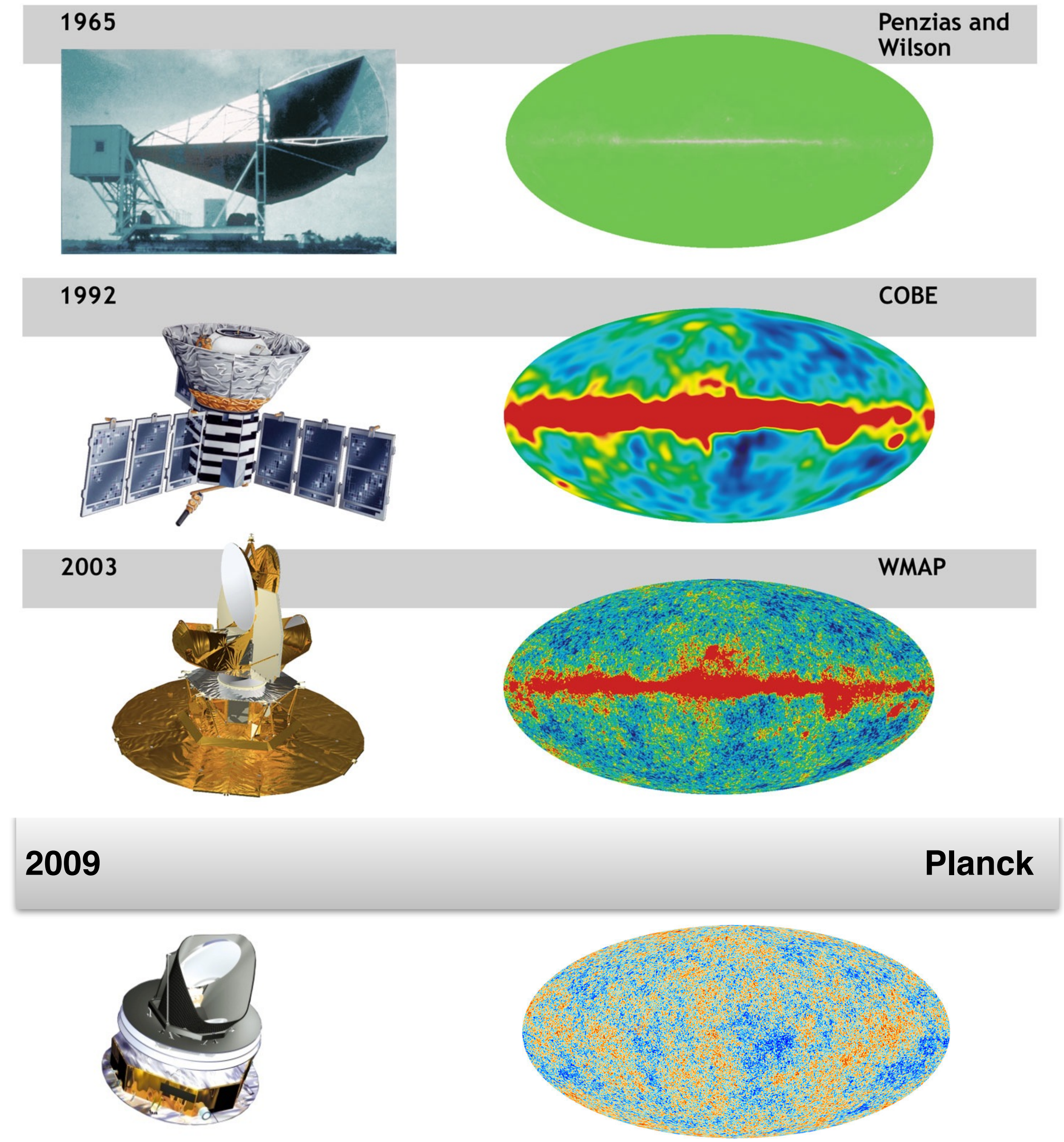
Outline

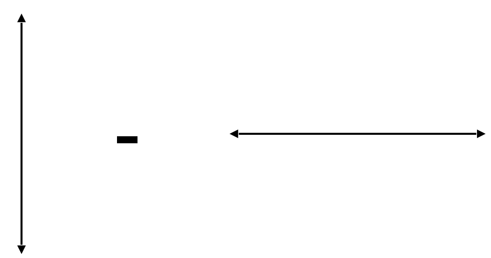
- Introduction
  - The cosmic microwave background, CMB
  - The polarized signal
  - The B-map
- Motivations
- Method
- Results on the temperature power spectrum
- Preliminary results on the B-mode power spectrum

CMB: A brief introduction

- Discovered in1965 by Penzias and Wilson
- Temperature anisotropies measured by COBE in 1992
- Improved measurements by WMAP and Planck

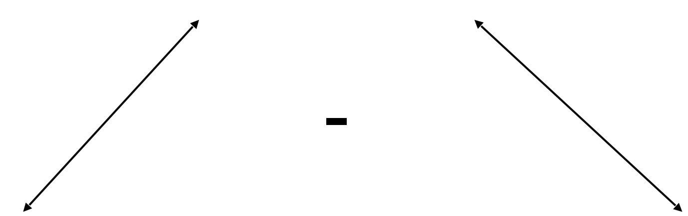


The polarization signal



Q

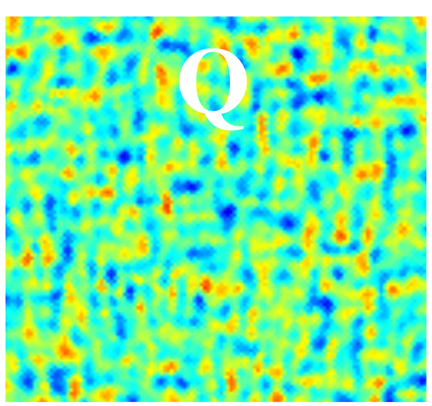
$Q \rightarrow -Q$, $U \rightarrow -U$ under 90 degree rotation

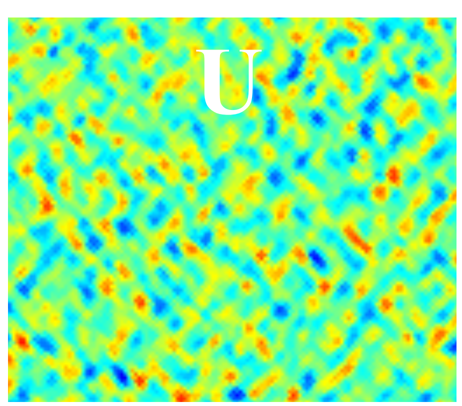


U

$Q \rightarrow U$, $U \rightarrow -Q$ under 45 degree rotation

- By rotating the local coordinate system, Q is rotated into U and vice-versa.





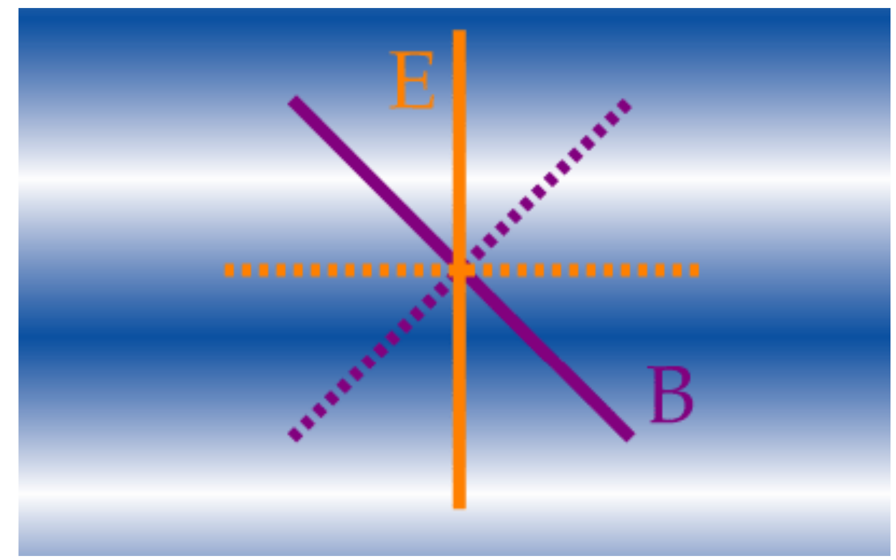
The linearly polarized CMB is completely described by a spin 2 and spin -2 field

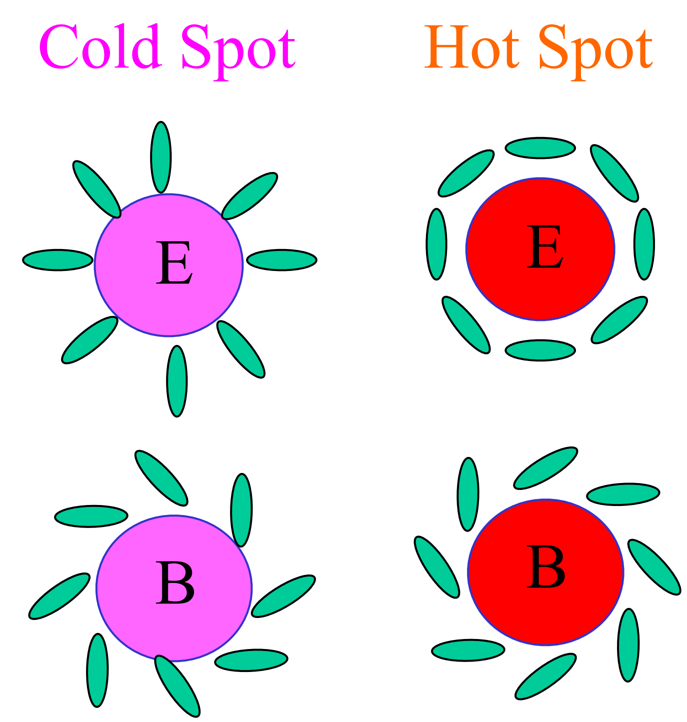
$$P_{\pm 2}(\vec{n}) = Q(\vec{n}) \pm iU(\vec{n})$$

E/B decomposition

- The description North-south and East-West (more formally, the stokes parameters) depends on the arbitrary choices of coordinates

- We then describe the polarization by its orientation relative to itself: E-mode and B-mode





Spherical harmonics expansion

$$P_{\pm}(\vec{n}) = \sum_{lm} a_{\pm 2,lm} \; {}_{\pm 2}Y_{lm}(\vec{n})$$

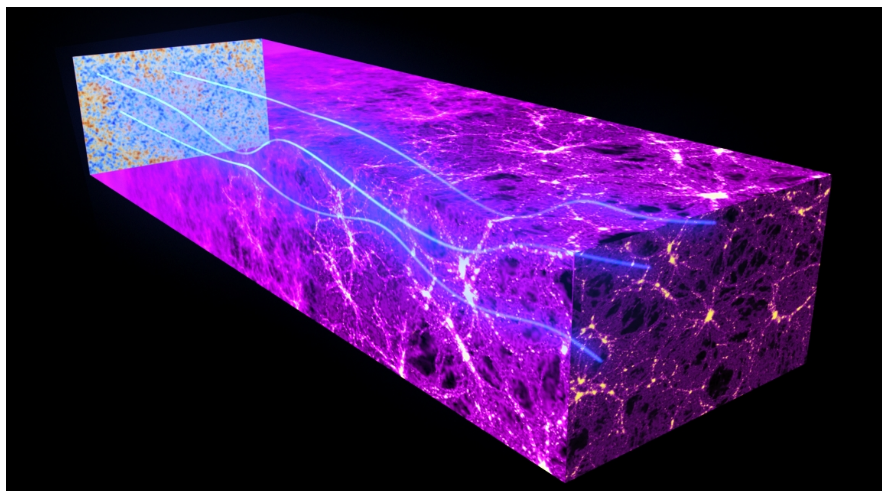
$$E_{lm} \equiv -\frac{1}{2}\left[a_{2,lm} + a_{-2,lm}\right]$$

