Primordial black holes and Hawking radiation

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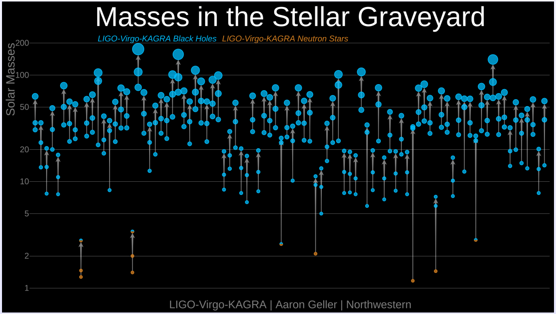
Spontaneous Workshop XIV

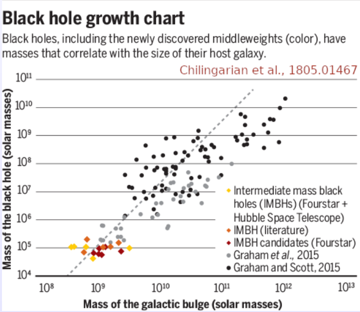
IESC Cargese, May 13th, 2022

Observed black holes

Three types of black holes have been discovered

- Stellar black holes
  BHs originated in the explosion of massive stars/supernovae, $\sim 3 - 200\,M_\odot$
- Intermediate mass black holes (IMBH)
  New class of recently discovered BHs, $\sim 10^3 - 10^6\,M_\odot$
- supermassive black holes (SMBH)
  BHs at the center of galaxies, $\sim 10^6 - 10^9\,M_\odot$

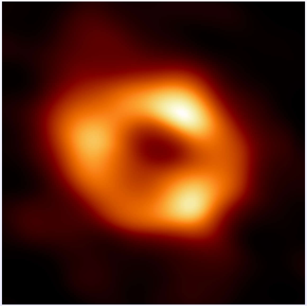




Imaging black holes

Supermassive black holes images by the Event Horizon Telescope

M87* (2019)



Sgr A* (yesterday)

Origin of primordial black holes

Multiple inflationary origins

- collapse of large primordial overdensities
- phase transitions
- collapse of cosmic strings, domain walls

Mass predictions

Assuming that one PBH can be formed in a Hubble volume in the early Universe, one gets

$$M_{\rm PBH} \sim M_{\rm Planck} \times \frac{t_0}{t_{\rm Planck}} \sim 10^{38}\ {\rm g}\ \times t_0({\rm s})$$

where t_0 is the creation time.

We get:

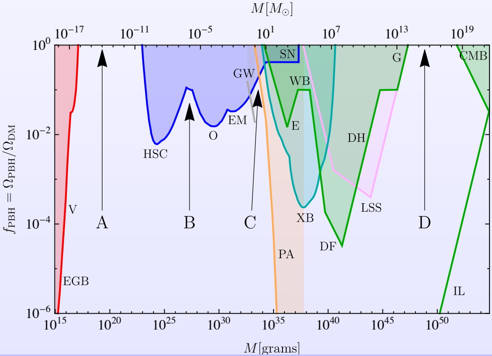
- $M \sim 10^{-5}$ g for $t_0 \sim 10^{-43}$ s $\rightarrow$ Planck black holes
- $M \sim 10^{15}$ g for $t_0 \sim 10^{-23}$ s $\rightarrow$ lightest black holes still (possibly) existing
- $M \sim 10^5\ M_\odot$ for $t_0 \sim 1$ s $\rightarrow$ IMHB? seeds for SMBH?

