Challenges in supersymmetric cosmology

I. Antoniadis

LPTHE, Sorbonne University, CNRS, Paris

Hot Topics in Modern Cosmology

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Problem of scales

- describe high energy (SUSY?) extension of the Standard Model

  unification of all fundamental interactions

- incorporate Dark Energy

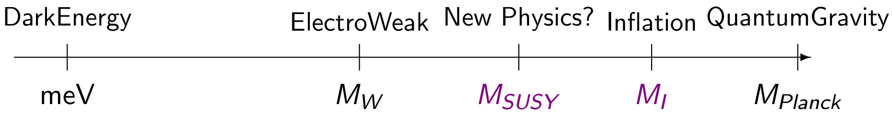
  simplest case: infinitesimal (tuneable) +ve cosmological constant

- describe possible accelerated expanding phase of our universe

  models of inflation (approximate de Sitter)

$\Rightarrow$ 3 very different scales besides M_W and M_{Planck} :

Physics behind the scales



1. they are independent
2. possible connections
   - M_I could be near the EW scale, such as in Higgs inflation

     but large non minimal coupling to explain
   - M_{Planck} could be emergent from the EW scale

     in models of low-scale gravity and TeV strings

→ • SUSY: well motivated proposal (e.g. stability of hierarchy)
   but no experimental indication of any BSM physics at LHC

→ • Inflation: theoretical paradigm consistent with observations
   but unknown origin/nature of the inflaton and scale

Inflation in supergravity: main problems

Inflaton: part of a chiral superfield X [11]

- slow-roll conditions: the eta problem $\Rightarrow$ fine-tuning of the potential

  $$\eta = V''/V\,, \quad V_F = e^K(|DW|^2 - 3|W|^2)\,, \quad DW = W' + K'W$$

  K: Kähler potential, W: superpotential   Planck units: $\kappa = 1$

  canonically normalised field: $K = X\bar{X} \Rightarrow \eta = 1 + \ldots$

- trans-Planckian initial conditions $\Rightarrow$ break validity of EFT

  no-scale type models that avoid the η-problem

  $$K = -3\ln(T + \bar{T})\,;\; W = W_0 \;\Rightarrow\; V_F = 0$$

- stabilisation of the (pseudo) scalar companion of the inflaton

  chiral multiplets $\Rightarrow$ complex scalars

- moduli stabilisation, de Sitter vacuum, ...

Starobinsky model of inflation

$$\mathcal{L} = \frac{1}{2}R + \alpha R^2$$

Lagrange multiplier ϕ : $\mathcal{L} = \frac{1}{2}(1+2\phi)R - \frac{1}{4\alpha}\phi^2$ $\qquad \phi = 2\alpha R$

Rescaling the metric to the Einstein frame $\Rightarrow$

equivalent to a scalar field with exponential potential:

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 \qquad M^2 = \frac{3}{4\alpha}$$

supersymmetric extension: need two Lagrange multipliers

$\Rightarrow$ two chiral superfields

one contains the inflaton ϕ and the other the goldstino [9]

Goldstone fermion of spontaneous supersymmetry breaking

SUSY extension of Starobinsky model

$$K = -3\ln(T + \bar{T} - C\bar{C}) \quad ; \quad W = MC(T - \frac{1}{2})$$

- T contains the inflaton: $\mathrm{Re}\, T = e^{\sqrt{\frac{2}{3}}\phi}$
- $C \sim \mathcal{R}$ is unstable during inflation

  $\Rightarrow$ add higher order terms to stabilize it

  e.g. $C\bar{C} \to h(C, \bar{C}) = C\bar{C} - \zeta(C\bar{C})^2$ Kallosh-Linde '13
- SUSY is broken during inflation with C the goldstino superfield

$\to$ model independent treatment in the decoupling sgoldstino limit

replace C by a constrained superfield X satisfying $X^2 = 0$

$\Rightarrow$ sgoldstino $=$ (goldstino)$^2/F$

$\Rightarrow$ minimal SUSY extension that evades stability problem

Non-linear supersymmetry $\Rightarrow$ goldstino mode χ

Volkov-Akulov '73

Effective field theory of SUSY breaking at low energies

Analog of non-linear σ-model $\Rightarrow$ constraint superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0 \Rightarrow$

$$X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa}\left\{\int d^2\theta X_{NL} + h.c.\right\} = \mathcal{L}_{Volkov-Akulov}$$