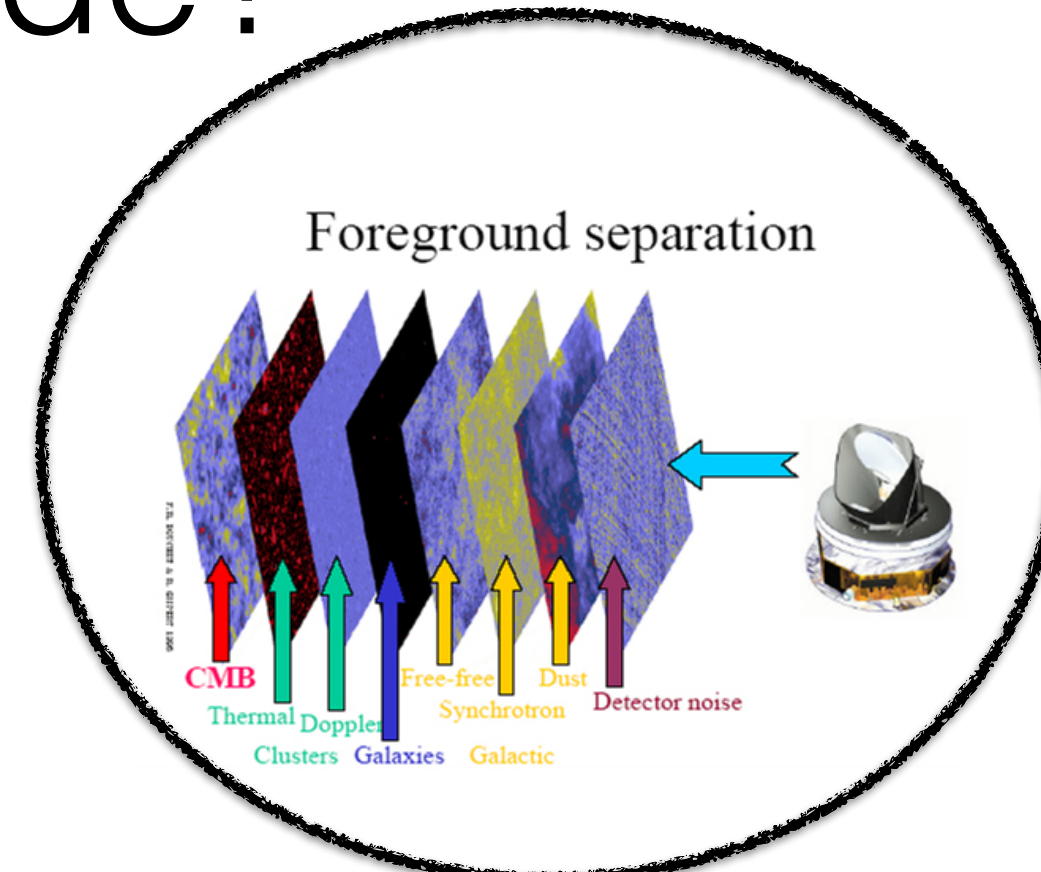
$$B_{lm} \equiv -\frac{1}{2i}\left[a_{2,lm} - a_{-2,lm}\right]$$

Our target

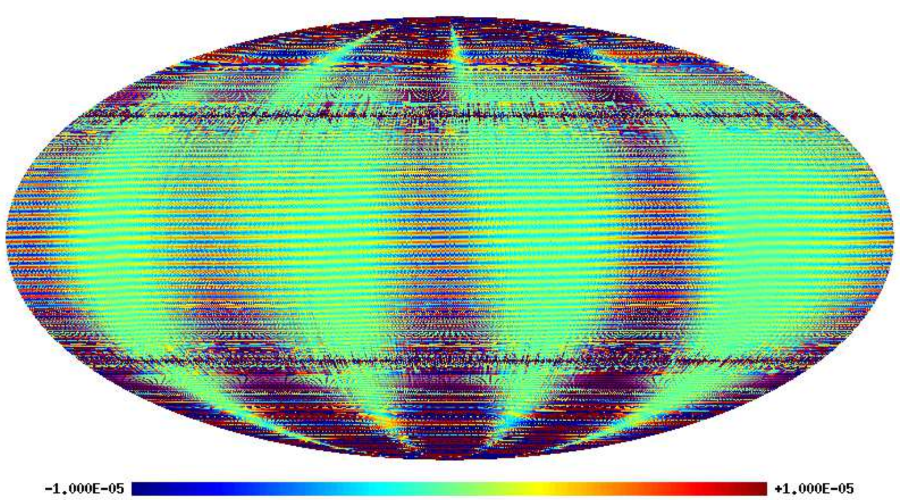
But, what can generate B-mode?

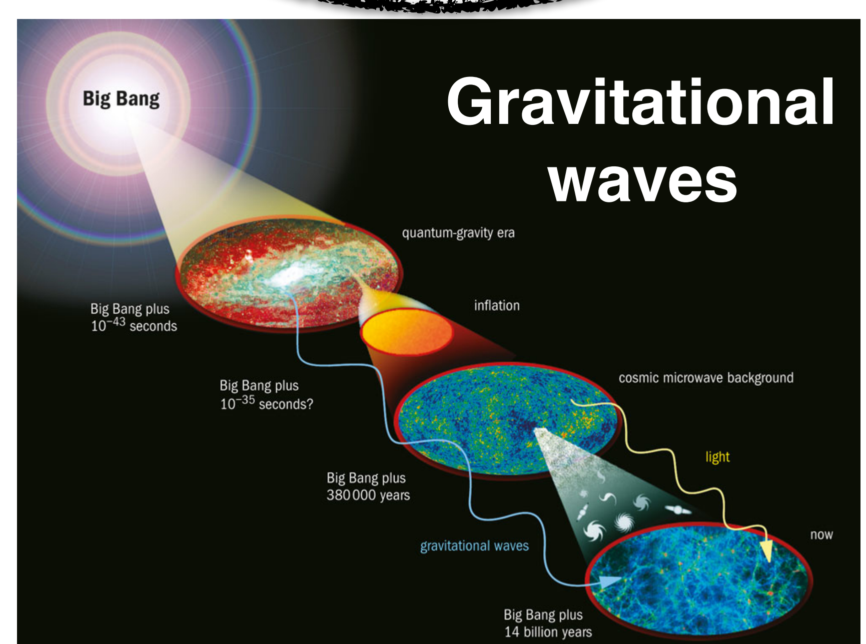
CMB lensing





E-to-B leakage





Motivations

- The accuracy of the CMB power spectra measurements is limited by several foregrounds
- Frequency dependence of foreground components is poorly known, leading to unwanted foreground residuals in the final cleaned map
- Blind foreground separation algorithms make no prior assumption on foregrounds and have been extensively addressed in literature
- However, most of the methods are computationally challenging.

- We use the analytical method of blind separation (ABS) presented recently by Zhang et al (2016)
    - The ABS method is based on the measured cross band power between different frequency bands and do not rely on any assumption about the foreground components
    - The method allows a fast numerical reconstruction of the CMB power spectra
- Here, we present our first results for the polarized signal using the ABS method of foreground separation.

The ABS method (Zhang, P. et al, 2016)

- The ABS approach blindly and analytically subtracts the foregrounds and recover the CMB signal in the spherical harmonic domain

- The measured data can be written as :

$$\mathcal{D}_{ij}^{\mathrm{obs}}(\ell) = f_i f_j \mathcal{D}^{\mathrm{cmb}}(\ell) + \mathcal{D}_{ij}^{\mathrm{fore}}(\ell) + \delta\mathcal{D}_{ij}^{\mathrm{noise}}(\ell)$$

- We can recover the CMB power spectrum by

Case without noise

$$\mathcal{D}^{\rm cmb} = \left(\sum_{\mu=1}^{M+1} G_\mu^2 \lambda_\mu^{-1} \right)^{-1}$$

$$G_\mu = \mathbf{f} \cdot \mathbf{E}^\mu$$

$$\mathbf{E}^\mu \cdot \mathbf{E}^\nu = \delta_{\mu\nu}.$$

Case with noise

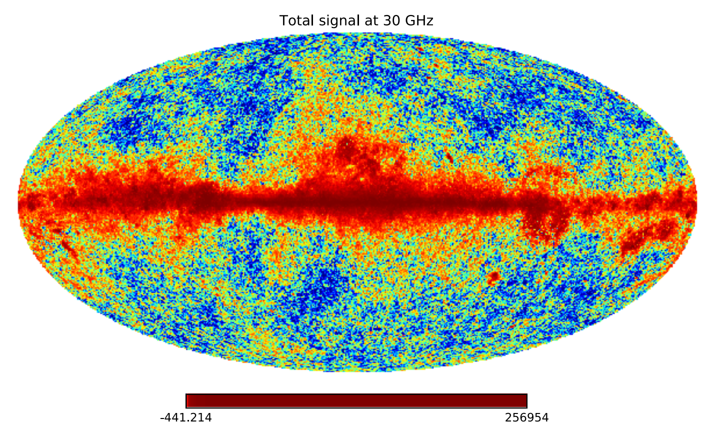
$$\hat{\mathcal{D}}^{\rm cmb} = \left(\sum^{\tilde{\lambda}_\mu \geq \lambda_{\rm cut}} \tilde{G}_\mu^2 \tilde{\lambda}_\mu^{-1} \right)^{-1} - \mathcal{S}$$

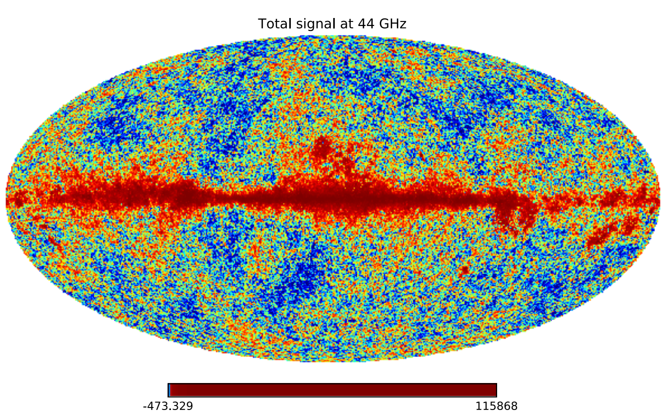
$$\tilde{\mathcal{D}}_{ij}^{\rm obs} \equiv \frac{\mathcal{D}_{ij}^{\rm obs}}{\sqrt{\sigma_{\mathcal{D},i}^{\rm noise} \sigma_{\mathcal{D},j}^{\rm noise}}} + \tilde{f}_i \tilde{f}_j \mathcal{S} \,,$$

$$\tilde{f}_i \equiv \frac{f_i}{\sqrt{\sigma_{\mathcal{D},i}^{\rm noise}}} \,, \quad \tilde{G}_\mu \equiv \tilde{\mathbf{f}} \cdot \tilde{\mathbf{E}}^\mu \,,$$

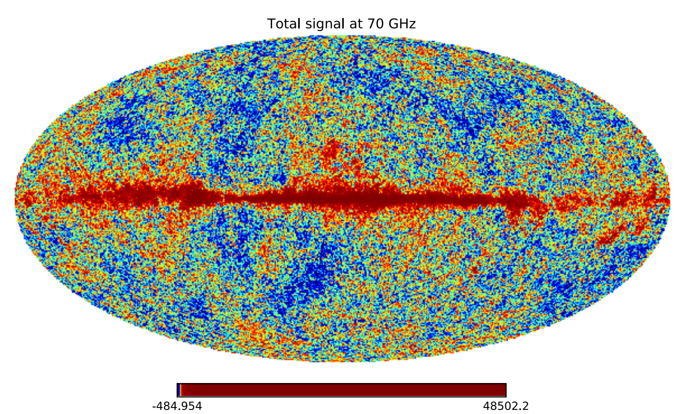
$$\mathbf{f} = (f_1, \ldots, f_{N_f})^T.$$

