

Purely virtual particles in primordial cosmology and quantum gravity

Damiano Anselmi

NICPB, Tallinn, University of Pisa, INFN

This talk is about the prediction

$$\frac{(n_s - 1)^2}{3} \leq r \leq 3(n_s - 1)^2$$

for the tensor-to-scalar ratio, how it comes from high-energy physics and why it is basically unique

- D. A., E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211 and [arXiv:2005.10293 \[hep-th\]](#)

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And other connections between perturbative quantum field theory and primordial cosmology, such as the possibility of viewing inflation as a renormalization-group flow

- D.A., Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, J. Cosmol. Astropart. Phys. 01 (2021) 048 and arXiv: 2007.15023 [hep-th]
- D.A., High-order corrections to inflationary perturbation spectra in quantum gravity, J. Cosmol. Astropart. Phys. 02 (2021) 029 and arXiv: 2010.04739 [hep-th]

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they allow us, among the other things, to formulate a consistent (i.e., local, unitary and renormalizable within perturbation theory) theory of quantum gravity

- D. A., **On the quantum field theory of the gravitational interactions**, J. High Energy Phys. 06 (2017) 086 and arXiv:1704.07728 [hep-th]

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which is experimentally testable due to its sharp prediction ($0.4 \leq 1000r \leq 3$) of the tensor-to-scalar ratio r in inflationary cosmology

- D. A., E. Bianchi and M. Piva, **Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term**, J. High Energy Phys. 07 (2020) 211 and arXiv:2005.10293 [hep-th]

They can also be used to search for new physics beyond the standard model, by evading common constraints in collider phenomenology

- D. A., K. Kannike, C. Marzo, L. Marzola, A. Melis, K. Mürsepp, M. Piva and M. Raidal, Phenomenology of a fake inert doublet model, J. High Energy Phys. 10 (2021) 132 and arXiv:2104.02071 [hep-ph]

and offering new possibilities to solve discrepancies with data

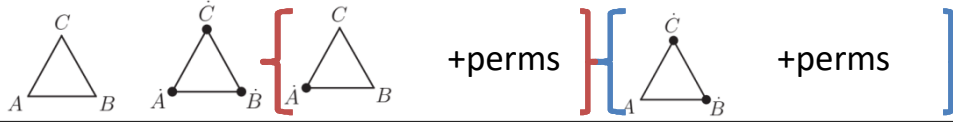
- D. A., K. Kannike, C. Marzo, L. Marzola, A. Melis, K. Mürsepp, M. Piva and M. Raidal, A fake doublet solution to the muon anomalous magnetic moment, Phys. Rev. D 104 (2021) 035009 and arXiv:2104.03249 [hep-ph]

Their diagrammatics can be implemented in softwares like FeynCalc, FormCalc, LoopTools and Package-X and used to make predictions in phenomenology

It is possible to describe inflation as a renormalization-group flow and calculate the power spectra to high orders in quantum gravity, in the presence of higher-derivative terms

- D.A., Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, *J. Cosmol. Astropart. Phys.* 01 (2021) 048 and arXiv: 2007.15023 [hep-th]
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Triangle diagram



Th\G	T_{ABC}^s	$T_{\dot{A}BC}^s$	$T_{A\dot{B}C}^s$	$T_{AB\dot{C}}^s$	$T_{\dot{A}B\dot{C}}^s$	$T_{A\dot{B}\dot{C}}^s$	$T_{\dot{A}B\dot{C}}^s$	$T_{\dot{A}\dot{B}C}^s$
—	$-i\mathcal{P}_{ABC}$	$i\mathcal{P}_{ABC}$	0	0	0	0	0	0
Δ^{23}	$-Q^{21}$	$-Q^{21}$	0	0	0	0	0	$2Q^{21}$
Δ^{12}	$-Q^{13}$	$-Q^{13}$	0	0	0	0	$2Q^{13}$	0
Δ^{31}	$-Q^{32}$	$-Q^{32}$	0	0	0	$2Q^{32}$	0	0
Δ^{32}	$-Q^{31}$	$-Q^{31}$	0	0	$2Q^{31}$	0	0	0
Δ^{21}	$-Q^{23}$	$-Q^{23}$	0	$2Q^{23}$	0	0	0	0
Δ^{13}	$-Q^{12}$	$-Q^{12}$	$2Q^{12}$	0	0	0	0	0
$\Delta^{12}\Delta^{13}$	i	$-i$	$-2i$	0	0	0	$2i$	0
$\Delta^{23}\Delta^{21}$	i	$-i$	0	$-2i$	0	0	0	$2i$
$\Delta^{31}\Delta^{32}$	i	$-i$	0	0	$-2i$	$2i$	0	0
$\Delta^{31}\Delta^{21}$	i	$-i$	0	$-2i$	0	$2i$	0	0
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$\Delta^{23}\Delta^{13}$	i	$-i$	$-2i$	0	0	0	0	$2i$