Constraints on Primordial Black Holes

Plausible dark matter candidates

- no need for Standard Model / General Relativity extension
- dynamically cold
- BH existence (somehow) proven
- mass ranges still available for BHs to represent all of dark matter

Constraints on PBHs – from Carr & Kuhnel, 2006.02838



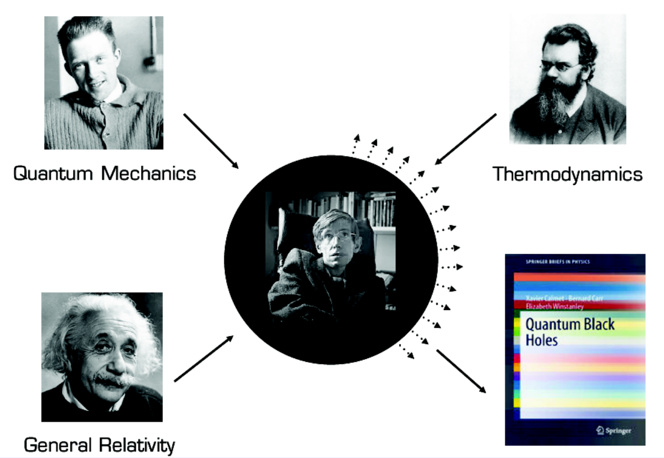
red: evaporation
blue: lensing
gray: gravitational waves
light blue: accretion
orange CMB distortions
green: dynamical effects
purple: large scale structure

A-D: possible open windows

Hawking radiation

Why are PBHs so special?

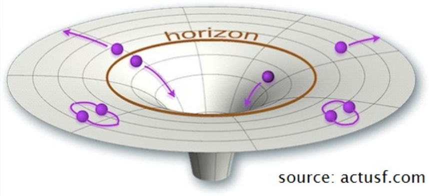
Light PBHs cannot be described only with General Relativity...



from B. Carr

... because they emit Hawking radiation and evaporate!

Black hole Hawking radiation



Fundamental equation for Kerr BHs

Rate of emission of Standard Model particles i at energy E by a BH of mass M and spin parameter a^*:

$$Q_i = \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \frac{1}{2\pi} \sum_{\mathrm{dof.}} \frac{\Gamma_i(M,E,a^*)}{e^{E'/T(M,a^*)} \pm 1}$$

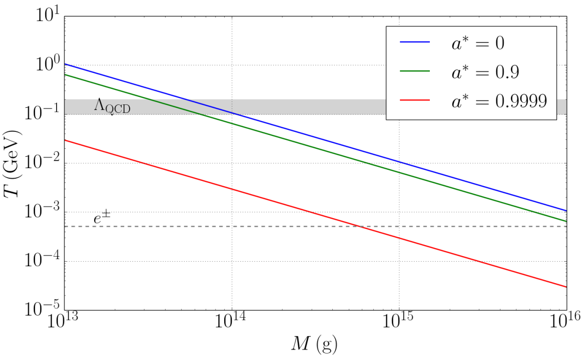
Γ_i is the greybody factor ($\sim$ absorption coefficient in Planck's black-body law)

Hawking temperature

Hawking temperature for Kerr BHs

$$T(M, a^*) = \frac{1}{4\pi M}\left(\frac{\sqrt{1-(a^*)^2}}{1+\sqrt{1-(a^*)^2}}\right) \xrightarrow[a^*=0]{\text{Schwarzschild}} \frac{1}{8\pi M}$$

Comparison with the $e^\pm$ rest mass and QCD scale $\Lambda_{\rm QCD}$



BHs get warmer when they lose energy via Hawking radiation.

Kerr Hawking radiation equations

Kerr metric

$$\mathrm{d}s^2 = \left(1 - \frac{2Mr}{\Sigma^2}\right)\mathrm{d}t^2 + \frac{4a^* M^2 r \sin^2\theta}{\Sigma^2}\mathrm{d}t\,\mathrm{d}\phi - \frac{\Sigma^2}{\Delta}\mathrm{d}r^2$$
$$- \Sigma^2\mathrm{d}\theta^2 - \left(r^2 + (a^*)^2 M^2 + \frac{2(a^*)^2 M^3 r \sin^2\theta}{\Sigma^2}\right)\sin^2\theta\mathrm{d}\phi^2$$

$\Sigma \equiv r^2 + (a^*)^2 M^2 \cos^2\theta$ and $\Delta \equiv r^2 - 2Mr + (a^*)^2 M^2$

Equations of motion in free space

$$\mathrm{Dirac:}\ (i\partial\!\!\!/ - \mu)\psi = 0\ \mathrm{(fermions)}$$
$$\mathrm{Proca:}\ (\Box + \mu^2)\phi = 0\ \mathrm{(bosons)}$$