Let’s go back one step: CMB temperature maps

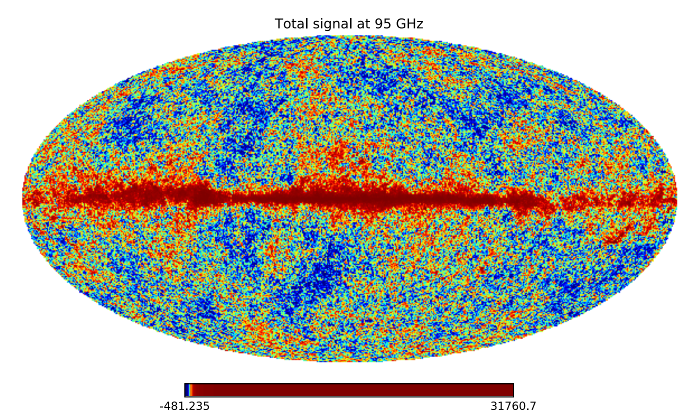




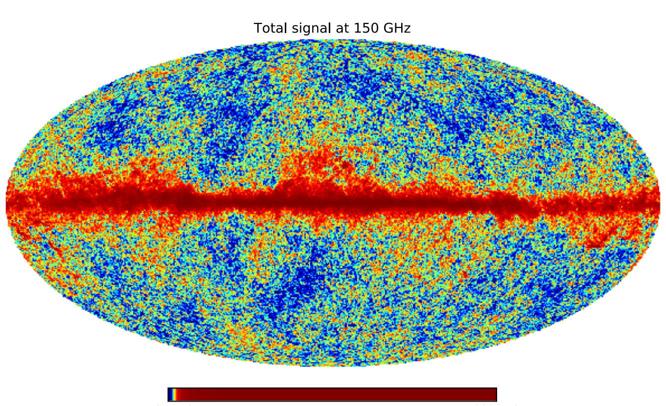
(b) 44GHz



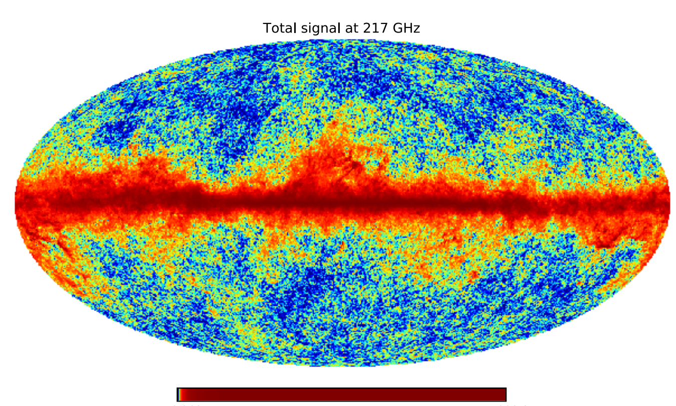
(c) 70GHz



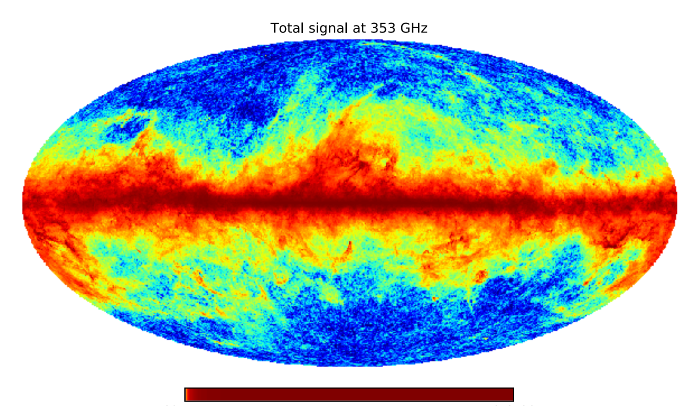
(d) 95GHz



(e) 150GHz



(f) 217GHz



(g) 353GHz

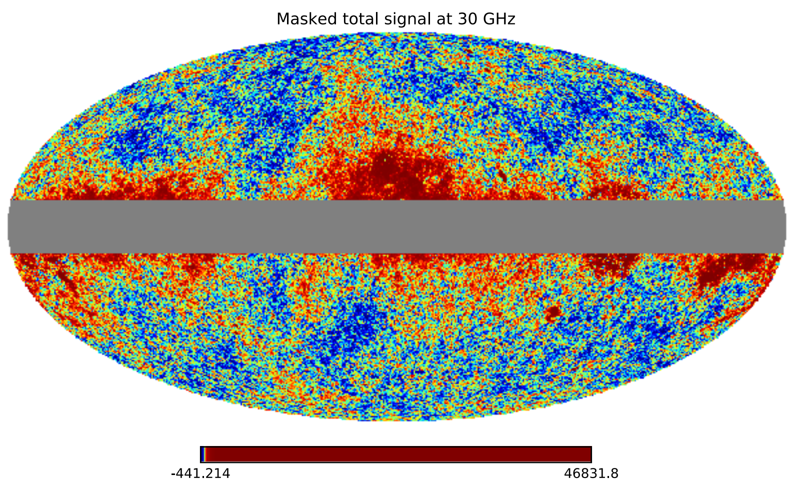
(Yao, J. et al 2018: arXiv 1807.07016)

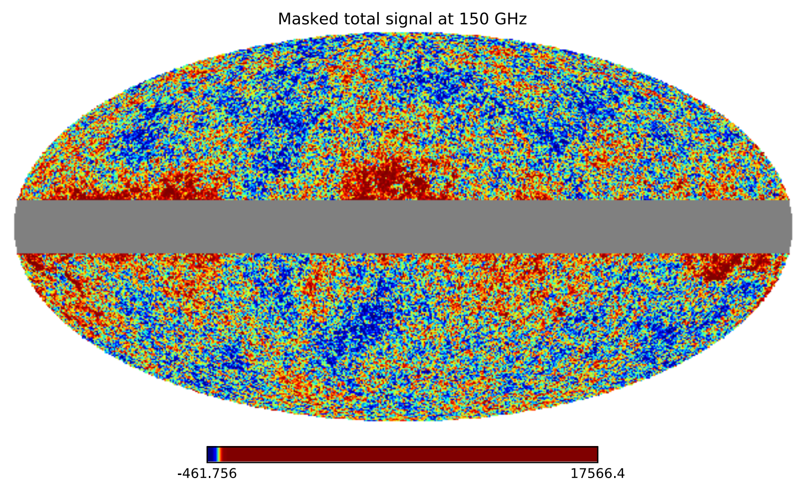
Recovered temperature power spectrum

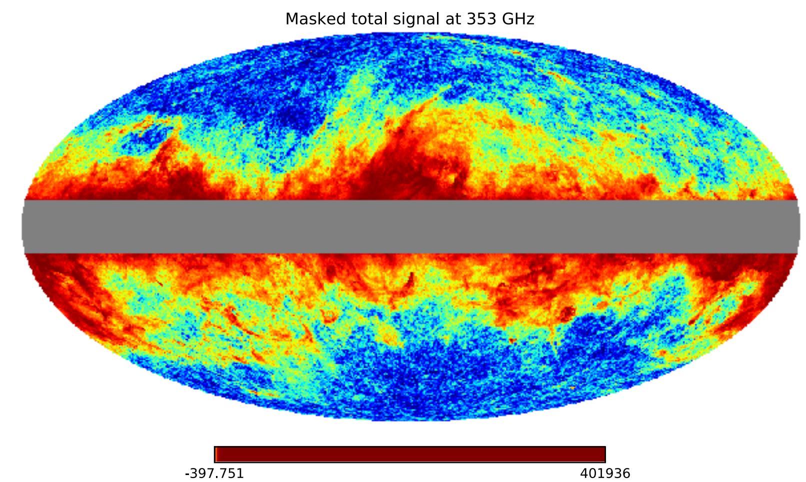
- For temperature we consider 4 different foregrounds
  - synchrotron, free-free, thermal dust and anomalous microwave emission
- For simplicity, in this case, we do not apply any primary beam pattern for any frequency channel
- The Gaussian noise level is assumed to be of the order of that of Planck

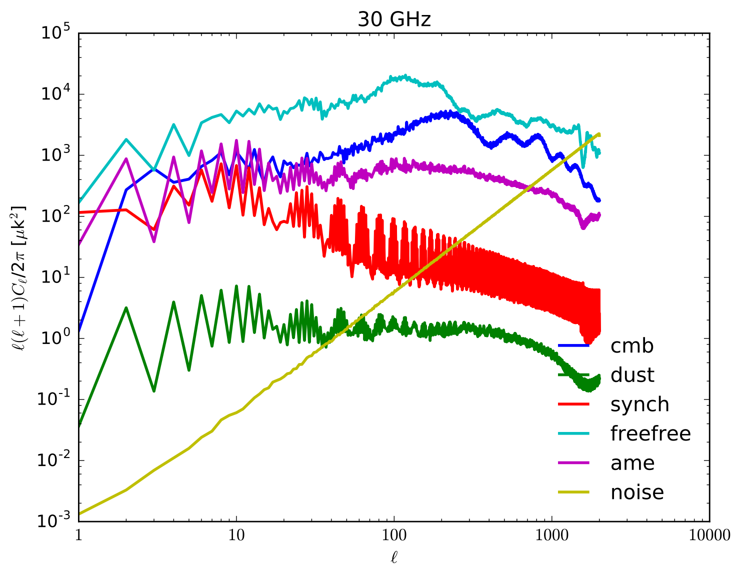
Channel	30 GHz	44 GHz	70 GHz	100 GHz	143 GHz	217 GHz	353 GHz
FWHM [arcmin]	33	24	14	10	7.1	5	5
$\sigma_{\rm hit}$ [$\mu K_{\rm RJ}$]	1030	1430.	2380	1250	754	610	425
$\sigma_{\rm hit}$ [$\mu K_{\rm CMB}$]	1050	1510	2700	1600	1250	1820	5470
Mean; Median hits per pixel	82; 64	170; 134	579; 455	1010; 790	2260; 1790	2010; 1580	2010; 1580
$N_\ell^{1/2}$ [$\mu K_{\rm CMB}$]	0.066	0.065	0.063	0.028	0.015	0.023	0.068

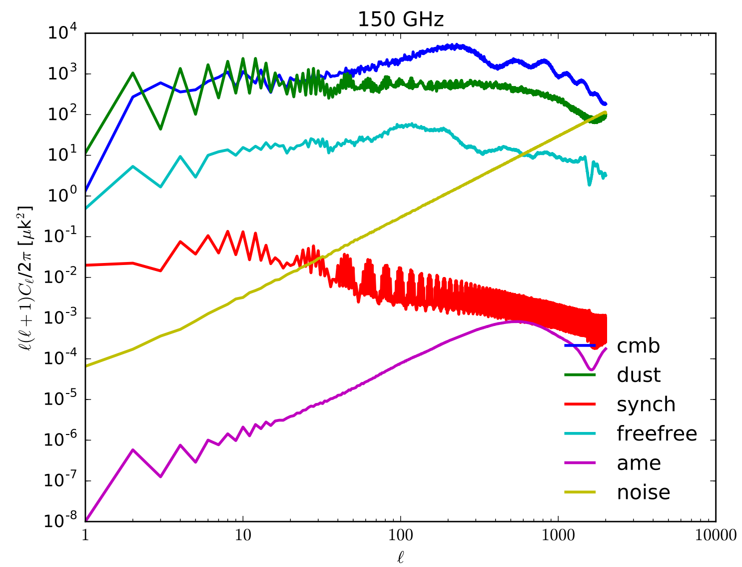
Power spectra components outside the mask