$$\frac{i}{p^2 - m^2 + i\epsilon} = \mathcal{P} \frac{i}{p^2 - m^2} + \pi\delta(p^2 - m^2)$$

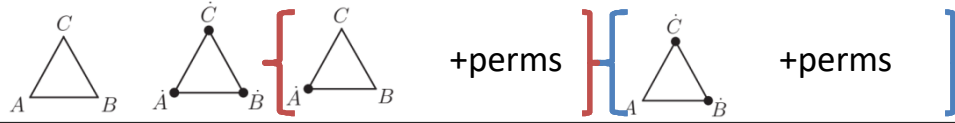
$$\frac{i}{x + i\epsilon} = \mathcal{P} \frac{i}{x} + \pi\delta(x)$$

$$\mathcal{P}^{ab} = \mathcal{P} \frac{1}{e_a - e_b - \omega_a - \omega_b},$$

$$Q^{ab} = \mathcal{P}^{ab} - \mathcal{P} \frac{1}{e_a - e_b - \omega_a + \omega_b}$$

$$\Delta^{ab} = \pi\delta(e_a - e_b - \omega_a - \omega_b)$$

Triangle diagram with a fakeon in leg 1 (AB)



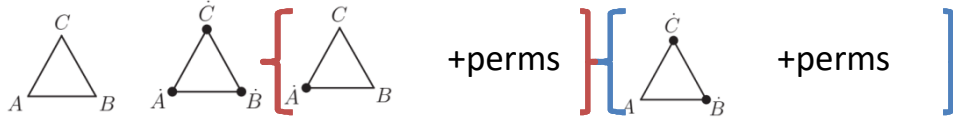
Th\G	T_{ABC}^s	$T_{\dot{A}BC}^s$	$T_{A\dot{B}C}^s$	T_{ABC}^s	T_{ABC}^s	T_{ABC}^s	T_{ABC}^s	T_{ABC}^s
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Quantum gravity

$$S_{\text{QG}}(g) = -\frac{M_{\text{Pl}}^2}{16\pi} \int d^4x \sqrt{-g} \left(R + \frac{1}{2m_\chi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2} \right)$$

Quantum gravity

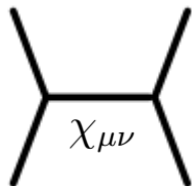
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It contains a triplet made of:

The **graviton**

The **inflaton** (spin 0, mass m_ϕ)

The “**purely virtual particle**” (or “**fake particle**” or “**fakeon**”) (spin 2, mass m_χ)



Quantum gravity

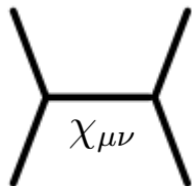
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This is NOT the true classical action in the fakeon approach, because it is unprojected

True classical action: collection of tree diagrams with no external fakeon legs, where the purely virtual mediator is treated in a suitable way (to be truly virtual)

Primordial cosmology

With a field redefinition, switch to

$$S_{\text{QG}} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R + \frac{1}{2m_\chi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right) + \frac{1}{2} \int d^4x \sqrt{-g} (D_\mu \phi D^\mu \phi - 2V(\phi)) ,$$

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Parametrizing the metric as $g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$

the Friedmann equations are not affected by C^2 and read

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = -4\pi G \dot{\phi}^2, \quad \frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3} \left(\dot{\phi}^2 + 2V(\phi) \right), \quad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -V'(\phi),$$

