# Effective potential of scalar-tensor gravity

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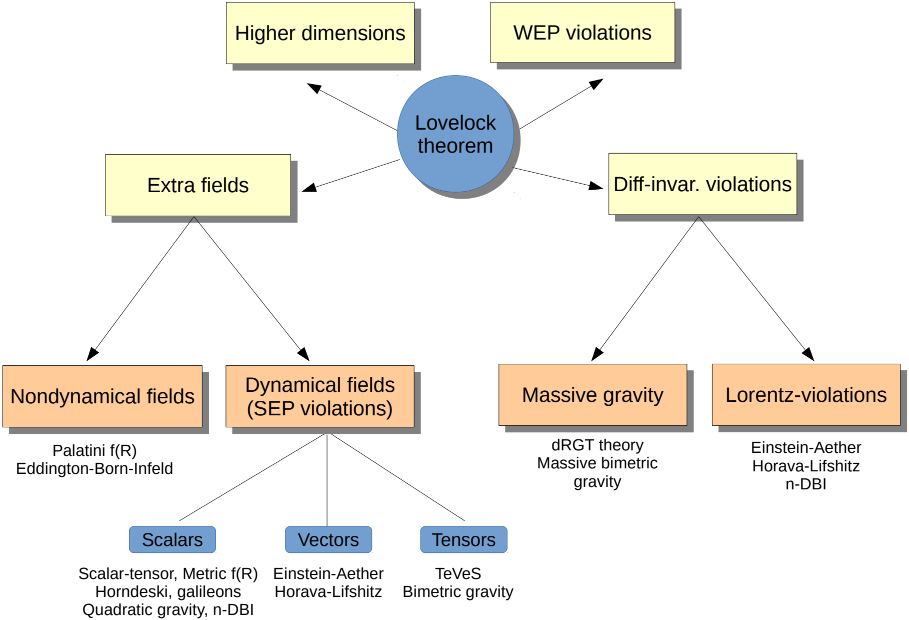
# Motivation

General relativity requires:

- 4-dimensional spacetime
- $g_{\mu\nu}$ describes gravity
- Second order field equations
- Diffeomorphism invariance

Lovelock, J.Math.Phys. 12 (1971) 498
Berti et al, Class.Quant.Grav. 32 (2015) 243001


Higher dimensions
WEP violations
Lovelock theorem
Extra fields
Diff-invar. violations
Nondynamical fields
Dynamical fields (SEP violations)
Massive gravity
Lorentz-violations
Palatini f(R)
Eddington-Born-Infeld
dRGT theory
Massive bimetric gravity
Einstein-Aether
Horava-Lifshitz
n-DBI
Scalars
Vectors
Tensors
Scalar-tensor, Metric f(R)
Horndeski, galileons
Quadratic gravity, n-DBI
Einstein-Aether
Horava-Lifshitz
TeVeS
Bimetric gravity

# Motivation

Why scalars?

- More simple
- Arise in almost all cases
- The Higgs boson in the SM
- Good for inflation
- Can be further generalized for other spins
- Theoretical aspects: UV behaviour

# Motivation

Why quantum effects?

Typically quantum effects are just small corrections to the (semi)classical approximaion. But they are certainly there and can lead to

- small but visible effects
- new interaction channels
- new types of interactions
- anomalies, i.e., breaking of a classical symmetry
- modification of the ground state (vacuum)
- provide hints for theory

# Motivation

Horndeski gravity

$$
\begin{aligned}
\mathcal{L}_2 =& G_2(\phi, X)\\
\mathcal{L}_3 =& G_3(\phi, X)\,\Box\phi\\
\mathcal{L}_4 =& G_4(\phi, X)\,R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_{\mu\nu}\phi)^2\right]\\
\mathcal{L}_5 =& G_5(\phi, X)\,G_{\mu\nu}\nabla^{\mu\nu}\phi\\
&- \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_{\mu\nu}\phi)^2 + 2(\nabla_{\mu\nu}\phi)^3\right]
\end{aligned}
$$