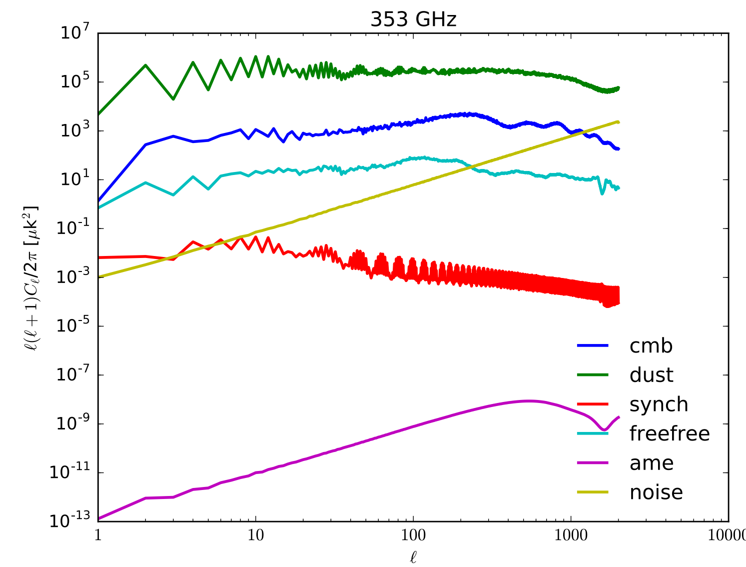




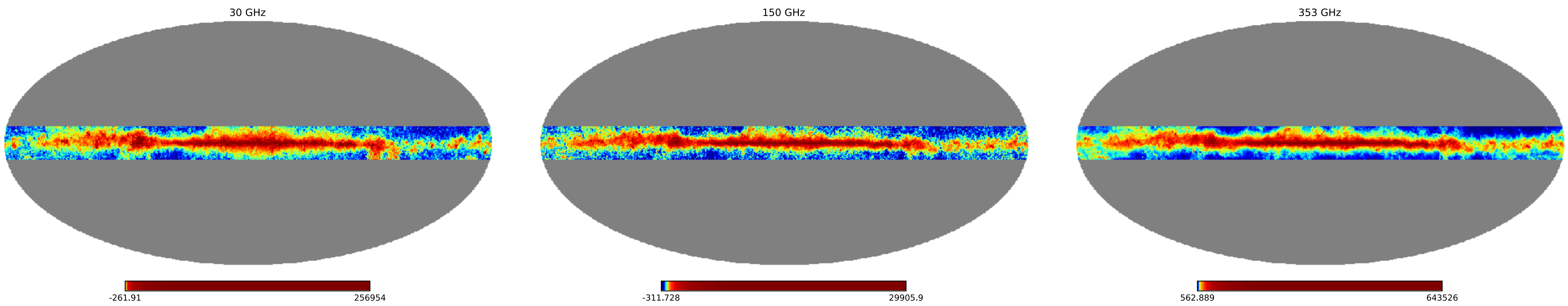


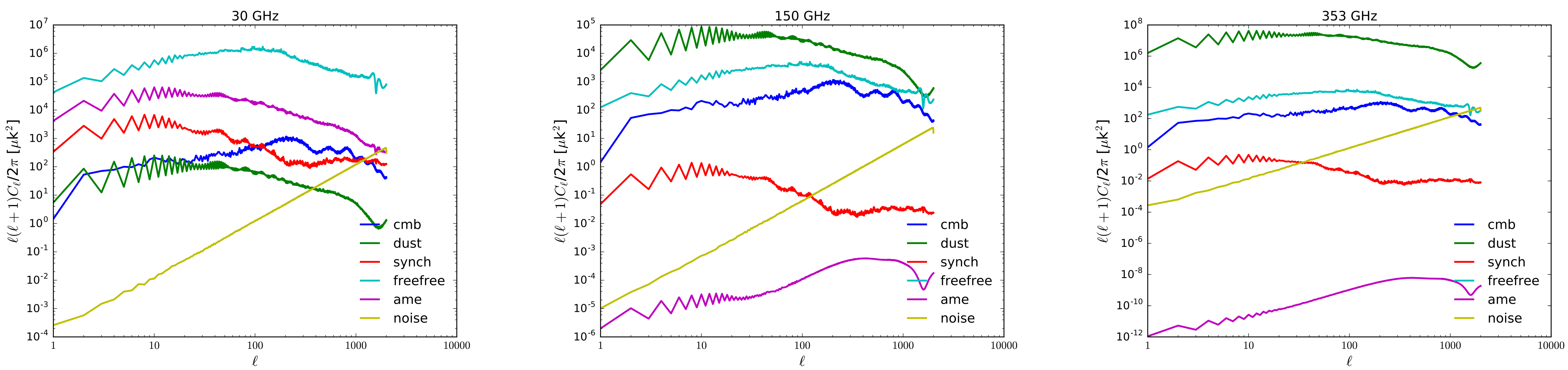




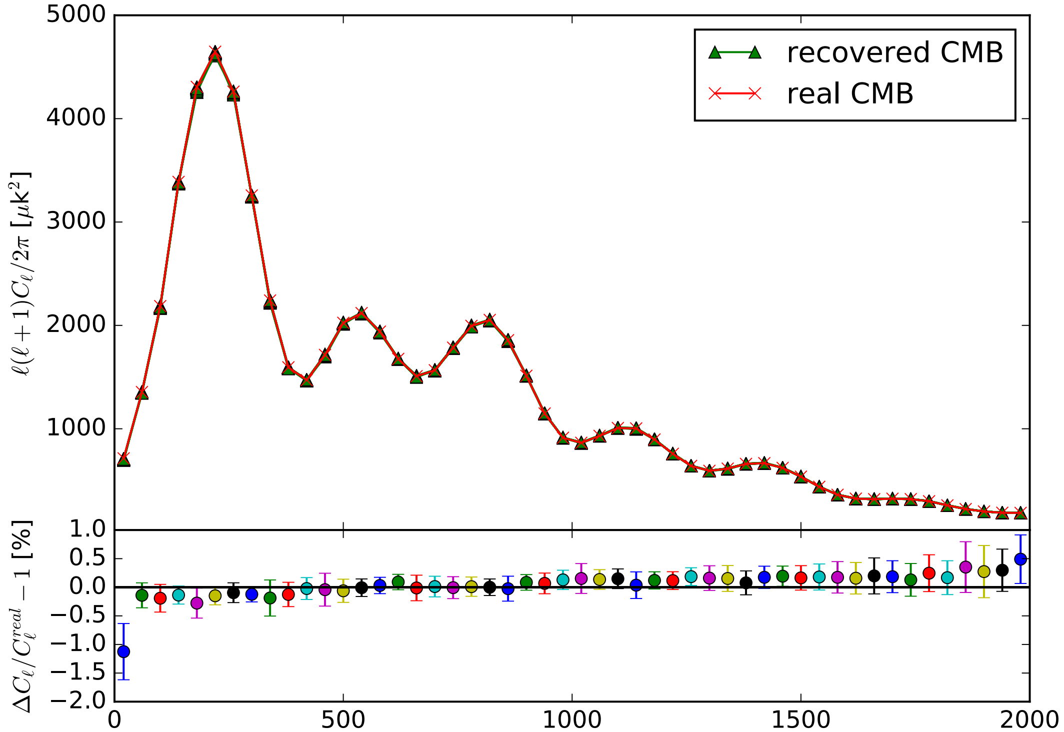


Power spectra components inside the mask

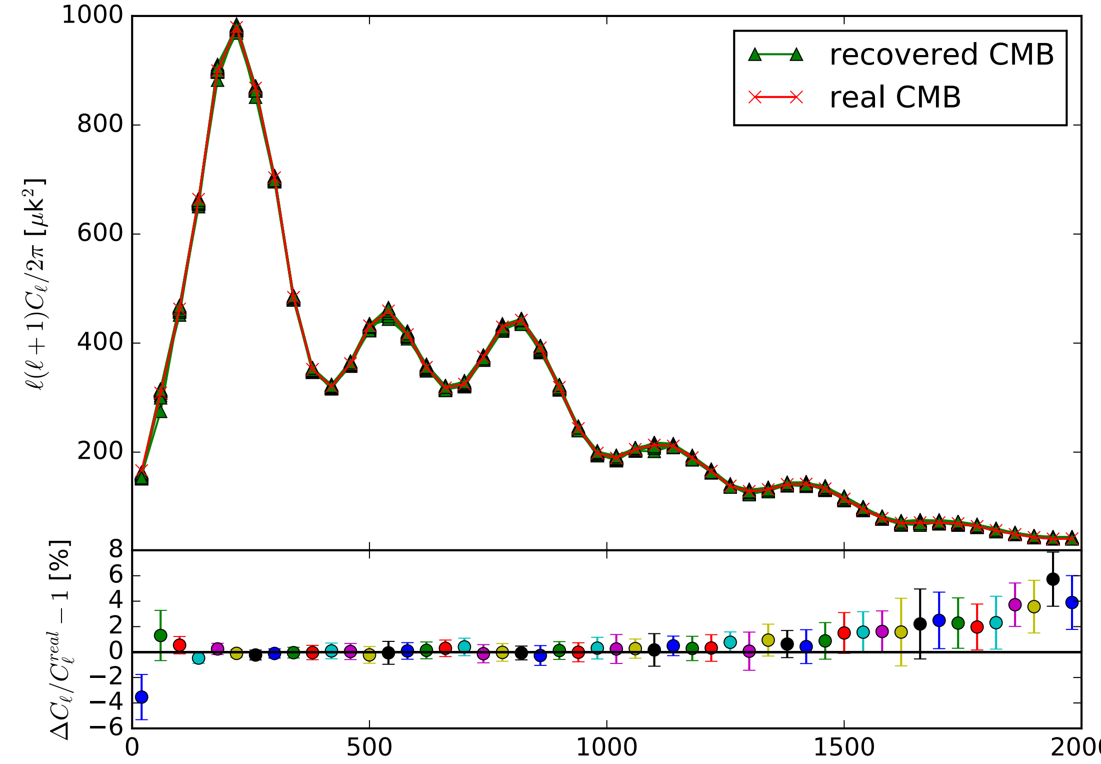




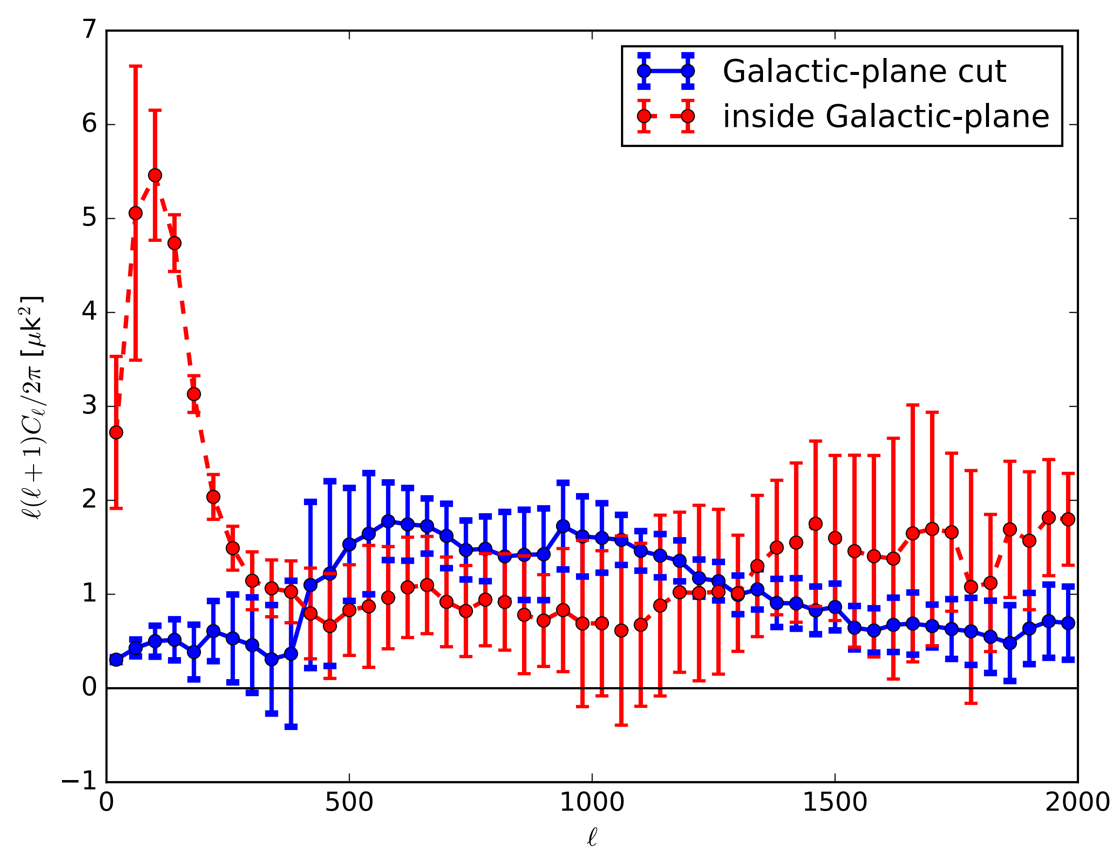
Outside the mask |b|<10



Inside the mask |b|<10



Null test



Conclusions

- The ABS estimator does not rely on any assumptions of foreground components, and it was applied to simulated multi-frequency Planck maps at 30, 44, 70, 95, 150, 217 and 353 GHz.

- The ABS estimator provide an unbiased and efficient estimate of underlying the CMB power spectrum well within 1-σ error bar at most scales

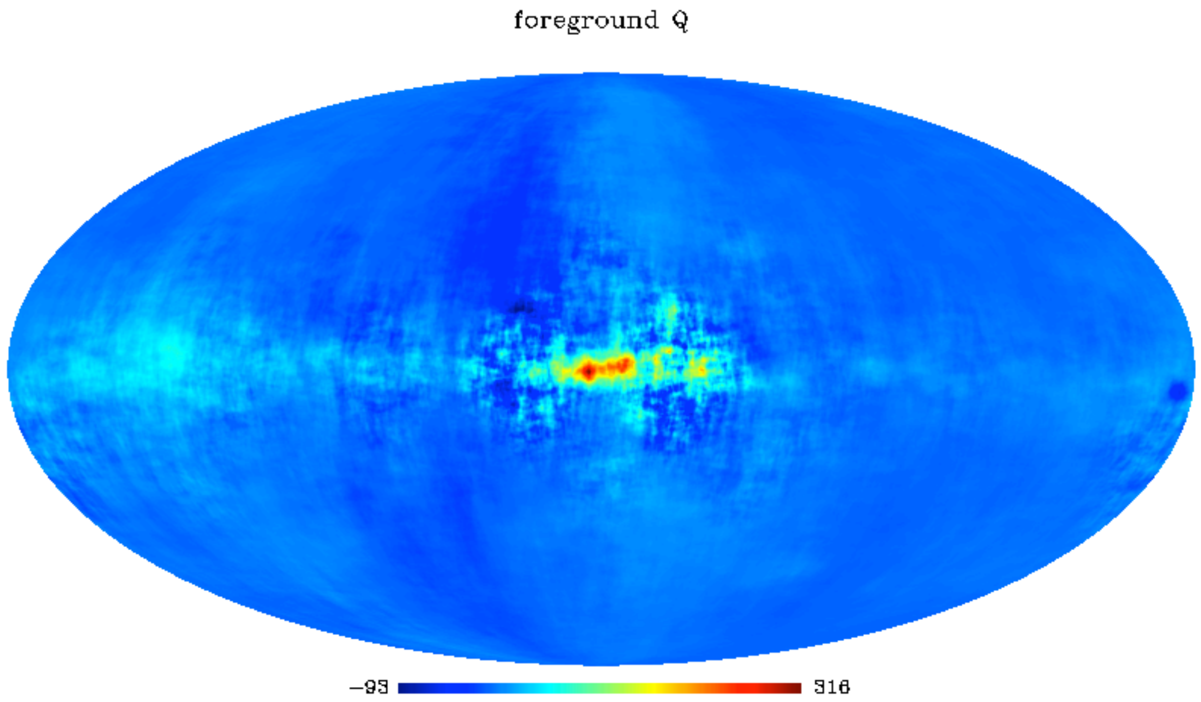
- When choosing a Galactic mask excluding the region $|b| < 10$, the CMB power spectrum is recovered with an accuracy at the level of less than 0.5% over all scales.