Then, parametrizing the fluctuations as

$$g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2) - 2a^2 \left(u\delta_\mu^1\delta_\nu^1 - u\delta_\mu^2\delta_\nu^2 + v\delta_\mu^1\delta_\nu^2 + v\delta_\mu^2\delta_\nu^1 \right), \\ + 2\text{diag}(\Phi, a^2\Psi, a^2\Psi, a^2\Psi) - \delta_\mu^0\delta_\nu^i\partial_i B - \delta_\mu^i\delta_\nu^0\partial_i B$$

where $u = u(t, z)$ and $v = v(t, z)$

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you can work out the spectra, tilts and tensor to scalar ratio r

$A_{\mathcal{R}}$	A_T	r	$n_{\mathcal{R}} - 1$	n_T
$\frac{m_\phi^2 N^2}{3\pi M_{\text{Pl}}^2}$	$\frac{8m_\chi^2 m_\phi^2}{\pi(m_\phi^2 + 2m_\chi^2)M_{\text{Pl}}^2}$	$\frac{24m_\chi^2}{N^2(m_\phi^2 + 2m_\chi^2)}$	$-\frac{2}{N}$	$-\frac{3m_\chi^2}{N^2(m_\phi^2 + 2m_\chi^2)}$

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$$r \simeq -8n_T \quad n_{\mathcal{R}} = 0.9649 \pm 0.0042$$

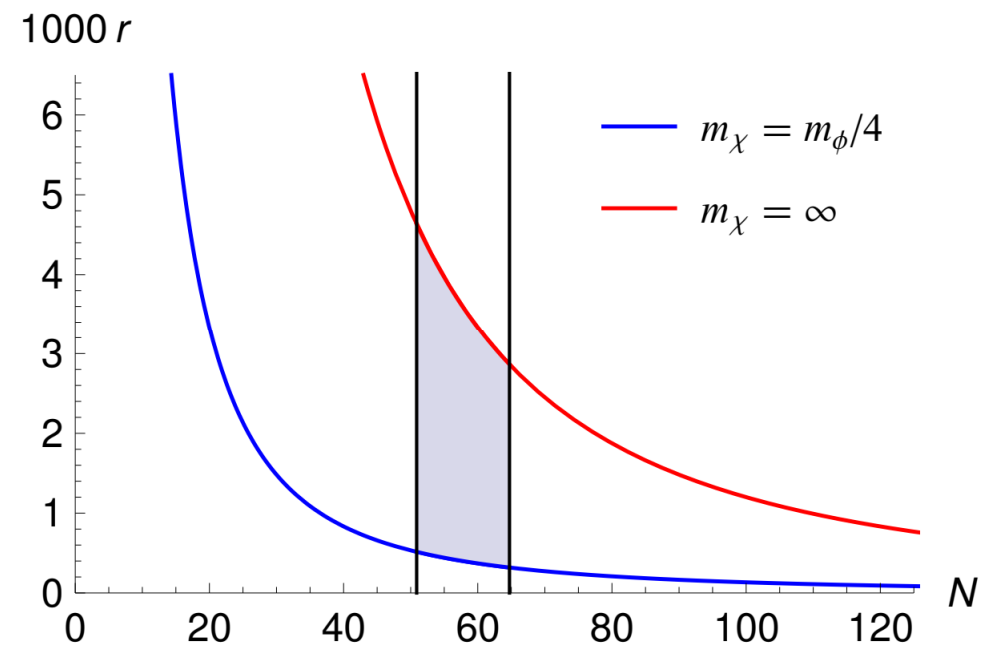
$$m_\phi = (2.99 \pm 0.37) \cdot 10^{13} \text{GeV}$$