Horndeski, Int.J.Theor.Phys. 10 (1974) 363
Kobayashi et al, Prog Theor Phys 126, 511 (2011)

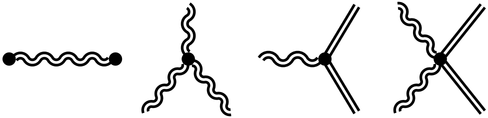
# Motivation

Perturbative quantization

$$\mathcal{Z} = \int \mathcal{D}[g_{\mu\nu}] \exp\left[i\mathcal{A}[g_{\mu\nu}]\right]$$
$$= \int \mathcal{D}[h_{\mu\nu}] \exp\left[i\mathcal{A}\left[\bar{g}_{\mu\nu} + \kappa h_{\mu\nu}\right]\right]$$

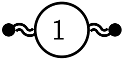
# Motivation

$$\mathcal{A}[\bar{g}+\kappa h] = \mathcal{A}[\bar{g}] + \frac{\delta \mathcal{A}[\bar{g}]}{\delta \bar{g}_{\mu\nu}} h_{\mu\nu} + \frac{\delta^2 \mathcal{A}[\bar{g}]}{\delta \bar{g}_{\mu\nu} \delta \bar{g}_{\alpha\beta}} h_{\mu\nu} h_{\alpha\beta}$$
$$+ \frac{\delta^3 \mathcal{A}[\bar{g}]}{\delta \bar{g}_{\mu_1\nu_1}\, \bar{g}_{\mu_1\nu_1}\, \bar{g}_{\mu_3\nu_3}} h_{\mu_1\nu_1}\, h_{\mu_2\nu_2}\, h_{\mu_3\nu_3} + \cdots$$

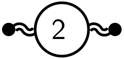


# Motivation

Non-renormalizability of general relativity




$$\rightarrow R^2 + R^2_{\mu\nu}$$




$$\rightarrow R^3_{\mu\nu\alpha\beta} + \cdots$$

Ghosts of higher-derivative gravity

$$\mathcal{L} = -\frac{1}{16\pi G}\left[R + \frac{1}{6m_0^2}\,R^2 - \frac{1}{2m_2^2}\,C^2\right]$$

Stelle, Phys.Rev.D 16 (1977) 953

# Motivation

Effective field theory

Theory is given by microscopic action $\mathcal{A}$
Theory is defined below the factorization scale $\mu < m_P$
Normalization is performed in the low energy limit
Provides a consistent (but restricted) approach to loop corrections
Remind, e.g., the Chiral Perturbative Theory

Donoghue, Phys.Rev.D 50 (1994) 3874
Burgess, Living Rev.Rel. 7 (2004) 5

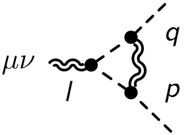
# Motivation

Example

Beyond Horndeski interactions, i.e., non-minimal interactions of the sacalr field with matter

$$\mathcal{A}_{\text{int}} = \int d^4x \sqrt{-g}\, [C(\phi, X)\, g_{\mu\nu} + D(\phi, X)\, \partial_\mu\phi\, \partial_\nu\phi]\, T^{\mu\nu}$$

are induced by one-loop corrections




$$= i\kappa^3\, l^2\, C_{\mu\nu\alpha\beta}\, p^\alpha q^\beta$$

B. Latosh, Mod.Phys.Lett.A 36 (2021) 37, 2150258

# Effective action

Effective action $\Gamma(\varphi)$

- Operates with classical fields $\varphi = \left\langle 0 \middle| \widehat{\phi} \middle| 0 \right\rangle$
- Minimal on classical configurations $\delta\Gamma/\delta\varphi = 0$
- Accounts for quantum effects

Buchbinder, Odintsov, Shapiro,
Effective action in quantum gravity, 1992.