μ = rest mass

Kerr Hawking radiation equations

Teukolsky radial equation

$$\frac{1}{\Delta^s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R}{\mathrm{d}r}\right)+\left(\frac{K^2+2i\,s(r-M)K}{\Delta}-4i\,sEr-\lambda_{slm}-\mu^2r^2\right)R=0$$

R radial component of ψ/ϕ

$K\equiv(r^2+a^2)E+a\,m$, s = spin, l = angular momentum and m = projection

Transformation into a Schrödinger equation

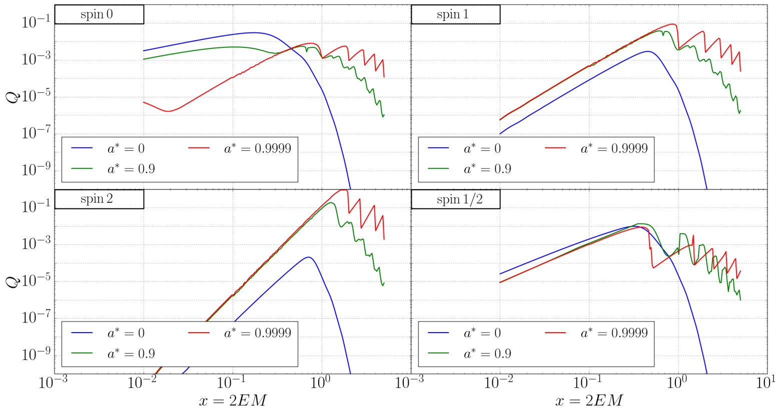
Change $\psi/\phi\longrightarrow Z$ and $r\longrightarrow r^*$ (generalized Eddington-Finkelstein coordinate system) (Chandrasekhar & Detweiler 1970s)

$$\frac{\mathrm{d}^2Z}{\mathrm{d}r^{*2}}+(E^2-V(r^*))Z=0$$

Solved with purely outgoing solution $Z\underset{r^*\to-\infty}{\longrightarrow}e^{-i\,Er^*}$

Transmission coefficient $\Gamma\equiv|Z^{+\infty}_{\mathrm{out}}/Z^{\mathrm{horizon}}_{\mathrm{out}}|^2$

Hawking radiation of particles



All particles can be emitted by a black hole!

Including gravitons / gravitational waves... and even new physics particles!

Hawking radiation is enhanced for particles of spin 1 or 2.

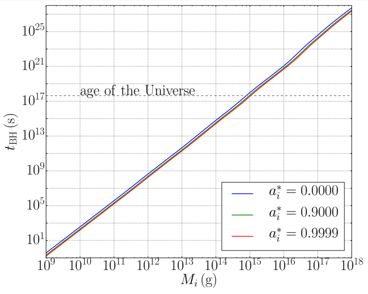
Black hole lifetime

Evolution equations

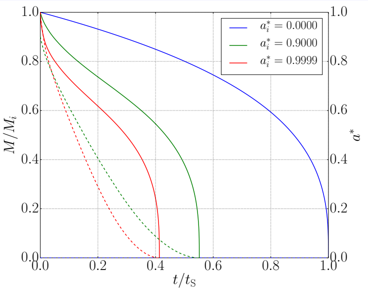
$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2} \qquad f \sim \int_E ener. \times emiss.$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*) - g(M,a^*))}{M^3} \qquad g \sim \int_E ang.\ mom. \times emiss.$$

BH lifetime



BH mass (solid) and spin (dotted) evolution



AA, J. Auffinger, J. Silk, MNRAS 494 (2020) 1257

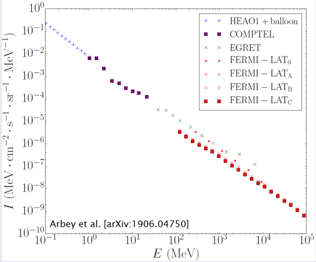
Constraints on primordial black holes

Isotropic gamma ray background (IGRB) constraints

Origin

Diffuse background +

- Active galactic nuclei
- Gamma ray bursts
- DM annihilation/decay?
- Hawking radiation?