No tachyon condition

$$m_\chi > \frac{m_\phi}{4}$$

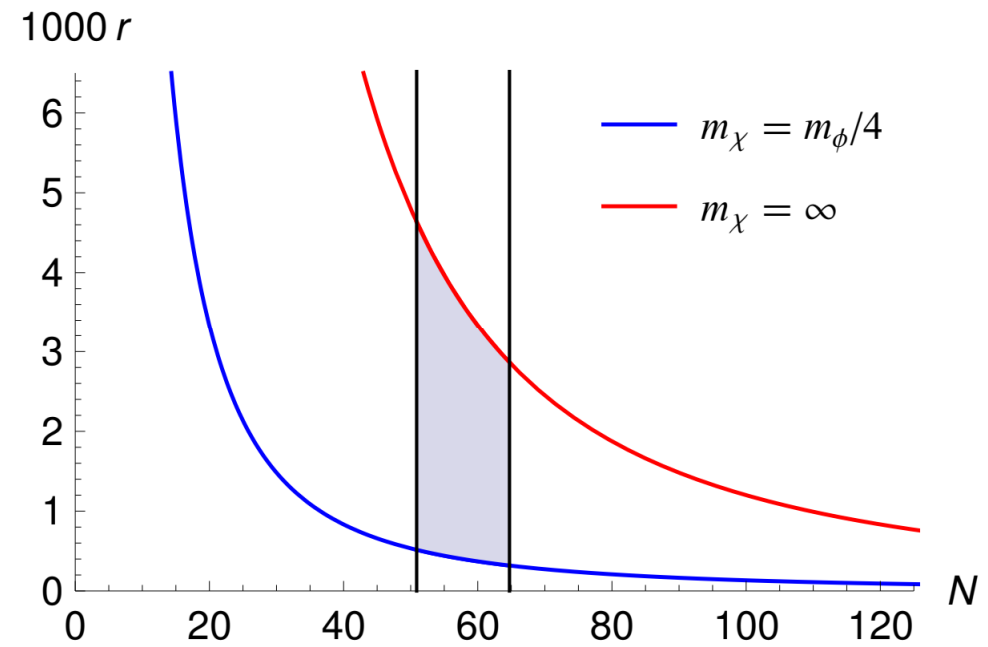
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No tachyon condition

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$$0.4 \lesssim 1000r \lesssim 3, \quad -0.4 \lesssim 1000n_T \lesssim -0.05$$

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-D.A., Quantum gravity, fakeons and
microcausality, J. High Energy Phys. 11
(2018) 21 and [arXiv: 1806.03605 \[hep-th\]](#)

Consider the higher-derivative classical Lagrangian

$$\mathcal{L}_{\text{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2 \ddot{x}^2) - V$$

$$V = \frac{m}{2}\omega^2 x^2 + \frac{\lambda}{4!}x^4$$

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The equations of motion are

where

$$m \left(\frac{d^2}{dt^2} + \Omega^2 \right) \left(\frac{d^2}{dt^2} + \tilde{\Omega}^2 \right) x = -\frac{\tilde{\lambda}}{3!}x^3, \qquad \tilde{\lambda} = \frac{\lambda}{m}$$

$$\Omega = \frac{1}{\tau\sqrt{2}}\sqrt{1 - \sqrt{1 - 4\tau^2\omega^2}}, \qquad \tilde{\Omega} = \frac{1}{\tau\sqrt{2}}\sqrt{1 + \sqrt{1 - 4\tau^2\omega^2}},$$

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We want to project the higher-derivatives away

NOTE THE **NO-TACHYON CONDITION**

$$\omega > \frac{1}{2\tau}$$

Now that we have understood that THERE IS a condition, we proceed with the simpler case $\omega = 0$, where formulas can be more easily visualized

$$\frac{d^2}{dt^2} \left(1 + \tau^2 \frac{d^2}{dt^2} \right) x = -\frac{\tilde{\lambda}}{3!} x^3, \quad \tilde{\lambda} = \frac{\lambda}{m}$$

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$$\begin{aligned} \ddot{x} = & -\frac{\tilde{\lambda}x}{6} (x^2 - 6\tau^2 \dot{x}^2) - \frac{\tilde{\lambda}^2 \tau^2 x}{12} (x^4 - 48\tau^2 x^2 \dot{x}^2 + 372\tau^4 \dot{x}^4) \\ & - \frac{\tilde{\lambda}^3 \tau^4 x}{6} (x^6 - 156\tau^2 x^4 \dot{x}^2 + 4572\tau^4 x^2 \dot{x}^4 - 31152\tau^6 \dot{x}^6) + \mathcal{O}(\tilde{\lambda}^4) \end{aligned}$$

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$$\frac{\mathcal{L}}{m} = \frac{\dot{x}^2}{2} - \frac{\tilde{\lambda}x^2}{4!} (x^2 + 12\tau^2 \dot{x}^2) + \frac{\tau^2 \tilde{\lambda}^2 x^2}{72} (x^4 - 54\tau^2 x^2 \dot{x}^2 + 372\tau^4 \dot{x}^4) + \mathcal{O}(\tilde{\lambda}^3)$$