R-symmetry with $[\theta]_R = [\chi]_R = 1$ and $[X]_R = 2$ $\quad F = \frac{1}{\sqrt{2}\kappa} + \dots$

Non-linear SUSY in supergravity

I.A.-Dudas-Ferrara-Sagnotti '14

$$K = X\bar{X} \quad ; \quad W = f\,X + W_0$$

$X \equiv X_{NL}$ nilpotent goldstino superfield

$$X_{NL}^2 = 0 \;\Rightarrow\; X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\Rightarrow \quad V = |f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- V can have any sign contrary to global NL SUSY
- NL SUSY in flat space $\Rightarrow f = \sqrt{3}\,m_{3/2}M_p$
- R-symmetry is broken by W_0

Non-linear Starobinsky supergravity [6][6]

I.A.-Dudas-Ferrara-Sagnotti '14

Non-linear supersymmetry limit: one field decouples: [5]

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\phi}(\partial a)^2 - \frac{M^2}{18}e^{-2\sqrt{\frac{2}{3}}\phi}a^2$$

- no eta-problem but

  initial conditions require trans-planckian values for ϕ $(\phi > 1)$

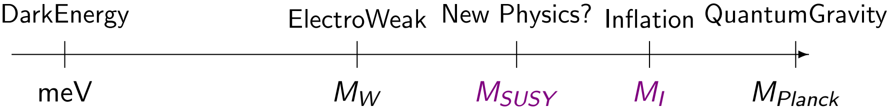
- pseudoscalar a much heavier than ϕ during inflation, decouples:

  $m_\phi = \frac{M}{3}e^{-\sqrt{\frac{2}{3}}\phi_0} << m_a = \frac{M}{3}$

- inflation scale M independent from supersymmetry breaking scale

  $\Rightarrow$ compatible with low energy supersymmetry

Problem of scales: connections



Direct connection of inflation and supersymmetry breaking:

identify the inflaton with the partner of the goldstino

Goldstone fermion of spontaneous supersymmetry breaking

while accommodating observed vacuum energy

Inflation from supersymmetry breaking

I.A.-Chatrabhuti-Isono-Knoops '16, '17, '19

Inflaton : goldstino superpartner in the presence of a gauged R-symmetry

- linear superpotential $W = f\,X \Rightarrow$ no η-problem [4]

$$\begin{aligned} V_F &= e^K \left(|DW|^2 - 3|W|^2\right) \\ &= e^K \left(|1 + K_X X|^2 - 3|X|^2\right)|f|^2 \qquad K = X\bar{X} \\ &= e^{|X|^2} \left(1 - |X|^2 + \mathcal{O}(|X|^4\right)|f|^2 = \mathcal{O}(|X|^4) \quad \Rightarrow \eta = 0 + \ldots \end{aligned}$$

  linear W guaranteed by an R-symmetry

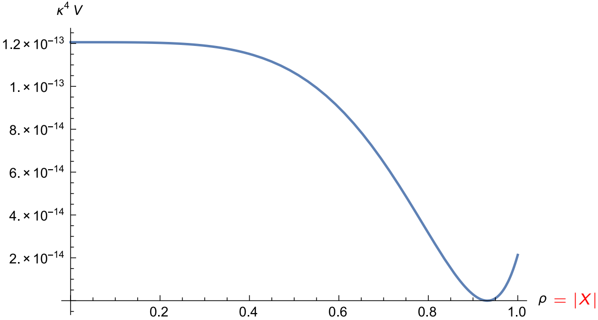
- gauge R-symmetry: (pseudo) scalar absorbed by the $U(1)_R$
- inflation around a maximum of scalar potential (hill-top) $\Rightarrow$ small field

  no large field initial conditions

- vacuum energy at the minimum: tuning between V_F and V_D

Two classes of models

- Case 1: R-symmetry is restored during inflation (at the maximum)