Flux estimation for BHs

AA *et al.*, arXiv:1906.04750

$$I \approx \frac{1}{4\pi} E \int_{t_{\rm CMB}}^{t_{\rm today}} (1 + z(t))$$

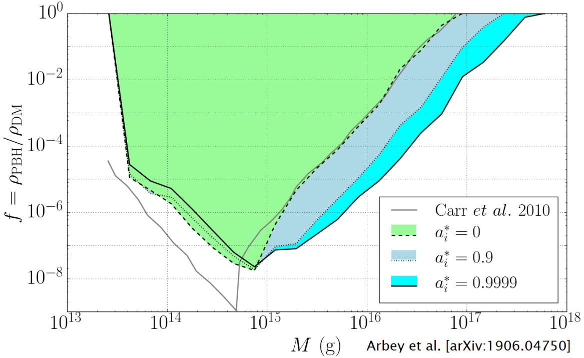
$$\times \int_M \left[\frac{\mathrm{d}n}{\mathrm{d}M} \frac{\mathrm{d}^2 N}{\mathrm{d}t \mathrm{d}E} (M, (1 + z(t))E) \, \mathrm{d}M \right] \mathrm{d}t$$

IGRB and Kerr PBHs: monochromatic mass distributions

Main spin effects

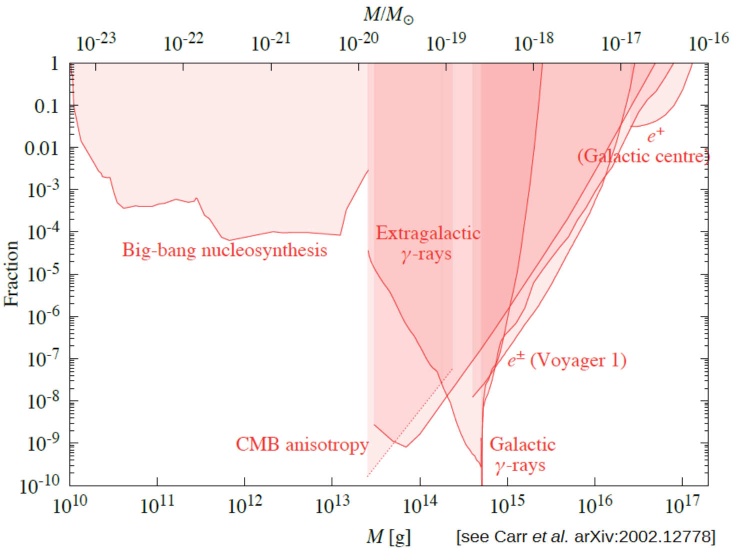
- enhanced luminosity ⇒ stronger constraints
- reduced temperature ⇒ reduced emission energy ⇒ weaker constraints

Monochromatic constraints from the IGRB



Arbey et al. [arXiv:1906.04750]

Complementary constraints



[see Carr *et al.* arXiv:2002.12778]

Non-standard black holes

The case of static spherically-symmetric metrics

AA, J. Auffinger, M. Geiller, E. Livine, F. Sartini, arXiv:2101.02951 + 2107.03293

$$\mathrm{d}s^2 = -G(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + H(r)\mathrm{d}\Omega^2$$

Equations of motion:

$$\partial^2_{r^*} Z + \Big(\omega^2 - V\big(r(r^*)\big)\Big) Z = 0$$

where $\mathrm{d}r^*/\mathrm{d}r \equiv 1/\sqrt{FG}$ and the spin-dependent potentials V are related to F, G, H.

Transmission coefficient: $\Gamma_i \equiv |Z^{+\infty}_{\rm out}/Z^{\rm horizon}_{\rm out}|^2$

Rate of emission of one degree of freedom i per unit time t and energy E:

$$\frac{\mathrm{d}^2 N_i}{\mathrm{d}t\,\mathrm{d}E} = \sum_{l,m} \frac{1}{2\pi} \frac{\Gamma_i(E, M, x_j)}{e^{E/T} - (-1)^{2s_i}}\,,$$

where s_i is the spin of the particle i and T is its Hawking temperature given by

$$T = \frac{1}{4\pi} \left.\frac{F^{1/2} G'}{G^{1/2}}\right|_{\rm hor}$$

where "hor" denotes the horizon $r = r_{\rm H}$.