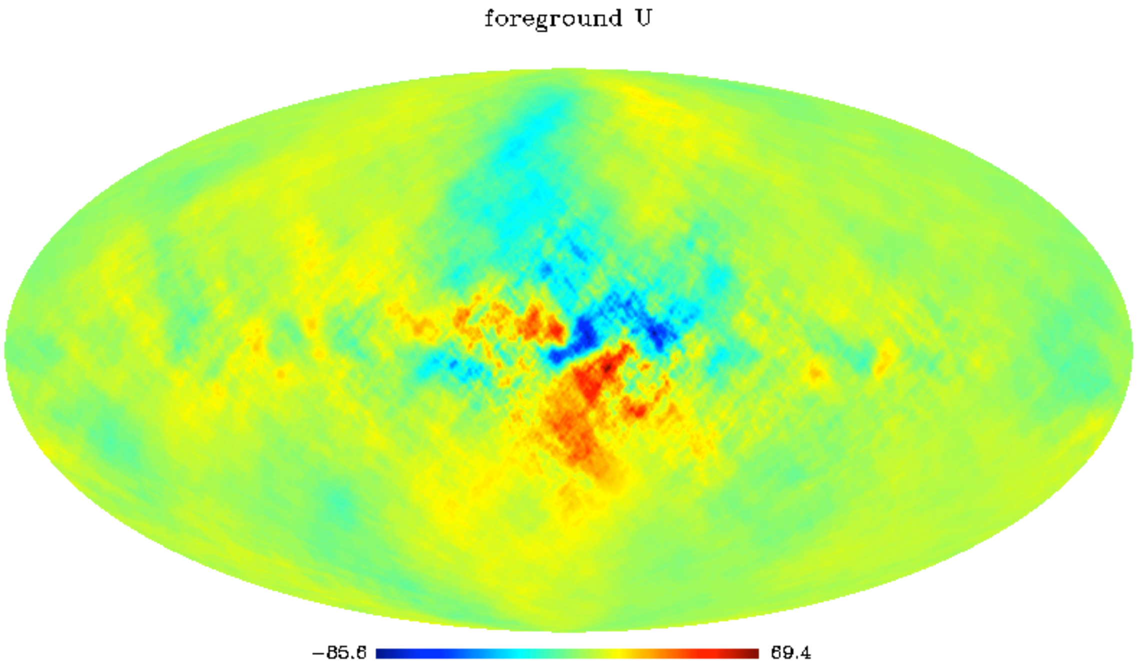
The polarization simulations

Hypothetical future experiment with10 frequency channels: 30, 43, 75, 90, 108, 129, 100, 155, 223, 268, 321 GHz

- **CMB**: We used CAMB to generate the lensed CMB power spectra
  - Planck best fit parameters for the LCDM model for r=0 and r=0.05
- These power spectra are the input of lenspix/healpix to produce T, Q and U maps with nside=1024

- **Foregrounds**: Synchrotron and thermal dust
    - Nominal PySM model (Thorne et al. 2017) as the fiducial foreground models

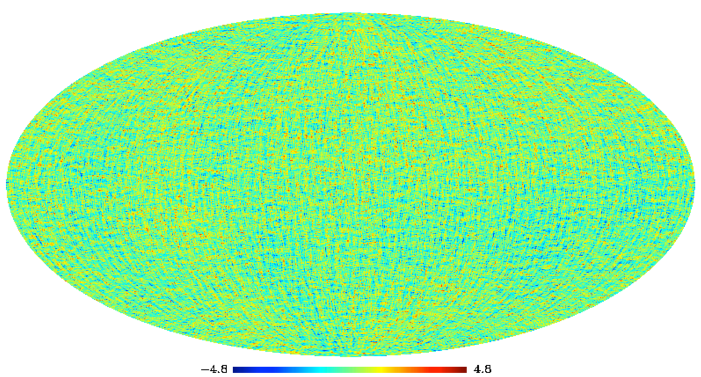




- **Noise**: 50 realizations of uncorrelated Gaussian white noise

- **Total**: CMB + foregrounds + noise_k = 50 maps for each frequency band.

Q:



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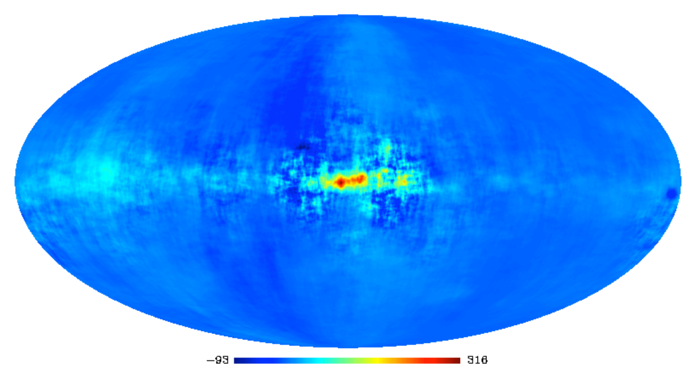

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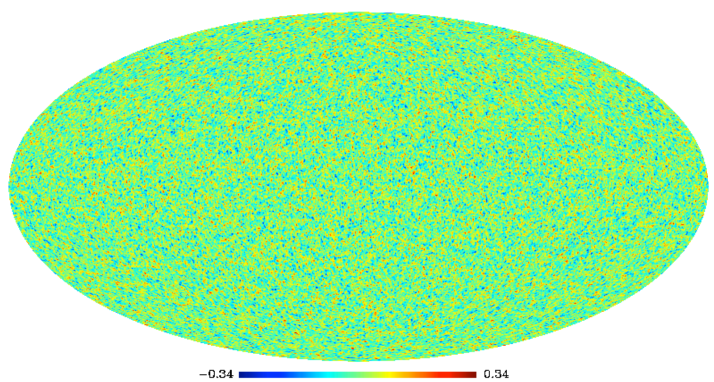
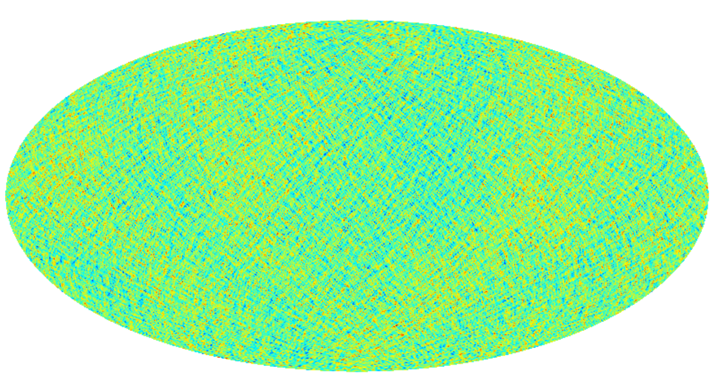


U:



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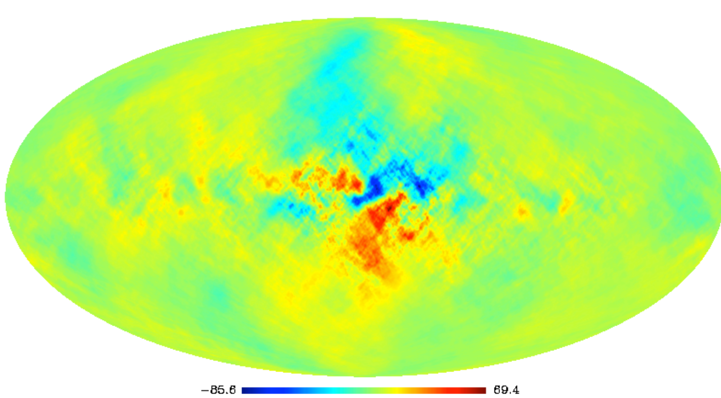

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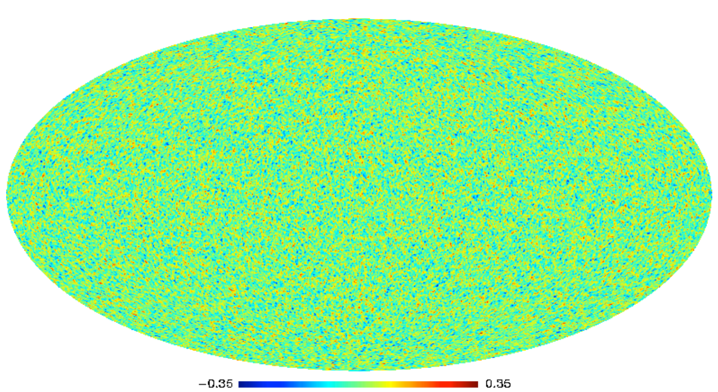
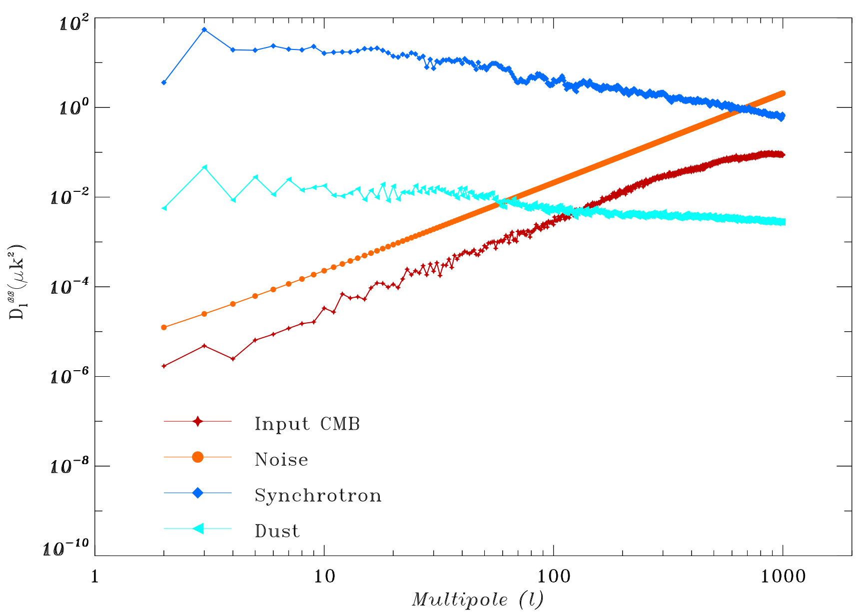


Experimental setup

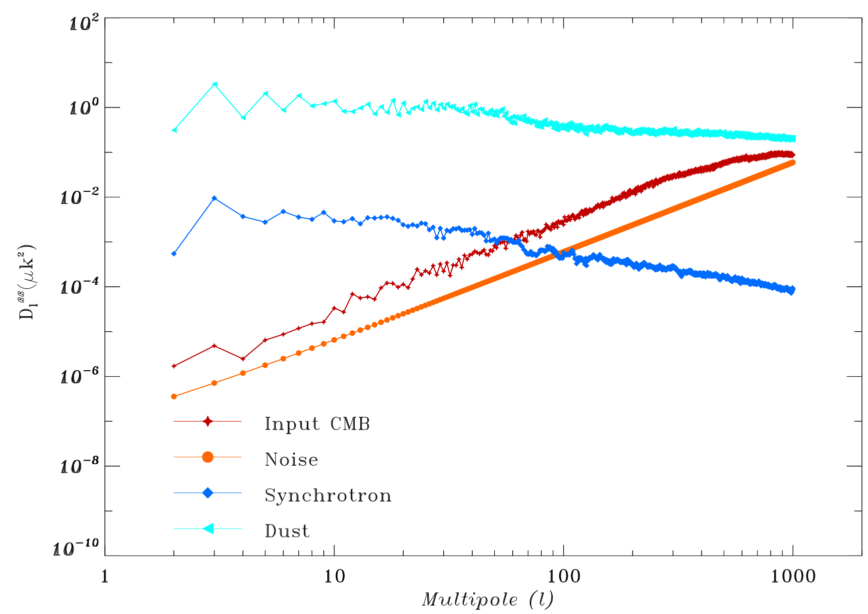
Band center (GHz)	Beam FWHM (arcmin)	noise level (μK_{CMB}- arcmin)
030	28.3	12.4
043	22.2	7.9
075	10.7	4.2
090	9.5	2.8
108	7.9	2.3
129	7.4	2.1
155	6.2	1.8
223	3.6	4.5
268	3.2	3.1
321	2.6	4.2