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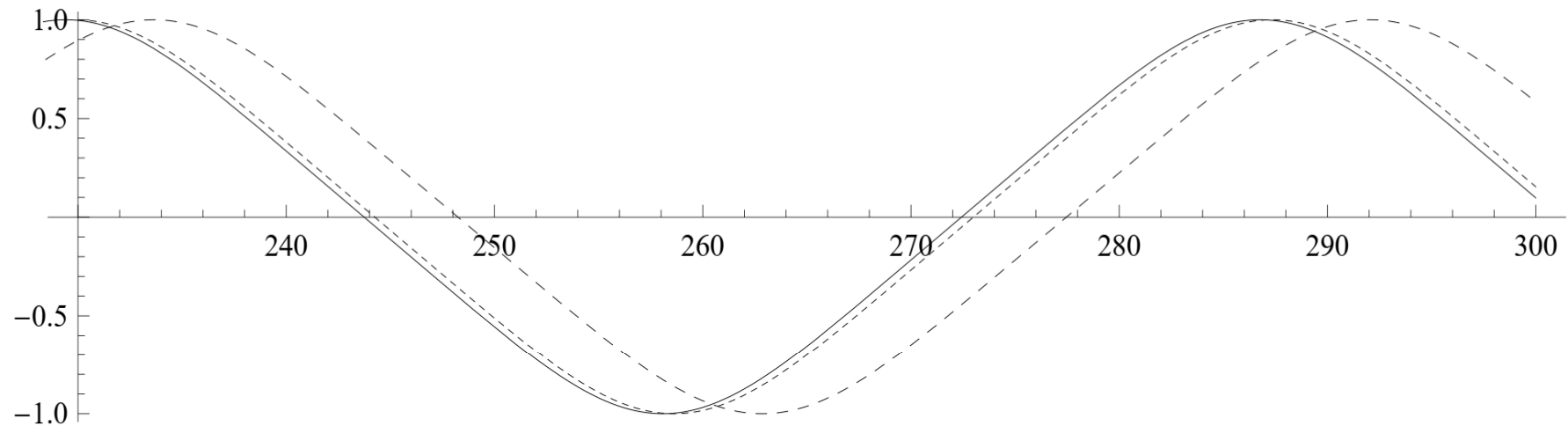


Figure 3: Solution $x(t)$ of the truncated equation (3.5) for $x(0) = 1$, $\dot{x}(0) = 0$, $m = \tau = 1$, $\lambda = 1/10$. The sparsely dashed line is $n = 1$. The densely dashed line is $n = 2$, while the continuous line is $n = 3$. The solution remains stable from $n = 3$ to $n = 10$.

Doing this in primordial cosmology, you find the no-tachyon condition

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The bound makes the theory predictive even before knowing the actual value of m_χ

Inflation as a “cosmic” RG flow

Quantum field theory

Inflationary cosmology

Quantum field theory Inflationary cosmology
RG flow \leftrightarrow slow roll

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couplings $\alpha, \lambda \dots$	\leftrightarrow	slow-roll parameters $\epsilon, \delta \dots$

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dimensional transmutation	\rightarrow	τ drops out from the spectra, “replaced” by k
running coupling	\rightarrow	ok
resummation of leading logs	\rightarrow	ok
??	\leftarrow	potential $V(\phi)$
anomalous dimensions	\rightarrow	0

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \frac{1}{2} \int d^4x \sqrt{-g} (D_\mu \phi D^\mu \phi - 2V(\phi))$$

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R + \frac{1}{2m_\chi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right) + \frac{1}{2} \int d^4x \sqrt{-g} (D_\mu \phi D^\mu \phi - 2V(\phi))$$

For a classification of the allowed potentials, see

D. A., F. Fruzza and M. Piva, [Renormalization-group techniques for single-field inflation in primordial cosmology and quantum gravity](#), *Class. Quantum Grav.* **38** (2021) 225011 and [arXiv: 2103.01653 \[hep-th\]](#)