Extra-dimensional PBHs

Metric:

$$F(r) = G(r) \equiv 1 - \left(\frac{r_{\rm H}}{r}\right)^{n+1}, \quad H(r) = r^2$$

where $n > 0$ is the number of extra dimensions.

Horizon radius is given by

$$r_{\rm H} = \frac{1}{\sqrt{\pi} M_*} \left(\frac{M}{M_*}\right)^{1/(n+1)} \left(\frac{8\Gamma\left((n+3)/2\right)}{n+2}\right)^{1/(n+1)}$$

where Γ is the Euler gamma function and the rescaled Planck mass is

$$M_{\rm Pl}^2 = M_*^{n+2} R^n$$

with R the radius of the extra-dimension(s).

Temperature:

$$T_n = \frac{\kappa_n}{2\pi} = \frac{n+1}{4\pi r_{\rm H}}$$

Remark: Schwarzschild metric retrieved for $n = 0$.

Extra-dimensional PBHs

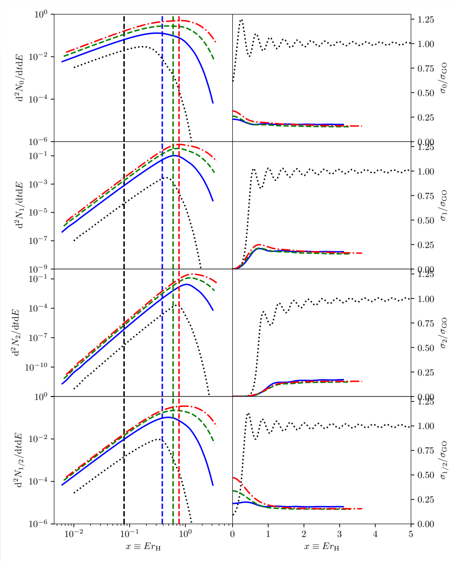
Hawking radiation spectra

dotted black: Schwarzschild

blue: $n = 2$

green: $n = 4$

red: $n = 6$



LQG-inspired PBHs

Schwarzschild BHs are singular at the center → non-physical → quantum gravity?

Loop Quantum gravity (LQG): structure of spacetime described by spin networks.

Metric inspired by LQG, regularising the naked singularity at the center of the Schwarzschild metric:

$$G = \frac{(r-r_+)(r-r_-)(r+\sqrt{r_+r_-})^2}{r^4+a_0^2}\,, \quad F = \frac{(r-r_+)(r-r_-)r^4}{(r+\sqrt{r_+r_-})^2(r^4+a_0^2)}\,, \quad H = r^2 + \frac{a_0^2}{r^2}\,.$$

Two roots: $r_+ = \dfrac{2M}{(1+P)^2}$ and $r_- = \dfrac{2MP(\varepsilon)^2}{(1+P)^2}$

M: (ADM) mass

$P(\varepsilon) = \dfrac{\sqrt{1+\varepsilon^2}-1}{\sqrt{1+\varepsilon^2}+1}$: polymerization factor

a_0: minimal area in loop quantum gravity, typically of the Planck scale

$\varepsilon \geq 0$: typical scale of the geometry fluctuations (a priori independent parameter)