Polarized power spectra components contribution in full sky

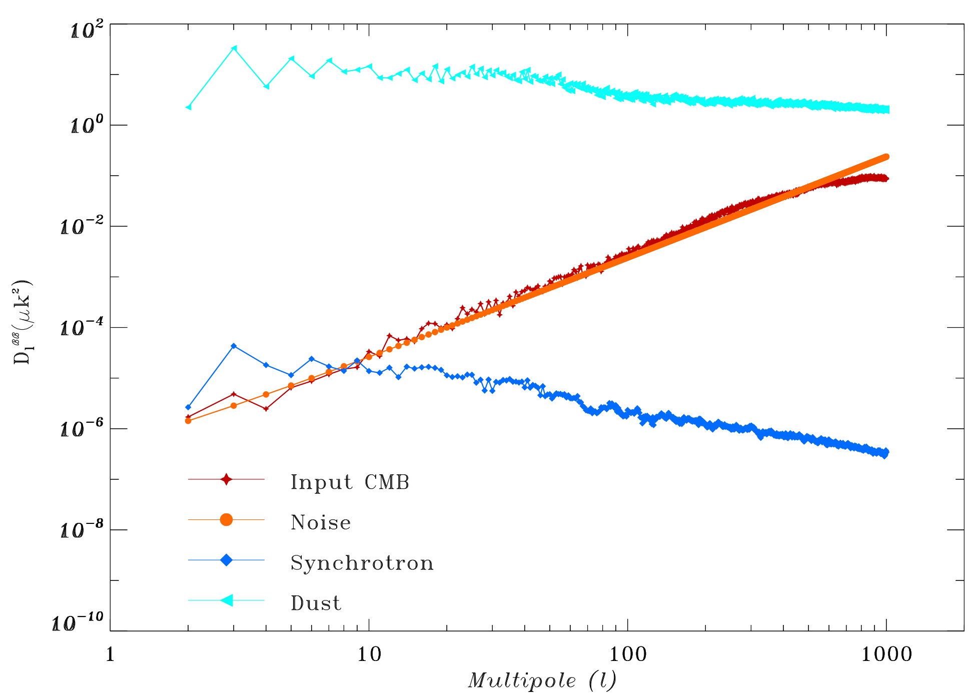
030 GHz



129 GHz



321 GHz



The CMB polarization: A partial sky approach

- In the ideal case (full sky map), we directly derive E and B from Q and U

- However, even for satellite surveys we will not get a full-sky map. Why?

- We must mask out the unavoidable Galactic foreground

E/B decomposition in partial sky

$$\mathcal{B}(\hat{n}) = -\frac{1}{2i}\left[\bar{\eth}\bar{\eth}P_{+}(\hat{n}) - \eth\eth P_{-}(\hat{n})\right]$$

$$\eth f \equiv -\sin^{s}\theta\left(\frac{\partial}{\partial\theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right)(f\sin^{-s}\theta),$$

$$\bar{\eth} f \equiv -\sin^{-s}\theta\left(\frac{\partial}{\partial\theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right)(f\sin^{s}\theta).$$

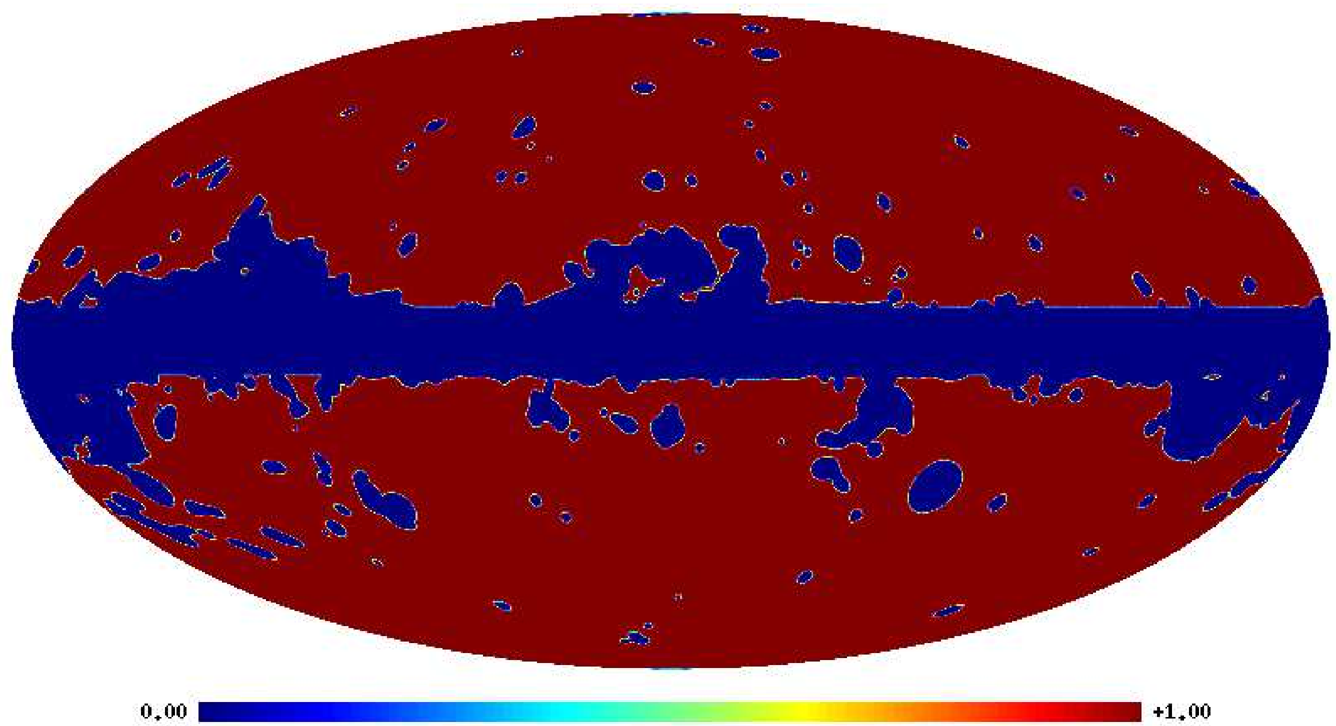
$$\mathcal{B}(\hat{n}) \equiv \sum_{\ell,m}\mathcal{B}_{\ell m}Y_{\ell m}(\hat{n}) \longrightarrow \mathcal{B}_{\ell m} = \int \mathcal{B}(\hat{n})Y^{*}_{\ell m}(\hat{n})d\hat{n} = N_{\ell,2}B_{\ell m}$$

$$N_{\ell,s} = \sqrt{(\ell+s)!/(\ell-s)!}$$

$$\boxed{\mathcal{B}^{\text{pure}}_{lm} \equiv -\frac{1}{2i}\int d\hat{n}\left\{P_{+}(\hat{n})\left[\bar{\eth}\bar{\eth}\left(W(\hat{n})Y_{lm}(\hat{n})\right)\right]^{*} - P_{-}(\hat{n})\left[\eth\eth\left(W(\hat{n})Y_{lm}(\hat{n})\right)\right]^{*}\right\}}$$

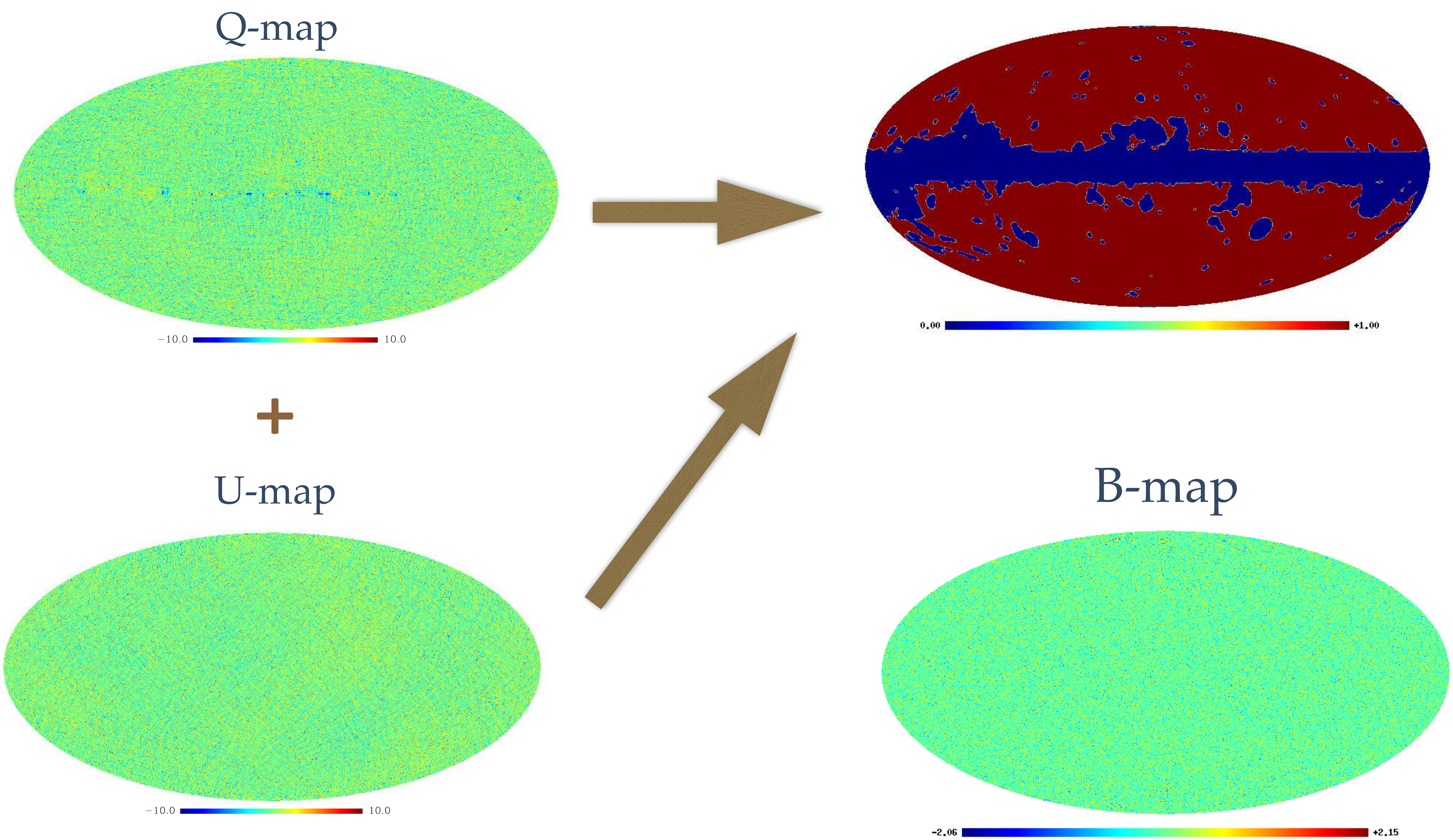
Sky apodization
(Planck polarization mask)

Yi-Fan Wang, Kai Wang , Wen Zhao, arXiv:1511.01220



$$W_i = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\delta_i - \frac{\delta_c}{2}}{\sqrt{2}\sigma} \right) \quad \delta_i < \delta_c \qquad\qquad W_i = 1 \quad \delta_i > \delta_c$$

B-map separation



E/B decomposition numerical method

- SZ approach [Smith, (2006); Smith and Zaldarriaga (2007)]: the most efficient method for estimating the CMB B-mode power spectrum in partial sky [Fertè et al. (2013)]

- Step 1: To compute the spin-0, spin-1 and spin-2 rendition of the window function

$$W_0 = W \qquad W_1 = \partial W \qquad W_2 = \partial\partial W$$

$$\partial W = -\frac{\partial W}{\partial \theta} - \frac{i}{\sin\theta}\frac{\partial W}{\partial \phi}$$

$$\partial\partial W = -\cot\theta\frac{\partial W}{\partial \theta} + \frac{\partial^2 W}{\partial \theta^2} - \frac{1}{\sin^2\theta}\frac{\partial^2 W}{\partial \phi^2} - \frac{2i\cot\theta}{\sin\theta}\frac{\partial W}{\partial \phi} + \frac{2i}{\sin\theta}\frac{\partial^2 W}{\partial\theta\partial\phi}$$

