Gravitational birefringence of light

at cosmological scales

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To the memory of Christian Duval

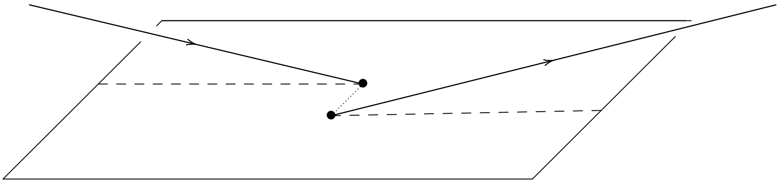

Figure: The Fedorov (1955) Imbert (1972) effect for reflection: A plane glass surface reflects an incoming, circularly polarized light beam. The dashed lines indicate the orthogonal projections of incoming and reflected light beams onto the glass surface. The dotted line (between the blobs) is the offset between incoming and reflected beams. It is of the order of the wavelength of the light beam.

O. Hosten, P. Kwiat, “Observation of the Spin Hall Effect of Light via weak measurements”, Science **319** (2008) 787–790.

K. Yu. Bliokh, A. Niv, V. Kleinert, E. Hasman, “Geometrodynamics of spinning light”, Nature Photonics **2** (2008) 748.

Generalizing the geodesic equation

Let $X^\mu(\tau)$ be the trajectory of a massless spinless particle with 4-velocity $dX^\mu/d\tau$ and 4-momentum P^μ. Its equations of motion,

$$\begin{aligned}
\frac{d}{d\tau} X^\mu &= P^\mu + 2\,\frac{S^\mu{}_\nu R^\nu{}_{\beta\rho\sigma} S^{\rho\sigma} P^\beta}{R_{\alpha\beta\rho\sigma} S^{\alpha\beta} S^{\rho\sigma}}\,, \\
\frac{D}{d\tau} P^\mu &= 0 - s\,\frac{\sqrt{-\det(R^\alpha{}_{\beta\rho\sigma} S^{\rho\sigma})}}{R_{\alpha\beta\rho\sigma} S^{\alpha\beta} S^{\rho\sigma}}\, P^\mu\,, \\
\frac{D}{d\tau} S^{\mu\nu} &= P^\mu \frac{d}{d\tau} X^\nu - P^\nu \frac{d}{d\tau} X^\mu\,,
\end{aligned}$$

have one conserved quantity, $P^\mu P_\mu = 0$. Souriau 1974 & Saturnini 1976 add spin using the *antisymmetric* spin tensor $S^{\mu\nu}$.

Now there are three more conserved quantities:

$$P_\mu \frac{d}{d\tau} X^\mu = 0\,,$$

the Tulczyjew condition 1959,

$$S^\mu{}_\nu P^\nu = 0\,,$$

and the *“scalar spin”* s,

$$-\tfrac{1}{2} S^\mu{}_\nu S^\nu{}_\mu =: s^2\,.$$

Photons have

$$s = \pm\hbar.$$

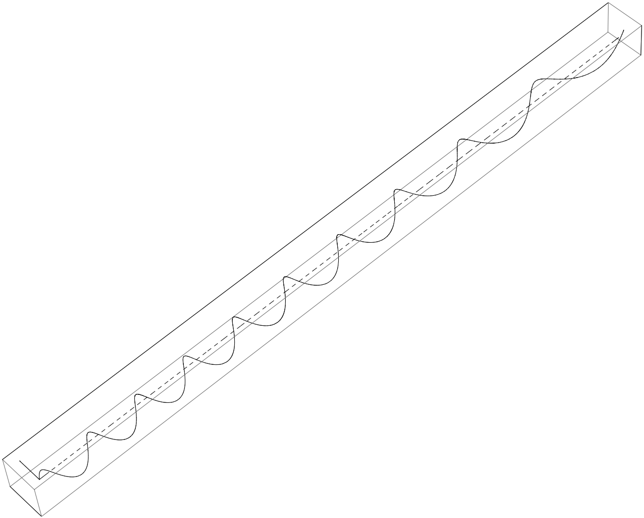

Figure: The trajectory of photons, $\mathbf{x}(t)$, in a flat Robertson-Walker universe in comoving coordinates is the helix. The dashed line is the null geodesic. The transverse spin $\mathbf{s}_e^\perp$ at emission time t_e is indicated by the short arrow at the left.

The spin 3-vector $\mathbf{s}$ precesses in lockstep around the null geodesic and the norm of its transverse part is almost conserved:

$$|\mathbf{s}_\perp|\,\sqrt{a'^2+K} \;=\; constant, \quad \text{with} \quad K := \tfrac{1}{6}\,{}^{(3)}\!R.$$

Denote by $\sim$ linearization in $|\mathbf{s}_e^\perp|\,\lambda_e/(2\pi\,\hbar\,a_e)$, λ_e being the wave-length at emission.
Then the instantaneous period of the helix is

$$T_{\rm helix}(t) \sim \frac{a(t)}{a_e}\,\frac{\lambda_e}{1+Q(t)} \quad \text{with} \quad Q := \frac{-a\,a''}{a'^2+K}\,,\; K := \tfrac{1}{6}\,{}^{(3)}\!R.$$

The radius of the helix is

$$R_{\rm helix}(t) \sim \frac{a(t)}{a_e}\,\frac{\lambda_e}{2\pi},$$

implying $|d\mathbf{x}/dt| \;\sim\; \sqrt{2}\,c$.

An exotic definition of redshift

The conventional definition of redshift z is in terms of the atomic period T_e of light emitted by an excited atom and the same period T_0 at reception today:

$$z+1 = \frac{T_0}{T_e} = \frac{a(t_0)}{a(t_e)}.$$

In order to address the dark matter problem, we imagine that the photon on its trajectory from a supernova to us counts the number of its precessions:

$$z+1 = \frac{T_{\rm helix}(t_0)}{T_{\rm helix}(t_e)} = \frac{a(t_0)}{a(t_e)}\,\frac{1+Q(t_e)}{1+Q(t_0)}.$$

Fitting this exotic formula to the Hubble diagram of the 740 type Ia supernovae of the Joint Light curve Analysis yields indeed a much lower mass density of our universe (1σ errors) and at the same time solves the cosmological constant problem (if you believe that there is one):

$$\Omega_{m0} = -0.15 \pm 0.07, \quad \Omega_{\Lambda 0} = (-3 \pm 2)\,10^{-4}.$$

Conclusions and questions

- The gravitational field of an expanding universe produces birefringence of light.
- This birefringence carries information on the acceleration of the universe.
- Can this birefringence be measured?