- Case 2: R-symmetry is (spontaneously) broken everywhere

  and restored at infinity   example: $S = \ln X$

Case 1: R-symmetry restored during inflation

maximum at the origin with small η by a correction to the Kähler potential

$$\mathcal{K}(X,\bar{X}) = \kappa^{-2}X\bar{X} + \kappa^{-4}A\,(X\bar{X})^2 \qquad A > 0 \qquad [15][18]$$

$$W(X) = \kappa^{-3}f\,X \qquad \Rightarrow$$

$$f(X) = 1 \qquad (+\beta \ln X \text{ to cancel anomalies but } \beta \text{ very small})$$

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D$$

$$\mathcal{V}_F = \kappa^{-4}f^2 e^{X\bar{X}(1+AX\bar{X})}\left[-3X\bar{X} + \frac{\left(1+X\bar{X}(1+2AX\bar{X})\right)^2}{1+4AX\bar{X}}\right]$$

$$\mathcal{V}_D = \kappa^{-4}\frac{q^2}{2}\left[1+X\bar{X}(1+2AX\bar{X})\right]^2$$

Assume inflation happens around the maximum $|X| \equiv \rho \simeq 0 \quad \Rightarrow$

Predictions

slow-roll parameters $(q \simeq 0)$

$$\eta = \frac{1}{\kappa^2}\left(\frac{V''}{V}\right) = -4A + \mathcal{O}(\rho^2)$$

$$\epsilon = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 = 16A^2\rho^2 + \mathcal{O}(\rho^4) \simeq \eta^2\rho^2 << |\eta|$$

η naturally small since A is a correction

inflation starts with an initial condition for $\phi = \phi_*$ near the maximum

and ends when $|\eta| = 1$

$\Rightarrow$ number of e-folds $N = \int_{end}^{start} \frac{V}{V'} = \kappa \int \frac{1}{\sqrt{2\epsilon}} \simeq \frac{1}{|\eta_*|} \ln\left(\frac{\rho_{\rm end}}{\rho_*}\right)$

Planck '15 data : $\eta \simeq -0.02 \Rightarrow N \gtrsim 50$ naturally

Predictions

$$\text{amplitude of density perturbations} \quad A_s = \frac{\kappa^2 H_*^2}{8\pi^2 \epsilon_*}$$

$$\text{spectral index} \quad n_s \simeq 1 + 2\eta_*$$

$$\text{tensor} - \text{to} - \text{scalar ratio} \quad r = 16\epsilon_*$$

Planck '15 data : $\eta_* \simeq -0.02$, $A_s \simeq 2.2 \times 10^{-9}$, $N \gtrsim 50$

$$\Rightarrow r \lesssim 10^{-4},\ H_* \lesssim 10^{12}\ \text{GeV}$$

Question: can a 'nearby' minimum exist with a tiny +ve vacuum energy?

Answer: Yes [13]

need an extra correction to the kinetic terms

Microscopic Model

Fayet-Iliopoulos model based on a $U(1)$ R-symmetry in supergravity

two chiral multiplets $\Phi_\pm$ of charges $q_\pm$ and mass m and FI parameter ξ

$$W = m\,\Phi_+\Phi_-$$

R-symmetry $\Rrightarrow$ $q_+ + q_- \neq 0$

Higgs phase: $\langle\Phi_-\rangle = v \neq 0$

Limit of small SUSY breaking compared to the $U(1)$ mass: $m^2 << q_-^2 v^2$

integrate out gauge superfield $\rightarrow$ EFT for the goldstino superfield Φ_+

$$W = mv\Phi_+ \quad ; \quad K = \bar\Phi_+\Phi_+ + A\,(\bar\Phi_+\Phi_+)^2 + B\,(\bar\Phi_+\Phi_+)^3 + \cdots$$

parameter space allows realistic inflation

and a nearby minimum with tuneable energy

Coupling with MSSM

I.A.-Aldabergenov-Chatrabhuti-Isono '21

Two distinct cases:

- Standard Model superfields ϕ neutral under $U(1)_R \Rightarrow (\kappa = 1)$

$$W(X,\phi) = [f + w(\phi)]\, X \qquad w(\phi) : \mathrm{MSSM\, superpotential}$$

- SM particles neutral and superpartners charged $\Rightarrow U(1)_R \supset R$-parity:

$$W(X,\phi) = f\, X + w(\phi)$$

I.A.-Knoops '16

Both cases lead to similar results [20]