# Effective potential

Generating functional $Z$
Green functions $G$
$\downarrow$
Connected generating functional $W$
Connected Green functions $G_c$
$\downarrow$
Effective action $\Gamma$
Vertex functions $\Gamma_n$

# CW potential

Generalization of Coleman-Weinberg model

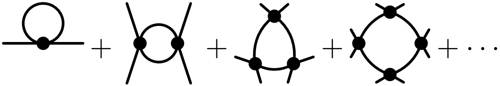
$$\mathcal{A} = \int d^4x \left[ -\frac{1}{2}\phi\,(\Box + m^2)\phi - \frac{\lambda}{4!}\phi^4 \right]$$

$$\mathcal{G} = (\Box + m^2)^{-1} = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{-k^2 + m^2}$$

Coleman, Weinberg, Phys.Rev.D 7 (1973) 1888

# CW potential

Effective potential series



# CW potential

Effective potential

$$
\begin{aligned}
V_{\text{eff}} &= -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left[1-\frac{\frac{\lambda}{2}\phi^2}{k^2-m^2}\right] \\
&= \frac{m^2}{2}\varphi^2\left(\frac{\lambda}{16\pi^2}\right)\left[\frac{1}{d-4}-\frac{3}{4}+\frac{1}{2}\gamma-\frac{3}{2}\ln 2\pi-\frac{1}{2}\ln 2+\frac{1}{2}\ln\frac{\frac{\lambda}{2}\varphi^2+m^2}{\mu^2}\right] \\
&+ \frac{\lambda}{4!}\varphi^4\left(\frac{3\lambda}{16\pi^2}\right)\left[\frac{1}{d-4}-\frac{3}{4}+\frac{1}{2}\gamma-\frac{3}{2}\ln 2\pi-\frac{1}{2}\ln 2+\frac{1}{2}\ln\frac{\frac{\lambda}{2}\varphi^2+m^2}{\mu^2}\right] \\
&+ \frac{m^4}{64\pi^2}\ln\left[1+\frac{\frac{\lambda}{2}\varphi^2}{m^2}\right]+\mathcal{O}(d-4)
\end{aligned}
$$

+ (re)normalization conditions

# Minimal ST gravity

Minimal model

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}g^{\mu\nu}\,\nabla_\mu\phi\nabla_\nu\phi - \frac{m^2}{2}\phi^2\right]$$

$$= \int d^4x\Big[-\frac{1}{2}h^{\mu\nu}\mathcal{D}_{\mu\nu\alpha\beta}\,\Box h^{\alpha\beta} - \frac{1}{2}\phi(\Box + m^2)\phi$$

$$-\frac{\kappa}{4}h^{\mu\nu}\left[C_{\mu\nu\alpha\beta}\partial^\alpha\phi\partial^\beta\phi + \eta_{\mu\nu}\,m^2\phi^2\right]$$

$$+\,\text{irrelevant terms}$$

# Minimal ST gravity

Gauge-fixing term

$$\mathcal{A}_{\text{gf}} = \int d^4x \left( \partial_\mu h^{\mu\nu} - \frac{1}{2} \partial^\nu h \right)^2$$

Gravitational coupling

$$\kappa^2 = 32\pi\, G$$

Tensor structure

$$C_{\mu\nu\alpha\beta} = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}$$

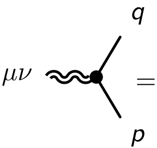
# Minimal ST gravity

Feynman rules

$$\mu\nu \quad \alpha\beta = i\,\frac{\frac{1}{2}\,C_{\mu\nu\alpha\beta}}{k^2}$$



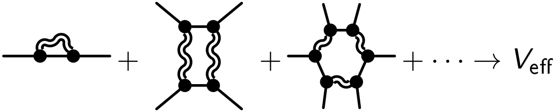
$$= \frac{i}{k^2 - m^2}$$




$$= i\,\frac{\kappa}{4}\left[C_{\mu\nu\alpha\beta}\,p^\alpha q^\beta + m^2\eta_{\mu\nu}\right]$$

Only mass-term contributes to the effective potential!