Choose the metric

$$g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$$

$$H = \dot{a}/a$$

You obtain the equations

$$\hat{\kappa} = \sqrt{16\pi G/3}$$

$$\dot{H} = -4\pi G\dot{\phi}^2, \quad H^2 = \frac{4\pi G}{3} \left(\dot{\phi}^2 + 2V(\phi) \right), \quad \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0,$$

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Define the **coupling** $\alpha = \frac{\hat{\kappa}\dot{\phi}}{2H} = \sqrt{-\frac{\dot{H}}{3H^2}}$

Eliminating V and $\dot{\phi}$ by means of the first two equations of (2.4) and $\ddot{\phi}$ from the last equation, it is easy to show that α satisfies

$$\dot{\alpha} = m_\phi \sqrt{1 - \alpha^2} - H(2 + 3\alpha)(1 - \alpha^2).$$

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$$\beta_\alpha = \frac{d\alpha}{d \ln|\tau|} = -2\alpha^2 \left[1 + \frac{5}{6}\alpha + \frac{25}{9}\alpha^2 + \frac{383}{27}\alpha^3 + \frac{8155}{81}\alpha^4 + \frac{72206}{81}\alpha^5 + \frac{2367907}{243}\alpha^6 + \mathcal{O}(\alpha^7) \right]$$



de Sitter free

The spectra satisfy Callan-Symanzik equations at superhorizon scales:

$$\frac{d\mathcal{P}}{d \ln |\tau|} = \left(\frac{\partial}{\partial \ln |\tau|} + \beta_\alpha(\alpha) \frac{\partial}{\partial \alpha} \right) \mathcal{P} = 0.$$

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 running coupling

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Without Weyl squared:

$$\mathcal{P}_T(k) = \frac{4Gm_\phi^2}{\pi} \left[1 - 3\alpha_k + (47 - 24\gamma_M) \frac{\alpha_k^2}{4} - \left(\frac{307}{6} + 12\gamma_M^2 - 42\gamma_M - \pi^2 \right) \alpha_k^3 + \mathcal{O}(\alpha_k^4) \right],$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{Gm_\phi^2}{12\pi\alpha_k^2} \left[1 + (5 - 4\gamma_M)\alpha_k - \frac{67}{12}\alpha_k^2 + (12\gamma_M^2 - 40\gamma_M + 7\pi^2) \frac{\alpha_k^2}{3} + \mathcal{O}(\alpha_k^3) \right]$$

With Weyl squared

$$\mathcal{P}_T(k) = \frac{4m_\phi^2 \zeta G}{\pi} \left[1 - 3\zeta \alpha_k \left(1 + 2\alpha_k \gamma_M + 4\gamma_M^2 \alpha_k^2 - \frac{\pi^2 \alpha_k^2}{3} \right) + \frac{\zeta^2 \alpha_k^2}{8} (94 + 11\xi) \right. \\ \left. + 3\gamma_M \zeta^2 \alpha_k^3 (14 + \xi) - \frac{\zeta^3 \alpha_k^3}{12} (614 + 191\xi + 23\xi^2) + \mathcal{O}(\alpha_k^4) \right].$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{Gm_\phi^2}{12\pi\alpha_k^2} \left[1 + (5 - 4\gamma_M)\alpha_k + \left(4\gamma_M^2 - \frac{40}{3}\gamma_M + \frac{7}{3}\pi^2 - \frac{67}{12} - \frac{\xi}{2}F_s(\xi) \right) \alpha_k^2 + \mathcal{O}(\alpha_k^3) \right]$$

$$F_s(\xi) = 1 + \frac{\xi}{4} + \frac{\xi^2}{8} + \frac{\xi^3}{8} + \frac{7\xi^4}{32} + \frac{19}{32}\xi^5 + \frac{295}{128}\xi^6 + \frac{1549}{128}\xi^7 + \frac{42271}{512}\xi^8 + \mathcal{O}(\xi^9)$$

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \quad \zeta = \left(1 + \frac{\xi}{2} \right)^{-1}, \quad \tilde{\gamma}_M = \gamma_M - \frac{i\pi}{2}, \quad \gamma_M = \gamma_E + \ln 2,$$

Conclusions

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It is possible to study inflation as a renormalization-group flow, where the spectra play the roles of correlation functions and obey Callan-Symanzik equations in the super-horizon limit. The techniques imported from high-energy physics allow us to gain in understanding and compute high-order corrections more easily

Thanks!