Kinetic terms: $K(X,\bar{X},\phi,\bar{\phi}) = \sum \left[1 + \Delta_\phi(X\bar{X})\right] \phi\bar{\phi} + J(X\bar{X})$

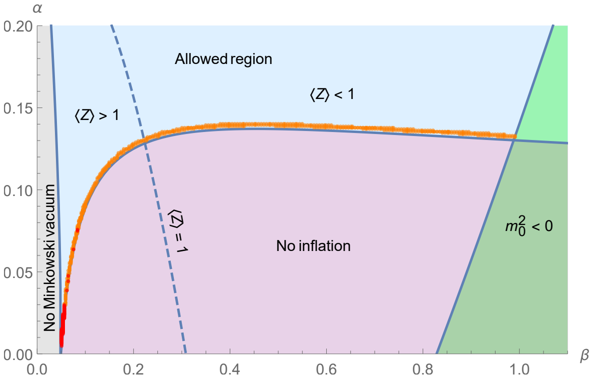
$$J = X\bar{X} + \alpha(X\bar{X})^2 + \beta(X\bar{X})^3$$

Coupling with MSSM: constraints

- Viable inflation and (nearly vanishing) vacuum energy $\Rightarrow q \gtrsim 0.8f$ [13]
- Positive scalar masses: $m_0^2 = m_{3/2}^2 - \frac{1}{2}\langle 1 + \Delta_\phi \rangle \langle \mathcal{D}_R \rangle^2 \geq 0$

$\Rightarrow \langle \Delta_\phi \rangle \lesssim 0.15$

$Z \equiv X$



Spectrum

Gaugino masses from $U(1)_R$ anomaly cancellation:

$$e^{-1}\mathcal{L} \supset \frac{1}{8} \sum_{A=R,1,2,3} \mathrm{Im}(f_A)\, F^A \tilde{F}^A \quad ; \quad f_A = 1 + \beta_A \ln X$$

with $\quad \beta_R = -\frac{g^2}{3\pi^2}\,, \quad \beta_1 = -\frac{11 g_1^2}{8\pi^2}\,, \quad \beta_2 = -\frac{5 g_2^2}{8\pi^2}\,, \quad \beta_3 = -\frac{3 g_3^2}{8\pi^2}$

Typical spectrum:

$$\alpha = 0.139\,, \quad \beta = 0.6\,, \quad g/f = 0.7371\,, \quad f = 2.05 \times 10^{-7} \quad \Rightarrow$$

$m_z\,, m_R$	m_ζ	$m_{3/2}$	m_0	m_1	m_2	m_3
1.25×10^{12}	6.15×10^{11}	7.51×10^{11}	2.68×10^{11}	1.03×10^{10}	6.54×10^{9}	5.84×10^{9}

masses (in GeV) of inflaton, inflatino, gravitino, and MSSM sparticles

$(H_{\mathrm{inf}} = 3 \times 10^{11}$ GeV$)$

$\{m_z, m_{3/2}, m_0\} > \{m_1, m_2, m_3\}$

Inflaton decay and reheating

Dominant decay to scalars and inflatino

$$\Gamma^{\rm tot}_{z\to\phi\phi} = 5.8\times 10^{-3}\ {\rm GeV}$$
$$\Gamma^{\rm tot}_{z\to\lambda\lambda}\approx\Gamma_{z\to\zeta\zeta} = 4.7\times 10^{-4}\ {\rm GeV}$$

$\Rightarrow T_{\rm reh}\simeq\sqrt{M_P\Gamma_{\rm tot}} = 1.26\times 10^{8}\ {\rm GeV}$

Possible dark matter candidate: superheavy LSP

$m_{\rm LSP}\sim 10^{10}$ GeV with $T_{\rm reh}/m_{\rm DM}\sim 10^{-3}$ Chung-Kolb-Riotto '99

Conclusions

General class of models with inflation from SUSY breaking:

identify inflaton with goldstino superpartner

- (gauged) R-symmetry restored

  small field, avoids the η-problem, no (pseudo) scalar companion

  a nearby minimum can have tuneable positive vacuum energy

- inflaton sector can be coupled to MSSM

  with gauge $U(1)_R$ containing the R-parity

- D-term inflation is also possible using a new FI term

  it can lead to large r of primordial gravitational waves

Open question: string theory realisation