## Effective potential




$$
\begin{aligned}
V_{\text{eff}} =& \frac{m^2\varphi^2}{2}\,\frac{m^2\kappa^2}{8\pi^2}\left[-\frac{1}{d-4}-\frac{\gamma}{2}+\frac{1}{2}\ln(8\pi)-\frac{1}{2}\ln\frac{m^2}{\mu^2}\right]-\frac{m^4\ln(2)}{64\pi^2}\\
&+\frac{m^4}{128\pi^2}\left(1-\sqrt{1-4\kappa^2\varphi^2}-2\kappa^2\varphi^2\right)\left[1-\sqrt{1-4\kappa^2\varphi^2}\right]\\
&+\frac{m^4}{128\pi^2}\left(1+\sqrt{1-4\kappa^2\varphi^2}-2\kappa^2\varphi^2\right)\left[1+\sqrt{1-4\kappa^2\varphi^2}\right]
\end{aligned}
$$

# Minimal ST gravity

Renormalized effective potential

$$
\begin{aligned}
V_{\text{eff,ren}} =& -\frac{m^4 \ln(2)}{64\pi^2} + \frac{m^2\varphi^2}{2}\left[1 + \frac{m^2\kappa^2}{32\pi^2}\left(1+\ln(4)\right)\right] \\
&+ \frac{m^4}{128\pi^2}\left(1 - \sqrt{1-4\kappa^2\varphi^2} - 2\kappa^2\varphi^2\right)\left[1 - \sqrt{1-4\kappa^2\varphi^2}\right] \\
&+ \frac{m^4}{128\pi^2}\left(1 + \sqrt{1-4\kappa^2\varphi^2} - 2\kappa^2\varphi^2\right)\left[1 + \sqrt{1-4\kappa^2\varphi^2}\right]
\end{aligned}
$$

- No new minima
- Applicable even for $\varphi \geq 2/\kappa$
- Can drive inflation

Arbuzov, Latosh, Class.Quant.Grav. 38 (2021) 1, 015012

# Non-minimal scalar-tensor gravity

## Non-minimal model

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[ -\frac{2}{\kappa^2} R + \frac{1}{2} \left( g^{\mu\nu} + \beta G^{\mu\nu} \right) \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} \left( m^2 + \lambda R \right) \phi^2 \right]$$

$$\Delta V_{\text{eff}} = \frac{1}{2} \ln \left[ 1 + \frac{3}{2} \lambda^2 \kappa^2 \varphi^2 \right] \left( \frac{d_E^4 k}{(2\pi)^4} \right)$$
$$\to \Delta V_{\text{eff,ren}} = \frac{1}{2} \ln \left[ 1 + \frac{3}{2} \lambda^2 \kappa^2 \varphi^2 \right] \left( \frac{2 m_{\text{obs}}^2}{3 \kappa^2 \lambda^2} \right)$$

- Power-like factorization scale sensitivity
- Requires infinitely many counter-terms
- Can drive slow-roll inflation

Arbuzov, Latosh, Class.Quant.Grav. 38 (2021) 1, 015012

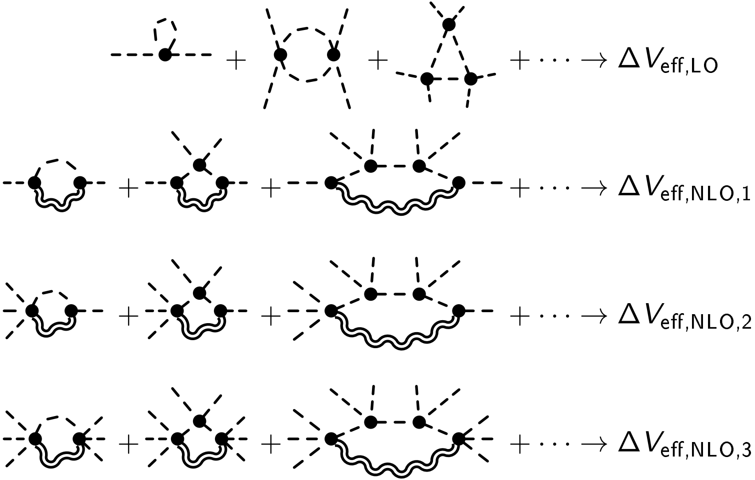
# Minimal $\phi^4$ scalar-tensor gravity