Temperature: $T_{\rm LQG} = \dfrac{r_+^2(r_+-r_-)}{4\pi(r_+^4+a_0^2)}$

LQG-inspired PBHs

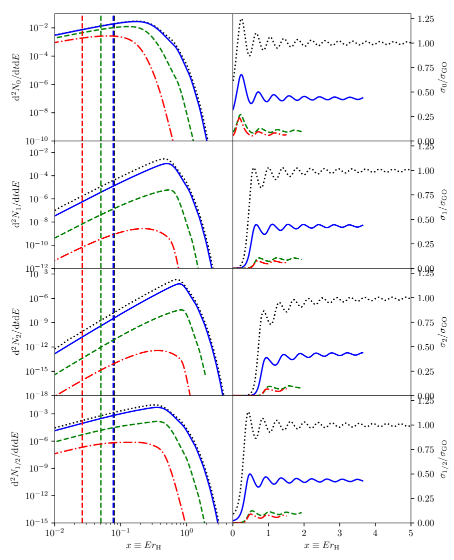
Hawking radiation spectra

dotted black: Schwarzschild

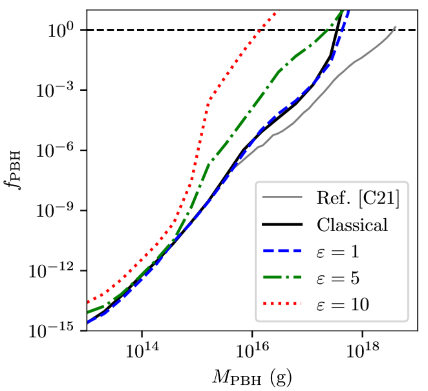
blue: $\epsilon = 1$

green: $\epsilon = 4$

red: $\epsilon = 10$



LQG-inspired PBHs



Predictions for the sensitivity of the AMEGO gamma-ray experiment to classical and polymerised LQG-inspired PBHs

ε: scale of deformation of spacetime due to polymerisation in Loop Quantum Gravity

AA, J. Auffinger, M. Geiller, E. Livine, F. Sartini, arXiv:2101.02951 + 2107.03293

→ Theories beyond General Relativity predict very different signals of Hawking radiation.

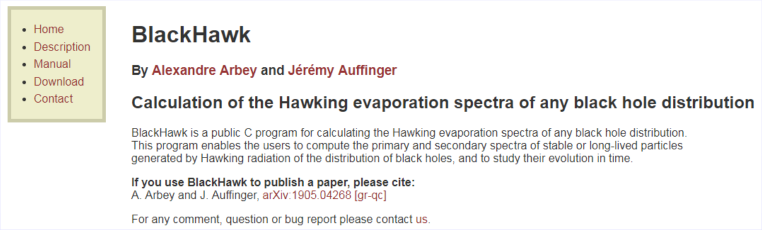
BlackHawk

Public C code computing Hawking radiation:

- Schwarzschild & Kerr PBHs + non-standard BHs
- primary spectra of all Standard Model fundamental particles
- secondary spectra of stable particles
- extended mass and spin functions
- time evolution of the PBHs

Download: `http://blackhawk.hepforge.org`

Manual: arXiv:1905.04268, Eur.Phys.J. C79, 693
+ arXiv:2108.02737, Eur.Phys.J. C81, 10



BlackHawk

Secondary spectra

Particles produced via Hawking radiation are not all stable:
hadronisation, decay, fragmentation...

Schematical description

$$\text{Metric} \xrightarrow{\texttt{BlackHawk}} \text{Primary spectra} \xrightarrow[\text{particle physics codes}]{\text{Interface with}} \text{Secondary spectra}$$

Interfaces

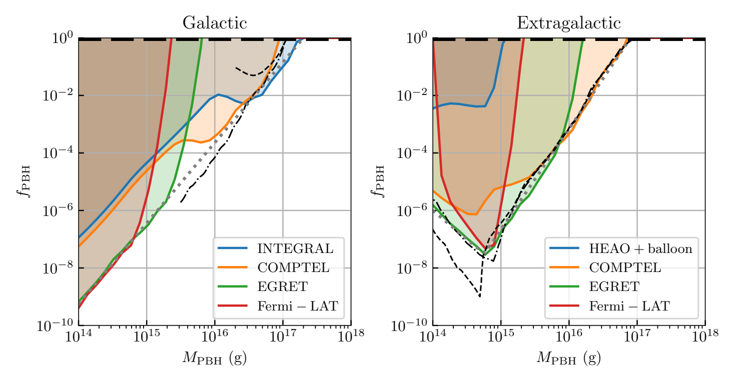
Low energies		Collider energies (LHC)		High energies
$E_{\rm cdm} \sim 1\,\text{keV} - 5\,\text{GeV}$	$<$	$E_{\rm cdm} \sim 5\,\text{GeV} - 100\,\text{TeV}$	$<$	$E_{\rm cdm} \sim 100\,\text{TeV} - E_{\rm P}$
$\rightarrow$ `Hazma`		$\rightarrow$ `PYTHIA`, `HERWIG`		$\rightarrow$ `HDMSpectra`