- Step 2: Construct 3 apodized maps

$$P_{\pm 2} = W_0 P_{\pm 2} \qquad P_{\pm 1} = W_{\mp 1} P_{\pm 2} \qquad P_{\pm 0} = W_{\mp 2} P_{\pm 2}$$

- Step 3: Generating the new B_{lm} and finally the B-map

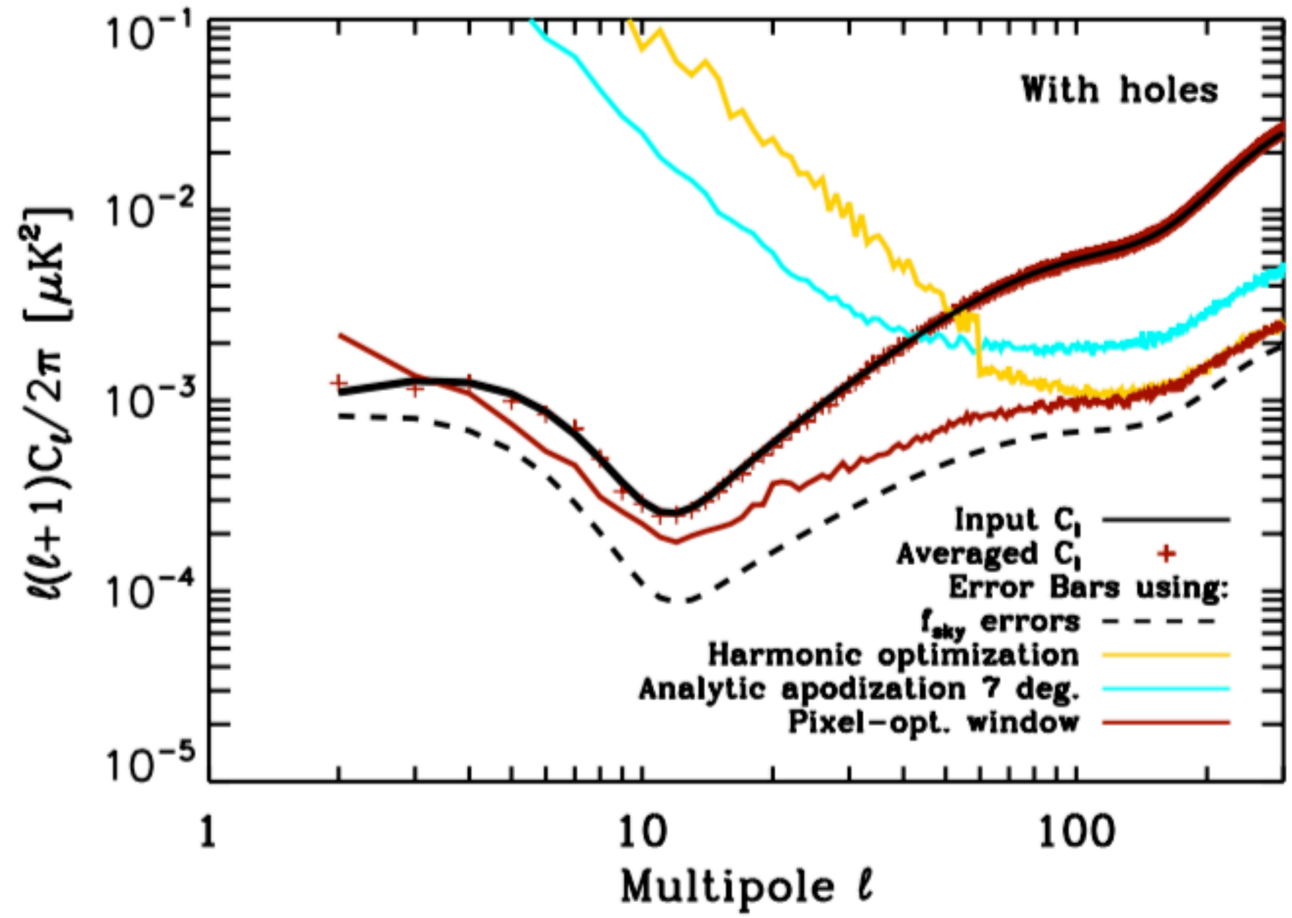
$$\tilde{B}_{lm} = \left(B_{0,lm} + 2N_{l,1}B_{1,lm} + N_{l,2}B_{2,lm}\right)$$

The Pseudo Cls

$$\langle D_\ell \rangle = \sum_{\ell'} M_{\ell\ell'} B^2_{\ell'} C^{\mathcal{BB}}_{\ell'} = \sum_{\ell'} M_{\ell\ell'} N^2_{\ell'} B^2_{\ell'} C^{BB}_{\ell'}.$$

$$M_{\ell_1\ell_2} = (2\ell_2+1)\sum_{\ell_3} \frac{(2\ell_3+1)}{4\pi} w'_{\ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^2$$

Ferte et al (2013)

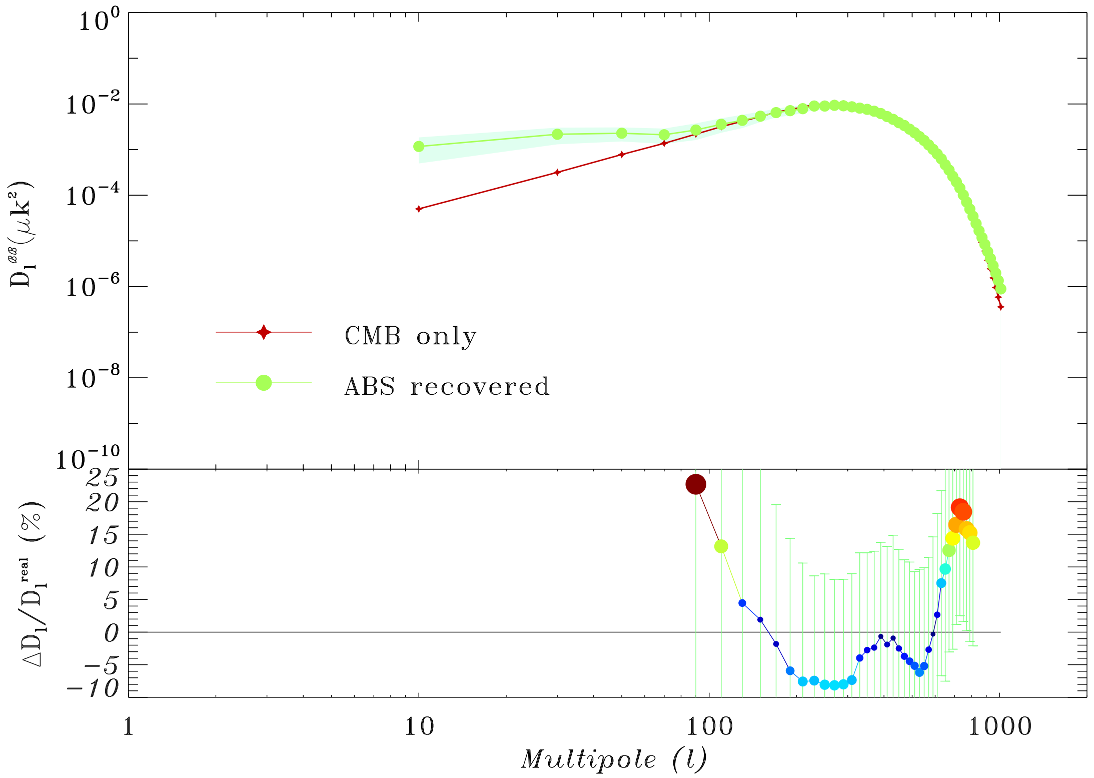


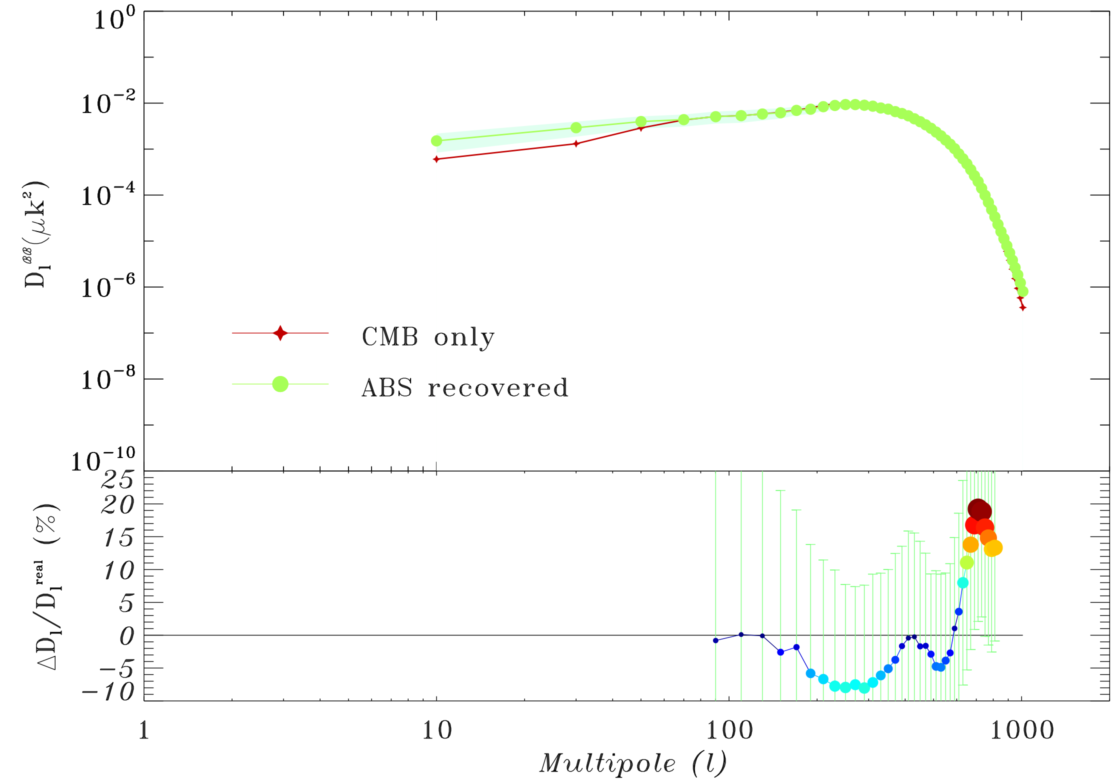
- The performance on the SZ B-mode reconstruction method deteriorates rapidly below l=100 for analytically apodized windows, especially containing holes.