$\phi^4$ scalar-tensor model

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]$$

$$\begin{aligned}
V_{\text{eff}} =& \frac{m^4}{64\pi^2}\ln\left[1 + \frac{\frac{\lambda}{2}\varphi^2}{m^2}\right] \\
&+ \frac{m^2}{2}\varphi^2\left[\left(\frac{\lambda}{16\pi^2} - \frac{\kappa^2 m^2}{8\pi^2}\right)\left(\frac{1}{d-4} + \frac{1}{2}\ln\frac{\frac{\lambda}{2}\varphi^2 + m^2}{\mu^2}\right) + \mathcal{F}_1\right] \\
&+ \frac{\lambda}{4!}\varphi^4\left[\left(\frac{3\lambda}{16\pi^2} - \frac{\kappa^2 m^2}{2\pi^2}\right)\left(\frac{1}{d-4} + \frac{1}{2}\ln\frac{\frac{\lambda}{2}\varphi^2 + m^2}{\mu^2}\right) + \mathcal{F}_2\right] \\
&+ \frac{\kappa^2}{6!}\varphi^6\left[-\left(\frac{5\lambda^2}{4\pi^2}\right)\left(\frac{1}{d-4} + \frac{1}{2}\ln\frac{\frac{\lambda}{2}\varphi^2 + m^2}{\mu^2}\right) + \mathcal{F}_3\right] \\
&+ \mathcal{O}(d-4)
\end{aligned}$$

# Minimal $\phi^4$ scalar-tensor gravity



# Minimal $\phi^4$ scalar-tensor gravity

$$
\begin{aligned}
V_{\text{eff,ren}} = & \ln\left[1+\frac{\frac{\lambda}{2}\varphi^2}{m^2}\right]\left\{\frac{m^4}{64\pi^2}+\frac{m^2}{2}\varphi^2\frac{\lambda-2m^2\kappa^2}{32\pi^2}+\frac{\lambda}{4!}\varphi^4\,\frac{3\lambda-8m^2\kappa^2}{32\pi^2}-\frac{\kappa^2}{6!}\varphi^6\frac{5\lambda^2}{8\pi^2}\right\} \\
& +\frac{m^2}{2}\left[1-\frac{\lambda}{64\pi^2}\right]\varphi^2+\frac{\lambda}{4!}\left[1-\frac{3(3\lambda-8\kappa^2m^2)}{64\pi^2}\right]\varphi^4 \\
& +\frac{g}{6!}\varphi^6\,\left[1-\frac{15\lambda^2}{32\pi^2}\frac{\lambda-2m^2\kappa^2}{gm^2}\right]
\end{aligned}
$$

- Logarithmically UV sensitive
- Generates $\varphi^6$ interaction
- Admits an instability region in $\phi \gtrsim m_P$
- Can drive inflation

Arbuzov, Latosh, Nikitenko, Class.Quant.Grav. 39 (2022) 5, 055003

# Application to inflation

Set up for inflation

$$S = \int d^4x \sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}g^{\mu\nu}\,\nabla_\mu\varphi\,\nabla_\nu\varphi - V(\varphi)\right]$$

Friedmann equations

$$\begin{cases} H^2 = \dfrac{\kappa^2}{12}\left[(\dot{\varphi})^2 + V\right] \\ \ddot{\varphi} + 3H\dot{\varphi} - V' = 0 \end{cases}$$

Slow-roll parameters

$$\begin{cases} \varepsilon = \dfrac{2}{3\kappa^2}\left(\dfrac{V'}{V}\right)^2 \ll 1 \\ \eta = \dfrac{4}{3\kappa^2}\dfrac{V''}{V} \end{cases}$$

# Application to inflation

Inflationary scenario is successful if

- Potential has area with $\varepsilon, \eta \ll 1$
- Inflation makes at least 60 e-foldings
- Tensor-to-scalar ratio $r$ is small

Number of e-foldings

$$N = \frac{\kappa^2}{4} \int\limits_{\varphi_{\text{end}}}^{\varphi_{\text{init}}} \frac{V(\varphi)}{V'(\varphi)} d\varphi$$