A BlackHawk tool: Isatis

Isatis

Aims at computing automatically the PBH constraints from `BlackHawk` Hawking radiation spectra. Embeds a `BlackHawk` launcher as well as the technical characteristics of various (prospective) optical instruments.

Manual: J. Auffinger, arXiv:2201.01265, Eur.Phys.J. C82, 384

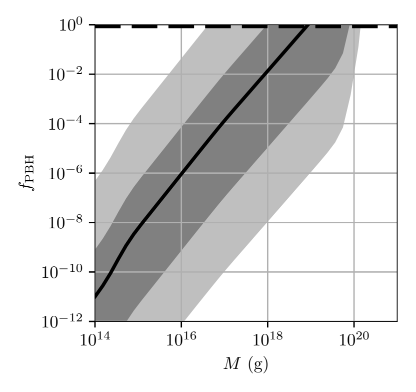


A BlackHawk tool: Isatis

Application

Isatis can be used to explore the impact of various assumptions and uncertainties on the PBH constraints:

- instrument characteristics
- (extra)galactic photon flux
- statistical treatment
- Hawking radiation



Summary

Main results

- Study of the evolution of PBHs and experimental constraints
- Extension non-standard PBH
- Public code `BlackHawk` to compute Hawking radiation
- Public code `Isatis` to assess constraints

Perspectives

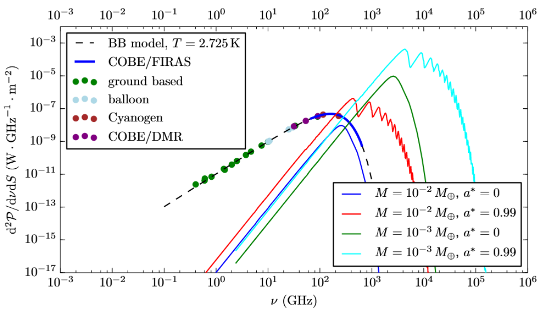
- Closing the remaining PBH mass windows for all dark matter into PBHs?
- Other types of non-standard PBHs?
- Constraints from extrasolar planet searches?
- Other constraints...

Backup

Backup

A primordial black hole in the Solar System

Looking for ~~Planet~~ Black Hole 9



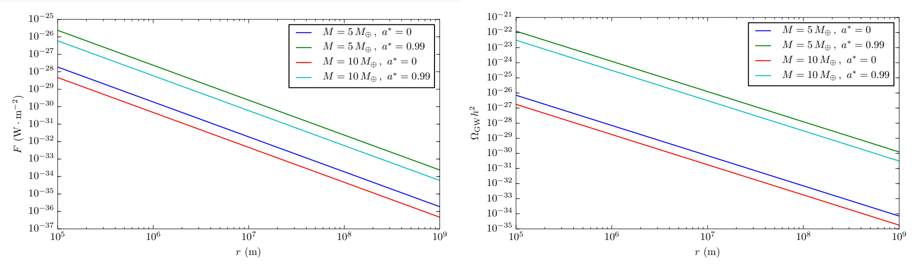
AA, J. Auffinger, 2006.02944

Comparison of radiation by a planet-mass PBH and by the Cosmic Microwave Background

→ A PBH with mass similar to a planet in our Solar System would emit a theoretically measurable *in-situ* Hawking radiation.

Towards Planet 9...

Hawking radiation too weak to be seen from Earth



AA, J. Auffinger, 2006.02944

$\rightarrow$ need to send a probe in orbit to study the emitted radio waves (and why not gravitational waves)

($\rightarrow$ Breakthrough Starshot project, proof-of-concept for a fleet of light sail spacecrafts)