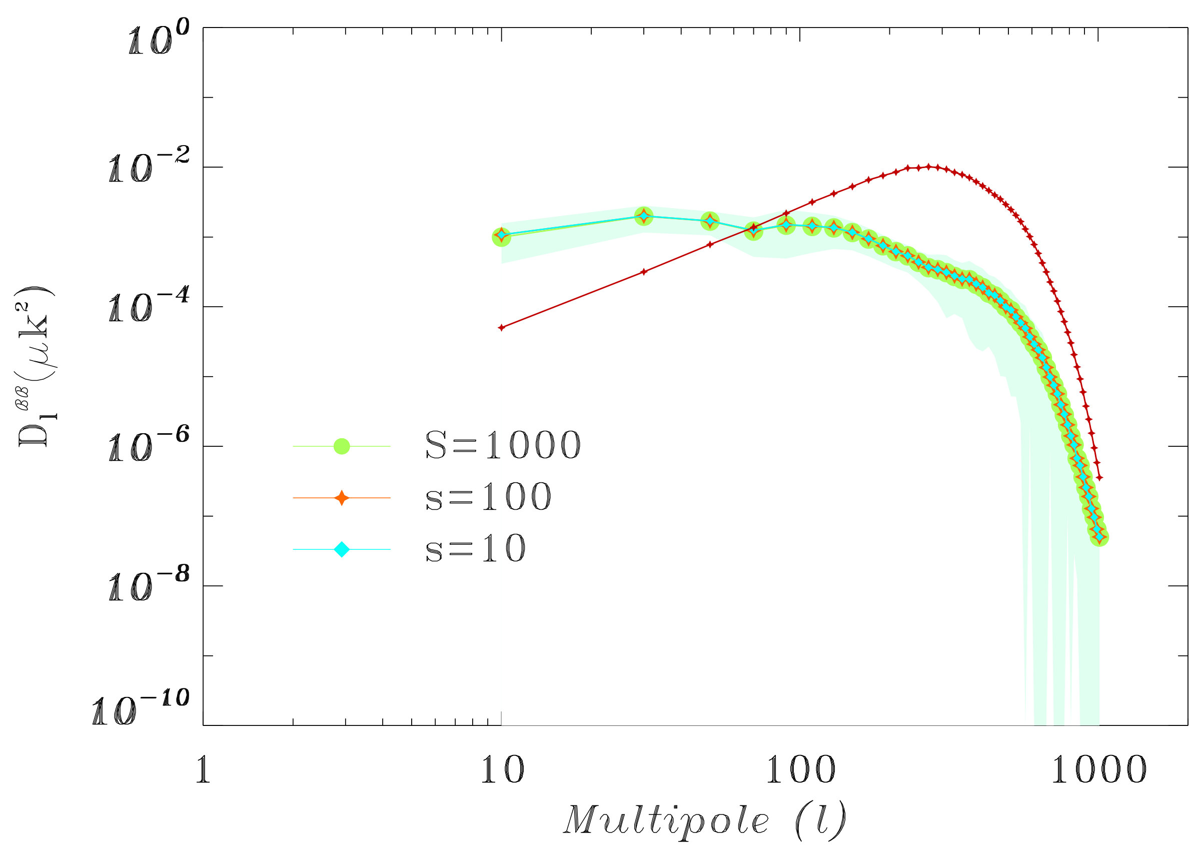
A recap: step by step

- $Q_f \longrightarrow$ CMB + 2fore + noise_k =50 Q_{maps}
- $U_f \longrightarrow$ CMB + 2fore + noise_k =50 U_{maps}
- SZ method for reconstructing the 50 B_f
- Apply the ABS method 50 times using B_{030}, B_{043}, B_{075}, B_{090}, B_{108}, B_{129}, B_{155}, B_{223}, B_{268}, B_{321}
- Compute the pseudo power spectrum
- Finally, calculate mean and sttdev

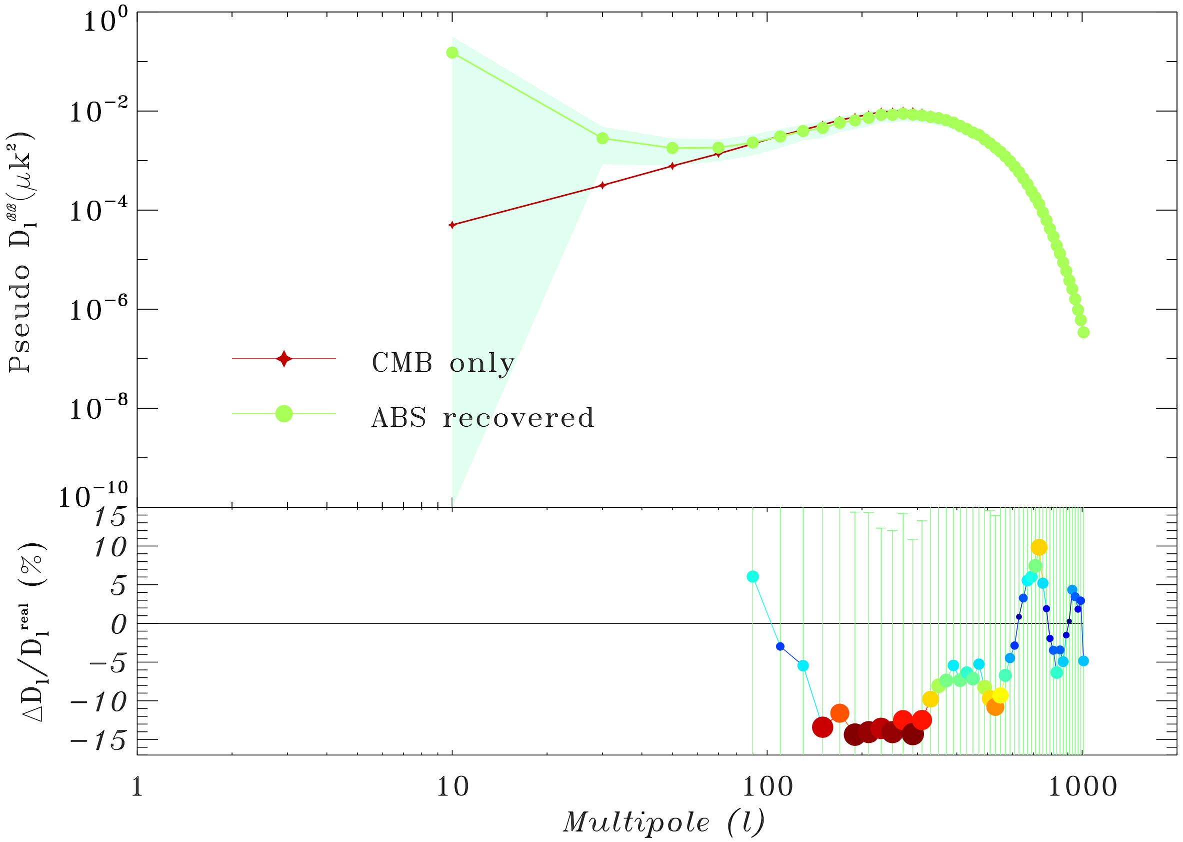
Results: full sky



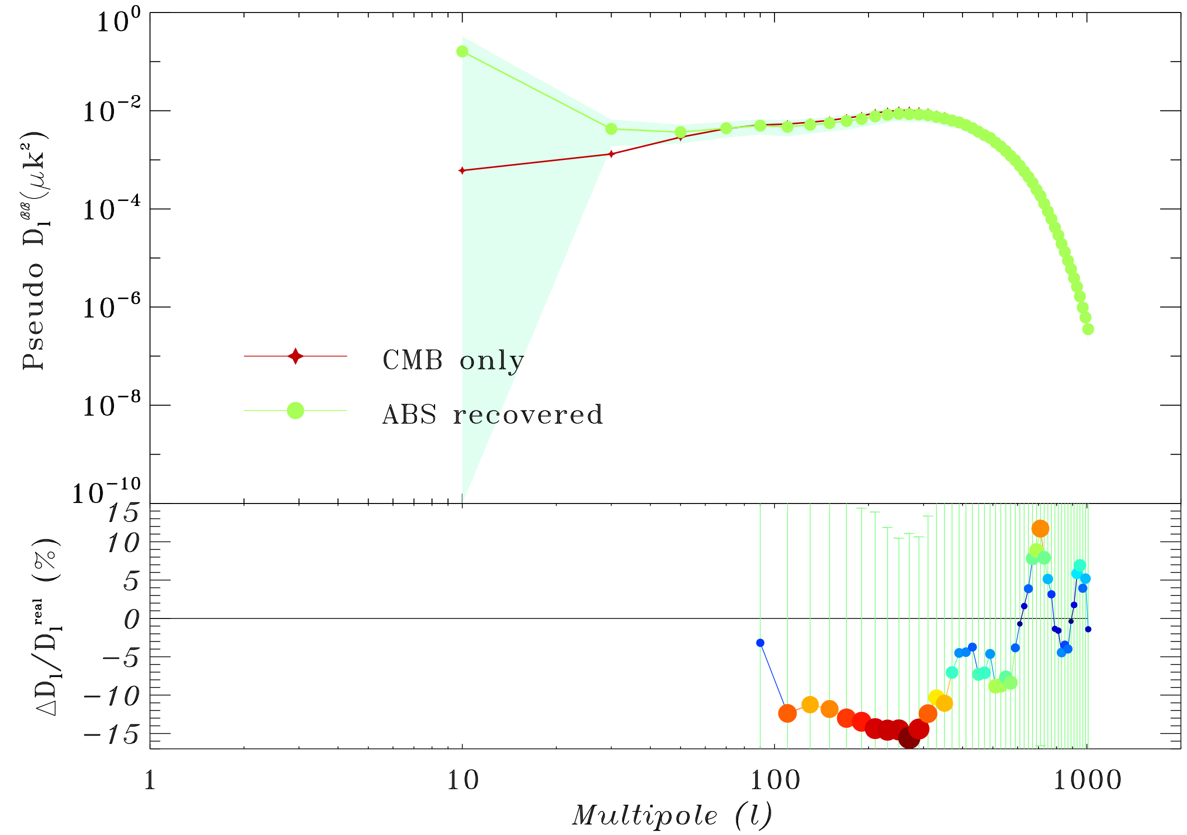




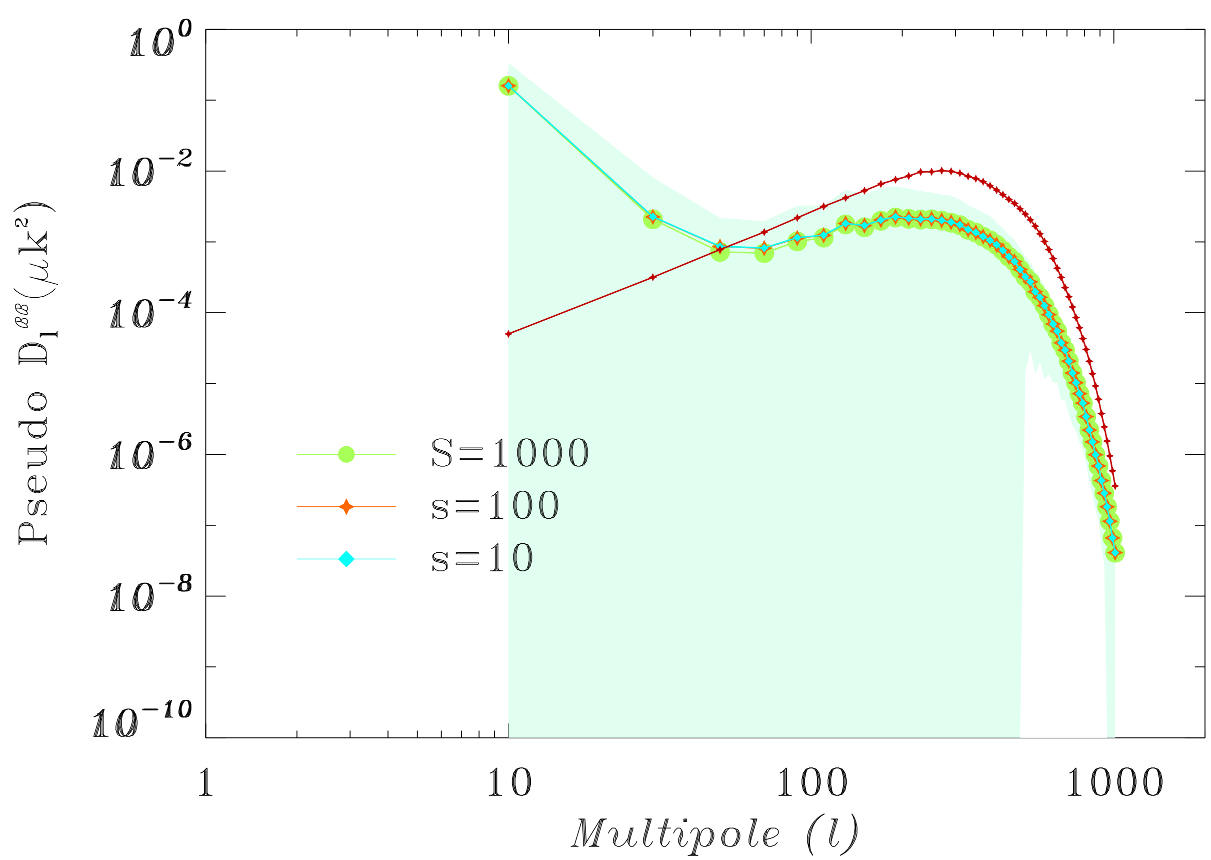
Partial sky



CMB only
ABS recovered
Pseudo $D_l^{BB}(\mu k^2)$
$\Delta D_l/D_l^{real}$ (%)
Multipole (l)



CMB only
ABS recovered
Pseudo $D_l^{BB}(\mu k^2)$
$\Delta D_l/D_l^{real}$ (%)
Multipole (l)



S=1000
s=100
s=10
Pseudo $D_l^{BB}(\mu k^2)$
Multipole (l)

Conclusions

- TheABS method is able to estimate the CMB B-mode PS within 1-sigma error bar at most scales for l>90
- Full sky: The PS is recovered with an accuracy of less than 25% for 90 < l < 800
- Partial sky: The PS in recovered within 15% accuracy for l > 90
- In both cases, for low multipoles, the ABS results diverge from the input CMB spectrum
- The ABS methodology MUST be improved in order to resolve the possible primordial B-mode signal in future CMB polarization data.