# Application to inflation

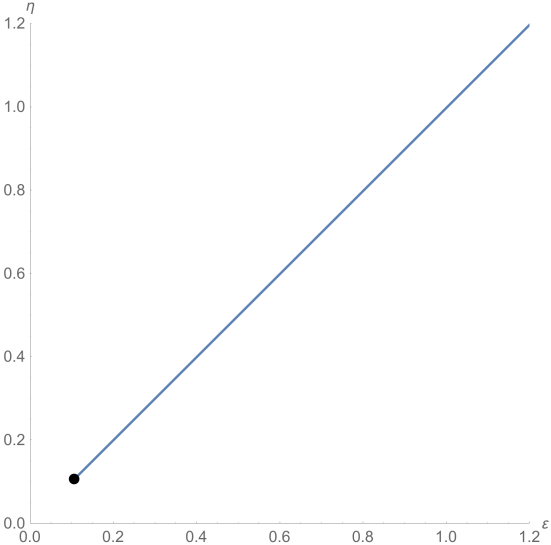
Minimal model in $\kappa$ units

$$V_{\text{eff,ren}} = -\frac{m^4 \ln(2)}{64\pi^2} + \frac{m^2\varphi^2}{2}\left[1 + \frac{m^2}{32\pi^2}(1+\ln(4))\right] + \frac{m^4}{128\pi^2}\left(1 - \sqrt{1-4\varphi^2} - 2\varphi^2\right)\left[1 - \sqrt{1-4\varphi^2}\right] + \frac{m^4}{128\pi^2}\left(1 + \sqrt{1-4\varphi^2} - 2\varphi^2\right)\left[1 + \sqrt{1-4\varphi^2}\right]$$

Inflation is weakly dependent on $m$

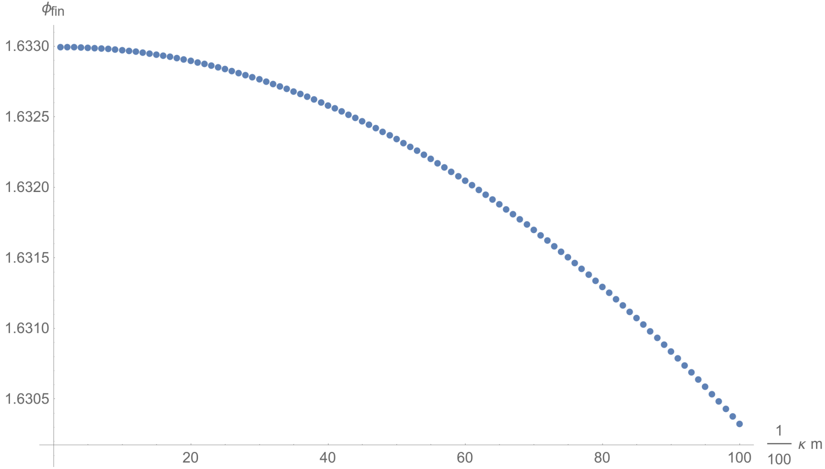
# Application to inflation

## Slow-roll parameters



# Application to inflation

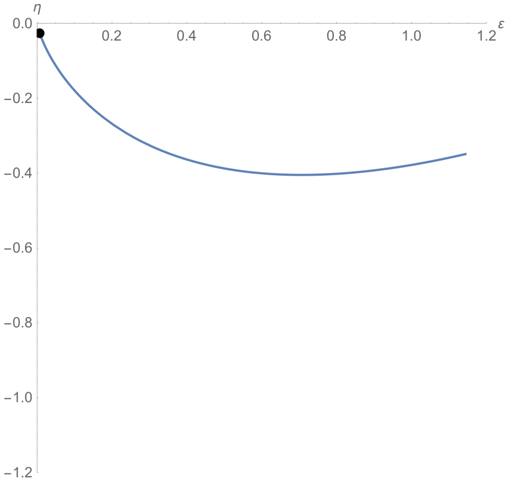
Inflation initial condition is $\varphi \sim 7.5 \times 10^{19}$ GeV
Inflation final condition:



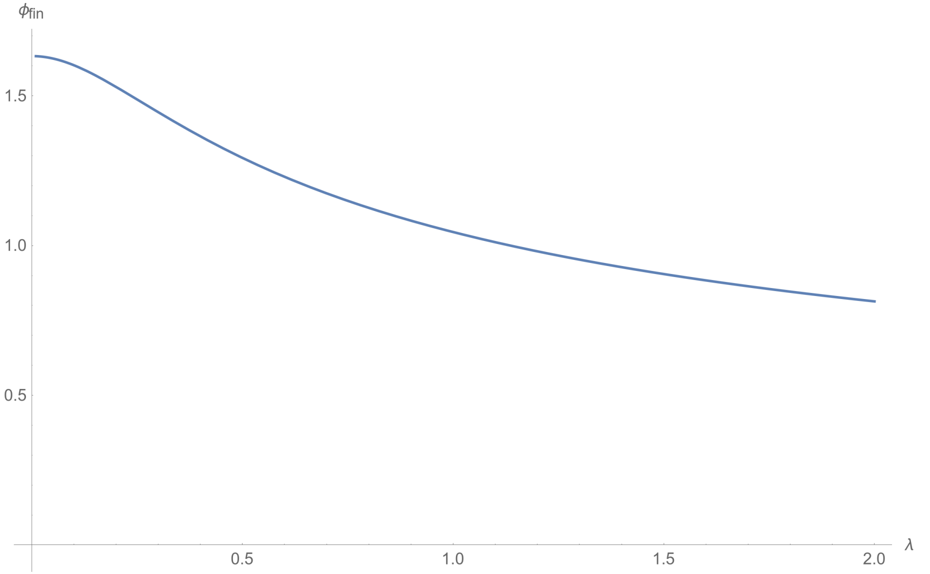
# Application to inflation

## Non-minimal model in $\kappa$ units

$$V_{\text{eff,new}} = \frac{1}{2}\ln\left[1+\frac{3}{2}\lambda^2\varphi^2\right]\mathfrak{V}$$

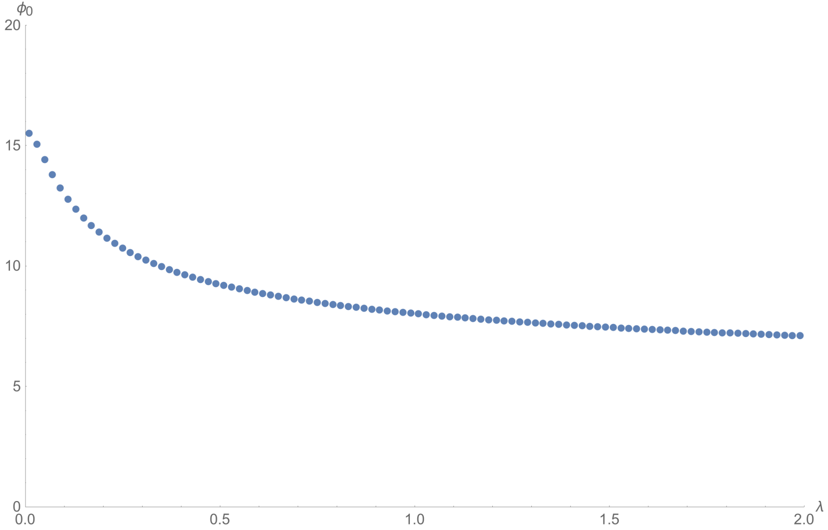


## Inflation finial condition



# Application to inflation

Inflation initial condition ($\kappa\phi_0 = 15 \leftrightarrow \phi_0 = 1.8 \times 10^{19}$ GeV)



# Conclusions

- Effective potential of a scalar field can account for quantum gravity corrections
- Effective potentials can be extended deep in the Planck region
- Minimal couplings appear to admit logarithmic factorization scale dependency
- Effective potential appears to provide viable inflationary scenarios

# Conclusions

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- Minimal couplings appear to admit logarithmic factorization scale dependency
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**Thank you for attention!**